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Soft Neutrosophic Bigroup and Soft Neutrosophic N-Group

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Abstract. Soft neutrosophic group and soft neutrosophic subgroup are generalized to soft neutrosophic bigroup and soft neutrosophic N-group respectively in this paper. Different kinds of soft neutrosophic bigroup and soft neutrosophic N-group are given. The structural properties and theorems have been discussed with a lot of examples to disclose many aspects of this beautiful man made structure.

Keywords: Neutrosophic bigroup, Neutrosophic N-group, soft set, soft group, soft subgroup, soft neutrosophic bigroup, soft neutrosophic subbigroup, soft neutrosophic N-group, soft neutrosophic sub N-group.

1 Introduction

Neutrosophy is a new branch of philosophy which is in fact the birth stage of neutrosophic logic first found by Florentin Smarandache in 1995. Each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set $[1]$, intuitionistic fuzzy set $[2]$ and interval valued fuzzy set $[3]$. This mathematical tool is handling problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures in $[11]$. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

Molodtsov in $[11]$ laid down the stone foundation of a richer structure called soft set theory which is free from the parameterization inadequacy, syndrome of fuzzy se theory, rough set theory, probability theory and so on. In many areas it has been successfully applied such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. Recently soft set theory has attained much attention since its appearance and the work based on several operations of soft sets introduced in $[2,9,10]$. Some more exciting properties and algebra may be found in $[1]$. Feng et al. introduced the soft semirings $[5]$. By means of level soft sets an adjustable approach to fuzzy soft sets based decision making can be seen in $[6]$. Some other new concept combined with fuzzy sets and rough sets was presented in $[7,8]$. AygÄunoglu et al. introduced the Fuzzy soft groups $[4]$. This paper is a mixture of neutrosophic bigroup,neutrosophic N-group and soft set theory which is infact a generalization of soft neutrosophic group. This combination gave birth to a new and fantastic approach called "Soft Neutrosophic Bigroup and Soft Neutrosophic N-group".

2.1 Neutrosophic Bigroup and N-Group

Definition 1 Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a non empty subset with two binary operations on $B_N(G)$ satisfying the following conditions:

1) $B_N(G)=\{B(G_1)\cup B(G_2)\}$ where $B(G_1)$ and $B(G_2)$ are proper subsets of $B_N(G)$.
2) $(B(G_1),*_1)$ is a neutrosophic group.
3) $(B(G_2),*_2)$ is a group .

Then we define $(B_N(G),*_1,*_2)$ to be a neutrosophic bigroup. If both $B(G_1)$ and $B(G_2)$ are neutrosophic groups. We say $B_N(G)$ is a strong neutrosophic bigroup. If both the groups are not neutrosophic group, we say $B_N(G)$ is just a bigroup.

Example 1 Let $B_N(G)=\{B(G_1)\cup B(G_2)\}$ where $B(G_1)=\{g/g^9=1\}$ be a cyclic group of order 9 and $B(G_2)=\{1,2,I,2I\}$ neutrosophic group under multiplication modulo 3. We call $B_N(G)$ a neutrosophic bigroup.

Example 2 Let $B_N(G)=\{B(G_1)\cup B(G_2)\}$ Where $B(G_1)=\{1,2,3,4,I,2I,3I,4I\}$ a neutrosophic group under multiplication modulo 5.

$B(G_2)=\{0,1,2,I,2I,1+I,2+I,1+2I,2+2I\}$ is a neutrosophic group under multiplication modulo 3. Clearly $B_N(G)$ is a strong neutrosophic bigroup.

Definition 2 Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. A proper subset $P=\{P_1\cup P_2,*_1,*_2\}$ is a neutrosophic subbigroup of $B_N(G)$ if the following conditions are satisfied $P=\{P_1\cup P_2,*_1,*_2\}$ is a neutrosophic bigroup under the operations $*_1,*_2$ i.e. $(P_1,*_1)$ is a neutrosophic subgroup of $(B_1,*_1)$ and $(P_2,*_2)$ is a subgroup of $(B_2,*_2)$.

$P_1=P\cap B_1$ and $P_2=P\cap B_2$ are subgroups of B_1 and B_2 respectively. If both of P_1 and P_2 are not neutrosophic then we call $P=P_1\cup P_2$ to be just a bigroup.

Definition 3 Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. If both $B(G_1)$ and $B(G_2)$ are commutative groups, then we call $B_N(G)$ to be a commutative bigroup.

Definition 4 Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. If both $B(G_1)$ and $B(G_2)$ are cyclic, we call $B_N(G)$ a cyclic bigroup.

Definition 5 Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic bigroup. $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ is said to be a neutrosophic normal subbigroup of $B_N(G)$ if $P(G)$ is a neutrosophic subbigroup and both $P(G_1)$ and $P(G_2)$ are normal subgroups of $B(G_1)$ and $B(G_2)$ respectively.

Definition 6 Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup of finite order. Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic subbigroup of $B_N(G)$. If $o(P(G))/o(B_N(G))$ then we call $P(G)$ a Lagrange neutrosophic subbigroup, if every neutrosophic subbigroup P is such that $o(P)/o(B_N(G))$ then we call $B_N(G)$ to be a Lagrange neutrosophic bigroup.

Definition 7 If $B_N(G)$ has atleast one Lagrange neutrosophic subbigroup then we call $B_N(G)$ to be a weak Lagrange neutrosophic bigroup.

Definition 8 If $B_N(G)$ has no Lagrange neutrosophic subbigroup then $B_N(G)$ is called Lagrange free neutrosophic bigroup.

Definition 9 Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup. Suppose $P=\{P(G_1)\cup P(G_2),*_1,*_2\}$ and

$K=\{K(G_1)\cup K(G_2),*_1,*_2\}$ be any two neutrosophic subbigroups. we say P and K are conjugate if each $P(G_i)$ is conjugate with $K(G_i), i=1,2$, then we say P and K are neutrosophic conjugate subbigroups of $B_N(G)$.

Definition 10 A set $(\langle G\cup I\rangle,+,\circ)$ with two binary operations ` $+$ ' and ` $\circ$ ' is called a strong neutrosophic bigroup if

1) $\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle$,
2) $(\langle G_1\cup I\rangle,+)$ is a neutrosophic group and
3) $(\langle G_2\cup I\rangle,\circ)$ is a neutrosophic group.

Example 3 Let $\{\langle G\cup I\rangle,*_1,*_2\}$ be a strong neutrosophic bigroup where $\langle G\cup I\rangle=\langle Z\cup I\rangle\cup\{0,1,2,3,4,I,2I,3I,4I\}$. $\langle Z\cup I\rangle$ under ` $+$ ' is a neutrosophic group and $\{0,1,2,3,4,I,2I,3I,4I\}$ under multiplication modulo 5 is a neutrosophic group.

Definition 11 A subset $H\neq\phi$ of a strong neutrosophic bigroup $(\langle G\cup I\rangle,*,\circ)$ is called a strong neutrosophic subbigroup if H itself is a strong neutrosophic bigroup under ` $*$ ' and ` $\circ$ ' operations defined on $\langle G\cup I\rangle$.

Definition 12 Let $(\langle G\cup I\rangle,*,\circ)$ be a strong neutrosophic bigroup of finite order. Let $H\neq\phi$ be a strong neutrosophic subbigroup of $(\langle G\cup I\rangle,*,\circ)$. If $o(H)/o(\langle G\cup I\rangle)$ then we call H, a Lagrange strong neutrosophic subbigroup of $\langle G\cup I\rangle$. If every strong neutrosophic subbigroup of $\langle G\cup I\rangle$ is a Lagrange strong neutrosophic subbigroup then we call $\langle G\cup I\rangle$ a Lagrange strong neutrosophic bigroup.

Definition 13 If the strong neutrosophic bigroup has at least one Lagrange strong neutrosophic subbigroup then we call $\langle G\cup I\rangle$ a weakly Lagrange strong neutrosophic bigroup.

Definition 14 If $\langle G\cup I\rangle$ has no Lagrange strong neutrosophic subbigroup then we call $\langle G\cup I\rangle$ a Lagrange free strong neutrosophic bigroup.

Definition 15 Let $(\langle G\cup I\rangle,+,\circ)$ be a strong neutrosophic bigroup with $\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle$. Let $(H,+,\circ)$ be a neutrosophic subbigroup where $H=H_1\cup H_2$. We say H is a neutrosophic normal subbigroup of G if both H_1 and H_2 are neutrosophic normal subgroups of $\langle G_1\cup I\rangle$ and $\langle G_2\cup I\rangle$ respectively.

Definition 16 Let $G=\langle G_1\cup G_2,*,\otimes\rangle$, be a neutrosophic bigroup. We say two neutrosophic strong subbigroups $H=H_1\cup H_2$ and $K=K_1\cup K_2$ are conjugate neutrosophic subbigroups of $\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle$ if H_1 is conjugate to K_1 and H_2 is conjugate to K_2 as neutrosophic subgroups of $\langle G_1\cup I\rangle$ and $\langle G_1\cup I\rangle$ respectively.

Definition 17 Let $(\langle G\cup I\rangle,*_1,...,*_N)$ be a nonempty set with N -binary operations defined on it. We say $\langle G\cup I\rangle$ is a strong neutrosophic N -group if the following conditions are true.

1) $\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup...\cup\langle G_N\cup I\rangle$ where $\langle G_i\cup I\rangle$ are proper subsets of $\langle G\cup I\rangle$.
2) $(\langle G_i\cup I\rangle,*_i)$ is a neutrosophic group, $i=1,2,...,N$.
3) If in the above definition we have
   a. $\langle G\cup I\rangle=G_1\cup\langle G_2\cup I\rangle\cup...\cup\langle G_k\cup I\rangle\cup\langle G_{k+1}\cup I\rangle\cup...\cup G_N$
   b. $(G_i,*_i)$ is a group for some i or
4) $(\langle G_j\cup I\rangle,*_j)$ is a neutrosophic group for some j . Then we call $\langle G\cup I\rangle$ to be a neutrosophic N -group.

Example 4 Let $\langle G\cup I\rangle=(\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup\langle G_3\cup I\rangle\cup\langle G_4\cup I\rangle,*_1,*_2,*_3,*_4)$ be a neutrosophic 4 -group where $\langle G_1\cup I\rangle=\{1,2,3,4,I,2I,3I,4I\}$ neutrosophic group under multiplication modulo 5 . $\langle G_2\cup I\rangle=\{0,1,2,I,2I,1+I,2+I,1+2I,2+2I\}$ a neutrosophic group under multiplication modulo 3 ,

$\langle G_3 \cup I \rangle = \langle Z \cup I \rangle$, a neutrosophic group under addition and $\langle G_4 \cup I \rangle = \{(a,b) : a,b \in \{1, I, 4, 4I\}\}$, component-wise multiplication modulo $5\}$.

Hence $\langle G \cup I \rangle$ is a strong neutrosophic 4 -group.

