Neutrosophic Sets and Systems, Vol. 50, 2022
University of New Mexico

MADM Technique Using Tangent Trigonometric SvNN Aggregation Operators for the Teaching Quality Assessment of Teachers

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Abstract: In current Chinese higher education, the teaching quality assessment (TQA) of teachers in colleges/universities is an essential way to promotes the improvement of teacher teaching quality in the teaching process. In the TQA process of teachers, the evaluation information of experts/decision makers implies incompleteness, uncertainty and inconsistency corresponding to experts' cognition and judgment on evaluation indicators. Neutrosophic multiple attribute decision making (MADM) is one of key research topics in indeterminate and inconsistent decision-making problems. This paper presents a novel MADM technique using tangent trigonometric aggregation operators for single-valued neutrosophic numbers (SvNNs) to assess the teaching quality of teachers. First, we propose novel operational laws of tangent trigonometric SvNNs based on tangent trigonometric function. In view of the tangent trigonometric SvNN operational laws, we present tangent trigonometric SvNN weighted averaging and geometric operators to aggregate tangent trigonometric SvNNs. Then, we establish the MADM technique using the proposed two aggregation operators to perform MADM problems, and provide an actual example about the TQA of teachers and the comparison of existing related MADM techniques in the SvNN environment to reveal the efficiency and suitability of the proposed technique.

Keywords: single-valued neutrosophic number; tangent trigonometric operation; tangent trigonometric aggregation operator; decision making; teaching quality assessment

1. Introduction

The teaching quality of teachers reveals significance in the training and competition of modern talents in current Chinese higher education. In this case, the teaching quality assessment (TQA) mechanism in colleges and universities plays an important role in the teaching process and promotes the improvement of teachers' teaching quality. Since the TQA of teachers contains many evaluation indicators/attributes, such as teaching level and skill, teaching means and method, teaching attitude, TQA is a multiple attribute decision-making (MADM) issue. Then, in the TQA process of teachers, the evaluation information of experts implies incompleteness, uncertainty, and inconsistency corresponding to experts' cognition and judgment on evaluation indicators.

In the environment of incompleteness, uncertainty and inconsistency, a neutrosophic set (NS) was presented by Smarandache [1] in view of the true, false and indeterminate membership functions subject to a non-standard interval $]^-0, 1^+[$. Then, some researchers introduced an interval NS (INS) [2], a single-valued NS (SvNS) [3], and a simplified NS [4] based on a real-standard interval [0, 1] to suit the application of engineering and science fields. Due to the merit of simplified NSs, including INSs and SvNSs, the simplified NSs have been used for many MADM issues [5-8]. Recently, neutrosophic MADM models have been applied to the TQA of teachers [9-11].

Neutrosophic decision making is a current research hotspot. It is very vital to establish reasonable information representations and operations in decision-making models. It is worth noting that the neutrosophic aggregation operation plays an important role in neutrosophic MADM issues. Some researchers [12-14] proposed the operational laws of logarithmic single-valued neutrosophic numbers (SvNNs) and sine trigonometric SvNNs (ST-SvNNs) and their weighted aggregation operators for MADM problems in the SvNS setting. Then, the merit of the sine trigonometric function is its periodicity and symmetry about the origin, which meets the preference of decision-makers for multiple time phase parameters. Another periodic function, except the sine trigonometric function, is the tangent trigonometric function. In terms of the superiority of the tangent trigonometric function, this paper needs to build up some new operational laws of tangent trigonometric SvNNs (TT-SvNNs) and studies their aggregation operators, then establishes the MADM technique to perform the assessment mechanism of teaching quality in Shaoxing University in China under the environment of SvNSs.

The remainder of this article is arranged as follows. Section 2 introduces some preliminaries related to SvNNs. In Section 3, we give the definition of TT-SvNN and some novel operational laws of TT-SvNNs. In Section 4, we propose the TT-SvNN weighted averaging (TT-SvNNWA) and TT-SvNN weighted geometric (TT-SvNNWG) operators, along with the related proof of their properties. Section 5 establishes the MADM technique in terms of the TT-SvNNWA and TT-SvNNWG operators to perform MADM problems with SvNN information. Section 6 applies the established MADM technique to an actual example about the TQA problem of teachers in Shaoxing University in China and conducts the comparison of existing related MADM techniques to show the efficiency and suitability of the established MADM technique in the environment of SvNSs. The article ends with conclusions and future research in Section 7.

2. Some Preliminaries of SvNNs

2.1 Operations and sorting rules of SvNNs

The SvNS S_N in a universe set Y is denoted as $S_N = \{<y, Nt(y), Nu(y), Nf(y)>|\, y \in Y\}$ [3], where $Nt(y), Nu(y), Nf(y) \in [0, 1]$ are the true, indeterminate, and false membership functions subject to $0 \le Nt(y) + Nu(y) + Nf(y) \le 3$ for $y \in Y$. Then, $<y, Nt(y), Nu(y), Nf(y)>$ in S_N is denoted as $Ns = <Nt, Nu, Nf>$ for simplicity, which is named SvNN.

