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A Study on Neutrosophic Algebra

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Abstract: The notion of neutrosophic algebra, ideal of neutrosophic algebra, kernel and neutrosophic quotient algebra have been proposed in this paper. We characterize some properties of neutrosophic algebra and proved that every quotient neutrosophic algebra is quotient algebra. Also we proved that every neutrosophic algebra is algebra and direct product of neutrosophic algebras over a neutrosophic field is algebra.

Key words: Neutrosophic set, neutrosophic algebra, neutrosophic algebra isomorphism, ideals of neutrosophic algebra, quotient neutrosophic algebra, neutrosophic subalgebra.

1. Introduction

A fuzzy set A in X is characterized by a membership function which is associated with each element in X to a real number in the unit interval [0, 1]. In 1965 L. A. Zadeh [24] introduced the concept of fuzzy set theory. This novel concept is used successfully in modeling uncertainty in many fields of real life. Fuzzy sets and its applications have been extensively studied in different aspects. In 1998, Neutrosophic set was introduced by Florentin Smarandache [17, 18], where each element associated with three defining functions, namely the membership function (T), the non-membership function (F) and the indeterminacy function (I) defined on the universe of discourse X, the three functions are completely independent. Relative to the natural problems sometimes one may not be able to decide.

In 2004 W. B. Vasantha Kandaswamy and Florentin Smarandache [**23**] introduced a neutrosophic structure based on indeterminacy I only, which they called I-neutrosophic algebraic structures. Algebraic structure based sets of neutrosophic numbers of the form $a+bI$ where a,b are real (or complex) and Indeterminacy I with $I^2 = I$. This I is different from the imaginary $i=\sqrt{-1}$. After the development of the Neutrosophic set theory, one can easily take decision and indeterminacy function of the set is the nondeterministic part of the situation. The applications of the theory have been found in various fields for dealing with indeterminate and consistent information in real world. The neutrosophic set generalizes the concept of classical fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set and so on. Broumi Said et al. [**6, 7**] studied the notion of intuitionistic Neutrosophic Soft Set and Rough neutrosophic sets. The branch of neutrosophic theory is the theory of neutrosophic algebraic structures. Abobala [4, 5] introduced Some Special Substructures of Neutrosophic Rings and AH-Substructures in n-Refined Neutrosophic Vector Spaces**.** The authors in [1, 2] studied the notion of Neutrosophic vector spaces and Mamouni Dhar, Said Broumi and Florentin Smarandache [10] introduced Square Neutrosophic Fuzzy Matrices. In [**3**] Abdel Nasser Hussian, Mai Mohamed, Mohamed Abdel-Baset and Florentin smarandache studied the Neutrosophic Linear Programming Problems. W. B. Vasanntha Kandasamy and F. Smarandache [20, 21] introduced the concept of neutrosophic algebraic structure and neutrosophic N-algebraic structures. P. Narasimha Swamy et al. [14, 15] studied the notion of Fuzzy quasi-ideals of near algebra and Anti Fuzzy Gamma Near-Algebras. T. Nagaiah et al. [11, 12, 13, 16] introduced Partially ordered Gamma semi groups, near-rings, direct product and strongest interval valued anti fuzzy ideals of Gamma near-rings and special class of ring structure. T. Srinivas, T. Nagaiah and P.Narasimha Swamy [**19**] initiated the concept of Anti Fuzzy Ideals of Gamma Near-rings.Hatip and Abobala [9] studied the notion of AH-substructuresin strong refined models. Bijan Davvaz [8] introduced Neutrosophic ideals of Neutrosophic KU-algebra. Since then several researcher have been study the concept of neutrosophic theory and its application in varies branches. In this paper our main aim is to introduce the concept of neutrosophic algebra and its application in varies branches of Mathematics. Also we proved that every neutrosophic algebra is algebra and direct product of neutrosophic algebras over a neutrosophic field is algebra.

This paper is organized into four sections. The first section is introductory. The second section presents the basic concepts needed to make this paper a self-contained one. Section three discusses and describes the neutrosophic algebra and its examples. Final section gives ideal of neutrosophic algebra, neutrosophic quotient algebra and studied their properties.