Example 5 Let

$\left(\langle G \cup I \rangle = \langle G_1 \cup I \rangle \cup \langle G_2 \cup I \rangle \cup G_3 \cup G_4, *_1, *_2, *_3, *_4\right)$

be a neutrosophic 4 -group, where

$\langle G_1 \cup I \rangle = \{1, 2, 3, 4, I, 2I, 3I, 4I\}$ a neutrosophic group under multiplication modulo 5.

$\langle G_2 \cup I \rangle = \{0, 1, I, 1+I\}$,a neutrosophic group under multiplication modulo 2. $G_3 = S_3$ and $G_4 = A_5$, the alternating group. $\langle G \cup I \rangle$ is a neutrosophic 4 -group.

Definition 18 Let

$\left(\langle G \cup I \rangle = \langle G_1 \cup I \rangle \cup \langle G_2 \cup I \rangle \cup ... \cup \langle G_N \cup I \rangle, *_1, ..., *_N\right)$

be a neutrosophic N -group. A proper subset $(P, *_1, ..., *_N)$ is said to be a neutrosophic sub N -group of $\langle G \cup I \rangle$ if $P = (P_1 \cup ... \cup P_N)$ and each $(P_i, *_i)$ is a neutrosophic subgroup (subgroup) of $(G_i, *_i), 1 \leq i \leq N$.

It is important to note $(P, *_i)$ for no i is a neutrosophic group.

Thus we see a strong neutrosophic N -group can have 3 types of subgroups viz.

1) Strong neutrosophic sub N -groups.
2) Neutrosophic sub N -groups.
3) Sub N -groups.

Also a neutrosophic N -group can have two types of sub N -groups.

1) Neutrosophic sub N -groups.
2) Sub N -groups.

Definition 19 If $\langle G \cup I \rangle$ is a neutrosophic N -group and if $\langle G \cup I \rangle$ has a proper subset T such that T is a neutrosophic sub N -group and not a strong neutrosophic sub N -group and $o(T) / o(\langle G \cup I \rangle)$ then we call T a Lagrange sub N -group. If every sub N -group of $\langle G \cup I \rangle$ is a Lagrange sub N -group then we call $\langle G \cup I \rangle$ a Lagrange N -group.

Definition 20 If $\langle G \cup I \rangle$ has atleast one Lagrange sub N -group then we call $\langle G \cup I \rangle$ a weakly Lagrange neutrosophic N-group.

Definition 21 If $\langle G \cup I \rangle$ has no Lagrange sub N -group then we call $\langle G \cup I \rangle$ to be a Lagrange free N -group.

Definition 22 Let

$\left(\langle G \cup I \rangle = \langle G_1 \cup I \rangle \cup \langle G_2 \cup I \rangle \cup ... \cup \langle G_N \cup I \rangle, *_1, ..., *_N\right)$

be a neutrosophic N -group. Suppose

$H = \{H_1 \cup H_2 \cup ... \cup H_N, *_1, ..., *_N\}$ and

$K = \{K_1 \cup K_2 \cup ... \cup K_N, *_1, ..., *_N\}$ are two sub N -groups of $\langle G \cup I \rangle$, we say K is a conjugate to H or H is conjugate to K if each H_i is conjugate to K_i $(i = 1, 2, ..., N)$ as subgroups of G_i .

2.2 Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A \subset E$. Molodtsov defined the soft set in the following manner:

Definition 23 A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U . For $x \in A$, $F(x)$ may be considered as the set of x -elements of the soft set (F, A) , or as the set of e-approximate elements of the soft set.

Example 6 Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or sentence. Let

$$E = \left\{ \begin{array}{l} \text{high rent,normal rent,} \\ \text{in good condition,in bad condition} \end{array} \right\}.$$

Let us consider a soft set (F, A) which describes the attractiveness of shops that Mr. Z is taking on rent. Suppose that there are five houses in the universe $U = \{s_1, s_2, s_3, s_4, s_5\}$ under consideration, and that $A = \{x_1, x_2, x_3\}$ be the set of parameters where

x_1 stands for the parameter 'high rent,

x_2 stands for the parameter 'normal rent,

x_3 stands for the parameter 'in good condition.

Suppose that

$$F(x_1) = \{s_1, s_4\},$$
$$F(x_2) = \{s_2, s_5\},$$
$$F(x_3) = \{s_3, s_4, s_5\}.$$

The soft set (F,A) is an approximated family $\{F(e_i), i = 1,2,3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Then (F,A) is a soft set as a collection of approximations over U, where

$$F(x_1) = high\ rent = \{s_1, s_2\},$$
$$F(x_2) = normal\ rent = \{s_2, s_5\},$$
$$F(x_3) = in\ good\ condition = \{s_3, s_4, s_5\}.$$

Definition 24 For two soft sets (F,A) and (H,B) over U, (F,A) is called a soft subset of (H,B) if

1. $A \subseteq B$ and
2. $F(x) \subseteq H(x)$, for all $x \in A$.

This relationship is denoted by $(F,A) \subset (H,B)$. Similarly (F,A) is called a soft superset of (H,B) if (H,B) is a soft subset of (F,A) which is denoted by $(F,A) \supset (H,B)$.

Definition 25 Two soft sets (F,A) and (H,B) over U are called soft equal if (F,A) is a soft subset of (H,B) and (H,B) is a soft subset of (F,A).

Definition 26 Let (F,A) and (K,B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F,A) \cap_R (K,B) = (H,C)$ where (H,C) is defined as $H(c) = F(c) \cap \mathrm{K}(c)$ for all $c \in C = A \cap B$.

Definition 27 The extended intersection of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B, \\ G(c) & \text{if } c \in B - A, \\ F(c) \cap G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F,A) \cap_\varepsilon (K,B) = (H,C)$.

Definition 28 The restricted union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as $H(c) = F(c) \cup G(c)$ for all $c \in C$. We write it as $(F,A) \cup_R (K,B) = (H,C).$

Definition 29 The extended union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - B, \\ G(c) & \text{if } c \in B - A, \\ F(c) \cup G(c) & \text{if } c \in A \cap B. \end{cases}$$

We write $(F,A) \cup_\varepsilon (K,B) = (H,C)$.

2.3 Soft Groups

Definition 30 Let (F,A) be a soft set over G. Then (F,A) is said to be a soft group over G if and only if $F(x) \prec G$ for all $x \in A$.

Example 7 Suppose that $G = A = S_3 = \{e,(12),(13),(23),(123),(132)\}.$ Then (F,A) is a soft group over S_3 where

$$F(e) = \{e\},$$
$$F(12) = \{e,(12)\},$$
$$F(13) = \{e,(13)\},$$
$$F(23) = \{e,(23)\},$$
$$F(123) = F(132) = \{e,(123),(132)\}.$$

Definition 31 Let (F,A) be a soft group over G. Then

1. (F,A) is said to be an identity soft group over G if $F(x) = \{e\}$ for all $x \in A$, where e is the identity element of G and
2. (F,A) is said to be an absolute soft group if $F(x) = G$ for all $x \in A$.

3 Soft Neutrosophic Bigroup

Definition 32 Let

$$B_N(G) = \{B(G_1) \cup B(G_2), *_1, *_2\}$$

be a neutrosophic bigroup and let (F,A) be a soft set over $B_N(G)$. Then (F,A) is said to be soft neutrosophic bigroup over $B_N(G)$ if and only if $F(x)$ is a subbigroup of $B_N(G)$ for all $x \in A$.

Example 8 Let

$$B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$$

be a neutrosophic bigroup, where

$$B(G_1)=\{0,1,2,3,4,I,2I,3I,4I\}$$

is a neutrosophic group under multiplication modulo 5. $B(G_2)=\{g : g^{12}=1\}$ is a cyclic group of order 12.

Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic subbigroup where $P(G_1)=\{1,4,I,4I\}$ and $P(G_2)=\{1,g^2,g^4,g^6,g^8,g^{10}\}$.

Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another neutrosophic subbigroup where $Q(G_1)=\{1,I\}$ and $Q(G_2)=\{1,g^3,g^6,g^9\}$.

Then (F,A) is a soft neutrosophic bigroup over $B_N(G)$, where

$$F(e_1)=\{1,4,I,4I,1,g^2,g^4,g^6,g^8,g^{10}\}$$

$$F(e_2)=\{1,I,1,g^3,g^6,g^9\}.$$

Theorem 1 Let (F,A) and (H,A) be two soft neutrosophic bigroup over $B_N(G)$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic bigroup over $B_N(G)$.

Proof Straight forward.

Theorem 2 Let (F,A) and (H,B) be two soft neutrosophic bigroups over $B_N(G)$ such that $A\cap B=\phi$, then their union is soft neutrosophic bigroup over $B_N(G)$.

Proof Straight forward.

Proposition 1 The extended union of two soft neutrosophic bigroups (F,A) and (K,D) over $B_N(G)$ is not a soft neutrosophic bigroup over $B_N(G)$.

To prove it, see the following example.

Example 9 Let $B_N(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$, where $B(G_1)=\{1,2,3,4I,2I,3I,4I\}$ and $B(G_2)=S_3$.

Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic subbigroup where $P(G_1)=\{1,4,I,4I\}$ and $P(G_2)=\{e,(12)\}$.

Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another neutrosophic subbigroup where $Q(G_1)=\{1,I\}$ and $Q(G_2)=\{e,(123),(132)\}$.

Then (F,A) is a soft neutrosophic bigroup over $B_N(G)$, where

$$F(x_1)=\{1,4,I,4I,e,(12)\}$$

$$F(x_2)=\{1,I,e,(123),(132)\}.$$

Again let $R(G)=\{R(G_1)\cup R(G_2),*_1,*_2\}$ be another neutrosophic subbigroup where $R(G_1)=\{1,4,I,4I\}$ and $R(G_2)=\{e,(13)\}$.

Also $T(G)=\{T(G_1)\cup T(G_2),*_1,*_2\}$ be a neutrosophic subbigroup where $T(G_1)=\{1,I\}$ and $T(G_2)=\{e,(23)\}$.

Then (K,D) is a soft neutrosophic bigroup over $B_N(G)$, where

$$K(x_2)=\{1,4,I,4I,e,(13)\},$$

$$K(x_3)=\{1,I,e,(23)\}.$$

The extended union $(F,A)\cup_\varepsilon(K,D)=(H,C)$ such that $C=A\cup D$ and for $x_2\in C$, we have

$$H(x_2)=F(x_2)\cup K(x_2)=\{1,4,I,4I,e,(13)(123),(132)\}$$

is not a subbigroup of $B_N(G)$.

Proposition 2 The extended intersection of two soft neutrosophic bigroups (F,A) and (K,D) over $B_N(G)$ is again a soft neutrosophic bigroup over $B_N(G)$.

Proposition 3 The restricted union of two soft neutrosophic bigroups (F,A) and (K,D) over $B_N(G)$ is not a soft neutrosophic bigroup over $B_N(G)$.

Proposition 4 The restricted intersection of two soft neutrosophic bigroups (F,A) and (K,D) over $B_N(G)$ is a soft neutrosophic bigroup over $B_N(G)$.