Set two SvNNs as $Ns_1 = <Nt_1, Nu_1, Nf_1>$ and $Ns_2 = <Nt_2, Nu_2, Nf_2>$ with $h > 0$. Then, their operations are defined below [4, 7]:

(1) $Ns_1 \supseteq Ns_2 \Leftrightarrow Nt_1 \ge Nt_2$, $Nu_1 \le Nu_2$, and $Nf_1 \le Nf_2$;

(2) $Ns_1 = Ns_2 \Leftrightarrow Ns_1 \supseteq Ns_2$ and $Ns_2 \supseteq Ns_1$;

(3) $Ns_1 \bigcup Ns_2 = \langle Nt_1 \vee Nt_2, Nu_1 \wedge Nu_2, Nf_1 \wedge Nf_2 \rangle$;

(4) $Nt_1 \bigcap Nt_2 = \langle Nt_1 \wedge Nt_2, Nu_1 \vee Nu_2, Nf_1 \vee Nf_2 \rangle$;

(5) $(Ns_1)^c = \langle Nf_1, 1 - Nu_1, Nt_1 \rangle$ (Complement of Ns_1);

(6) $Ns_1 \oplus Ns_2 = \langle Nt_1 + Nt_2 - Nt_1 Nt_2, Nu_1 Nu_2, Nf_1 Nf_2 \rangle$;

(7) $Ns_1 \otimes Ns_2 = \langle Nt_1 Nt_2, Nu_1 + Nu_2 - Nu_1 Nu_2, Nf_1 + Nf_2 - Nf_1 Nf_2 \rangle$;

(8) $h \cdot Ns_1 = \left\langle 1-(1-Nt_1)^h, Nu_1^h, Nf_1^h \right\rangle$;

(9) $Ns_1^h = \left\langle Nt_1^h, 1-(1-Nu_1)^h, 1-(1-Nf_1)^h \right\rangle$.

Set Ns_g = <Nt_g, Nu_g, Nf_g> (g = 1, 2, …, n) as a group of SvNNs with their weight vector $\boldsymbol{H} = (h_1, h_2, \ldots, h_n)$ subject to $0 \le h_g \le 1$ and $\sum_{g=1}^{n} h_g = 1$. Then, the weighted averaging and geometric operators of SvNNs are denoted as SvNNWA and SvNNWG and defined by the following equations [7]:

$$SvNNWA(Ns_1, Ns_2, \ldots, Ns_n) = \sum_{g=1}^{n} h_g Ns_g = \left\langle 1-\prod_{g=1}^{n}\left(1-Nt_g\right)^{h_g}, \prod_{g=1}^{n}\left(Nu_g\right)^{h_g}, \prod_{g=1}^{n}\left(Nf_g\right)^{h_g} \right\rangle, \quad (1)$$

$$SvNNWG(Ns_1, Ns_2, \ldots, Ns_n) = \prod_{g=1}^{n}\left(Ns_g\right)^{h_g} = \left\langle \prod_{g=1}^{n}\left(Nt_g\right)^{h_g}, 1-\prod_{g=1}^{n}\left(1-Nu_g\right)^{h_g}, 1-\prod_{g=1}^{n}\left(1-Nf_g\right)^{h_g} \right\rangle. \quad (2)$$

To compare two SvNNs Ns_g = <Nt_g, Nu_g, Nf_g> (g = 1, 2), the score and accuracy functions of SvNNs are defined as follows [7]:

$$F(Ns_g) = (2 + Nt_g - Nu_g - Nf_g)/3 \text{ for } F(Ns_g) \in [0,1], \quad (3)$$

$$G(Ns_g) = Nt_g - Nf_g \text{ for } G(Ns_g) \in [-1,1]. \quad (4)$$

In terms of the score and accuracy functions, a sorting method of two SvNNs is defined by the following rules:

(1) If $F(Ns_1) > F(Ns_2)$, then $Ns_1 > Ns_2$;

(2) If $F(Ns_1) = F(Ns_2)$ and $G(Ns_1) > G(Ns_2)$, then $Ns_1 > Ns_2$;

(3) If $F(Ns_1) = F(Ns_2)$ and $G(Ns_1) = G(Ns_2)$, then $Ns_1 \cong Ns_2$.

2.2 Operational Laws of ST-SvNNs

Set SvNN as Ns = <Nt, Nu, Nf>. Then, ST-SvNN is defined as $\sin(Ns) = \left\langle \sin(\frac{\pi}{2}Nt), 1-\sin(\frac{\pi}{2}-Nu), 1-\sin(\frac{\pi}{2}-Nf) \right\rangle$ [14], where the true, indeterminate, and false membership degrees are $\sin(\frac{\pi}{2}Nt): Y \to [0,1]$, $1-\sin(\frac{\pi}{2}-Nu): Y \to [0,1]$, and $1-\sin(\frac{\pi}{2}-Nf): Y \to [0,1]$, respectively.

