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TOPSIS for Single Valued Neutrosophic Soft Expert Set Based Multi-attribute Decision Making Problems

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Abstract. In the paper, we propose Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) technique for solving single valued neutrosophic soft set expert based multi-attribute decision making problems. Single valued neutrosophic soft expert sets are combination of single valued neutrosophic sets and soft expert sets. In the decision making process, the ratings of alternatives with respect to the parameters are expressed in terms of single valued neutrosophic soft expert sets to deal with imprecise or vague information. The unknown weights of the parameters are derived from maximizing deviation method. Then, we determine the rank of the alternatives and choose the best one by using TOPSIS method. Finally, a numerical example for teacher selection is presented to demonstrate the applicability and effectiveness of the proposed approach.

Keywords: Single valued neutrosophic sets, single valued neutrosophic soft expert sets, TOPSIS, multi-attribute decision making.

1 Introduction

Hwang and Yoon [1] grounded the technique for order preferece by similarity to ideal solution (TOPSIS) method for solving conventional multi-attribute decision making (MADM) problems. The basic concept of TOPSIS is straightforward. It is developed from the concept of a displaced ideal point from which the compromise solution has the shortest distance. Hwang and Yoon [1] proposed that the ranking of alternatives would be based on the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). TOPSIS approac simultaneously considers the distances to both PIS and NIS, and a preference order is ranked based on their relative closeness, and a combination of these two distance measures.

MADM is the process of identifying the most suitable alternative from a finite set of feasible alternatives with respect to numerous usually conflicting attributes. MADM has been applied to various practical problems such as learning management system evaluation [2], project portfolio selection [3], electric utility resource planning [4], economics, military affairs, etc. However, in practical decision making situation, the information about the rating of the alternative with respect to the attributes cannot be assessed due to imprecise source of information. So, traditional MADM methods are not capable to solve these types of problems.

In 1965, Zadeh [5] proposed fuzzy set which is characterized by membership function to deal with problems with imprecise information. Atanassov [6] defined intuitionistic fuzzy set by incorporating non-membership function. However, for proper description of an object in uncertain and complex environment, we require to deal with indeterminate and inconsistent information. So, Smarandache [7, 8, 9, 10] extended the concept of Atanassov [6] by introducing indeterminacy membership function as an independent component and defined neutrosophic set for dealing with the problems with incomplete, imprecise, inconsistent information. Thereafter, Wang et al. [11] defned single valued neutrosophic set (SVNS) as an instance of neutrosophic set for dealing with real scientific and engineering problems.

Molodtsov [12] initiated the concept of soft set theory for dealing with uncertainty and vagueness in 1999. Soft set is free from the limitation of variety of theories such as probability theory, fuzzy theory, rough set theory, vague set theory and it is easy to implement in practical problems. After the pioneering work of Molodtsov [12], many researchers developed diverse mathematical hybrid models such as fuzzy soft sets [13, 14, 15], intuitionistic fuzzy soft set theory [16, 17, 18], possibility fuzzy soft set [19], generalized fuzzy soft sets [20, 21], generalized intuitionistic fuzzy soft [22], possibility intuitionistic fuzzy soft set [23], vague soft set [24], possibility vague soft set [25], neutrosophic soft set [26], weighted neutrosophic soft sets [27], generalized neutrosophic soft set [28], intuitionistic neutrosophic soft set [29, 30], etc in order to solve different practical problems. However, most of the models consider only one expert and this creates difficulties for the researchers who employ questionnaires for his/her works and studies [31]. In order to overcome the difficulties, Alkhazaleh and Salleh [31] developed soft expert sets in 2011 where the researcher can observe the opinions of all experts in one model without any operations. They defined basic operations of soft expert sets and studied some of their properties and then applied the concept in decision making problem. Alkhazaleh and Salleh [32] also defined fuzzy soft expert set which is a hybridization of soft expert set and fuzzy set. Hazaymey et al. [33] introduced generalized fuzzy soft expert set by combining soft expert set due to Alkhazaleh and Salleh [31] and generalized soft set due to Majumdar and Samanta [21]. Hazaymey et al. [34] also incorporated fuzzy parameterized fuzzy soft expert set by extending the concept of fuzzy soft expert set by providing a membership value of each parameter in a set of parameters. Later, many authors have developed soft expert sets in different environment to form different structures such as vague soft expert set [35], generalized vague soft expert set [36], fuzzy parameterized soft expert set [37], possibility fuzzy soft expert set [38], intuitionistic fuzzy soft expert set [39], etc and the concepts of soft expert sets are applied to different practical problems [40, 41, 42]. Recently, Şahin et al. [43] incorporated neutrosophic soft expert set as a combination of neutrosophic set and soft expert set to deal with indeterminate and inconsistent information. Later, Broumi and Smarandache [44] explored the concept of single valued neutrosophic soft expert set (SVNSES) which is an extension of fuzzy soft expert sets and intuitionistic fuzzy soft expert sets and they investigated some related properties with supporting proofs.