2. Preliminaries

In this section we recall some basic concepts of neutrosophic set, proposed by W.B. Vasantha Kandasamy and University of New Mexico professor F. Smarandache in their monograph [21, 22].

Definition 2.1 Let U be an universe of discourse then the neutrosophic set A is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ U} where the functions T, I, F: U → $]^-0, 1^+[$ define respectively the degree of membership (or truthness), the degree of indeterminacy, and the degree of non-membership (or falsehood)of the element x in U to the set A with the condition. $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems. Let X be a non-empty set. A set $X(I)=\langle X,I\rangle$ generated by X and I is called a neutrosophic set. The elements of X (I) are of the form (a,bI), where a,b are elements of X .

Definition 2.2: Algebra Y over a field X is a linear space Y over a field X on whicha multiplication is defined such that

i. Y forms a semi group under multiplication,

ii. multiplication is distributive over addition

iii. $\lambda (x y) = (\lambda x) y = x (\lambda y)$, for all $x, y \in Y$ and $\lambda \in X$.

3. Neutrosophic algebra

In this section we define the neutrosophic algebra and provide examples. Also define neutrosophic subalgebra and characterize their properties. Excepted otherwise stated, all strong neutrosophic algebras in this paper will be considering neutrosophic algebra.

Definition 3.1: Let Y be algebra over a field X . The set generated by Y and I is denoted by $\langle Y \cup I\rangle$ = Y(I) = $\{a + bI : a , b \epsilon Y\}$ is called a weak neutrosophic algebra over a field X . If Y(I) is a ~~neutrosophic algebra over a neutrosophic field $X(I)$ then~~

Y(I) is called a strong neutrosophic algebra. The elements of Y(I) are calledneutrosophic vectors and the elements of X (I) are called neutrosophic scalars.

Examples 3.2:

i. $\mathbb{R}(I)$ is a weak neutrosophic algebra over a field $\mathbb{Q}$ and it is strongneutrosophic algebra over a neutrosophic field $\mathbb{Q}(I)$, where $\mathbb{Q}(I) = \{a+bI : a,b \in Q\}$

ii. $\mathbb{C}(I)$ is a weak neutrosophic algebra over a field $\mathbb{R}$ and it is a strongneutrosophic algebra over a neutrosophic field $\mathbb{R}(I)$

iii. $\mathbb{R}^n(I)$ is a weak neutrosophic algebra over a field $\mathbb{R}$ and it is a strong neutrosophic algebra over a neutrosophic field $\mathbb{R}(I)$

iv. $M_{m\times n}(I) = \{[a_{ij} + b_{ij}I] ; a_{ij} , b_{ij} \epsilon \mathbb{Q}\}$ is strong neutrosophic algebra over a neutrosohic field $\mathbb{Q}(I)$ and it is weak neutrosophic algebra over a field $\mathbb{Q}$.

Definition 3.3: Let Y (I) be neutrosophic algebra over a neutrosophic field X (I).

The non-empty subset W(I) of Y(I) is called a neutrosophic subalgebra over a field X (I), if $W(I)$ is itself a neutrosophic algebra over a neutrosophic field X (I).

Theorem 3.4: Every strong neutrosophic algebra is weak neutrosophic algebra.

Proof: Suppose $Y(I)$ is strong neutrosophic algebra over a neutrosophic field$X(I)$. Since $X \subseteq X(I)$, so that $Y(I)$ is weak neutrosophic algebra over a field X. Hence every strong neutrosophic algebra is weak neutrosophic algebra.

Theorem 3.5: Every strong (weak) neutrosophic algebra is neutrosophic vector space.

Proof:- Suppose $Y(I)$ is a strong neutrosophic algebra over a neutrosophic field $X(I)$.
This implies that Y is algebra over a field X. So that Y is a vector space over a field X.
From Theorem 3.4, this show $Y(I)$ is a neutrosophic vector space over a field X. Similarly we can prove, if $Y(I)$ is a weak neutrosophic algebra over a field X, then Y is an algebra over a field X and hence Y is a vector space over a field X. Hence $Y(I)$ is a neutrosophic vector space.