Set two ST-SvNNs as $\sin(Ns_1) = \left\langle \sin(\frac{\pi}{2}Nt_1), 1-\sin(\frac{\pi}{2}-Nu_1), 1-\sin(\frac{\pi}{2}-Nf_1) \right\rangle$ and $\sin(Ns_2) = \left\langle \sin(\frac{\pi}{2}Nt_2), 1-\sin(\frac{\pi}{2}-Nu_2), 1-\sin(\frac{\pi}{2}-Nf_2) \right\rangle$ with $h > 0$. Then, their operational laws are defined below [14]:

(1) $$\sin(Ns_1) \oplus \sin(Ns_2) = \left\langle \begin{matrix} 1-(1-\sin(\frac{\pi}{2}Nt_1))(1-\sin(\frac{\pi}{2}Nt_2)), \\ (1-\sin(\frac{\pi}{2}-Nu_1))(1-\sin(\frac{\pi}{2}-Nu_2)), \\ (1-\sin(\frac{\pi}{2}-Nf_1))(1-\sin(\frac{\pi}{2}-Nf_2)) \end{matrix} \right\rangle,$$

(2) $$\sin(Ns_1) \otimes \sin(Ns_2) = \left\langle \begin{matrix} \sin(\frac{\pi}{2}Nt_1)\sin(\frac{\pi}{2}Nt_2), \\ 1-\sin(\frac{\pi}{2}-Nu_1)\sin(\frac{\pi}{2}-Nu_2), \\ 1-\sin(\frac{\pi}{2}-Nf_1)\sin(\frac{\pi}{2}-Nf_2) \end{matrix} \right\rangle,$$

(3) $h\cdot\sin(Ns_1)=\left\langle 1-(1-\sin(\frac{\pi}{2}Nt_1))^h,(1-\sin(\frac{\pi}{2}-Nu_1))^h,(1-\sin(\frac{\pi}{2}-Nf_1))^h\right\rangle$,

(4) $(\sin(Ns_1))^h=\left\langle(\sin(\frac{\pi}{2}Nt_1))^h,1-(\sin(\frac{\pi}{2}-Nu_1))^h,1-(\sin(\frac{\pi}{2}-N_{f1}))^h\right\rangle$.

Set Ns_g = <Nt_g, Nu_g, Nf_g> (g = 1, 2, …, n) as a group of SvNNs with their weight vector $\boldsymbol{H}$ = (h_1, h_2, …, h_n) subject to $0 \le h_g \le 1$ and $\sum_{g=1}^{n}h_g=1$. Then, the ST-SvNN weighted averaging and geometric operators are denoted as ST-SvNNWA and ST-SvNNWG and defined by the following equations [14]:

$$ST-SvNNWA(Ns_1,Ns_2,...,Ns_n)=\sum_{g=1}^{n}h_g\sin(Ns_g)$$
$$=\left(1-\prod_{g=1}^{n}\left(1-\sin\left(\frac{\pi}{2}Nt_g\right)\right)^{h_g},\prod_{g=1}^{n}\left(1-\sin\left(\frac{\pi}{2}-Nu_g\right)\right)^{h_g},\prod_{g=1}^{n}\left(1-\sin\left(\frac{\pi}{2}-Nf_g\right)\right)^{h_g}\right), \quad (5)$$

$$ST-SvNNWG(Ns_1,Ns_2,...,Ns_n)=\prod_{g=1}^{n}\left(\sin(Ns_g)\right)^{h_g}$$
$$=\left(\prod_{g=1}^{n}\left(\sin(\frac{\pi}{2}Nt_g)\right)^{h_g},1-\prod_{g=1}^{n}\left(\sin(\frac{\pi}{2}-Nu_g)\right)^{h_g},1-\prod_{g=1}^{n}\left(\sin(\frac{\pi}{2}-Nf_g)\right)^{h_g}\right). \quad (6)$$

3. Operational Laws of TT-SvNNs

This section defines TT-SvNN and some operational laws of TT-SvNNs.

First, we give the definition of TT-SvNN.

Definition 1. Set SvNN as Ns = <Nt, Nu, Nf>. Then, TT-SvNN is defined below:

$$\tan(Ns)=\left\langle\tan(\frac{\pi}{4}Nt),1-\tan(\frac{\pi}{4}(1-Nu),1-\tan(\frac{\pi}{4}(1-Nf)\right\rangle,$$

where the true, false, and indeterminate membership degrees are given, respectively, by

$$\tan(\frac{\pi}{4}Nt):Y\to[0,1],\quad 0\le\tan(\frac{\pi}{4}Nt)\le 1,$$

$$1-\tan(\frac{\pi}{4}(1-Nf)):Y\to[0,1],\quad 0\le 1-\tan(\frac{\pi}{4}(1-Nf))\le 1,$$

$$1-\tan(\frac{\pi}{4}(1-Nu)):Y\to[0,1],\quad 0\le 1-\tan(\frac{\pi}{4}(1-Nu))\le 1.$$

Definition 2. Set SvNN as Ns = <Nt, Nu, Nf>. If $\tan(Ns)=\left\langle\tan(\frac{\pi}{4}Nt),1-\tan(\frac{\pi}{4}(1-Nu)),1-\tan(\frac{\pi}{4}(1-Nf))\right\rangle$, then tan($Ns$) is named the tangent trigonometric operator and its value is named TT-SvNN.