In the paper, we have developed a new method for solving SVNSES based MADM problem through TOPSIS technique.

The content of the paper is constructed as follows. Section 2 presents some basic definitions concerning neutrosophic sets, SVNSs, soft sets, soft expert. Section 3 is devoted to present TOPSIS method for SVNSESs based MADM problems. Section 4 presents an algorithm of the proposed method. A hypothetical problem regarding teacher selection is solved in Section 5 to illustrate the applicability of the proposed method. Finally, Section 6 presents conclusions and future scope research.

2 Preliminaries

We present basic definitions regarding neutrosophic sets, soft sets, soft expert sets and SVNSESs in this Section as follows:

2.1 Neutrosophic Sets [7, 8, 9, 10]

Consider X be a space of objects with a generic element of X denoted by x. Then, a neutrosophic set N on X is defined as follows:

$$N = \{x, \langle \mathrm{T}_N(x), \mathrm{I}_N(x), \mathrm{F}_N(x) \rangle \mid x \in X\}$$

where, $\mathrm{T}_N(x)$, $\mathrm{I}_N(x)$, $\mathrm{F}_N(x)$: $X \rightarrow\ ]^-0, 1^+[$ represent respectively the degrees of truth-membership, indeterminacy-membership, and falsity-membership of a point $x \in X$ to the set N with the condition $^-0 \le \mathrm{T}_N(x) + \mathrm{I}_N(x) + \mathrm{F}_N(x) \le 3^+$.

2.2 Single valued neutrosophic Sets [11]

Let X be a universal space of points with a generic element of X denoted by x, then a SVNS S is presented as follows:

$$S = \{x, \langle \mathrm{T}_S(x), \mathrm{I}_S(x), \mathrm{F}_S(x) \rangle \mid x \in X\}$$

where, $\mathrm{T}_S(x)$, $\mathrm{I}_S(x)$, $\mathrm{F}_S(x)$: $X \rightarrow [0, 1]$ and $0 \le \mathrm{T}_S(x) + \mathrm{I}_S(x) + \mathrm{F}_S(x) \le 3$ for each point $x \in X$.

Definition 1 [45] The Hamming distance between two SVNSs $S_1 =$ $\{x_j, \langle \mathrm{T}_{S_1}(x_j), \mathrm{I}_{S_1}(x_j), \mathrm{F}_{S_1}(x_j) \rangle \mid x_j \in X\}$ and S_2 = $\{x_j, \langle \mathrm{T}_{S_2}(x_j), \mathrm{I}_{S_2}(x_j), \mathrm{F}_{S_2}(x_j) \rangle \mid x_j \in X\}$ is defined as follows:

$\mathrm{L}_{\mathrm{Ham}}(S_1, S_2) =$

$$\sum_{j=1}^{n} \left\{ \begin{array}{l} |\mathrm{T}_{S_1}(x_j) - \mathrm{T}_{S_2}(x_j)| + |\mathrm{I}_{S_1}(x_j) - \mathrm{I}_{S_2}(x_j)| + \\ |\mathrm{F}_{S_1}(x_j) - \mathrm{F}_{S_2}(x_j)| \end{array} \right\} \quad (1)$$

Definition 2 [45] The normalized Hamming distance between two SVNSs $S_1 =$ $\{x_j, \langle \mathrm{T}_{S_1}(x_j), \mathrm{I}_{S_1}(x_j), \mathrm{F}_{S_1}(x_j) \rangle \mid$

$x_j \in X\}$ and $S_2 = \{x_j, \langle T_{S_2}(x_j), I_{S_2}(x_j), F_{S_2}(x_j) \rangle \mid x_j \in X\}$ is defined as follows:

$^{N}L_{Ham}(S_1, S_2) =$

$$\frac{1}{3n}\sum_{j=1}^{n}\begin{Bmatrix} |T_{S_1}(x_j) - T_{S_2}(x_j)| + |I_{S_1}(x_j) - I_{S_2}(x_j)| + \\ |F_{S_1}(x_j) - F_{S_2}(x_j)| \end{Bmatrix} \quad (2)$$

Definition 3 [45] The Euclidean distance between two SVNSs $S_1 = \{x_j, \langle T_{S_1}(x_j), I_{S_1}(x_j), F_{S_1}(x_j) \rangle \mid x_j \in X\}$ and $S_2 = \{x_j, \langle T_{S_2}(x_j), I_{S_2}(x_j), F_{S_2}(x_j) \rangle \mid x_j \in X\}$ is defined as follows:

$L_{Euc}(S_1, S_2)$

$$= \sqrt{\sum_{j=1}^{n}\begin{Bmatrix} (T_{S_1}(x_j) - T_{S_2}(x_j))^2 + (I_{S_1}(x_j) - I_{S_2}(x_j))^2 + \\ (F_{S_1}(x_j) - F_{S_2}(x_j))^2 \end{Bmatrix}} \quad (3)$$

Definition 4 [45] The normalized Euclidean distance between two SVNSs $S_1 = \{x_j, \langle T_{S_1}(x_j), I_{S_1}(x_j), F_{S_1}(x_j) \rangle \mid x_j \in X\}$ and $S_2 = \{x_j, \langle T_{S_2}(x_j), I_{S_2}(x_j), F_{S_2}(x_j) \rangle \mid x_j \in X\}$ is defined as follows:

$^{N}L_{Euc}(S_1, S_2)$

$$= \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\begin{Bmatrix} (T_{S_1}(x_j) - T_{S_2}(x_j))^2 + (I_{S_1}(x_j) - I_{S_2}(x_j))^2 + \\ (F_{S_1}(x_j) - F_{S_2}(x_j))^2 \end{Bmatrix}} \quad (4).$$

2.3 Soft set [12]

Let X be a universal set and E be a set of parameters. Assume that P (X) represents power set of X. Also, let A be a non-empty set, where $A \subset E$. Then, a pair (M, A) is called a soft set over X, where M is a mapping given by M: $A \to P(X)$.

2.4 Neutrosophic soft set [26]

Let, X be an initial universal set. Also, let E be a set of parameters and A be a non-empty set such that $A \subset E$. NS (X) represents the set of all neutrosophic subsets of X. Then, a pair (M, A) is termed to be the neutrosophic soft set over X, where M is a mapping given by M: $A \to NS(X)$.

2.5 Soft expert set [31]

Consider X be an initial universal set, E be the set of parameters, Z be a set of experts (agents) and O = {agree = 1, disagree = 0} be a set of opinions. Let, $W = E \times Z \times O$, $A \subseteq W$. Then, a pair (M, A) is called soft expert set over X, where M is a mapping given by M: $A \to P(X)$, where P (X) represents power set of X.

Definition 5 [31] An agree-soft expert set $(M, A)_1$ over X is a soft expert subset of (M, A) is defined as follows:

$$(M, A)_1 = \{M(\beta): \beta \in E \times Z \times \{1\}\}.$$

Definition 6 [31] An disagree-soft expert set $(M, A)_0$ over X is a soft expert subset of (M, A) is defined as follows:
$(M, A)_0 = \{M(\beta): \beta \in E \times Z \times \{0\}\}$.

2.6 Single valued neutrosophic soft expert set [44]

Consider $X = \{x_1, x_2, \ldots, x_n\}$be a universal set of objects, $E = \{e_1, e_2, \ldots, e_n\}$ be the set of parameters, $Z = \{z_1, z_2, \ldots, z_n\}$ be a set of experts (agents) and O = {agree = 1, disagree = 0} be a set of opinions. Let, $W = E \times Z \times O$, and A be a non-empty set such that $A \subseteq W$. A pair (M, A) is said to be SVNSES over X, where M is a mapping given by M: $A \to$ SVNSES (X), where SVNSES (X) represents all single valued neytrosophic subsets of X.

Example: Let X be the set of objects under consideration and E be the set of parameters, where every parameter is a neutrosophic word or sentence concerning neutrosophic words. Suppose there are three objects in the universe X given by $X = \{x_1, x_2, x_3\}$, E = {costly, beautiful} = $\{e_1, e_2\}$ be the set of decision parameters and $Z = \{z_1, z_2\}$ be a set of experts. Suppose M: $A \to$ SVNSES (X) is defined as follows:

$M(e_1, z_1, 1) =$
$\{\langle x_1, 0.2, 0.5, 0.7\rangle, \langle x_2, 0.4, 0.2, 0.5\rangle, \langle x_3, 0.6, 0.3, 0.4\rangle\}$,

$M(e_2, z_1, 1) =$
$\{\langle x_1, 0.5, 0.1, 0.2\rangle, \langle x_2, 0.5, 0.2, 0.4\rangle, \langle x_3, 0.6, 0.2, 0.2\rangle\}$,

$M(e_1, z_2, 1) =$
$\{\langle x_1, 0.7, 0.1, 0.3\rangle, \langle x_2, 0.8, 0.3, 0.1\rangle, \langle x_3, 0.8, 0.2, 0.4\rangle\}$,

$M(e_2, z_2, 1) =$
$\{\langle x_1, 0.9, 0.1, 0.2\rangle, \langle x_2, 0.3, 0.3, 0.2\rangle, \langle x_3, 0.4, 0.3, 0.1\rangle\}$,

