



Slope Stability Assessment Method Using the Arctangent and Tangent Similarity Measure of Neutrosophic Numbers

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Abstract. Slope is a typical disaster in open-pit mining. Then it is a kind of non-continuous and uncertain natural geological body. In this case, common assessment approaches cannot assess the slope stability problems with both certain and uncertain information. Then, a neutrosophic number (NN) can easily represent the certain and uncertain information. Unfortunately NNs have not been applied in slope stability analysis so far. Therefore, this paper proposes the arctangent and tangent similarity measures of NNs and a slope stability assessment method using the arctangent and tangent similarity measures of NNs. By the similarity measure between the classification grades of slope stability and a slope sample with NNs, we can determine the assessment grade of the slope sample/case. Then, the assessment results of ten slope samples/cases demonstrate the same as actual grades of the ten slope cases, which indicate the effectiveness and feasibility of the proposed slope stability assessment method. The advantages are that the slope stability assessment method based on the arctangent and tangent similarity measures of NNs is simple and suitable and can assess slope stability problems in NN setting.

Keywords: Neutrosophic number, Arctangent similarity measure, Tangent similarity measure, Slope stability, Classification grade

1 Introduction

Geological disasters caused by open-pit mining have been gradually increased, while one of them is the slope instability of open-pit mining. Therefore, one is convinced of the importance of slope stability evaluation. The majority of slope stability evaluation performed in practical engineering still use deterministic analytical methods, such as traditional limit equilibrium methods [1-4] and finite element methods [5,6]. However, the slope is a kind of non-continuous and uncertain natural geological body along with the effect of external various factors, such as geological environment, topography, hydrology, and climate, which will affect the analysis of slope stability. However, the slope data usually imply imprecise and indeterminate information because of uncertain natural geological body and various external factors. In real situations, the slope data usually contain the indeterminate and determinate information. To represent it, a neutrosophic number (NN) was proposed firstly by Smarandache [7-9] and denoted by $N = c + eI$, which is composed of its certain part c and its uncertain part eI . Especially, NN is reduced to its certain part $N = c$ if $eI = 0$ (the best case) and its uncertain part $N = eI$ if $c = 0$ (the worst case). Clearly, NNs can show the advantage of representing the certain and uncertain information. Hence, the misfire fault diagnosis method of gasoline engines was presented by the cosine similarity measure of NNs [10]. Then, the fault diagnosis method of steam turbine was introduced by the exponential similarity measure of NNs [11]. A NN optimization model was used for the optimal design of truss structures [12]. Further, the multiple attribute decision making approach of clay-brick selection was provided by the projection model of NNs [13]. Recently, NNs have been also used for linear and nonlinear programming problems [14-17], neutrosophic traffic flow problems [18], as well as a neutrosophic neural network for NN function approximations [19]. Furthermore, NN functions and neutrosophic statistic methods were utilized for scale effect and anisotropy analyses of rock joint roughness coefficient in rock mechanics [20-22].

Slope is a typical disaster in open-pit mining. Then it is a kind of non-continuous and uncertain natural geological body. In this case, it is difficult to assess the slope stability with certain and uncertain information. However, existing slope stability assessment approaches [1-6] cannot handle the assessment problems of slope stability in certain and uncertain setting. As mentioned above, NNs can easily represent the advantage of certain and uncertain information. Unfortunately, NNs have not been applied in slope stability analysis so far. Therefore, we

need to develop a new slope stability assessment method for assessing slope stability problems with NNs in the geotechnical engineering field. To do so, this paper presents the arctangent and tangent similarity measures of NNs and their assessment method in slope stability problems with NN information.

Then, this study indicates the contribution of two new arctangent and tangent similarity measures of NNs and their slope stability assessment method for assessing slope stability problems in NN setting. Then, the main advantages are that the developed assessment approach is simpler and more suitable than existing common ones [1-6] and can assess slope stability problems with NN information, which traditional limit equilibrium methods [1-4] and finite element methods [5,6] cannot do.