Definition 3. Set two TT-SvNNs as $\tan(Ns_1)=\left\langle\tan(\frac{\pi}{4}Nt_1),1-\tan(\frac{\pi}{4}(1-Nu_1)),1-\tan(\frac{\pi}{4}(1-Nf_1))\right\rangle$ and $\tan(Ns_2)=\left\langle\tan(\frac{\pi}{4}Nt_2),1-\tan(\frac{\pi}{4}(1-Nu_2)),1-\tan(\frac{\pi}{4}(1-Nf_2))\right\rangle$. Then, their operational laws are defined below:

(1) $$\tan(Ns_1)\oplus\tan(Ns_2)=\left\langle\begin{matrix}1-(1-\tan(\frac{\pi}{4}Nt_1))(1-\tan(\frac{\pi}{4}Nt_2)),\\(1-\tan(\frac{\pi}{4}(1-Nu_1)))(1-\tan(\frac{\pi}{4}(1-Nu_2))),\\(1-\tan(\frac{\pi}{4}(1-Nf_1)))(1-\tan(\frac{\pi}{4}(1-Nf_2)))\end{matrix}\right\rangle,$$

(2) $\tan(Ns_1)\otimes\tan(Ns_2)=\left\langle\begin{array}{l}\tan(\frac{\pi}{4}Nt_1)\tan(\frac{\pi}{4}Nt_2),\\ 1-\tan(\frac{\pi}{4}(1-Nu_1))\tan(\frac{\pi}{4}(1-Nu_2)),\\ 1-\tan(\frac{\pi}{4}(1-Nf_1))\tan(\frac{\pi}{4}(1-Nf_2))\end{array}\right\rangle$,

(3) $h\cdot\tan(Ns_1)=\left\langle 1-(1-\tan(\frac{\pi}{4}Nt_1))^h,(1-\tan(\frac{\pi}{4}(1-Nu_1)))^h,(1-\tan(\frac{\pi}{4}(1-Nf_1)))^h\right\rangle$,

(4) $(\tan(Ns_1))^h=\left\langle(\tan(\frac{\pi}{4}Nt_1))^h,1-(\tan(\frac{\pi}{4}(1-Nu_1)))^h,1-(\tan(\frac{\pi}{4}(1-Nf_1)))^h\right\rangle$.

4. TT-SvNN Aggregation Operators

This section proposes two weighted aggregation operators of TT-SvNNWA and TT-SvNNWG in terms of the proposed operational laws of TT-SvNNs and indicates their properties.

3.1 TT-SvNNWA Operator

Definition 4. Set Ns_g = <Nt_g, Nu_g, Nf_g> (g = 1, 2, …, n) as a group of SvNNs with their weight vector ***H*** = (h_1, h_2, …, h_n) subject to 0 ≤ h_g ≤ 1 and $\sum_{g=1}^{n}h_g=1$. Then, the TT-SvNN weighted averaging operator is denoted by TT-SvNNWA and defined below:

$$\begin{aligned}&TT-SvNNWA(Ns_1,Ns_2,...,Ns_n)=h_g\tan(Ns_1)\oplus h_g\tan(Ns_2)\oplus...\oplus h_n\tan(Ns_n)\\&=\sum_{g=1}^{n}h_g\tan(Ns_g)\end{aligned}. \quad (7)$$

Theorem 1. Set Ns_g = <Nt_g, Nu_g, Nf_g> (g = 1, 2, …, n) as a group of SvNNs with their weight vector ***H*** = (h_1, h_2, …, h_n) subject to 0 ≤ h_g ≤ 1 and $\sum_{g=1}^{n}h_g=1$. Then, the value of the TT-SvNNWA operator is obtained by the following equation:

$$\begin{aligned}&TT-SvNNWA(Ns_1,Ns_2,...,Ns_n)=\sum_{g=1}^{n}h_g\tan(Ns_g)\\&=\left(1-\prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}Nt_g\right)\right)^{h_g},\prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nu_g)\right)\right)^{h_g},\prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nf_g)\right)\right)^{h_g}\right)\end{aligned}. \quad (8)$$

Proof: We can verify Theorem 1 in view of mathematical induction and Definition 3.

For n = 2, we obtain the operational result:

$$\begin{aligned}&TT-SvNNWA(Ns_1,Ns_2)=\sum_{g=1}^{2}h_g\tan(Ns_g)\\&=h_1\tan(Ns_1)\oplus h_2\tan(Ns_2)=\begin{pmatrix}1-(1-\tan(\frac{\pi}{4}Nt_1))^{h_1},\\(1-\tan(\frac{\pi}{4}(1-Nu_1)))^{h_1},\\(1-\tan(\frac{\pi}{4}(1-Nf_1)))^{h_1}\end{pmatrix}\oplus\begin{pmatrix}1-(1-\tan(\frac{\pi}{4}Nt_2))^{h_2},\\(1-\tan(\frac{\pi}{4}(1-Nu_2)))^{h_2},\\(1-\tan(\frac{\pi}{4}(1-Nf_2)))^{h_2}\end{pmatrix}\\&=\left(1-\prod_{g=1}^{2}(1-\tan(\tfrac{\pi}{4}Ns_g))^{h_g},\prod_{g=1}^{2}(1-\tan(\tfrac{\pi}{4}(1-Nu_g)))^{h_g},\prod_{g=1}^{2}(1-\tan(\tfrac{\pi}{4}(1-Nf_g)))^{h_g}\right).\end{aligned}$$