Theorem 3.6: Every neutrosophic algebra is algebra.

Proof: Let $Y(I)$ be the neutrosophic algebra over a neutrosophic field $X(I)$. By Theorem 3.5, $Y(I)$ is a neutrosophic vector space over a field $X(I)$.
This implies $Y(I)$ is a vector space over a field $X(I)$. It is easy to verify that all the algebra properties of $Y(I)$ over a field $X(I)$.i.e., (i) $x(yz) = (xy)z$ (ii)$x \cdot (y+z) = x \cdot y + x \cdot z, (x+y) \cdot z = x \cdot z + y \cdot z$ and $\lambda(xy) = (\lambda x)y = x(\lambda y), \forall x,y,z \epsilon Y(I)\ and\ \lambda \epsilon X(I)$.
We can easy to see that Y (I) is a neutrosophic algebra over a field X (I).

Theorem 3.7: Let $M_1(I)$ and $M_2(I)$ be neutrosophic algebras over a neutrosophic field $X(I)$. Then the direct product
$M_1(I) \times M_2(I) = \{(u_1,u_2): u_1 \in M_1(I), u_2 \in M_2(I)\}$ is algebra over a neutrosophic field $X(I)$, where addition, multiplication and scalar multiplication is defined by

(i) $(u_1,u_2) + (v_1,v_2) = (u_1+v_1, u_2+v_2)$
(ii) $(u_1,u_2) \cdot (v_1,v_2) = (u_1v_1, u_2v_2)$
(iii) $\alpha(u_1,u_2) = (\alpha u_1, \alpha u_2), \forall \alpha \epsilon X(I)$ and $(u_1,u_2),(v_1,v_2) \epsilon M_1(I) \times M_2(I)$.

Proof: Let $M_1(I)\ and\ M_2(I)$ be two neutrosophic algebras over a neutrosophic field $X(I)$. In view of Theorem 3.5, $M_1(I)\ and\ M_2(I)$ are linear spaces over a field $X(I)$.
This implies $M_1(I) \times M_2(I)$ is a linear space over a field $X(I)$.
Let $x = (u_1,u_2), y = (v_1,v_2), z = (w_1,w_2) \epsilon M_1(I) \times M_2(I)$.

Consider $x(yz) = (u_1,u_2)\big((v_1,v_2)(w_1,w_2)\big)$
$= (u_1,u_2)(v_1w_1, v_2w_2)$
$= \big(u_1(v_1w_1), u_2(v_2w_2)\big)$

$= ((u_1v_1)w_1, (u_2v_2)w_2)$
$= (u_1v_1, u_2v_2)(w_1, w_2)$
$= ((u_1v_1, u_2v_2))(w_1, w_2) = (xy)z$

This show $M_1(I) \times M_2(I)$ is a semi group under multiplication.

Now $x(y+z) = (u_1, u_2) \cdot [(v_1, v_2) + (w_1, w_2)]$
$= (u_1, u_2) \cdot [(v_1 + w_1, v_2 + w_2)]$
$= (u_1 \cdot (v_1 + w_1)\,,\; u_2 \cdot (v_2 + w_2))$
$= (u_1 \cdot v_1 + u_1 \cdot w_1\,,\; u_2 \cdot v_2 + u_2 \cdot w_2)$
$= (u_1 \cdot v_1\,,\; u_2 \cdot v_2) + (u_1 \cdot w_1\,,\; u_2 \cdot w_2)$
$= (u_1, u_2) \cdot (v_1, v_2) + (u_1, u_2) \cdot (w_1, w_2)$
$= x \cdot y + x \cdot z$

Also $(x+y)z = (u_1 + v_1\,, u_2 + v_2) \cdot (w_1, w_2)$
$= ((u_1 + v_1) \cdot w_1\,, (u_2 + v_2) \cdot w_2)$
$= (u_1 \cdot w_1 + v_1 \cdot w_1\,,\; u_2 \cdot w_2 + v_2 \cdot w_2)$
$= (u_1 \cdot w_1\,,\; u_2 \cdot w_2) + (v_1 \cdot w_1\,, v_2 \cdot w_2)$
$= (u_1, u_2) \cdot (w_1, w_2) + (v_1, v_2) \cdot (w_1, w_2)$
$= x\,z + y\,z.$

Let $\alpha \epsilon\, X(I)$ and $x, y \epsilon M_1(I) \times M_2(I)$.