Proposition 5 The AND operation of two soft neutrosophic bigroups over $B_N(G)$ is again soft neutrosophic bigroup over $B_N(G)$.

Proposition 6 The OR operation of two soft neutrosophic bigroups over $B_N(G)$ may not be a soft nuetrosophic bigroup.

Definition 33 Let (F,A) be a soft neutrosophic bigroup over $B_N(G)$. Then

1) (F,A) is called identity soft neutrosophic bigroup if $F(x)=\{e_1,e_2\}$ for all $x\in A$, where e_1 and e_2 are the identities of $B(G_1)$ and $B(G_2)$ respectively.
2) (F,A) is called Full-soft neutrosophic bigroup if $F(x)=B_N(G)$ for all $x\in A$.

Theorem 3 Let $B_N(G)$ be a neutrosophic bigroup of prime order P, then (F,A) over $B_N(G)$ is either identity soft neutrosophic bigroup or Full-soft neutrosophic bigroup.

Definition 34 Let (F,A) and (H,K) be two soft neutrosophic bigroups over $B_N(G)$. Then (H,K) is soft neutrosophi subbigroup of (F,A) written as $(H,K)\prec(F,A)$, if

1) $K\subset A$,
2) $K(x)\prec F(x)$ for all $x\in A$.

Example 10 Let

$B(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ where

$$B(G_1)=\begin{Bmatrix}0,1,2,3,4,I,2I,3I,4I,1+I,2+I,3+I,4+I,\\ 1+2I,2+2I,3+2I,4+2I,1+3I,2+3I,\\ 3+3I,4+3I,1+4I,2+4I,3+4I,4+4I\end{Bmatrix}$$

be a neutrosophic group under multiplication modulo 5 and $B(G_2)=\{g:g^{16}=1\}$ a cyclic group of order 16. Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic subbigroup where

$P(G_1)=\{0,1,2,3,4,I,2I,3I,4I\}$

and

be another neutrosophic subbigroup where

$P(G_2)=\{g^2,g^4,g^6,g^8,g^{10},g^{12},g^{14},1\}$.

Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$

$Q(G_1)=\{0,1,4,I,4I\}$

and

$Q(G_2)=\{g^4,g^8,g^{12},1\}$.

Again let $R(G)=\{R(G_1)\cup R(G_2),*_1,*_2\}$ be a neutrosophic subbigroup where

$R(G_1)=\{0,1,I\}$ and $R(G_2)=\{1,g^8\}$.

Let (F,A) be a soft neutrsophic bigroup over $B_N(G)$ where

$F(x_1)=\{0,1,2,3,4,I,2I,3I,4I,g^2,g^4,g^6,g^8,g^{10},g^{12},g^{14},1\}$,

$F(x_2)=\{0,1,4,I,4I,g^4,g^8,g^{12},1\}$,

$F(x_3)=\{0,1,I,g^8,1\}$.

Let (H,K) be another soft neutrosophic bigroup over $B_N(G)$, where

$H(x_1)=\{0,1,2,3,4,g^4,g^8,g^{12},1\}$,

$H(x_2)=\{0,1,I,g^8,1\}$.

Clearly $(H,K)\prec(F,A)$.

Definition 35 Let $B_N(G)$ be a neutrosophic bigroup. Then (F,A) over $B_N(G)$ is called commutative soft neutrosophic bigroup if and only if $F(x)$ is a commutative subbigroup of $B_N(G)$ for all $x\in A$.

Example 11 Let $B(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup where $B(G_1)=\{g:g^{10}=1\}$ be a cyclic group of order 10 and $B(G_2)=\{1,2,3,4,I,2\mathrm{I},3I,4I\}$ be a neutrosophic group under mltiplication modulo 5.
Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a commutative neutrosophic subbigroup where $P(G_1)=\{1,g^5\}$ and $P(G_2)=\{1,4,I,4I\}$. Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another commutative neutrosophic subbigroup where $Q(G_1)=\{1,g^2,g^4,g^6,g^8\}$ and $Q(G_2)=\{1,I\}$.
Then (F,A) is commutative soft neutrosophic bigroup over $B_N(G)$, where

$$F(x_1)=\{1,g^5,1,4,I,4I\},$$
$$F(x_2)=\{1,g^2,g^4,g^6,g^8,1,I\}.$$

Theorem 4 Every commutative *soft neutrosophic bigroup* (F,A) over $B_N(G)$ is a soft *neutrosophic bigroup* but the converse is not true.

Theorem 5 If $B_N(G)$ is commutative neutrosophic bigroup. Then (F,A) over $B_N(G)$ is commutative soft neutrosophic bigroup but the converse is not true.

Theorem 6 If $B_N(G)$ is cyclic neutrosophic bigroup. Then (F,A) over $B_N(G)$ is commutative soft neutrosophic bigroup.

Proposition 7 Let (F,A) and (K,D) be two commutative soft neutrosophic bigroups over $B_N(G)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(K,D)$ over $B_N(G)$ is not commutative soft neutrosophic bigroup over $B_N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,D)$ over $B_N(G)$ is commutative soft neutrosophic bigroup over $B_N(G)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $B_N(G)$ is not commutative soft neutrosophic bigroup over $B_N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $B_N(G)$ is commutative soft neutrosophic bigroup over $B_N(G)$.

Proposition 8 Let (F,A) and (K,D) be two commutative soft neutrosophic bigroups over $B_N(G)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is commutative soft neutrosophic bigroup over $B_N(G)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not commutative soft neutrosophic bigroup over $B_N(G)$.

Definition 36 Let $B_N(G)$ be a neutrosophic bigroup. Then (F,A) over $B_N(G)$ is called cyclic soft neutrosophic bigroup if and only if $F(x)$ is a cyclic subbigroup of $B_N(G)$ for all $x\in A$.

Example 12 Let $B(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup where $B(G_1)=\{g:g^{10}=1\}$ be a cyclic group of order 10 and $B(G_2)=\{0,1,2,I,2I,1+I,2+I,1+2I,2+2I\}$ be a neutrosophic group under multiplication modulo 3.
Le $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a cyclic neutrosophic subbigroup where $P(G_1)=\{1,g^5\}$ and $\{1,1+I\}$.
Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another cyclic neutrosophic subbigroup where $Q(G_1)=\{1,g^2,g^4,g^6,g^8\}$ and $Q(G_2)=\{1,2+2I\}$.
Then (F,A) is cyclic soft neutrosophic bigroup over $B_N(G)$, where

$$F(x_1)=\{1,g^5,1,1+I\},$$
$$F(x_2)=\{1,g^2,g^4,g^6,g^8,1,2+2I\}.$$

Theorem 7 If $B_N(G)$ is a cyclic neutrosophic soft bigroup, then (F,A) over $B_N(G)$ is also cyclic soft neutrosophic bigroup.

Theorem 8 Every cyclic soft neutrosophic bigroup (F,A) over $B_N(G)$ is a soft neutrosophic bigroup but the converse is not true.

Proposition 9 Let (F,A) and (K,D) be two cyclic soft neutrosophic bigroups over $B_N(G)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(K,D)$ over $B_N(G)$ is not cyclic soft neutrosophic bigroup over $B_N(G)$.
2) Their extended intersection $(F,A)\bigcap_{\varepsilon}(K,D)$ over $B_N(G)$ is cyclic soft neutrosophic bigroup over $B_N(G)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $B_N(G)$ is not cyclic soft neutrosophic bigroup over $B_N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $B_N(G)$ is cyclic soft neutrosophic bigroup over $B_N(G)$.

Proposition 10 Let (F,A) and (K,D) be two cyclic soft neutrosophic bigroups over $B_N(G)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is cyclic soft neutrosophic bigroup over $B_N(G)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not cyclic soft neutrosophic bigroup over $B_N(G)$.

Definition 37 Let $B_N(G)$ be a neutrosophic bigroup. Then (F,A) over $B_N(G)$ is called normal soft neutrosophic bigroup if and only if $F(x)$ is normal subbigroup of $B_N(G)$ for all $x\in A$.

Example 13 Let $B(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup, where

$$B(G_1)=\left\{\begin{array}{l} e,y,x,x^2,xy,x^2y,I, \\ Iy,Ix,Ix^2,Ixy,Ix^2y \end{array}\right\}$$

is a neutrosophic group under multiplaction and $B(G_2)=\{g:g^6=1\}$ is a cyclic group of order 6.

Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a normal neutrosophic subbigroup where $P(G_1)=\{e,y\}$ and

$$P(G_2)=\{1,g^2,g^4\}.$$

Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another normal neutrosophic subbigroup where

$Q(G_1)=\{e,x,x^2\}$ and $Q(G_2)=\{1,g^3\}$.

Then (F,A) is a normal soft neutrosophic bigroup over $B_N(G)$ where

$$F(x_1)=\{e,y,1,g^2,g^4\},$$
$$F(x_2)=\{e,x,x^2,1,g^3\}.$$

Theorem 9 Every normal soft neutrosophic bigroup (F,A) over $B_N(G)$ is a soft neutrosophic bigroup but the converse is not true.

Theorem 10 If $B_N(G)$ is a normal neutrosophic bigroup. Then (F,A) over $B_N(G)$ is also normal soft neutrosophic bigroup.

Theorem 11 If $B_N(G)$ is a commutative neutrosophic bigroup. Then (F,A) over $B_N(G)$ is normal soft neutrosophic bigroup.

Theorem 12 If $B_N(G)$ is a cyclic neutrosophic bigroup. Then (F,A) over $B_N(G)$ is normal soft neutrosophic bigroup.

Proposition 11 Let (F,A) and (K,D) be two normal soft neutrosophic bigroups over $B_N(G)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(K,D)$ over $B_N(G)$ is not normal soft neutrosophic bigroup over

$B_N(G)$.

2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ over $B_N(G)$ is normal soft neutrosophic bigroup over $B_N(G)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $B_N(G)$ is not normal soft neutrosophic bigroup over $B_N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $B_N(G)$ is normal soft neutrosophic bigroup over $B_N(G)$.

Proposition 12 Let (F,A) and (K,D) be two normal soft neutrosophic bigroups over $B_N(G)$. Then

1) Their *AND* operation $(F,A)\wedge(K,D)$ is normal soft neutrosophic bigroup over $B_N(G)$.
2) Their *OR* operation $(F,A)\vee(K,D)$ is not normal soft neutrosophic bigroup over $B_N(G)$.

Definition 38 Let (F,A) be a soft neutrosophic bigroup over $B_N(G)$. If for all $x\in A$ each $F(x)$ is a Lagrange subbigroup of $B_N(G)$, then (F,A) is called Lagrange soft neutosophic bigroup over $B_N(G)$.

Example 14 Let $B(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup, where

$$B(G_1)=\begin{Bmatrix} e,y,x,x^2,xy,x^2y,I, \\ Iy,Ix,Ix^2,Ixy,Ix^2y \end{Bmatrix}$$

is a neutrosophic symmetric group of and $B(G_2)=\{0,1,I,1+I\}$ be a neutrosophic group under addition modulo 2. Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic subbigroup where $P(G_1)=\{e,y\}$ and $P(G_2)=\{0,1\}$.

Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another neutrosophic subbigroup where $Q(G_1)=\{e,Iy\}$ and $Q(G_2)=\{0,1+I\}$.

Then (F,A) is Lagrange soft neutrosophic bigroup over $B_N(G)$, where

$F(x_2)=\{e,y,0,1\}$,

$F(x_2)=\{e,yI,0,1+I\}$.

Theorem 13 If $B_N(G)$ is a Lagrange neutrosophic bigroup, then (F,A) over $B_N(G)$ is Lagrange soft neutrosophic bigroup.

Theorem 14 Every Lagrange soft *neutrosophic bigroup* (F,A) over $B_N(G)$ is a soft *neutrosophic bigroup* but the converse is not true.

Proposition 13 Let (F,A) and (K,D) be two Lagrange soft neutrosophic bigroups over $B_N(G)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ over $B_N(G)$ is not Lagrange soft neutrosophic bigroup over $B_N(G)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ over $B_N(G)$ is not Lagrange soft neutrosophic bigroup over $B_N(G)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $B_N(G)$ is not Lagrange soft neutrosophic bigroup over $B_N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $B_N(G)$ is not Lagrange soft neutrosophic bigroup over $B_N(G)$.

Proposition 14 Let (F,A) and (K,D) be two Lagrange soft neutrosophic bigroups over $B_N(G)$. Then

1) Their *AND* operation $(F,A)\wedge(K,D)$ is not Lagrange soft neutrosophic bigroup over $B_N(G)$.
2) Their *OR* operation $(F,A)\vee(K,D)$ is not Lagrange soft neutrosophic bigroup over $B_N(G)$.

Definition 39 Let (F,A) be a soft neutrosophic bigroup over $B_N(G)$. Then (F,A) is called weakly Lagrange soft neutosophic bigroup over $B_N(G)$ if at-least one $F(x)$ is a Lagrange subbigroup of $B_N(G)$, for some $x \in A$.

Example 15 Let $B(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup, where

$$B(G_1)=\begin{Bmatrix} 0,1,2,3,4,I,2I,3I,4I,1+I,2+I,3+I,4+I, \\ 1+2I,2+2I,3+2I,4+2I,1+3I,2+3I, \\ 3+3I,4+3I,1+4I,2+4I,3+4I,4+4I \end{Bmatrix}$$

is a neutrosophic group under multiplication modulo 5 and $B(G_2)=\{g:g^{10}=1\}$ is a cyclic group of order 10. Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic subbigroup where $P(G_1)=\{0,1,4,I,4I\}$ and $P(G_2)=\{g^2,g^4,g^6,g^8,1\}$. Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another neutrosophic subbigroup where $Q(G_1)=\{0,1,4,I,4I\}$ and $Q(G_2)=\{g^5,1\}$. Then (F,A) is a weakly Lagrange soft neutrosophic bigroup over $B_N(G)$, where

$$F(x_1)=\{0,1,4,I,4I,g^2,g^4,g^6,g^8,1\},$$
$$F(x_2)=\{0,1,4,I,4I,g^5,1\}.$$

Theorem 15 Every weakly Lagrange *soft neutrosophic bigroup* (F,A) over $B_N(G)$ is a soft *neutrosophic bigroup* but the converse is not true.

Proposition 15 Let (F,A) and (K,D) be two weakly Lagrange soft neutrosophic bigroups over $B_N(G)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(K,D)$ over $B_N(G)$ is not weakly Lagrange soft neutrosophic bigroup over $B_N(G)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,D)$ over $B_N(G)$ is not weakly Lagrange soft neutrosophic bigroup over $B_N(G)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $B_N(G)$ is not weakly Lagrange soft neutrosophic bigroup over $B_N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $B_N(G)$ is not weakly Lagrange soft neutrosophic bigroup over $B_N(G)$.

Proposition 16 Let (F,A) and (K,D) be two weakly Lagrange soft neutrosophic bigroups over $B_N(G)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is not weakly Lagrange soft neutrosophic bigroup over $B_N(G)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not weakly Lagrange soft neutrosophic bigroup over $B_N(G)$.

Definition 40 Let (F,A) be a soft neutrosophic bigroup over $B_N(G)$. Then (F,A) is called Lagrange free soft neutrosophic bigroup if each $F(x)$ is not Lagrange subbigroup of $B_N(G)$, for all $x \in A$.

Example 16 Let $B(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a neutrosophic bigroup, where $B(G_1)=\{0,1,I,1+I\}$ is a neutrosophic group under addition modulo 2 of order 4 and $B(G_2)=\{g:g^{12}=1\}$ is a cyclic group of order 12. Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic subbigroup where $P(G_1)=\{0,I\}$ and $P(G_2)=\{g^4,g^8,1\}$. Also $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another neutrosophic subbigroup where $Q(G_1)=\{0,1+I\}$ and $Q(G_2)=\{1,g^3,g^6,g^9\}$. Then (F,A) is Lagrange free soft neutrosophic bigroup over $B_N(G)$, where

$$F(x_1)=\{0,I,1,g^4,g^8\},$$
$$F(x_2)=\{0,1+I,1,g^3,g^6,g^9\}.$$

Theorem 16 If $B_N(G)$ is Lagrange free neutrosophic bigroup, and then (F,A) over $B_N(G)$ is Lagrange free soft neutrosophic bigroup.

Theorem 17 Every Lagrange free *soft neutrosophic bigroup* (F,A) over $B_N(G)$ is a soft *neutrosophic bigroup* but the converse is not true.

Proposition 17 Let (F,A) and (K,D) be two Lagrange free soft neutrosophic bigroups over $B_N(G)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ over $B_N(G)$ is not Lagrange free soft neutrosophic bigroup over $B_N(G)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ over $B_N(G)$ is not Lagrange free soft neutrosophic bigroup over $B_N(G)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $B_N(G)$ is not Lagrange free soft neutrosophic bigroup over $B_N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $B_N(G)$ is not Lagrange free soft neutrosophic bigroup over $B_N(G)$.

Proposition 18 Let (F,A) and (K,D) be two Lagrange free soft neutrosophic bigroups over $B_N(G)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is not Lagrange free soft neutrosophic bigroup over $B_N(G)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not Lagrange free soft neutrosophic bigroup over $B_N(G)$.

Definition 41 Let $B_N(G)$ be a neutrosophic bigroup. Then (F,A) is called conjugate soft neutrosophic bigroup over $B_N(G)$ if and only if $F(x)$ is neutrosophic conjugate subbigroup of $B_N(G)$ for all $x\in A$.

Example 17 Let $B(G)=\{B(G_1)\cup B(G_2),*_1,*_2\}$ be a soft neutrosophic bigroup, where

$$B(G_1)=\{e,y,x,x^2,xy,x^2y\}$$

is Klien 4 -group and

$$B(G_2)=\begin{Bmatrix}0,1,2,3,4,5,I,2I,3I,4I,5I,\\ 1+I,2+I,3+I,...,5+5I\end{Bmatrix}$$

be a neutrosophic group under addition modulo 6.
Let $P(G)=\{P(G_1)\cup P(G_2),*_1,*_2\}$ be a neutrosophic subbigroup of $B_N(G)$, where $P(G_1)=\{e,y\}$ and $P(G_2)=\{0,3,3I,3+3I\}$. Again let $Q(G)=\{Q(G_1)\cup Q(G_2),*_1,*_2\}$ be another neutrosophic subbigroup of $B_N(G)$, where $Q(G_1)=\{e,x,x^2\}$ and $Q(G_2)=\{0,2,4,2+2I,4+4I,2I,4I\}$. Then (F,A) is conjugate soft neutrosophic bigroup over $B_N(G)$, where

$$F(x_1)=\{e,y,0,3,3I,3+3I\},$$
$$F(x_2)=\{e,x,x^2,0,2,4,2+2I,4+4I,2I,4I\}.$$

Theorem 18 If $B_N(G)$ is conjugate neutrosophic bigroup, then (F,A) over $B_N(G)$ is conjugate soft neutrosophic bigroup.

Theorem 19 Every conjugate soft neutrosophic bigroup (F,A) over $B_N(G)$ is a soft neutrosophic bigroup but the converse is not true.

Proposition 19 Let (F,A) and (K,D) be two conjugate soft neutrosophic bigroups over $B_N(G)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ over $B_N(G)$ is not conjugate soft neutrosophic bigroup over $B_N(G)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ over $B_N(G)$ is conjugate soft neutrosophic bigroup over $B_N(G)$.

3) Their restricted union $(F,A)\cup_R(K,D)$ over $B_N(G)$ is not conjugate soft neutrosophic bigroup over $B_N(G)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $B_N(G)$ is conjgate soft neutrosophic bigroup over $B_N(G)$.

Proposition 20 Let (F,A) and (K,D) be two conjugate soft neutrosophic bigroups over $B_N(G)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is conjugate soft neutrosophic bigroup over $B_N(G)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not conjugate soft neutrosophic bigroup over $B_N(G)$.

3.3 Soft Strong Neutrosophic Bigroup

Definition 42 Let $(\langle G\cup I\rangle,*_1,*_2)$ be a strong neutrosophic bigroup. Then (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is called soft strong neutrosophic bigroup if and only if $F(x)$ is a strong neutrosophic subbigroup of $(\langle G\cup I\rangle,*_1,*_2)$ for all $x\in A$.

Example 18 Let $(\langle G\cup I\rangle,*_1,*_2)$ be a strong neutrosophic bigroup, where $\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle$ with $\langle G_1\cup I\rangle=\langle Z\cup I\rangle$, the neutrosophic group under addition and $\langle G_2\cup I\rangle=\{0,1,2,3,4,I,2I,3I,4I\}$ a neutrosophic group under multiplication modulo 5. Let $H=H_1\cup H_2$ be a strong neutrosophic subbigroup of $(\langle G\cup I\rangle,*_1,*_2)$, where $H_1=\{\langle 2Z\cup I\rangle,+\}$ is a neutrosophic subgroup and $H_2=\{0,1,4,I,4I\}$ is a neutrosophic subgroup. Again let $K=K_1\cup K_2$ be another strong neutrosophic subbigroup of $(\langle G\cup I\rangle,*_1,*_2)$, where $K_1=\{\langle 3Z\cup I\rangle,+\}$ is a neutrosophic subgroup and $K_2=\{0,1,I,2I,3I,4I\}$ is a neutrosophic subgroup. Then clearly (F,A) is a soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$, where

$$F(x_1)=\{0,\pm2,\pm4,\ldots,1,4,I,4I\},$$
$$F(x_2)=\{0,\pm3,\pm6,\ldots,1,I,2I,3I,4I\}.$$

Theorem 20 Every soft strong neutrosophic bigroup (F,A) is a soft neutrosophic bigroup but the converse is not true.