$M(e_1, z_1, 0) =$
$\{\langle x_1, 0.3, 0.5, 0.1\rangle, \langle x_2, 0.5, 0.2, 0.1\rangle, \langle x_3, 0.4, 0.3, 0.2\rangle\}$,

$M(e_2, z_1, 0) =$
$\{\langle x_1, 0.7, 0.1, 0.5\rangle, \langle x_2, 0.6, 0.3, 0.4\rangle, \langle x_3, 0.6, 0.5, 0.4\rangle\}$,

$M(e_1, z_2, 0) =$
$\{\langle x_1, 0.2, 0.1, 0.4\rangle, \langle x_2, 0.6, 0.5, 0.4\rangle, \langle x_3, 0.5, 0.6, 0.3\rangle\}$,

M (e_2, z_2, 0) =

$\{\langle x_1, 0.8, 0.4, 0.2\rangle, \langle x_2, 0.7, 0.5, 0.4\rangle, \langle x_3, 0.5, 0.3, 0.3\rangle\}$.

Then, (M, A) is a SVNSES over the soft universe.

Definition 7 [44]**:** Let (M_1, A) and (M_2, B) be two SVNSESs over a common soft universe. Then, (M_1, A) is said to be single valued neutrosophic soft expert subset of (M_2, B) if

(i). $B \subseteq A$

(ii). $M_1(\delta)$ is a single valued neutrosophic subset $M_2(\delta), \forall \delta \in A$.

Definition 8 [44]**:** A null SVNSES (φ, A) is defined as follows:

$$(\varphi, A) = M(\beta) \text{ where } \beta \in W.$$

Where $M(\beta)$ = <0, 0, 1>, that is $T_{M(\beta)} = 0$, $I_{M(\beta)} = 0$, $F_{M(\beta)} = 1$, $\forall \beta \in W$.

Definition 9 [44]: The complement of a SVNSES (M, A) is defined as follows:

$$(M, A)^C = \tilde{C}(M(\beta)) \; \forall \beta \in X.$$

Where, $\tilde{C}$ represents single valued neutrosophic complement.

Definition 10 [44]: Consider (M_1, A) and (M_2, B) be two SVNSESs over a common soft universe. The union (M_1, A) and (M_2, B) is denoted by (M_1, A) $\cup$ (M_2, B) = (M_3, C), where $C = A \cup B$ and is defined as follows:

$$M_3(\beta) = M_1(\beta) \cup M_2(\beta), \forall \beta \in C.$$

$$\text{Where, } M_3(\beta) = \begin{cases} M_1(\beta), \beta \in A - B \\ M_2(\beta), \beta \in B - A \\ M_1(\beta) \cup M_2(\beta), \beta \in A \cap B \end{cases}$$

where $M_1(\beta) \cup M_2(\beta)$ = {<x, Max $\{T_{M_1(\beta)}, T_{M_2(\beta)}\}$, Min $\{I_{M_1(\beta)}, I_{M_2(\beta)}\}$, Min $\{F_{M_1(\beta)}, F_{M_2(\beta)}\}$>: $x \in X$}.

Definition 11 [44]: Suppose (M_1, A) and (M_2, B) are two SVNSESs over a common soft universe. The intersection (M_1, A) and (M_2, B) is denoted by (M_1, A) $\cap$ (M_2, B) = (M_4, D), where $D = A \cap B$ and is defined as follows:

$$M_4(\beta) = M_1(\beta) \cap M_2(\beta), \forall \beta \in D.$$

$$\text{Here, } M_4(\beta) = \begin{cases} M_1(\beta), \beta \in A - B \\ M_2(\beta), \beta \in B - A \\ M_1(\beta) \cap M_2(\beta), \beta \in A \cap B \end{cases}$$

where $M_1(\beta) \cap M_2(\beta)$ = {<x, Min $\{T_{M_1(\beta)}, T_{M_2(\beta)}\}$, Max $\{I_{M_1(\beta)}, I_{M_2(\beta)}\}$, Max $\{F_{M_1(\beta)}, F_{M_2(\beta)}\}$>: $x \in X$}

3 TOPSIS method for MADM with single valued neutrosophic soft expert information

Let $C = \{C_1, C_2, \ldots, C_m\}$, ($m \geq 2$) be a discrete set of m feasible alternatives, $A = \{a_1, a_2, \ldots, a_n\}$, ($n \geq 2$) be a set of parameters under consideration and $w = (w_1, w_2, \ldots, w_n)^T$ be the unknown weight vector of the attributes with $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. Let, $Z = \{z_1, z_2, \ldots, z_t\}$ be a set of t experts (agents), where we consider the weights of the experts are equal and O = {agree = 1, disagree = 0} be a set of opinions. The rating of performance value of alternative C_i, (i = 1, 2, …, m) with respect to the parameters is presented by the experts and they can be expressed in terms of SVNSs. Therefore, the proposed methodology for solving single valued neutrosophic soft expert MADM problem based on TOPSIS method is presented as follows:

Step 1. Formulation of decision matrix with SVNSs

Let, the rating of alternative C_i (i = 1, 2, …, m) with respect to the parameter provided by the experts is represented by SVNSES (M, A) and they can be presented in matrix form as follows:

$$D_{SVNSES} = \langle d_{ij} \rangle_{m \times q} = \begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1q} \\ d_{21} & d_{22} & \ldots & d_{2q} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ d_{m1} & d_{m2} & \ldots & d_{mq} \end{bmatrix}$$

Here, $d_{ij} = (T_{ij}, I_{ij}, F_{ij})$ where $T_{ij}, I_{ij}, F_{ij} \in [0, 1]$ and $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3$, i = 1, 2, …, m; j = 1, 2, …, q, where $q = n \times t \times 2$.

Step 2. Determination of unknown weights of the parameters

In the selection process, we assume that the importance (weight) of the attributes is not same and the weights of the attributes are completely unknown. Therefore, we employ maximizing deviation method due to Yang [46] in order to to obtain the unknown weights. The deviation values of alternative C_i (i = 1, 2, …, m) to all other alternatives under the attribute A_j (j = 1, 2, …, q) can be defined as $Y_{ij}(w_j) = \sum_{k=1}^{m} y(d_{ij}, d_{kj}) w_j$, then $Y_j(w_j) = \sum_{i=1}^{m} Y_{ij} w_j = \sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj}) w_j$ denotes the total deviation values of all alternatives to the other alternatives for the attribute A_j (j = 1, 2, …, q). Now $Y(w_j) = \sum_{j=1}^{n} Y_j(w_j) = \sum_{j=1}^{q}\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj}) w_j$ presents the deviation of all attributes for all alternatives to the other alternatives. The maximizing deviation model [47] is formulated as follows:

$$\text{Max } Y(w_j) = \sum_{j=1}^{q}\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj}) w_j$$

Subject to $\sum_{j=1}^{q} w_j^2 = 1$, $w_j \geq 0$, j = 1, 2, …, q.

Solving the above model, we obtain

$$w_j^* = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj})}{\sqrt{\left(\sum_{j=1}^{q}\sum_{i=1}^{m}\sum_{k=1}^{m} y^2(d_{ij}, d_{kj})\right)}} \quad (5)$$

Finally, we get normalized attribute weight based on the above model as follows:

$$w_j^* = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj})}{\sum_{j=1}^{q}\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj})} \quad (6)$$

Step 3. Construction of weighted decision matrix

We obtain aggregated weighted decision matrix by multiplying weights (w_j) [48] of the parameters and aggregated decision matrix $\langle d_{ij}^{w_j} \rangle_{m\times q}$ is presented as follows:

$$D_{SVNSES}^{w} = D_{SVNSES} \otimes w = \langle d_{ij} \rangle_{m\times q} \otimes w_j$$

$$= \langle d_{ij}^{w_j} \rangle_{m\times q} = \begin{bmatrix} d_{11}^{w_1} & d_{12}^{w_2} & \dots & d_{1q}^{w_q} \\ d_{21}^{w_1} & d_{22}^{w_2} & \dots & d_{2q}^{w_q} \\ . & . & \dots & . \\ . & . & \dots & . \\ d_{m1}^{w_1} & d_{m2}^{w_2} & \dots & d_{mq}^{w_q} \end{bmatrix}$$

Here, $d_{ij}^{w_j} = \langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ where $T_{ij}^{w_j}$, $I_{ij}^{w_j}$, $F_{ij}^{w_j} \in [0, 1]$ and $0 \leq T_{ij}^{w_j} + I_{ij}^{w_j} + F_{ij}^{w_j} \leq 3$, i = 1, 2, …, m; j = 1, 2, …, q.

Step 4. Determination of single valued neutrosophic relative positive ideal solution (SVNRPIS) and single valued neutrosophic relative negative ideal solution (SVNRNIS)

The parameters are generally classified into two categories namely benefit type attributes (α_1) and cost type attributes (α_2). Consider $R_{SVNRPIS}^{w+}$ and $R_{SVNRNIS}^{w-}$ be the single valued neutrosophic relative positive ideal solution (SVNRPIS) and single valued neutrosophic relative negative ideal solution (SVNRNIS). Then, $R_{SVNRPIS}^{w+}$ and $R_{SVNRNIS}^{w-}$ can be defined as follows:

$R_{SVNRPIS}^{w+} = \left(\langle T_1^{w_1+}, I_1^{w_1+}, F_1^{w_1+} \rangle, \langle T_2^{w_2+}, I_2^{w_2+}, F_2^{w_2+} \rangle, \dots, \langle T_q^{w_q+}, I_q^{w_q+}, F_q^{w_q+} \rangle \right)$

$R_{SVNRNIS}^{w-} = \left(\langle T_1^{w_1-}, I_1^{w_1-}, F_1^{w_1-} \rangle, \langle T_2^{w_2-}, I_2^{w_2-}, F_2^{w_2-} \rangle, \dots, \langle T_q^{w_q-}, I_q^{w_q-}, F_q^{w_q-} \rangle \right)$ where

$\langle T_j^{w_j+}, I_j^{w_j+}, F_j^{w_j+} \rangle = < [\{ \underset{i}{Max}(T_{ij}^{w_j}) \mid j \in \alpha_1 \}; \{ \underset{i}{Min}(T_{ij}^{w_j}) \mid j \in \alpha_2 \}], [\{ \underset{i}{Min}(I_{ij}^{w_j}) \mid j \in \alpha_1 \}; \{ \underset{i}{Max}(I_{ij}^{w_j}) \mid j \in \alpha_2 \}], [\{ \underset{i}{Min}(F_{ij}^{w_j}) \mid j \in \alpha_1 \}; \{ \underset{i}{Max}(F_{ij}^{w_j}) \mid j \in \alpha_2 \}] >$, j = 1, 2, …, q,

$\langle T_j^{w_j-}, I_j^{w_j-}, F_j^{w_j-} \rangle = < [\{ \underset{i}{Min}(T_{ij}^{w_j}) \mid j \in \alpha_1 \}; \{ \underset{i}{Max}(T_{ij}^{w_j}) \mid j \in \alpha_2 \}], [\{ \underset{i}{Max}(I_{ij}^{w_j}) \mid j \in \alpha_1 \}; \{ \underset{i}{Min}(I_{ij}^{w_j}) \mid j \in \alpha_2 \}], [\{ \underset{i}{Max}(F_{ij}^{w_j}) \mid j \in \alpha_1 \}; \{ \underset{i}{Min}(F_{ij}^{w_j}) \mid j \in \alpha_2 \}] >$, j = 1, 2, …, q.

Step 5. Computation of distance measure of each alternative from RPIS and RNIS

The normalized Euclidean measure of each alternative $\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ from the SVNRPIS $\langle T_j^{w_j+}, I_j^{w_j+}, F_j^{w_j+} \rangle$ for i = 1, 2, …, m; j = 1, 2, …., q can be defined as follows:

$$L_N^{i+}(d_{ij}^{w_j}, d_j^{w_j+}) = \sqrt{\frac{1}{3q}\sum_{j=1}^{q}\left\{ \begin{array}{l} (T_{ij}^{w_j}(x_j) - T_j^{w_j+}(x_j))^2 + (I_{ij}^{w_j}(x_j) - I_j^{w_j+}(x_j))^2 + \\ (F_{ij}^{w_j}(x_j) - F_j^{w_j+}(x_j))^2 \end{array} \right\}} \quad (7)$$

Similarly, normalized Euclidean measure of each alternative $\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \rangle$ from the SVNRNIS $\langle T_j^{w_j-}, I_j^{w_j-}, F_j^{w_j-} \rangle$ for i = 1, 2, …, m; j = 1, 2, …., q can be written as follows:

$$L_N^{i-}(d_{ij}^{w_j}, d_j^{w_j-}) = \sqrt{\frac{1}{3q}\sum_{j=1}^{q}\left\{\begin{array}{l}(T_{ij}^{w_j}(x_j) - T_j^{w_j-}(x_j))^2 + (I_{ij}^{w_j}(x_j) - I_j^{w_j-}(x_j))^2 + \\ (F_{ij}^{w_j}(x_j) - F_j^{w_j-}(x_j))^2\end{array}\right\}} \quad (8)$$

Step 6. Calculation of the relative closeness co-efficient to the neutrosophic ideal solution

The relative closeness co-efficient of each alternative C_i, (i = 1, 2, …, m) with respect to the SVNRPIS is defined as follows:

$$\tau_i^* = \frac{L_N^{i-}(d_{ij}^{w_j}, d_j^{w_j-})}{L_N^{i+}(d_{ij}^{w_j}, d_j^{w_j+}) + L_N^{i-}(d_{ij}^{w_j}, d_j^{w_j-})} \quad (9)$$

where, $0 \le \tau_i^* \le 1$, i = 1, 2, …, m.

Step 7. Rank the alternatives

We rank the alternatives according to the values of τ_i^*, i = 1, 2, …, m and bigger value of τ_i^*,(i = 1, 2, …, m) reflects the best alternative.