Consider $\alpha(xy) = \alpha[(u_1, u_2) \cdot (v_1, v_2)]$
$= \alpha(u_1 \cdot v_1\,, \quad u_2 \cdot v_2)$
$= (\alpha(u_1 \cdot v_1)\,, \alpha(u_2 \cdot v_2))$
$= ((\alpha u_1) \cdot v_1\,, (\alpha u_2) \cdot v_2) = (\alpha u_1\,, \alpha u_2) \cdot (v_1, v_2)$
$= (\alpha(u_1, u_2)) \cdot (v_1, v_2) = (\alpha x)y$

Also we prove that $\alpha(xy) = x(\alpha y)$, for all $x, y \epsilon M_1(I) \times M_2(I)$, $\alpha \epsilon\, X(I)$.

Hence $M_1(I) \times M_2(I)$ is algebra over a neutrosophic field $X(I)$.

Theorem 3.8:- Let W_1(I), W_2(I), …………W_n(I) be a neutrosophic algebra over a neutrosophic field $X(I)$. Then $W_1(I) \times W_2(I) \times \dots \dots W_n(I) = \{(u_1, u_2, \dots \dots u_n) : u_i \in W_i, for\ 1 \le i \le n\}$ is algebra over a neutrosophic field $X(I)$, where addition, multiplication and scalar multiplication defined as follows:

(i) $(u_1, u_2, \dots\dots u_n) + (v_1, v_2, \dots\dots v_n) = (u_1 + v_1, u_2 + v_2, \dots . u_n + v_n)$
(ii) $(u_1, u_2, \dots\dots u_n)(v_1, v_2, \dots\dots v_n) = (u_1v_1,\ u_2v_2, \dots \dots . u_nv_n)$
(iii) $\alpha(u_1, u_2, \dots\dots u_n) = (\alpha u_1, \alpha u_2, \dots\dots \alpha u_n)$.

Proof: Proof of this theorem is similar to theorem 3.7.

4. Ideal of neutrosophic algebra and neutrosophic quotient algebra

In this section we define the ideal of neutrosophic algebra and neutrosophic quotient algebra and studied their algebraic properties. Also we define Neutrosophic algebra homomorphism, Neutrosophic algebra isomorphism and kernel of neutrosophic algebra. We proved that every quotient neutrosophic algebra is quotient algebra.

Definition 4.1: A non-empty subset $W(I)$ of a neutrosophic algebra $Y(I)$ over a neutrosophic field $X(I)$ is an ideal of a neutrosophic algebra $Y(I)$ if

i. $W(I)$ is a subspace of a vector space $Y(I)$
ii. $\alpha u \epsilon W(I)$ for every $u \epsilon\, W(I), \alpha \epsilon X(I)$ and
iii. $v(u + \alpha) - vu\ \epsilon\, W(I)$ for every $u, v \epsilon\, W(I), \alpha \epsilon X(I)$.

If $W(I)$ satisfies (i) and (ii) then $W(I)$ is called a right ideal of neutrosophic algebra and if $W(I)$ satisfies (i) and (iii) then $W(I)$ is called a left ideal of neutrosophic algebra over a neutrosophic field$X(I)$.

Definition 4.2: Let $M_1(I)$ *and* $M_2(I)$ be two neutrosophic algebras over a neutrosophic field X(I). A mapping $\varphi: M_1$(I)→ $M_2(I)$ is called neutrosophic algebra homomorphism if the following conditions hold:

i. $\varphi(u+v) = \varphi(u) + \varphi(v)$
ii. $\varphi(uv) = \varphi(u)\,\varphi(v)$
iii. $\varphi(\alpha\, u) = \alpha\, \varphi(u)$
iv. $\varphi(I) = I$, for all u, v in $M_1(I)$, α in X(I) and I is a neutrosophic element of $M_1(I)$.