Assume that Eq. (8) holds for n = p as follows:

$$TT-SvNNWA(Ns_1, Ns_2, ..., Ns_p) = \sum_{g=1}^{p} h_g \tan(Ns_g)$$

$$= \left(1 - \prod_{g=1}^{p}\left(1-\tan\left(\tfrac{\pi}{4} Nt_g\right)\right)^{h_g}, \prod_{g=1}^{p}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nu_g)\right)\right)^{h_g}, \prod_{g=1}^{p}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nf_g)\right)\right)^{h_g} \right).$$

For $n = p + 1$, we have the following result:

$$TT-SvNNWA(Ns_1, Ns_2, ..., Ns_p, Ns_{p+1}) = \sum_{g=1}^{p} h_g \tan(Ns_g) \oplus h_{g+1} \tan(Ns_{p+1})$$

$$= \left(1 - \prod_{g=1}^{p}\left(1-\tan\left(\tfrac{\pi}{4} Nt_g\right)\right)^{h_g}, \prod_{g=1}^{p}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nu_g)\right)\right)^{h_g}, \prod_{g=1}^{p}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nf_g)\right)\right)^{h_g} \right)$$

$$\oplus \left(1-(1-\tan(\tfrac{\pi}{4} Ns_{p+1}))^{h_{p+1}}, (1-\tan(\tfrac{\pi}{4}(1-Nu_{p+1})))^{h_{p+1}}, (1-\tan(\tfrac{\pi}{4}(1-Nf_{p+1})))^{h_{p+1}}\right)$$

$$= \left(1 - \prod_{g=1}^{p+1}\left(1-\tan\left(\tfrac{\pi}{4} Nt_g\right)\right)^{h_g}, \prod_{g=1}^{p+1}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nu_g)\right)\right)^{h_g}, \prod_{g=1}^{p+1}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nf_g)\right)\right)^{h_g} \right).$$

Thus, Eq. (8) can hold for $n = p+1$.

Therefore, Eq. (8) also holds for any n. This proof is finished. □

Example 1. Set three SvNNs as Ns_1 = <0.76, 0.23, 0.2>, Ns_2 = <0.83, 0.3, 0.22>, and Ns_3 = <0.67, 0.12, 0.15> with the weight vector $\boldsymbol{H}$ = <0.35, 0.2, 0.45>. Using Eq. (8), we give the following calculational process:

$$TT-SvNNWA(Ns_1, Ns_2, Ns_3) = \sum_{g=1}^{3} h_g \tan(Ns_g)$$

$$= \left\langle \begin{array}{l} 1-\left(1-\tan\left(\tfrac{\pi}{4}\times 0.76\right)\right)^{0.35} \times \left(1-\tan\left(\tfrac{\pi}{4}\times 0.83\right)\right)^{0.2} \times \left(1-\tan\left(\tfrac{\pi}{4}\times 0.67\right)\right)^{0.45}, \\ \left(1-\tan\left(\tfrac{\pi}{4}(1-0.23)\right)\right)^{0.35} \times \left(1-\tan\left(\tfrac{\pi}{4}(1-0.3)\right)\right)^{0.2} \times \left(1-\tan\left(\tfrac{\pi}{4}(1-0.12)\right)\right)^{0.45}, \\ \left(1-\tan\left(\tfrac{\pi}{4}(1-0.2)\right)\right)^{0.35} \times \left(1-\tan\left(\tfrac{\pi}{4}(1-0.22)\right)\right)^{0.2} \times \left(1-\tan\left(\tfrac{\pi}{4}(1-0.15)\right)\right)^{0.45} \end{array} \right\rangle$$

$$=< 0.6596,\ 0.2488,\ 0.2478 >.$$

Theorem 2. The proposed TT-SvNNWA operator contains some properties based on the tangent trigonometric function as follows:

(i) Idempotency: If Ns_g = <Nt_g, Nu_g, Nf_g> = <Nt, Nu, Nf> = Ns (g = 1, 2, …, n), there is $TT-SvNNWA(Ns_1, Ns_2, ..., Ns_n) = \tan(Ns)$.

(ii) Boundedness: Set $Ns^- = \left\langle \min_g(Nt_g), \max_g(Nu_g), \max_g(Nf_g) \right\rangle$ and $Ns^+ = \left\langle \max_g(Nt_g), \min_g(Nu_g), \min_g(Nf_g) \right\rangle$ as the minimum SvNN and the maximum SvNN, respectively. Then, there is $\tan(Ns^-) \le TT-SvNNWA(Ns_1, Ns_2, ..., Ns_n) \le \tan(Ns^+)$.