4 Proposed algorithm for MADM problem with single valued neutrosophic soft expert information

An algorithm for MADM problem with single valued neutrosophic soft expert information through TOPSIS method is given using the following steps.

Step 1. Construct the decision matrix D_{SVNSES}.

Step 2. Determine the unknown weight (w_j) of the attributes by using Eq (6).

Step 3. Formulate the weighted aggregated decision matrix $D_G^w = \langle d_{ij}^{w_j} \rangle_{m\times q}$.

Step 4. Recognize the SVNRPIS ($R_{SVNRPIS}^{w+}$) and SVNRNIS ($R_{SVNRNIS}^{w-}$).

Step 5. Calculate the distance of each alternative from SVNRPIS ($R_{SVNRPIS}^{w+}$) and SVNRNIS ($R_{SVNRNIS}^{w-}$) using Eqs. (7) and (8) respectively.

Step 6. Determine the relative closeness co-efficient τ_i^* (i = 1, 2, …, m) using Eq. (9) of each alternative C_i.

Step 7. Rank the preference order of alternatives in accordance with the order of their relative closeness.

5 A numerical example

In this section, we solve a hypothetical problem to show the effectiveness of the proposed approach. Suppose that a school authority is going to recruit an assistant teacher in Mathematics to fill the vacancy on contractual basis for six months. After preliminary screening, three candidates (alternatives) C_1, C_2, C_3 are short-listed for further assessment. A committee consisting of two members namely 'Senior Mathematics teacher (z_1) and 'an external expert on the relevant subject' (z_2) is formed to conduct the interview in order to select the most suitable teacher and O = {1 = agree, 0 = disagree} be the set of opinions of the selection committee members. The committee considers two parameters a_i, i = 1, 2, where a_1 denotes 'pedagogical knowledge' and a_2 denotes 'personality'. After the interview of the candidates, the select committee provides the following SVNSESs.

M (a_1, z_1, 1) =
$\{\langle x_1, 0.7, 0.5, 0.2\rangle, \langle x_2, 0.6, 0.2, 0.3\rangle, \langle x_3, 0.8, 0.3, 0.3\rangle\}$,
M (a_2, z_1, 1) =
$\{\langle x_1, 0.5, 0.1, 0.4\rangle, \langle x_2, 0.9, 0.2, 0.2\rangle, \langle x_3, 0.8, 0.1, 0.2\rangle\}$,
M (a_1, z_2, 1) =
$\{\langle x_1, 0.7, 0.3, 0.5\rangle, \langle x_2, 0.9, 0.2, 0.1\rangle, \langle x_3, 0.7, 0.1, 0.4\rangle\}$,
M (a_2, z_2, 1) =
$\{\langle x_1, 0.6, 0.2, 0.3\rangle, \langle x_2, 0.9, 0.1, 0.1\rangle, \langle x_3, 0.8, 0.3, 0.2\rangle\}$,
M (a_1, z_1, 0) =
$\{\langle x_1, 0.3, 0.4, 0.3\rangle, \langle x_2, 0.5, 0.3, 0.2\rangle, \langle x_3, 0.2, 0.3, 0.5\rangle\}$,
M (a_2, z_1, 0) =
$\{\langle x_1, 0.4, 0.1, 0.3\rangle, \langle x_2, 0.3, 0.3, 0.1\rangle, \langle x_3, 0.4, 0.3, 0.4\rangle\}$,
M (a_1, z_2, 0) =
$\{\langle x_1, 0.5, 0.1, 0.2\rangle, \langle x_2, 0.4, 0.2, 0.3\rangle, \langle x_3, 0.5, 0.1, 0.4\rangle\}$,
M (a_2, z_2, 0) =
$\{\langle x_1, 0.5, 0.2, 0.3\rangle, \langle x_2, 0.3, 0.3, 0.2\rangle, \langle x_3, 0.5, 0.2, 0.5\rangle\}$.

Then, the proposed procedure for solving the problem is provided using the following steps.

Step 1: Formulation of decision matrix

We present the SVNSESs in the tabular form(see the table 1) as given below (see Table 1)

Step 2. Calculation of the weights of the attributes

We use Hamming distance and obtained the weights of the parameters using Eq. (6) as follows:
$w_1 = 0.12$, $w_2 = 0.14$, $w_3 = 0.16$, $w_4 = 0.14$, $w_5 = 0.12$, $w_6 = 0.12$, $w_7 = 0.08$, $w_8 = 0.12$, where $\sum_{j=1}^{8} w_j = 1$.

Step 3. Construction of weighted decision matrix

The tabular form of the weighted decision matrix is presented the Table 2.