A neutrosophic algebra homomorphism φ is said to be neutrosophic algebra monomorphism if φ is injective.
A neutrosophic algebra homomorphism φ is said to be neutrosophic algebra epimorphism if φ is surjective.
A neutrosophic algebra homomorphism φ is said to be neutrosophic algebra isomorphism if φ is bijection. A bijective neutrosophic algebra homomorphism from $M_1(I)$ onto $M_1(I)$ is called a neutrosophic algebra automorphism.

Definition 4.3: Let $M_1(I)$and $M_2(I)$ be two neutrosophic algebras over a field $X(I)$. Let $\varphi: M_1(I) \to M_2(I)$ be a neutrosophic algebra homomorphism. Then the kernel of φ is denoted by Kerφ and is defined by $Ker\ \varphi = \{u \in M_1(I);\ \varphi(u) = 0'\}$, where $0' = 0 + 0I \epsilon\, M_2(I)$.

Definition 4.4: Let $M(I)$ be an ideal of neutosophic algebra $Y(I)$ over a field $X(I)$. Then the set of all co-sets of $M(I)$ in $Y(I)$ is denoted by $Y(I)\ /\ M(I)$ and defined by $Y(I)/M(I) = \{u + M(I):\ for\ every\ \ u\epsilon\, Y(I)\}$.

Addition, multiplication and scalar multiplication on $Y(I)\ /\ M(I)$ defined as

$$\big(a + M(I)\big) + \big(b + M(I)\big) = (a+b) + M(I)$$

$$\big(a + M(I)\big)\big(b + M(I)\big) = ab + M(I)$$

And $\alpha\big(a + M(I)\big) = \alpha a + M(I)$, $\forall \alpha\epsilon X(I), \big(a + M(I)\big), \big(b + M(I)\big)\epsilon Y(I)/M(I)$.

The set $Y(I)/M(I)$ form neutrosophic algebra over a neutrosophic field $X(I)$.

This neutrosophic algebra is called quotient neutrosophic algebra.

Theorem 4.5: Let $Y(I)$ be neutrosophic algebra over a neutrosophic field $X(I)$. The intersection of any collection of right ideals of neutrosophic algebra $Y(I)$ over a neutrosophic field $X(I)$ is a right ideal of $Y(I)$.

Proof: Let $\{W_\alpha(I)\}$ be the collection of right ideals of neutrosophic algebras $Y(I)$ over a neutrosophic field $X(I)$. Let $W(I) = \cap_\alpha W_\alpha(I)$ be their intersection. As $\cap_\alpha W_\alpha(I)$ is the collection of right ideals of $Y(I)$ over a neutrosophic field $X(I)$.This implies each $W_\alpha(I)$, for each α is a right ideal of $Y(I)$ over a field $X(I)$. For each α, $W_\alpha(I)$ is a subspace of a vector space $Y(I)$. *This implies that* $\cap_\alpha W_\alpha(I)$ is a subspace of a vector space$Y(I)$.

Therefore $W(I)$ is sub-space of $Y(I)$.

Let $\alpha\epsilon X(I)$ and any $u\epsilon\ \cap_\alpha W_\alpha(I)$.

$\Rightarrow u\epsilon W_\alpha(I)$ for each α.

$\Rightarrow \alpha u\epsilon W_\alpha(I)$, for each $\alpha \Rightarrow \alpha u\ \epsilon\ \cap_\alpha W_\alpha(I)$.

Hence $\cap_\alpha W_\alpha(I)$ is a right ideal of $Y(I)$ over a neutrosophic field $X(I)$.

Theorem 4.6: Every quotient neutrosophic algebra over a neutrosophic field is quotient algebra.

Proof: Let $M(I)/U(I)$ be quotient algebra over a neutrosophic field $X(I)$.

For proving of $M(I)/U(I)$ is algebra, it is enough to show that the following.