(iii) Monotonicity: Set $Ns_g = \left\langle Nt_g, Nu_g, Nf_g \right\rangle$ and $Ns^* = \left\langle Nt_g^*, Nu_g^*, Nf_g^* \right\rangle$ (g = 1, 2, …, n) as two groups of SvNNs. Then $TT-SvNNWA(Ns_1, Ns_2, \cdots, Ns_n) \le TT-SvNNWA(Ns_1^*, Ns_2^*, \cdots, Ns_n^*)$ exists when $Ns_g \le Ns_g^*$.

Proof:

(i) For $Ns_g = Ns$, using Eq. (8), we obtain

$$TT-SvNNWA(Ns_1, Ns_2, ..., Ns_n) = \sum_{g=1}^{n} h_g \tan(Ns_g)$$
$$= \left(1-\prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}Nt_g\right)\right)^{h_g}, \prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nu_g)\right)\right)^{h_g}, \prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nf_g)\right)\right)^{h_g}\right)$$
$$= \left(1-\prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}Nt\right)\right)^{h_g}, \prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nu)\right)\right)^{h_g}, \prod_{g=1}^{n}\left(1-\tan\left(\tfrac{\pi}{4}(1-Nf)\right)\right)^{h_g}\right)$$
$$= \left(1-\left(1-\tan\left(\tfrac{\pi}{4}Nt\right)\right)^{\sum_{g=1}^{n}h_g}, \left(1-\tan\left(\tfrac{\pi}{4}(1-Nu)\right)\right)^{\sum_{g=1}^{n}h_g}, \left(1-\tan\left(\tfrac{\pi}{4}(1-Nf)\right)\right)^{\sum_{g=1}^{n}h_g}\right)$$
$$= \left(\tan\left(\tfrac{\pi}{4}Nt\right), 1-\tan\left(\tfrac{\pi}{4}(1-Nu)\right), 1-\tan\left(\tfrac{\pi}{4}(1-Nf)\right)\right) = \tan(Ns).$$

(ii) When $Ns^- \le Ns_g \le Ns^+$, $\tan(Ns^-) \le \tan(Ns_g) \le \tan(Ns^+)$ exists since tan (x) for $0 \le x \le \pi/4$ is an increasing function. Then, there is also $\sum_{g=1}^{n} h_g \tan(Ns^-) \le \sum_{g=1}^{n} h_g \tan(Ns_g) \le \sum_{g=1}^{n} h_g \tan(Ns^+)$. Therefore, based on the property (i), there is $\tan(Ns^-) \le TT-SvNNWA(Ns_1, Ns_2, ..., Ns_n) \le \tan(Ns^+)$.

(iii) When $Ns_g \le Ns_g^*$, there is $\tan(Ns_g) \le \tan(Ns_g^*)$ since tan (x) for $0 \le x \le \pi/4$ is an increasing function. $\sum_{g=1}^{n} h_g \tan(Ns_g) \le \sum_{g=1}^{n} h_g \tan(Ns_g^*)$ can hold in view of the property (ii). Thus, $TT-SvNNWA(Ns_1, Ns_2, \cdots, Ns_n) \le TT-SvNNWA(Ns_1^*, Ns_2^*, \cdots, Ns_n^*)$ exists.

3.2 TT-SvNNWG Operator

Definition 5. Set Ns_g = <Nt_g, Nu_g, Nf_g> (g = 1, 2, …, n) as a group of SvNNs with their weight vector ***H*** = (h_1, h_2, …, h_n) subject to $0 \le h_g \le 1$ and $\sum_{g=1}^{n} h_g = 1$. Then, the TT-SvNN weighted geometric operator is denoted by TT-SvNNWG and defined below:

$$TT-SvNNWG(Ns_1, Ns_2, ..., Ns_n) = \left(\tan(Ns_1)\right)^{h_1} \otimes \left(\tan(Ns_2)\right)^{h_2} \otimes ... \otimes \left(\tan(Ns_n)\right)^{h_n} = \prod_{g=1}^{n}\left(\tan(Ns_g)\right)^{h_g}. \quad (9)$$

Theorem 3. Set Ns_g = <Nt_g, Nu_g, Nf_g> (g = 1, 2, …, n) as a group of SvNNs with their weight vector ***H*** = (h_1, h_2, …, h_n) subject to $0 \le h_g \le 1$ and $\sum_{g=1}^{n} h_g = 1$. Then, the value of the TT-SvNNWG operator is obtained by the following equation:

$$TT-SvNNWG(Ns_1, Ns_2, ..., Ns_n) = \prod_{g=1}^{n}\left(\tan(Ns_g)\right)^{h_g}$$
$$= \left(\prod_{g=1}^{n}\left(\tan(\tfrac{\pi}{4}Nt_g)\right)^{h_g}, 1-\prod_{g=1}^{n}\left(\tan(\tfrac{\pi}{4}(1-Nu_g))\right)^{h_g}, 1-\prod_{g=1}^{n}\left(\tan(\tfrac{\pi}{4}(1-Nf_g))\right)^{h_g}\right). \quad (10)$$

In view of the similar proof process of Theorem 1, we can easily verify Theorem 3, which is omitted.