The structure of this paper is given as the following framework. Some concepts of NNs are described in Section 2. Section 3 presents the arctangent and tangent similarity measures of NNs. A slope stability assessment method is established by the arctangent and tangent similarity measures, and then the effectiveness and feasibility are indicated by ten slope samples/cases in Section 4. Lastly, Section 5 presents conclusions and future work.

2 Some NN concepts

The NN presented by Smarandache [1] includes the certain part c and uncertain part eI , which is represented by $N = c + eI$ for $c, e \in R$ (all real numbers) and the indeterminacy $I \in [\inf I, \sup I]$. For instance, a NN is $N = 5 + 3I$, and then it is equivalent to $N \in [8, 11]$ for $I \in [1, 2]$ and $N \in [11, 14]$ for $I \in [2, 3]$. Generally, NN implies a changeable interval number regarding $I \in [\inf I, \sup I]$.

Let $N_1 = c_1 + e_1I$ and $N_2 = c_2 + e_2I$ be two NNs. Smarandache [7-9] introduced the operational relations of NNs:

- (1) $N_1 + N_2 = c_1 + c_2 + (e_1 + e_2)I$;
- (2) $N_1 - N_2 = c_1 - c_2 + (e_1 - e_2)I$;
- (3) $N_1 \times N_2 = c_1c_2 + (c_1e_2 + e_1c_2 + e_1e_2)I$;
- (4) $N_1^2 = (c_1 + e_1I)^2 = c_1^2 + (2c_1e_1 + e_1^2)I$;
- (5) $\frac{N_1}{N_2} = \frac{c_1 + e_1I}{c_2 + e_2I} = \frac{c_1}{c_2} + \frac{e_1c_2 - c_1e_2}{c_2(c_2 + e_2)} \cdot I$ for $c_2 \neq 0$ and $e_2 \neq -c_2$;

$$(6) \sqrt{N_1} = \sqrt{c_1 + e_1I} = \begin{cases} \sqrt{c_1} - (\sqrt{c_1} - \sqrt{c_1 + e_1})I \\ \sqrt{c_1} - (\sqrt{c_1} + \sqrt{c_1 + e_1})I \\ -\sqrt{c_1} + (\sqrt{c_1} + \sqrt{c_1 + e_1})I \\ -\sqrt{c_1} + (\sqrt{c_1} - \sqrt{c_1 + e_1})I \end{cases}$$

3 Arctangent and tangent similarity measures between NNs

This section presents similarity measures of NNs based on arctangent and tangent functions.

It is well known that arctangent and tangent functions, $\arctan(y)$ for $y \in [0, 1]$ and $\tan(y)$ for $y \in [0, \pi/4]$ are two increasing functions. If their function values are defined within $[0, 1]$, we can present the arctangent and tangent similarity measures between NNs.

Definition 1. Set $P = \{N_{P1}, N_{P2}, \dots, N_{Pn}\}$ and $Q = \{N_{Q1}, N_{Q2}, \dots, N_{Qn}\}$ as two sets of NNs, where $N_{Pj} = c_{Pj} + e_{Pj}I$ and $N_{Qj} = c_{Qj} + e_{Qj}I$ ($j = 1, 2, \dots, n$) for $c_{Pj}, e_{Pj}, c_{Qj}, e_{Qj} \geq 0, I \in [\inf I, \sup I]$, and $N_{Pj}, N_{Qj} \subseteq [0, 1]$. Then, the arctangent and tangent similarity measures between P and Q are defined by

$$AT(P, Q) = 1 - \frac{4}{n\pi} \sum_{j=1}^n \arctan \left[\frac{1}{2} \left(\frac{|c_{Pj} + e_{Pj} \inf I - c_{Qj} - e_{Qj} \inf I|}{|c_{Pj} + e_{Pj} \sup I - c_{Qj} - e_{Qj} \sup I|} \right) \right], \tag{1}$$

$$T(P, Q) = 1 - \frac{1}{n} \sum_{j=1}^n \left\{ \tan \left[\frac{\pi}{8} \left(\frac{|c_{Pj} + e_{Pj} \inf I - c_{Qj} - e_{Qj} \inf I|}{|c_{Pj} + e_{Pj} \sup I - c_{Qj} - e_{Qj} \sup I|} \right) \right] \right\}. \tag{2}$$