Theorem 21 If $(\langle G\cup I\rangle,*_1,*_2)$ is a strong neutrosophic bigroup, then (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is soft strong neutrosophic bigroup.

Proposition 21 Let (F,A) and (K,D) be two soft strong neutrosophic bigroups over $(\langle G\cup I\rangle,*_1,*_2)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.

Proposition 22 Let (F,A) and (K,D) be two soft strong neutrosophic bigroups over $(\langle G\cup I\rangle,*_1,*_2)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.

Definition 43 Let $(\langle G\cup I\rangle,*_1,*_2)$ be a strong neutrosophic bigroup. Then (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is called Lagrange soft strong neutrosophic bigroup if and only if $F(x)$ is Lagrange subbigroup of $(\langle G\cup I\rangle,*_1,*_2)$ for all $x\in A$.

Example 19 Let $(\langle G\cup I\rangle,*_1,*_2)$ be a strong neutrosophic bigroup of order 15, where $\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle$ with $\langle G_1\cup I\rangle=\{0,1,2,1+I,I,2I,2+I,2+2I,1+2I\}$, the neutrosophic group under mltiplication modulo 3 and $\langle G_2\cup I\rangle=\langle A_3\cup I\rangle=\{e,x,x^2,I,xI,x^2I\}$. Let $H=H_1\cup H_2$ be a strong neutrosophic subbigroup of $(\langle G\cup I\rangle,*_1,*_2)$, where $H_1=\{1,2+2I\}$ is a neutrosophic subgroup and $H_2=\{e,x,x^2\}$ is a neutrosophic subgroup. Again let $K=K_1\cup K_2$ be another strong neutrosophic subbigroup of $(\langle G\cup I\rangle,*_1,*_2)$, where $K_1=\{1,1+I\}$ is a neutrosophic subgroup and $K_2=\{I,xI,x^2I\}$ is a neutrosophic subgroup. Then clearly (F,A) is Lagrange soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$, where

$$F(x_1)=\{1,2+2I,e,x,x^2\},$$
$$F(x_2)=\{1,1+I,I,xI,x^2I\}.$$

Theorem 22 Every Lagrange soft strong neutrosophic bigroup (F,A) is a soft neutrosophic bigroup but the converse is not true.

Theorem 23 Every Lagrange soft strong neutrosophic bigroup (F,A) is a soft strong neutrosophic bigroup but the converse is not true.

Theorem 24 If $(\langle G\cup I\rangle,*_1,*_2)$ is a Lagrange strong neutrosophic bigroup, then (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is a Lagrange soft strong neutrosophic soft bigroup.

Proposition 23 Let (F,A) and (K,D) be two Lagrange soft strong neutrosophic bigroups over $(\langle G\cup I\rangle,*_1,*_2)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not Lagrange soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not Lagrange soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not Lagrange soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not Lagrange soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.

Proposition 24 Let (F,A) and (K,D) be two Lagrange soft strong neutrosophic bigroups over $(\langle G\cup I\rangle,*_1,*_2)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is not Lagrange soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not Lagrange soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.

Definition 44 Let $(\langle G\cup I\rangle,*_1,*_2)$ be a strong neutrosophic bigroup. Then (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is called weakly Lagrange soft strong neutrosophic bigroup if atleast one $F(x)$ is a Lagrange subbigroup of $(\langle G\cup I\rangle,*_1,*_2)$ for some $x\in A$.

Example 20 Let $(\langle G\cup I\rangle,*_1,*_2)$ be a strong neutrosophic bigroup of order 15, where $\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle$ with $\langle G_1\cup I\rangle=\{0,1,2,1+I,I,2I,2+I,2+2I,1+2I\}$, the neutrosophic under mltiplication modulo 3 and

$\langle G_2 \cup I \rangle = \{e, x, x^2, I, xI, x^2I\}$. Let $H = H_1 \cup H_2$ be a strong neutrosophic subbigroup of $(\langle G \cup I \rangle, *_1, *_2)$, where $H_1 = \{1, 2, I, 2I\}$ is a neutrosophic subgroup and $H_2 = \{e, x, x^2\}$ is a neutrosophic subgroup. Again let $K = K_1 \cup K_2$ be another strong neutrosophic subbigroup of $(\langle G \cup I \rangle, *_1, *_2)$, where $K_1 = \{1, 1+I\}$ is a neutrosophic subgroup and $K_2 = \{e, I, xI, x^2I\}$ is a neutrosophic subgroup.

Then clearly (F, A) is weakly Lagrange soft strong neutrosophic bigroup over $(\langle G \cup I \rangle, *_1, *_2)$, where

$$F(x_1) = \{1, 2, I, 2I, e, x, x^2\},$$

$$F(x_2) = \{1, 1+I, e, I, xI, x^2I\}.$$

Theorem 25 Every weakly Lagrange soft strong neutrosophic bigroup (F, A) is a soft neutrosophic bigroup but the converse is not true.

Theorem 26 Every weakly Lagrange soft strong neutrosophic bigroup (F, A) is a soft strong neutrosophic bigroup but the converse is not true.

Proposition 25 Let (F, A) and (K, D) be two weakly Lagrange soft strong neutrosophic bigroups over $(\langle G \cup I \rangle, *_1, *_2)$. Then

1) Their extended union $(F, A) \cup_\varepsilon (K, D)$ over $(\langle G \cup I \rangle, *_1, *_2)$ is not weakly Lagrange soft strong neutrosophic bigroup over $(\langle G \cup I \rangle, *_1, *_2)$.
.
2) Their extended intersection $(F, A) \cap_\varepsilon (K, D)$ over $(\langle G \cup I \rangle, *_1, *_2)$ is not weakly Lagrange soft strong neutrosophic bigroup over $(\langle G \cup I \rangle, *_1, *_2)$.
3) Their restricted union $(F, A) \cup_R (K, D)$ over $(\langle G \cup I \rangle, *_1, *_2)$ is not weakly Lagrange soft strong neutrosophic bigroup over $(\langle G \cup I \rangle, *_1, *_2)$. .
4) Their restricted intersection $(F, A) \cap_R (K, D)$ over $(\langle G \cup I \rangle, *_1, *_2)$ is not weakly Lagrange soft strong neutrosophic bigroup over $(\langle G \cup I \rangle, *_1, *_2)$.

Proposition 26 Let (F, A) and (K, D) be two weakly Lagrange soft strong neutrosophic bigroups over $(\langle G \cup I \rangle, *_1, *_2)$. Then

1) Their AND operation $(F, A) \wedge (K, D)$ is not weakly Lagrange soft strong neutrosophic bigroup over $(\langle G \cup I \rangle, *_1, *_2)$. .
2) Their OR operation $(F, A) \vee (K, D)$ is not weakly Lagrange soft strong neutrosophic bigroup over $(\langle G \cup I \rangle, *_1, *_2)$.

Definition 45 Let $(\langle G \cup I \rangle, *_1, *_2)$ be a strong neutrosophic bigroup. Then (F, A) over $(\langle G \cup I \rangle, *_1, *_2)$ is called Lagrange free soft strong neutrosophic bigroup if and only if $F(x)$ is not Lagrange subbigroup of $(\langle G \cup I \rangle, *_1, *_2)$ for all $x \in A$.

Example 21 Let $(\langle G \cup I \rangle, *_1, *_2)$ be a strong neutrosophic bigroup of order 15, where $\langle G \cup I \rangle = \langle G_1 \cup I \rangle \cup \langle G_2 \cup I \rangle$ with $\langle G_1 \cup I \rangle = \{0, 1, 2, 3, 4, I, 2I, 3I, 4I\}$, the neutrosophic under mltiplication modulo 5 and $\langle G_2 \cup I \rangle = \{e, x, x^2, I, xI, x^2I\}$, a neutrosophic symmetric group. Let $H = H_1 \cup H_2$ be a strong neutrosophic subbigroup of $(\langle G \cup I \rangle, *_1, *_2)$, where $H_1 = \{1, 4, I, 4I\}$ is a neutrosophic subgroup and $H_2 = \{e, x, x^2\}$ is a neutrosophic subgroup. Again let $K = K_1 \cup K_2$ be another strong neutrosophic subbigroup of $(\langle G \cup I \rangle, *_1, *_2)$, where $K_1 = \{1, I, 2I, 3I, 4I\}$ is a neutrosophic subgroup and $K_2 = \{e, x, x^2\}$ is a neutrosophic subgroup.

Then clearly (F, A) is Lagrange free soft strong neutrosophic bigroup over $(\langle G \cup I \rangle, *_1, *_2)$, where

$$F(x_1)=\{1,4,I,4I,e,x,x^2\},$$
$$F(x_2)=\{1,I,2I,3I,4I,e,x,x^2\}.$$

Theorem 27 Every Lagrange free soft strong neutrosophic bigroup (F,A) is a soft neutrosophic bigroup but the converse is not true.

Theorem 28 Every Lagrange free soft strong neutrosophic bigroup (F,A) is a soft strong neutrosophic bigroup but the converse is not true.

Theorem 29 If $(\langle G\cup I\rangle,*_1,*_2)$ is a Lagrange free strong neutrosophic bigroup, then (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is also Lagrange free soft strong neutrosophic bigroup.

Proposition 27 Let (F,A) and (K,D) be weakly Lagrange free soft strong neutrosophic bigroups over $(\langle G\cup I\rangle,*_1,*_2)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not Lagrange free soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not Lagrange free soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not Lagrange free soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not Lagrange free soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.

Proposition 28 Let (F,A) and (K,D) be two Lagrange free soft strong neutrosophic bigroups over $(\langle G\cup I\rangle,*_1,*_2)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is not Lagrange free soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not Lagrange free soft strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.

Definition 46 Let $(\langle G\cup I\rangle,*_1,*_2)$ be a strong neutrosophic bigroup. Then (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is called soft normal strong neutrosophic bigroup if and only if $F(x)$ is normal strong neutrosophic subbigroup of $(\langle G\cup I\rangle,*_1,*_2)$ for all $x\in A$.

Theorem 30 Every soft normal *strong neutrosophic bigroup* (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is a soft *neutrosophic bigroup* but the converse is not true.

Theorem 31 Every soft normal *strong neutrosophic bigroup* (F,A) over $(\langle G\cup I\rangle,*_1,*_2)$ is a soft strong *neutrosophic bigroup* but the converse is not true.

Proposition 29 Let (F,A) and (K,D) be two soft normal strong neutrosophic bigroups over $(\langle G\cup I\rangle,*_1,*_2)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not soft normal strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is soft normal strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is not soft normal strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ over $(\langle G\cup I\rangle,*_1,*_2)$ is soft normal strong neutrosophic bigroup over $(\langle G\cup I\rangle,*_1,*_2)$.

Proposition 30 Let (F,A) and (K,D) be two soft normal strong neutrosophic bigroups over $\left(\langle G\cup I\rangle,*_1,*_2\right)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is soft normal strong neutrosophic bigroup over $\left(\langle G\cup I\rangle,*_1,*_2\right)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft normal strong neutrosophic bigroup over $\left(\langle G\cup I\rangle,*_1,*_2\right)$.