Example 2. Set three SvNNs as Ns_1 = <0.8, 0.2, 0.1>, Ns_2 = <0.7, 0.2, 0.2>, and Ns_3 = <0.9, 0.1, 0.1> with the weight vector ***H*** = (0.35, 0.25, 0.4). Using Eq. (10), we give the following calculational process:

$$TT-SvNNWG(Ns_1, Ns_2, Ns_3) = \prod_{g=1}^{3}\left(\tan(Ns_g)\right)^{h_g}$$

$$= \left\langle \begin{array}{l} \left(\tan\left(\frac{\pi}{4}\times 0.8\right)\right)^{0.35}\times\left(\tan\left(\frac{\pi}{4}\times 0.7\right)\right)^{0.25}\times\left(\tan\left(\frac{\pi}{4}\times 0.9\right)\right)^{0.4}, \\ 1-\left(\tan\left(\frac{\pi}{4}(1-0.2)\right)\right)^{0.35}\times\left(\tan\left(\frac{\pi}{4}(1-0.2)\right)\right)^{0.25}\times\left(\tan\left(\frac{\pi}{4}(1-0.1)\right)\right)^{0.4}, \\ 1-\left(\tan\left(\frac{\pi}{4}(1-0.1)\right)\right)^{0.35}\times\left(\tan\left(\frac{\pi}{4}(1-0.2)\right)\right)^{0.25}\times\left(\tan\left(\frac{\pi}{4}(1-0.1)\right)\right)^{0.4} \end{array} \right\rangle$$

$$=< 0.7428, 0.2249, 0.1798 > .$$

Theorem 4. The proposed TT-SvNNWG operator contains some properties based on the tangent trigonometric function as follows:

(i) Idempotency: If Ns_g = $<Nt_g, Nu_g, Nf_g>$ = $<Nt, Nu, Nf>$ = Ns (g = 1, 2, …, n), there is $TT-SvNNWG(Ns_1, Ns_2, ..., Ns_n) = \tan(Ns)$.

(ii) Boundedness: Set $Ns^- = \left\langle \min_g(Nt_g), \max_g(Nu_g), \max_g(Nf_g)\right\rangle$ and $Ns^+ = \left\langle \max_g(Nt_g), \min_g(Nu_g), \min_g(Nf_g)\right\rangle$ as the minimum SvNN and the maximum SvNN, respectively. Then, there is $\tan(Ns^-) \le TT-SvNNWG(Ns_1, Ns_2, ..., Ns_n) \le \tan(Ns^+)$.

(iii) Monotonicity: Set $Ns_g = \left\langle Nt_g, Nu_g, Nf_g\right\rangle$ and $Ns^* = \left\langle Nt_g^*, Nu_g^*, Nf_g^*\right\rangle$ (g = 1, 2, …, n) as two groups of SvNNs. Then $TT-SvNNWG(Ns_1, Ns_2, \cdots, Ns_n) \le TT-SvNNWG(Ns_1^*, Ns_2^*, \cdots, Ns_n^*)$ exists when $Ns_g \le Ns_g^*$.

Obviously, the proof process of Theorem 4 is similar to that of Theorem 2, which is omitted.

5. MADM Technique

This section establishes a MADM technique using the proposed TT-SvNNWA and TT-SvNNWG operators in the SvNS setting.

MADM problems usually contain a set of p alternatives $K = \{K_1, K_2, …, K_p\}$ and a set of n attributes $L = \{L_1, L_2, …, L_n\}$ and then indicate decision matrix $M = (Ns_{kg})_{p\times n}$, where Ns_{kg} (k = 1, 2, …, p; g = 1, 2, …, n) are SvNNs corresponding to satisfactory assessments of an alternative K_k over attributes L_g given by decision makers. The weight vector of the attributes is presented by $\boldsymbol{H} = (h_1, h_2, …, h_n)$ subject to $0 \le h_g \le 1$ and $\sum_{g=1}^{n} h_g = 1$. Regarding MADM problems with SvNN information, the MADM algorithm is composed of the following steps.

Step 1: In view of the satisfactory levels of each teacher with respect to the teaching quality indicators, the experts give the decision matrix of SvNNs $M = (Ns_{kg})_{p\times n}$.

Step 2: The aggregated values Ns_k for K_k (k = 1, 2, …, p) are calculated by the following TT-SvNNWA or TT-SvNNWG operator:

$$Ns_k = TT-SvNNWA(Ns_{k1}, Ns_{k2}, ..., Ns_{kn}) = \sum_{g=1}^{n} h_g \tan(Ns_{kg})$$
$$= \left(1-\prod_{g=1}^{n}\left(1-\tan\left(\frac{\pi}{4}Nt_{kg}\right)\right)^{h_g}, \prod_{g=1}^{n}\left(1-\tan\left(\frac{\pi}{4}(1-Nu_{kg})\right)\right)^{h_g}, \prod_{g=1}^{n}\left(1-\tan\left(\frac{\pi}{4}(1-Nf_{kg})\right)\right)^{h_g} \right). \quad (11)$$

$$Ns_k = TT - SvNNWG(Ns_{k1}, Ns_{k2}, ..., Ns_{kn}) = \prod_{g=1}^{n} \left(\tan(Ns_{kg}) \right)^{h_g}$$

or

$$= \left(\prod_{g=1}^{n} \left(\tan(\tfrac{\pi}{4} Nt_{kg}) \right)^{h_g}, 1 - \prod_{g=1}^{n} \left(\tan(\tfrac{\pi}{4}(1 - Nu_{kg})) \right)^{h_g}, 1 - \prod_{g=1}^{n} \left(\tan(\tfrac{\pi}{4}(1 - Nf_{kg})) \right)^{h_g} \right). \quad (12)$$

Step 3: The score values of $F(Ns_k)$ (the accuracy values of $G(Ns_k)$) are calculated by Eq. (3) (Eq. (4)).