Obviously, the arctangent and tangent similarity measures should satisfy the following properties [23]:

- (1) $0 \leq AT(P, Q) \leq 1$ and $0 \leq T(P, Q) \leq 1$;
- (2) $AT(P, Q)=1$ and $T(P, Q)=1$ if and only if $P = Q$;
- (3) $AT(P, Q) = AT(Q, P)$ and $T(P, Q) = T(Q, P)$;
- (4) If $S = \{N_{S1}, N_{S2}, \dots, N_{Sn}\}$ is a set of NNs, $AT(P, S) \leq AT(P, Q)$, $AT(P, S) \leq AT(Q, S)$, $T(P, S) \leq T(P, Q)$, and $T(P, S) \leq T(Q, S)$.

In practical applications, the importance of each element is considered in the sets of NNs. If we assume that the weight of elements N_{Pj} nad N_{Qj} is w_j for $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$, then the weighted arctangent and tangent similarity measures between P and Q can be introduced below:

$$AT_w(P, Q) = 1 - \frac{4}{\pi} \sum_{j=1}^n w_j \arctan \left[\frac{1}{2} \left(\frac{|c_{Pj} + e_{Pj} \inf I - c_{Qj} - e_{Qj} \inf I|}{|c_{Pj} + e_{Pj} \sup I - c_{Qj} - e_{Qj} \sup I|} \right) \right], \tag{3}$$

$$T_w(P, Q) = 1 - \sum_{j=1}^n \left\{ w_j \tan \left[\frac{\pi}{8} \left(\frac{|c_{Pj} + \inf(e_{Pj}I) - c_{Qj} - \inf(e_{Qj}I)|}{|c_{Pj} + \sup(e_{Pj}I) - c_{Qj} - \sup(e_{Qj}I)|} \right) \right] \right\}. \tag{4}$$

Clearly, the weighted arctangent and tangent similarity measures should satisfy the above properties (1)-(4).

4 Slope stability assessment method using the similarity measures

Because of the complexity of practical engineering, the assessment information of slope stability often is incomplete and indeterminate. By using the evaluation method of slope stability regarding determinate information, however, it is difficult to reasonably evaluate whether the slope is unstable/stable due to missing indeterminate information in certain and uncertain setting.

Then, how can we give a proper evaluation method of slop stability with indeterminate and determinate information? A slop stability assessment method is presented to help engineers' proper evaluation of slop stability problems in NN setting.

4.1 Classification grades of slope stability

First, the stability of slopes is divided into four classification grades: stability (I), basic stability (II), relative instability (III), and instability (IV). Among them, the stability (I) means the slope is in a safe state, while the basic stability (II) means the slope may imply a possible safe state, and other two grades imply unsafe states. Then, we choose some slope samples as actual cases from the south central area of Zhejiang province in China. Through the investigation and statistics of the slopes in this area, a set of main impact factors is selected for the slope stability assessment. In this study, we choose eight impact factors, including the structural surface occurrence (h_1), the degree of weathering (h_2), the integrity of rock mass (h_3), the slope angle (h_4), the slope height (h_5), the degree of vegetation coverage (h_6), the annual rainfall (h_7), and the degree of human activities (h_8). However, these actual values obtained from slope samples/cases need to be normalized and shown in Table 1.

h_j	I	II	III	IV
h_1	[0,0.3]	[0.3,0.5]	[0.5,0.7]	[0.7,1]
h_2	[0,0.2]	[0.2,0.6]	[0.6,0.8]	[0.8,1]
h_3	[0,0.25]	[0.25,0.45]	[0.45,0.65]	[0.65,1]
h_4	[0,0.33]	[0.33,0.5]	[0.5,0.67]	[0.67,1]
h_5	[0,0.33]	[0.33,0.5]	[0.5,0.67]	[0.67,1]
h_6	[0,0.3]	[0.3,0.6]	[0.6,0.8]	[0.8,1]
h_7	[0,0.25]	[0.25,0.5]	[0.5,0.75]	[0.75,1]
h_8	[0,0.3]	[0.3,0.6]	[0.6,0.8]	[0.8,1]

Table 1: Data of slope stability between the eight impact factors and the four classification grades.