(i) $M(I)/U(I)$ is a vector space over a field $X(I)$.
(ii) $M(I)/U(I)$ form a semigroup under multiplication
(iii) $x \cdot (y+z) = x\cdot y + x \cdot z$ and
$(x+y)\cdot z = x\cdot z + y\cdot z, \forall\, x, y\epsilon\, M(I)/U(I)$
(iv) $\alpha(xy) = (\alpha x)y = x(\alpha y)$, $\forall\ \alpha\epsilon\ X(I)\ and\ \ x, y\epsilon\, M(I)/U(I)$.

We have $M(I)/U(I) = \{m + U(I)/for\ every\ m\ \epsilon\ M(I)\}$. Since $M(I)/U(I)$ is quotient neutrosophic algebra then $U(I)$ is an ideal of a neutrosophic algebra $M(I)$. From the definition of ideal, it is clear that $U(I)$ is a subspace of a vector space $M(I)$ over $X(I)$. As we know every neutrosophic algebra is a vector space, so that $U(I)$ and $M(I)$ are vector spaces with $U(I)$ *is a subset of* $M(I)$. Therefore $M(I)/U(I)$ is a vector space over a neutrosophic field $X(I)$.

Let $x = m_1 + U(I), y = m_2 + U(I), z = m_3 + U(I)$ are in $M(I)/U(I)$, where $m_1, m_2, m_3 \epsilon M(I)$.

Consider $x(yz) = (m_1 + U(I))(m_2 + U(I) \cdot m_3 + U(I))$

$= (m_1 + U(I))(m_2\, m_3 + U(I)) = m_1(m_2\, m_3) + U(I)$

$= (m_1\, m_2)m_3 + U(I)$ ($\because M(I)$ is a neutrosophic algebra)

$= (m_1\, m_2 + U(I)) \cdot (m_3 + U(I))$

$= \big((m_1 + U(I))(m_2 + U(I))\big)(m_3 + U(I))$

$= (xy)z.$

Therefore $M(I)/U(I)$ form semigroup under multiplication.

Again consider$(x + y)z = [(m_1 + U(I)) + (m_2 + U(I))] \cdot (m_3 + U(I))$.

$= ((m_1 + m_2) + U(I)) \cdot (m_3 + U(I))$

$= (m_1 + m_2) \cdot m_3 + U(I)$

$= (m_1 \cdot m_3 + m_2 \cdot m_3) + U(I)$

$= (m_1 \cdot m_3 + U(I)) + (m_1 \cdot m_3 + U(I))$

$= (m_1 + U(I) \cdot m_3 + U(I)) + (m_2 + U(I) \cdot m_3 + U(I))$

$= x\,z + y\,z.$

Also let $X(I), x, y \epsilon M(I)/U(I)$.

Now $\alpha(xy) = \alpha(m_1 + U(I) \cdot m_2 + U(I))$

$= \alpha(m_1\, m_2 + U(I)) = \alpha(m_1\, m_2) + U(I)$

$= (\alpha m_1)\, m_2 + U(I)$

$= (\alpha m_1 + U(I))(m_2 + U(I))$

$= [\alpha(m_1 + U(I))](m_2 + U(I))$

$= (\alpha x)y.$

Also $x(\alpha y) = (m_1 + U(I))\big(\alpha(m_2 + U(I))\big)$

$= (m_1 + U(I))(\alpha m_2 + U(I))$

$= m_1(\alpha m_2) + U(I)$

$= \alpha(m_1\, m_2) + U(I)$

$= \alpha(m_1\, m_2 + U(I))$

$= \alpha[(m_1 + U(I))(m_2 + U(I))] = \alpha(xy)$

Hence complete the proof.

5. Conclusions

In this paper, we have introduced the notion of neutrosophic algebra, neutrosophic subalgebra, quotient neutrosophic algebra, homomorphism and isomorphism of neutrosophic algebra and ideals of neutrosophic algebra. We characterize some properties of neutrosophic algebra and proved that every quotient neutrosophic algebra is quotient algebra. Also we have proved that every neutrosophic algebra is algebra and direct product of neutrosophic algebras over a neutrosophic field is algebra. Several results and examples related to the neutrosophic algebra have been introduced. The concept of neutrosophic theory can be extend to near-algebra, Banach Algebra, C-algebra, Gamma near-algebra and near-modules.

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