Definition 47 Let $\left(\langle G\cup I\rangle,*_1,*_2\right)$ be a strong neutrosophic bigroup. Then (F,A) over $\left(\langle G\cup I\rangle,*_1,*_2\right)$ is called soft conjugate strong neutrosophic bigroup if and only if $F(x)$ is conjugate neutrosophic subbigroup of $\left(\langle G\cup I\rangle,*_1,*_2\right)$ for all $x\in A$.

Theorem 32 Every soft conjugate *strong neutrosophic bigroup* (F,A) over $\left(\langle G\cup I\rangle,*_1,*_2\right)$ is a soft *neutrosophic bigroup* but the converse is not true.

Theorem 33 Every soft conjugate *strong neutrosophic bigroup* (F,A) over $\left(\langle G\cup I\rangle,*_1,*_2\right)$ is a soft strong neutrosophic bigroup *but the converse is not true.*

Proposition 31 Let (F,A) and (K,D) be two soft conjugate strong neutrosophic bigroups over $\left(\langle G\cup I\rangle,*_1,*_2\right)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(K,D)$ over $\left(\langle G\cup I\rangle,*_1,*_2\right)$ is not soft conjugate strong neutrosophic bigroup over $\left(\langle G\cup I\rangle,*_1,*_2\right)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,D)$ over $\left(\langle G\cup I\rangle,*_1,*_2\right)$ is soft conjugate strong neutrosophic bigroup over $\left(\langle G\cup I\rangle,*_1,*_2\right)$.
3) Their restricted union $(F,A)\cup_{R}(K,D)$ over $\left(\langle G\cup I\rangle,*_1,*_2\right)$ is not soft conjugate strong neutrosophic bigroup over $\left(\langle G\cup I\rangle,*_1,*_2\right)$.
4) Their restricted intersection $(F,A)\cap_{R}(K,D)$ over $\left(\langle G\cup I\rangle,*_1,*_2\right)$ is soft conjugate strong neutrosophic bigroup over $\left(\langle G\cup I\rangle,*_1,*_2\right)$.

Proposition 32 Let (F,A) and (K,D) be two soft conjugate strong neutrosophic bigroups over $\left(\langle G\cup I\rangle,*_1,*_2\right)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is soft conjugate strong neutrosophic bigroup over $\left(\langle G\cup I\rangle,*_1,*_2\right)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft conjgate strong neutrosophic bigroup over $\left(\langle G\cup I\rangle,*_1,*_2\right)$.

4.1 Soft Neutrosophic N-Group

Definition 48 Let $\left(\langle G\cup I\rangle,*_1,\ldots,*_N\right)$ be a neutrosophic N -group. Then (F,A) over $\left(\langle G\cup I\rangle,*_1,\ldots,*_2\right)$ is called soft neutrosophic N -group if and only if $F(x)$ is a sub N -group of $\left(\langle G\cup I\rangle,*_1,\ldots,*_2\right)$ for all $x\in A$.

Example 22 Let $\left(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup\langle G_3\cup I\rangle,*_1,*_2,*_3\right)$ be a neutrosophic 3-group, where $\langle G_1\cup I\rangle=\langle Q\cup I\rangle$ a neutrosophic group under multiplication. $\langle G_2\cup I\rangle=\{0,1,2,3,4,I,2I,3I,4I\}$ neutrosophic group under multiplication modulo 5 and $\langle G_3\cup I\rangle=\{0,1,2,1+I,2+I,I,2I,1+2I,2+2I\}$ a neutrosophic group under multiplication modulo 3. Let

$$P=\left\{\left\{\left\langle\frac{1}{2^n},2^n,\frac{1}{(2I)^n},(2I)^n,I,1\right\rangle\right\},(1,4,I,4I),(1,2,I,2I)\right\},$$

$T=\{Q\setminus\{0\},\{1,2,3,4\},\{1,2\}\}$ and

$X=\{Q\setminus\{0\},\{1,2,I,2I\},\{1,4,I,4I\}\}$ are sub 3-groups.

Then (F,A) is clearly soft neutrosophic 3-group over $\left(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup\langle G_3\cup I\rangle,*_1,*_2,*_3\right)$, where

$F(x_1)=\left\{\left\{\left\langle \frac{1}{2^n},2^n,\frac{1}{(2I)^n},(2I)^n,I,1\right\rangle\right\},(1,4,I,4I),(1,2,I,2I)\right\},$

$F(x_2)=\{Q\setminus\{0\},\{1,2,3,4\},\{1,2\}\},$

$F(x_3)=\{Q\setminus\{0\},\{1,2,I,2I\},\{1,4,I,4I\}\}.$

Theorem 34 Let (F,A) and (H,A) be two soft neutrosophic N-groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Proof The proof is straight forward.

Theorem 35 Let (F,A) and (H,B) be two soft neutrosophic N-groups over $(\langle G\cup I\rangle,*_1,...,*_N)$ such that $A\cap B-=\phi,$ then their union is soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Proof The proof can be established easily.

Proposition 33 Let (F,A) and (K,D) be two soft neutrosophic N-groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ is not soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ is soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ is not soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ is soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Proposition 34 Let (F,A) and (K,D) be two soft neutrosophic N-groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Definition 49 Let (F,A) be a soft neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) (F,A) is called identity soft neutrosophic N-group if $F(x)=\{e_1,...,e_N\}$ for all $x\in A,$ where $e_1,...,e_N$ are the identities of $\langle G_1\cup I\rangle,...,\langle G_N\cup I\rangle$ respectively.
2) (F,A) is called Full soft neutrosophic N-group if $F(x)=(\langle G\cup I\rangle,*_1,...,*_N)$ for all $x\in A.$

Definition 50 Let (F,A) and (K,D) be two soft neutrosophic N-groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then (K,D) is soft neutrosophic sub N-group of (F,A) written as $(K,D)\prec(F,A)$, if

1) $D\subset A,$
2) $K(x)\prec F(x)$ for all $x\in A.$

Example 23 Let (F,A) be as in example 22. Let (K,D) be another soft neutrosophic soft N-group over $(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup\langle G_3\cup I\rangle,*_1,*_2,*_3)$, where

$K(x_1)=\left\{\left\{\left\langle\frac{1}{2^n},2^n\right\rangle\right\},\{1,4,I,4I\},\{1,2,I,2I\}\right\},$

$K(x_2)=\{Q\setminus\{0\},\{1,4\},\{1,2\}\}.$

Clearly $(K,D)\prec(F,A)$.

Thus a soft neutrosophic N-group can have two types of soft neutrosophic sub N-groups, which are following

Definition 51 A soft neutrosophic sub N-group (K,D) of a soft neutrosophic N-group (F,A) is called soft strong neutrosophic sub N-group if

1) $D\subset A,$
2) $K(x)$ is neutrosophic sub N-group of $F(x)$ for

all $x \in A$.

Definition 52 A soft neutrosophic sub N -group (K, D) of a soft neutrosophic N -group (F, A) is called soft sub N -group if

1) $D \subset A$,
2) $K(x)$ is only sub N -group of $F(x)$ for all $x \in A$.

Definition 53 Let $(\langle G \cup I \rangle, *_1, ..., *_N)$ be a neutrosophic N -group. Then (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is called soft Lagrange neutrosophic N -group if and only if $F(x)$ is Lagrange sub N -group of $(\langle G \cup I \rangle, *_1, ..., *_N)$ for all $x \in A$.

Example 24 Let $(\langle G \cup I \rangle = \langle G_1 \cup I \rangle \cup G_2 \cup G_3, *_1, *_2, *_3)$ be neutrosophic N -group, where $\langle G_1 \cup I \rangle = \{\langle Z_6 \cup I \rangle\}$ is a group under addition modulo 6, $G_2 = A_4$ and $G_3 = \langle g : g^{12} = 1 \rangle$, a cyclic group of order 12, $o(\langle G \cup I \rangle) = 60$.

Take $P = (\langle P_1 \cup I \rangle \cup P_2 \cup P_3, *_1, *_2, *_3)$, a neutrosophic sub 3-group where

$\langle T_1 \cup I \rangle = \{0, 3, 3I, 3 + 3I\}$,

$$P_2 = \left\{ \begin{pmatrix} 1234 \\ 1234 \end{pmatrix}, \begin{pmatrix} 1234 \\ 2143 \end{pmatrix}, \begin{pmatrix} 1234 \\ 4321 \end{pmatrix}, \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} \right\},$$

$P_3 = \{1, g^6\}$. Since P is a Lagrange neutrosophic sub 3-group where order of $P = 10$.

Let us Take $T = (\langle T_1 \cup I \rangle \cup T_2 \cup T_3, *_1, *_2, *_3)$, where $\langle T_1 \cup I \rangle = \{0, 3, 3I, 3 + 3I\}, T_2 = P_2$ and $T_3 = \{g^3, g^6, g^9, 1\}$ is another Lagrange sub 3-group where $o(T) = 12$.

Let (F, A) is soft Lagrange neutrosophic N -group over $(\langle G \cup I \rangle = \langle G_1 \cup I \rangle \cup G_2 \cup G_3, *_1, *_2, *_3)$, where

$$F(x_1) = \left\{ 0, 3, 3I, 3 + 3I, 1, g^6, \begin{pmatrix} 1234 \\ 1234 \end{pmatrix}, \begin{pmatrix} 1234 \\ 2143 \end{pmatrix}, \begin{pmatrix} 1234 \\ 4321 \end{pmatrix}, \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} \right\},$$

$$F(x_2) = \left\{ 0, 3, 3I, 3 + 3I, 1, g^3, g^6, g^9, \begin{pmatrix} 1234 \\ 1234 \end{pmatrix}, \begin{pmatrix} 1234 \\ 2143 \end{pmatrix}, \begin{pmatrix} 1234 \\ 4321 \end{pmatrix}, \begin{pmatrix} 1234 \\ 3412 \end{pmatrix} \right\}.$$

Theorem 36 Every soft Lagrange neutrosophic N -group (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is a soft neutrosophic N -group but the converse is not true.

Theorem 37 If $(\langle G \cup I \rangle, *_1, ..., *_N)$ is a Lagrange neutrosophic N -group, then (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is also soft Lagrange neutrosophic N -group.

Proposition 35 Let (F, A) and (K, D) be two soft Lagrange neutrosophic N -groups over $(\langle G \cup I \rangle, *_1, ..., *_N)$. Then

1) Their extended union $(F, A) \cup_{\varepsilon} (K, D)$ is not soft Lagrange neutrosophic N -group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, D)$ is not soft Lagrange neutrosophic N -group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
3) Their restricted union $(F, A) \cup_R (K, D)$ is not soft Lagrange neutrosophic N -group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
4) Their restricted intersection $(F, A) \cap_R (K, D)$ is not soft Lagrange neutrosophic N -group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.