Since NNs imply the changable interval values depending on ranges of the indeterminacy $I \in [\inf I, \sup I]$, they can represent indetermina data effectively and reasonably in indeterminate setting. Hence, the interval values in Table 1 can be transformed into NNs for $I \in [0, 0.25]$, as shown in Table 2.

h_j	I	II	III	IV
h_1	0+1.2I	0.3+0.8I	0.5+0.8I	0.7+1.2I
h_2	0+0.8I	0.2+0.16I	0.6+0.8I	0.8+0.8I
h_3	0+I	0.25+0.8I	0.45+0.8I	0.65+1.4I
h_4	0+1.32I	0.33+0.68I	0.5+0.68I	0.67+1.32I
h_5	0+1.32I	0.33+0.68I	0.5+0.68I	0.67+1.32I
h_6	0+1.2I	0.3+1.2I	0.6+0.8I	0.8+0.8I
h_7	0+I	0.25+I	0.5+I	0.75+I
h_8	0+1.2I	0.3+1.2I	0.6+0.8I	0.8+0.8I

Table 2: NNs of slope stability between the eight impact factors and the four classification grades.

4.2 Slope samples/cases with NNs

The actually measured data of the eight impact factors obtained by the slope samples are all the forms of single values, which can be also considered as special cases of NNs without the uncertain part. For instance, suppose there is a number 0.4, then it can be considered as NN $0.4 + 0I$ or $[0.4, 0.4]$. Thus, we choose ten slope samples/cases from the south central area of Zhejiang province in China, where the data are shown in Tables 3 and 4. The actual grades in Tables 3 and 4 are given by using the limit equilibrium method for convenient comparison.

h_j	T_1	T_2	T_3	T_4	T_5
h_1	0.35	0.75	0.85	0.45	0.15
h_2	0.4	0.7	0.6	0.4	0.2
h_3	0.4	0.8	0.85	0.4	0.3
h_4	0.76	0.78	0.81	0.62	0.56
h_5	0.81	0.79	1	0.59	0.3
h_6	0.5	0.4	0.25	0.35	0.4
h_7	0.71	0.71	0.86	0.86	0.86
h_8	0.4	0.2	0.2	0.4	0.3
Actual grade	II	IV	IV	II	I

Table 3: Data of slope samples/cases.

h_j	T_6	T_7	T_8	T_9	T_{10}
h_1	0.7	0.1	0.43	0.6	0.2
h_2	0.6	0.6	0.8	0.6	0.1
h_3	0.55	0.15	0.38	0.5	0.15
h_4	0.7	0.67	0.72	0.83	0.61
h_5	0.33	0.61	0.23	0.21	0.43
h_6	0.75	0.7	0.8	0.4	0.15
h_7	0.71	0.71	0.7	0.7	0.86
h_8	0.3	0.2	0.4	0.4	0.1
Actual grade	III	I	II	III	I

Table 4: Data of slope samples/cases.

4.3 Stability assessment of slope samples/cases based on the arctangent and tangent similarity measures

Set $G = \{G_1, G_2, G_3, G_4\} = \{I, II, III, IV\}$ as a set of four classification grades for slope stability assessment and $T = \{T_1, T_2, \dots, T_{10}\}$ as a set of ten slope samples/cases. If we consider $w = (0.35, 0.17, 0.39, 0.0115, 0.027, 0.0215, 0.005, 0.025)$ as the weight vector of the eight impact factors, the slope stability assessment method can be applied to the slope stability assessment problems with NNs.