Step 4. Determination of SVNRPIS and SVNRNIS

The SVNRPIS ($R^{w+}_{SVNRPIS}$) and SVNRNIS ($R^{w-}_{SVNRNIS}$) can be obtained from the weighted decision matrix (see Table 2) as follows:

$R^{w+}_{SVNRPIS}$ = < (0.176, 0.824, 0.824); (0.276, 0.724, 0.798); (0.308, 0.692, 0.692); (0.276, 0.724, 0.724); (0.08, 0.865, 0.824); (0.059, 0.758, 0.758); (0.054, 0.832, 0.879); (0.08, 0.824, 0.824) >,

$R^{w-}_{SVNRNIS}$ = < (0.104, 0.92, 0.865); (0.092, 0.798, 0.88); (0.175, 0.825, 0.895); (0.12, 0.845, 0.845); (0.026, 0.896, 0.92); (0.042, 0.865, 0.896); (0.04, 0.879, 0.929); (0.042, 0.865, 0.92) >.

Step 5. Compute the distance measure of each alternative from the SVNRPIS and SVNRNIS

The Euclidean distance measures of each alternative from the SVNRPIS are calculated by using Eq. (7) as follows:

$L^{1+}_N = 0.1542$, $L^{2+}_N = 0.0393$, $L^{3+}_N = 0.0753$.

Similarly, the Euclidean distance measures of each alternative from the SVNRNIS are determined by using Eq. (8) as follows:

$L^{1-}_N = 0.1736$, $L^{2-}_N = 0.1565$, $L^{3-}_N = 0.1542$.

Step 6. Calculation of the relative closeness coefficient

We calculate the relative closeness co-efficient τ^*_i (i = 1, 2, 3) by using Eq. (9) are shown as follows:

$\tau^*_1 = 0.5296$, $\tau^*_2 = 0.7993$, $\tau^*_3 = 0.6719$.

Step 7. Rank the alternatives

The ranking order of alternatives according to the relative closeness coefficient is presented as follows:

$C_2 \succ C_3 \succ C_1$.

Consequently, C_2 is the best candidate.

Table 1. Tabular form of the given SVNSESs

U_1	(0.7, 0.5, 0.2)	(0.5, 0.1, 0.4)	(0.7, 0.3, 0.5)	(0.6, 0.2, 0.3)	(0.3, 0.4, 0.3)	(0.4, 0.1, 0.3)	(0.5, 0.1, 0.2)	(0.5, 0.2, 0.3)
U_2	(0.6, 0.2, 0.3)	(0.9, 0.2, 0.2)	(0.9, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.5, 0.3, 0.2)	(0.3, 0.3, 0.1)	(0.4, 0.2, 0.3)	(0.3, 0.3, 0.2)
U_3	(0.8, 0.3, 0.3)	(0.8, 0.1, 0.2)	(0.9, 0.2, 0.1)	(0.7, 0.1, 0.4)	(0.8, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.4, 0.3, 0.4)	(0.5, 0.2, 0.5)

Table 2. Formulation of weighted decision matrix of the given SVNSESs

U_1	(0.134, 0.920, 0.824)	(0.092, 0.724, 0.880)	(0.175, 0.825, 0.895)	(0.120, 0.798, 0.845)	(0.042, 0.896, 0.865)	(0.059, 0.758, 0.865)	(0.054, 0.832, 0.879)	(0.08, 0.824, 0.865)
U_2	(0.104, 0.824, 0.865)	(0.276, 0.798, 0.798)	(0.308, 0.773, 0.692)	(0.276, 0.724, 0.724)	(0.080, 0.865, 0.824)	(0.042, 0.865, 0.758)	(0.04, 0.879, 0.908)	(0.042, 0.865, 0.824)
U_3	(0.176, 0.865, 0.920)	(0.202, 0.724, 0.798)	(0.175, 0.692, 0.864)	(0.202, 0.845, 0.798)	(0.026, 0.865, 0.920)	(0.054, 0.832, 0.929)	(0.04, 0.879, 0.908)	(0.042, 0.865, 0.824)

6 Conclusion

SVNSES is an effective and useful decision making tool to describe indeterminate and inconsistent information and it is also possible for a user to view the opinions of all experts in a single model. In this study, we have investigated a TOPSIS method for solving MADM problems with single valued neutrosophic soft expert information. The rating of performance values of the alternatives with respect to the parameters are presented in terms of SVNSs. We determine the weights of the parameters by maximizing deviation method and formulate weighted decision matrix. We identify SVNRPIS and SVNRNIS from the weighted decision matrix and normalized Euclidean distance measure is used to calculate distances of each alternative from SVNRPISs as well as SVNRNISs. Relative closeness co-efficient of each alternative is then calculated to select the most desirable alternative. Finally, an application of the proposed method for teacher selection is given.

In future, the proposed method can be used for dealing with interval-valued neutrosophic soft expert based MADM problems and different practical problems such as pattern recognition, medical diagnosis, information fusion, supplier selection, etc.

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