Proposition 36 Let (F, A) and (K, D) be two soft Lagrange neutrosophic N -groups over $(\langle G \cup I \rangle, *_1, ..., *_N)$. Then

1) Their AND operation $(F, A) \wedge (K, D)$ is not soft Lagrange neutrosophic N -group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
2) Their OR operation $(F, A) \vee (K, D)$ is not soft Lagrange neutrosophic N -group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.

Definition 54 Let $(\langle G\cup I\rangle,*_1,...,*_N)$ be a neutrosophic N -group. Then (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is called soft weakly Lagrange neutrosophic N -group if atleast one $F(x)$ is Lagrange sub N -group of $(\langle G\cup I\rangle,*_1,...,*_N)$ for some $x\in A$.

Examp 25 Let $(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup G_2\cup G_3,*_1,*_2,*_3)$ be neutrosophic N -group, where $\langle G_1\cup I\rangle=\{\langle Z_6\cup I\rangle\}$ is a group under addition modulo 6 , $G_2=A_4$ and $G_3=\langle g:g^{12}=1\rangle$, a cyclic group of order 12, $o(\langle G\cup I\rangle)=60$.

Take $P=(\langle P_1\cup I\rangle\cup P_2\cup P_3,*_1,*_2,*_3)$, a neutrosophic sub 3-group where

$\langle T_1\cup I\rangle=\{0,3,3I,3+3I\}$,

$$P_2=\left\{\begin{pmatrix}1234\\1234\end{pmatrix},\begin{pmatrix}1234\\2143\end{pmatrix},\begin{pmatrix}1234\\4321\end{pmatrix},\begin{pmatrix}1234\\3412\end{pmatrix}\right\},$$

$P_3=\{1,g^6\}$. Since P is a Lagrange neutrosophic sub 3-group where order of $P=10$.

Let us Take $T=(\langle T_1\cup I\rangle\cup T_2\cup T_3,*_1,*_2,*_3)$, where $\langle T_1\cup I\rangle=\{0,3,3I,3+3I\},T_2=P_2$ and $T_3=\{g^4,g^8,1\}$ is another Lagrange sub 3-group.

Then (F,A) is soft weakly Lagrange neutrosophic N -group over $(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup G_2\cup G_3,*_1,*_2,*_3)$, where

$$F(x_1)=\left\{0,3,3I,3+3I,1,g^6,\begin{pmatrix}1234\\1234\end{pmatrix},\begin{pmatrix}1234\\2143\end{pmatrix},\begin{pmatrix}1234\\4321\end{pmatrix},\begin{pmatrix}1234\\3412\end{pmatrix}\right\},$$

$$F(x_2)=\left\{0,3,3I,3+3I,1,g^4,g^8,\begin{pmatrix}1234\\1234\end{pmatrix},\begin{pmatrix}1234\\2143\end{pmatrix},\begin{pmatrix}1234\\4321\end{pmatrix},\begin{pmatrix}1234\\3412\end{pmatrix}\right\}.$$

Theorem 38 Every soft weakly Lagrange neutrosophic N -group (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is a soft neutrosophic N -group but the converse is not tue.

Theorem 39 If $(\langle G\cup I\rangle,*_1,...,*_N)$ is a weakly Lagrange neutrosophi N -group, then (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is also soft weakly Lagrange neutrosophic N -group.

Proposition 37 Let (F,A) and (K,D) be two soft weakly Lagrange neutrosophic N -groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1. Their extended union $(F,A)\cup_\varepsilon(K,D)$ is not soft weakly Lagrange neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2. Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ is not soft weakly Lagrange neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
3. Their restricted union $(F,A)\cup_R(K,D)$ is not soft weakly Lagrange neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
4. Their restricted intersection $(F,A)\cap_R(K,D)$ is not soft weakly Lagrange neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Proposition 38 Let (F,A) and (K,D) be two soft weakly Lagrange neutrosophic N -groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is not soft weakly Lagrange neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft weakly Lagrange neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Definition 55 Let $(\langle G\cup I\rangle,*_1,...,*_N)$ be a neutrosophic N -group. Then (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is called soft Lagrange free neutrosophic N-group if $F(x)$ is not Lagrange sub N-group of $(\langle G\cup I\rangle,*_1,...,*_N)$ for all $x\in A$.

Example 26 Let $(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup G_2\cup G_3,*_1,*_2,*_3)$ be neutrosophic 3-group, where $\langle G_1\cup I\rangle=\{\langle Z_6\cup I\rangle\}$ is a group under addition modulo 6, $G_2=A_4$ and $G_3=\langle g:g^{12}=1\rangle$, a cyclic group of order 12, $o(\langle G\cup I\rangle)=60$.

Take $P=(\langle P_1\cup I\rangle\cup P_2\cup P_3,*_1,*_2,*_3)$, a neutrosophic sub 3-group where

$P_1=\{0,2,4\}$,

$$P_2=\left\{\begin{pmatrix}1234\\1234\end{pmatrix},\begin{pmatrix}1234\\2143\end{pmatrix},\begin{pmatrix}1234\\4321\end{pmatrix},\begin{pmatrix}1234\\3412\end{pmatrix}\right\},$$

$P_3=\{1,g^6\}$. Since P is a Lagrange neutrosophic sub 3-group where order of $P=10$.

Let us Take $T=(\langle T_1\cup I\rangle\cup T_2\cup T_3,*_1,*_2,*_3)$, where $\langle T_1\cup I\rangle=\{0,3,3I,3+3I\},T_2=P_2$ and $T_3=\{g^4,g^8,1\}$ is another Lagrange sub 3-group.

Then (F,A) is soft Lagrange free neutrosophic 3-group over $(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup G_2\cup G_3,*_1,*_2,*_3)$, where

$$F(x_1)=\left\{0,2,4,1,g^6,\begin{pmatrix}1234\\1234\end{pmatrix},\begin{pmatrix}1234\\2143\end{pmatrix},\begin{pmatrix}1234\\4321\end{pmatrix},\begin{pmatrix}1234\\3412\end{pmatrix}\right\},$$

$$F(x_2)=\left\{0,3,3I,3+3I,1,g^4,g^8,\begin{pmatrix}1234\\1234\end{pmatrix},\begin{pmatrix}1234\\2143\end{pmatrix},\begin{pmatrix}1234\\4321\end{pmatrix},\begin{pmatrix}1234\\3412\end{pmatrix}\right\}$$

Theorem 40 Every soft Lagrange free neutrosophic N-group (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is a soft neutrosophic N-group but the converse is not true.

Theorem 41 If $(\langle G\cup I\rangle,*_1,...,*_N)$ is a Lagrange free neutrosophic N-group, then (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is also soft Lagrange free neutrosophic N-group.

Proposition 39 Let (F,A) and (K,D) be two soft Lagrange free neutrosophic N-groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ is not soft Lagrange free neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ is not soft Lagrange free neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ is not soft Lagrange free neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ is not soft Lagrange free neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Proposition 40 Let (F,A) and (K,D) be two soft Lagrange free neutrosophic N-groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is not soft Lagrange free neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft Lagrange free neutrosophic N-group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Definition 56 Let $(\langle G\cup I\rangle,*_1,...,*_N)$ be a neutrosophic N-group. Then (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is called soft normal neutrosophic N-group if $F(x)$ is normal sub N-group of $(\langle G\cup I\rangle,*_1,...,*_N)$ for all $x\in A$.

Example 27 Let $(\langle G_1\cup I\rangle=\langle G_1\cup I\rangle\cup G_2\cup\langle G_3\cup I\rangle,*_1,*_2,*_3)$ be a soft neutrosophic N-group, where $\langle G_1\cup I\rangle=\{e,y,x,x^2,xy,x^2y,I,yI,xI,x^2I,xyI,x^2yI\}$

is a neutrosophic group under multiplaction, $G_2=\{g:g^6=1\}$, a cyclic group of order 6 and $\langle G_3\cup I\rangle=\langle Q_8\cup I\rangle=\{\pm1,\pm i,\pm j,\pm k,\pm I,\pm iI,\pm jI,\pm kI\}$ is a group under multiplication. Let $P=(\langle P_1\cup I\rangle\cup P_2\cup\langle P_3\cup I\rangle,*_1,*_2,*_3)$, a normal sub 3-group where $P_1=\{e,y,I,yI\}$, $P_2=\{1,g^2,g^4\}$ and $P_3=\{1,-1\}$. Also $T=(\langle T_1\cup I\rangle\cup T_2\cup\langle T_3\cup I\rangle,*_1,*_2,*_3)$ be another normal sub 3-group where $\langle T_1\cup I\rangle=\{e,I,xI,x^2I\},T_2=\{1,g^3\}$ and $\langle T_3\cup I\rangle=\{\pm1,\pm i\}$. Then (F,A) is a soft normal neutrosophic N -group over $(\langle G_1\cup I\rangle=\langle G_1\cup I\rangle\cup G_2\cup\langle G_3\cup I\rangle,*_1,*_2,*_3)$, where

$$F(x_1)=\{e,y,I,yI,1,g^2,g^4,\pm1\},$$
$$F(x_2)=\{e,I,xI,x^2I,1,g^3,\pm1,\pm i\}.$$

Theorem 42 Every soft normal neutrosophic N -group (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is a soft neutrosophic N -group but the converse is not true.

Proposition 41 Let (F,A) and (K,D) be two soft normal neutrosophic N -groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ is not soft normal neutrosophic soft N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ is soft normal neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ is not soft normal neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ is soft normal neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Proposition 42 Let (F,A) and (K,D) be two soft normal neutrosophic N -groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is soft normal neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft normal neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Definition 56 Let $(\langle G\cup I\rangle,*_1,...,*_N)$ be a neutrosophic N -group. Then (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is called soft conjugate neutrosophic N -group if $F(x)$ is conjugate sub N -group of $(\langle G\cup I\rangle,*_1,...,*_N)$ for all $x\in A$.

Theorem 43 Every soft conjugate neutrosophic N -group (F,A) over $(\langle G\cup I\rangle,*_1,...,*_N)$ is a soft neutrosophic N -group but the converse is not true.

Proposition 43 Let (F,A) and (K,D) be two soft conjugate neutrosophic N -groups over $(\langle G\cup I\rangle,*_1,...,*_N)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ is not soft conjugate neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ is soft conjugate neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ is not soft conjugate neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ is soft conjugate neutrosophic N -group over $(\langle G\cup I\rangle,*_1,...,*_N)$.

Proposition 44 Let (F,A) and (K,D) be two soft conjugate neutrosophic N -groups over

$\left(\langle G\cup I\rangle,*_1,...,*_N\right)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is soft conjugate neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft conjugate neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.

4.2 Soft Strong Neutrosophic N-Group

Definition 57 Let $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ be a neutrosophic N -group. Then (F,A) over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ is called soft strong neutrosophic N -group if and only if $F(x)$ is a strong neutrosophic sub N -group for all $x\in A$.