Thus, we calculate the similarity measure between the slope sample $T_s (s = 1, 2, \dots, 10)$ and the classification grade $G_k (k = 1, 2, 3, 4)$ for the eight impact factors $h_j (j = 1, 2, \dots, 8)$ by the following formula:

$$AT_w(T_s, G_k) = 1 - \frac{4}{\pi} \sum_{j=1}^n w_j \arctan \left[\frac{1}{2} \left(\frac{|c_{sj} + e_{sj} \inf I - c_{kj} - e_{kj} \inf I|}{|c_{sj} + e_{sj} \sup I - c_{kj} - e_{kj} \sup I|} \right) \right], \tag{5}$$

$$\text{or } T_w(T_s, G_k) = 1 - \sum_{j=1}^8 \left\{ w_j \tan \left[\frac{\pi}{8} \left(\frac{|c_{sj} + e_{sj} \inf I - c_{kj} - e_{kj} \inf I|}{|c_{sj} + e_{sj} \sup I - c_{kj} - e_{kj} \sup I|} \right) \right] \right\}. \tag{6}$$

Then, the measure values of Eq. (5) or Eq. (6) are given in Tables 5 and 6. The maximum measure value indicates the corresponding assessment grade of slope stability. For the results in Tables 5 and 6, the arctangent and tangent similarity measures can carry out all the classification recognitions because all the obtained assessment grades are the same as the actual grades of the ten slope cases.

<i>s</i>	$AT_w(T_s, G_1)$	$AT_w(T_s, G_2)$	$AT_w(T_s, G_3)$	$AT_w(T_s, G_4)$	Assessment grade	Actual grade
1	0.6678	0.8347	0.7282	0.4592	II	II
2	0.3053	0.5455	0.7571	0.7637	IV	IV
3	0.2708	0.4988	0.6800	0.7405	IV	IV
4	0.6365	0.8428	0.7725	0.4898	II	II
5	0.7994	0.7809	0.5572	0.3041	I	I
6	0.4403	0.7076	0.8586	0.6965	III	III
7	0.7284	0.7025	0.5604	0.3194	I	I
8	0.5822	0.7972	0.7933	0.5536	II	II

Table 5: Results of the arctangent measures and the slope stability grades

<i>s</i>	$T_w(T_s, G_1)$	$T_w(T_s, G_2)$	$T_w(T_s, G_3)$	$T_w(T_s, G_4)$	Assessment grade	Actual grade
1	0.9963	0.9982	0.9970	0.9938	II	II
2	0.9916	0.9949	0.9973	0.9974	IV	IV
3	0.9910	0.9942	0.9965	0.9971	IV	IV
4	0.9960	0.9983	0.9975	0.9942	II	II
5	0.9978	0.9976	0.9950	0.9916	I	I
6	0.9935	0.9968	0.9985	0.9966	III	III
7	0.9969	0.9967	0.9950	0.9917	I	I
8	0.9952	0.9978	0.9977	0.9949	II	II

Table 6: Results of the tangent measures and the slope stability grades

Hence, the arctangent similarity measure and the tangent similarity measure can be suitable for handling slope stability assessment in these slope samples/cases. They demonstrate that the assessment results of slope stability corresponding to the arctangent and tangent similarity measures are the effectiveness and feasibility of the slope stability assessment method proposed in this paper.

Conclusion

Since existing assessment approaches [1-6] cannot cope with the evaluation problems of slope stability with NNs in uncertain setting, this paper put forward the arctangent and tangent similarity measures between NNs and their slope stability assessment method in NN setting. Further, Ten slope samples/cases with NN information were given to indicate the applicability of the slope stability assessment approach. It is obvious that the assessment results corresponding to the arctangent and tangent similarity measures demonstrated the same as actual grades of the ten slope cases, which indicated the effectiveness and feasibility of the developed slope stability assessment approach in NN setting. Then, the main advantages in this study are that the developed assessment approach is simpler and more effective than existing ones and can assess slope stability problems with NN information, which traditional limit equilibrium methods [1-4] and finite element methods [5,6] cannot do. For the future work, the proposed similarity measures of NNs will be extended to other applications, such as slope clustering analysis and slope failure recognition.

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