Example 28 Let $\left(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup\langle G_3\cup I\rangle,*_1,*_2,*_3\right)$ be a neutrosophic 3 -group, where $\langle G_1\cup I\rangle=\langle Z_2\cup I\rangle=\{0,1,I,1+I\}$, a neutrosophic group under multiplication modulo 2. $\langle G_2\cup I\rangle=\{O,1,2,3,4,I,2I,3I,4I\}$, neutrosophic group under multiplication modulo 5 and $\langle G_3\cup I\rangle=\{0,1,2,I,2I\}$, a neutrosophic group under multiplication modulo 3. Let

$$P=\left\{\left\{\left\langle\frac{1}{2^n},2^n,\frac{1}{(2I)^n},(2I)^n,I,1\right\rangle\right\},\{1,4,I,4I\},\{1,2,I,2I\}\right\},$$

and $X=\{Q\setminus\{0\},\{1,2,I,2I\},\{1,I\}\}$ are neutrosophic sub 3 -groups.

Then (F,A) is clearly soft strong neutrosophic 3 -group over

$$\left(\langle G\cup I\rangle=\langle G_1\cup I\rangle\cup\langle G_2\cup I\rangle\cup\langle G_3\cup I\rangle,*_1,*_2,*_3\right),$$

where

$$F(x_1)=\left\{\left\{\left\langle\frac{1}{2^n},2^n,\frac{1}{(2I)^n},(2I)^n,I,1\right\rangle\right\},\{1,4,I,4I\},\{1,I\}\right\},$$

$$F(x_2)=\{Q\setminus\{0\},\{1,2,I,2I\},\{1,I\}\}.$$

Theorem 44 Every soft strong neutrosophic soft N -group (F,A) is a soft neutrosophic N -group but the converse is not true.

Theorem 89 (F,A) over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ is soft strong neutrosophic N -group if $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ is a strong neutrosophic N -group.

Proposition 45 Let (F,A) and (K,D) be two soft strong neutrosophic N -groups over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$. Then

1) Their extended union $(F,A)\cup_\varepsilon(K,D)$ is not soft strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
2) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ is not soft strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
3) Their restricted union $(F,A)\cup_R(K,D)$ is not soft strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
4) Their restricted intersection $(F,A)\cap_R(K,D)$ is not soft strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.

Proposition 46 Let (F,A) and (K,D) be two soft strong neutrosophic N -groups over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is not soft strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.

Definition 58

Let (F,A) and (H,K) be two soft strong neutrosophic N -groups over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$. Then (H,K) is called soft strong neutrosophic sub $x\in A$ -group of (F,A) written as $(H,K)\prec(F,A)$, if

1) $K \subset A$,
2) $K(x)$ is soft neutrosophic soft sub N-group of $F(x)$ for all $x \in A$.

Theorem 45 If $(\langle G \cup I \rangle, *_1, ..., *_N)$ is a strong neutrosophic N-group. Then every soft neutrosophic sub N-group of (F, A) is soft strong neutosophic sub N-group.

Definition 59 Let $(\langle G \cup I \rangle, *_1, ..., *_N)$ be a strong neutrosophic N-group. Then (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is called soft Lagrange strong neutrosophic N-group if $F(x)$ is a Lagrange neutrosophic sub N-group of $(\langle G \cup I \rangle, *_1, ..., *_N)$ for all $x \in A$.

Theorem 46 Every soft Lagrange strong neutrosophic N-group (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is a soft neutrosophic soft N-group but the converse is not true.

Theorem 47 Every soft Lagrange strong neutrosophic N-group (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is a soft srtong neutrosophic N-group but the converse is not tue.

Theorem 48 If $(\langle G \cup I \rangle, *_1, ..., *_N)$ is a Lagrange strong neutrosophic N-group, then (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is also soft Lagrange strong neutrosophic N-group.

Proposition 47 Let (F, A) and (K, D) be two soft Lagrange strong neutrosophic N-groups over $(\langle G \cup I \rangle, *_1, ..., *_N)$. Then

1) Their extended union $(F, A) \cup_{\varepsilon} (K, D)$ is not soft Lagrange strong neutrosophic N-group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, D)$ is not soft Lagrange strong neutrosophic N-group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
3) Their restricted union $(F, A) \cup_R (K, D)$ is not soft Lagrange strong neutrosophic N-group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
4) Their restricted intersection $(F, A) \cap_R (K, D)$ is not soft Lagrange strong neutrosophic N-group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.

Proposition 48 Let (F, A) and (K, D) be two soft Lagrange strong neutrosophic N-groups over $(\langle G \cup I \rangle, *_1, ..., *_N)$. Then

1) Their AND operation $(F, A) \wedge (K, D)$ is not soft Lagrange strong neutrosophic N-group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
2) Their OR operation $(F, A) \vee (K, D)$ is not soft Lagrange strong neutrosophic N-group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.

Definition 60 Let $(\langle G \cup I \rangle, *_1, ..., *_N)$ be a strong neutrosophic N-group. Then (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is called soft weakly Lagrange strong neutrosophic soft N-group if atleast one $F(x)$ is a Lagrange neutrosophic sub N-group of $(\langle G \cup I \rangle, *_1, ..., *_N)$ for some $x \in A$.

Theorem 49 Every soft weakly Lagrange strong neutrosophic N-group (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is a soft neutrosophic soft N-group but the converse is not true.

Theorem 50 Every soft weakly Lagrange strong neutrosophic N-group (F, A) over $(\langle G \cup I \rangle, *_1, ..., *_N)$ is a soft strong neutrosophic N-group but the converse is not true.

Proposition 49 Let (F, A) and (K, D) be two soft weakly Lagrange strong neutrosophic N-groups over $(\langle G \cup I \rangle, *_1, ..., *_N)$. Then

1) Their extended union $(F, A) \cup_{\varepsilon} (K, D)$ is not soft weakly Lagrange strong neutrosophic N-group over $(\langle G \cup I \rangle, *_1, ..., *_N)$.
2) Their extended intersection $(F, A) \cap_{\varepsilon} (K, D)$ is not soft weakly Lagrange strong neutrosophic N-group over

$\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.

3) Their restricted union $(F, A) \cup_R (K, D)$ is not soft weakly Lagrange strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.
4) Their restricted intersection $(F, A) \cap_R (K, D)$ is not soft weakly Lagrange strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.

Proposition 50 Let (F, A) and (K, D) be two soft weakly Lagrange strong neutrosophic N -groups over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$. Then

1) Their AND operation $(F, A) \wedge (K, D)$ is not soft weakly Lagrange strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.
2) Their OR operation $(F, A) \vee (K, D)$ is not soft weakly Lagrange strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.

Definition 61 Let $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ be a strong neutrosophic N -group. Then (F, A) over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ is called soft Lagrange free strong neutrosophic N -group if $F(x)$ is not Lagrange neutrosophic sub N -group of $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ for all N .

Theorem 51 Every soft Lagrange free strong neutrosophic N -group (F, A) over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ is a soft neutrosophic N -group but the converse is not true.

Theorem 52 Every soft Lagrange free strong neutrosophic N -group (F, A) over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ is a soft strong neutrosophic N -group but the converse is not true.

Theorem 53 If $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ is a Lagrange free strong neutrosophic N -group, then (F, A) over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ is also soft Lagrange free strong neutrosophic N -group.

Proposition 51 Let (F, A) and (K, D) be two soft Lagrange free strong neutrosophic N -groups over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$. Then

1) Their extended union $(F, A) \cup_\varepsilon (K, D)$ is not soft Lagrange free strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.
2) Their extended intersection $(F, A) \cap_\varepsilon (K, D)$ is not soft Lagrange free strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.
3) Their restricted union $(F, A) \cup_R (K, D)$ is not soft Lagrange free strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.
4) Their restricted intersection $(F, A) \cap_R (K, D)$ is not soft Lagrange free strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.

Proposition 52 Let (F, A) and (K, D) be two soft Lagrange free strong neutrosophic N -groups over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$. Then

1) Their AND operation $(F, A) \wedge (K, D)$ is not soft Lagrange free strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.
2) Their OR operation $(F, A) \vee (K, D)$ is not soft Lagrange free strong neutrosophic N -group over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$.

Definition 62 Let N be a strong neutrosophic N -group. Then (F, A) over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ is called sofyt normal strong neutrosophic N -group if $F(x)$ is normal neutrosophic sub N -group of $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ for all $x \in A$.

Theorem 54 Every soft normal strong neutrosophic N -group (F, A) over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ is a soft neutrosophic N -group but the converse is not true.

Theorem 55 Every soft normal strong neutrosophic N -group (F, A) over $\left(\langle G \cup I \rangle, *_1, ..., *_N\right)$ is a soft strong neutrosophic N -group but the converse is not true.

Proposition 53 Let (F, A) and (K, D) be two soft normal strong neutrosophic N -groups over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(K,D)$ is not soft normal strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,D)$ is soft normal strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
3) Their restricted union $(F,A)\cup_{R}(K,D)$ is not soft normal strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
4) Their restricted intersection $(F,A)\cap_{R}(K,D)$ is soft normal strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.

Proposition 54 Let (F,A) and (K,D) be two soft normal strong neutrosophic N -groups over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is soft normal strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft normal strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.

Definition 63 Let $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ be a strong neutrosophic N -group. Then (F,A) over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ is called soft conjugate strong neutrosophic N -group if $F(x)$ is conjugate neutrosophic sub N -group of $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ for all $x\in A$.

Theorem 56 Every soft conjugate strong neutrosophic N -group (F,A) over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ is a soft neutrosophic N -group but the converse is not true.

Theorem 57 Every soft conjugate strong neutrosophic N -group (F,A) over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$ is a soft strong neutrosophic N -group but the converse is not true.

Proposition 55 Let (F,A) and (K,D) be two soft conjugate strong neutrosophic N -groups over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$. Then

1) Their extended union $(F,A)\cup_{\varepsilon}(K,D)$ is not soft conjugate strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
2) Their extended intersection $(F,A)\cap_{\varepsilon}(K,D)$ is soft conjugate strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
3) Their restricted union $(F,A)\cup_{R}(K,D)$ is not soft conjugate strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
4) Their restricted intersection $(F,A)\cap_{R}(K,D)$ is soft conjugate strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.

Proposition 56 Let (F,A) and (K,D) be two soft conjugate strong neutrosophic N -groups over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$. Then

1) Their AND operation $(F,A)\wedge(K,D)$ is soft conjugate strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.
2) Their OR operation $(F,A)\vee(K,D)$ is not soft conjugate strong neutrosophic N -group over $\left(\langle G\cup I\rangle,*_1,...,*_N\right)$.

Conclusion

This paper is about the generalization of soft neutrosophic groups. We have extended the concept of soft neutrosophic group and soft neutrosophic subgroup to soft neutrosophic bigroup and soft neutrosophic N-group. The notions of soft normal neutrosophic bigroup, soft normal neutrosophic N-group, soft conjugate neutrosophic bigroup and soft conjugate neutrosophic N-group are defined. We have given various examples and important theorems to illustrate the aspect of soft neutrosophic bigroup and soft neutrosophic N-group.

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