Step 4: Alternatives are sorted in descending order in terms of the score values (accuracy values) and the best one and the worst one are determined.

Step 5: End.

6. Actual Example about the TQA of Teachers

6.1. TQA Example of Teachers

In current Chinese higher education, the teaching quality of teachers is becoming more and more important in the training and competition of modern talents. In this case, the assessment mechanism of teaching quality in colleges and universities reveals its importance and necessary in the teaching process. Since the TQA problem of teachers contain many evaluation indicators/attributes, liking teaching level and skill, teaching means and methods, teaching attitude, etc. Therefore, TQA is a MADM issue, where evaluation data of the indicators/attributes contain incomplete, uncertain and inconsistent information in the evaluation process. This section applies the established MADM technique to an actual example about the TQA of teachers to show the efficiency and suitability of the established MADM technique in the environment of SvNSs.

Shaoxing University in China needs to establish the TQA system of teachers as an effective teaching management strategy. To evaluate the teaching quality of teachers, the teaching management department preliminarily chooses five teachers as the evaluated objects, which are denoted as a set of the five alternatives $K = \{K_1, K_2, K_3, K_4, K_5\}$. In the TQA process, they must evaluate the satisfactory levels of each teacher with respect to the indicators/attributes of teaching quality, including the teaching level and skill (L_1), the teaching means and method (L_2), the teaching attitude (L_3), the teaching innovation (L_4), and the satisfaction of students (L_5), which are denoted as a set of the attributes $L = \{L_1, L_2, L_3, L_4, L_5\}$. The weight vector of the five attributes is presented by $\boldsymbol{H}$ = (0.23, 0.2, 0.2, 0.17, 0.2).

In this MADM problem with SvNNs, the established MADM technique is applied to this actual example. Then, the decision steps are given below.

Step 1: In view of the satisfactory levels of each teacher with respect of the teaching quality indicators, the experts give the following decision matrix of SvNNs:

$$M = \begin{bmatrix} <0.6,0.2,0.3> & <0.7,0.3,0.2> & <0.7,0.3,0.3> & <0.8,0.1,0.2> & <0.6,0.1,0.4> \\ <0.7,0.1,0.2> & <0.6,0.1,0.1> & <0.9,0.4,0.3> & <0.9,0.2,0.2> & <0.8,0.2,0.4> \\ <0.8,0.1,0.2> & <0.9,0.3,0.3> & <0.6,0.1,0.3> & <0.8,0.4,0.3> & <0.8,0.3,0.3> \\ <0.9,0.1,0.2> & <0.7,0.4,0.2> & <0.9,0.4,0.5> & <0.7,0.1,0.5> & <0.6,0.4,0.4> \\ <0.7,0.4,0.5> & <0.8,0.3,0.3> & <0.9,0.3,0.3> & <0.6,0.3,0.1> & <0.9,0.1,0.2> \end{bmatrix}.$$

Step 2: Using Eq. (11) or Eq. (12), the aggregated values of Ns_k for K_k (k = 1, 2, …, p) are given below:

Ns_1 = <0.5960, 0.2491, 0.3569>, Ns_2 = <0.7361, 0.2346, 0.2906>, Ns_3 = <0.7289, 0.2649, 0.3574>, Ns_4 = <0.7332, 0.3020, 0.4074>, and Ns_5 = <0.7455, 0.3363, 0.3365>.

Or N_{S1} = <0.5827, 0.2794, 0.3710>, N_{S2} = <0.6909, 0.2745, 0.3244>, N_{S3} = <0.6990, 0.3150, 0.3627>, N_{S4} = <0.6812, 0.3735, 0.4503>, and N_{S5} = <0.7017, 0.3723, 0.3869>.

Step 3: Applying Eq. (3), the score values of $F(N_{Sk})$ are obtained as follows:

$F(N_{S1})$ = 0.6633, $F(N_{S2})$ = 0.7370, $F(N_{S3})$ = 0.7022, $F(N_{S4})$ = 0.6746, and $F(N_{S5})$ = 0.6909.

Or $F(N_{S1})$ = 0.6441, $F(N_{S2})$ = 0.6973, $F(N_{S3})$ = 0.6738 $F(N_{S4})$ = 0.6191, and $F(N_{S5})$ = 0.6475.

Step 4: The sorting order of the five teachers is $K_2 > K_3 > K_5 > K_4 > K_1$ or $K_2 > K_3 > K_5 > K_1 > K_4$, and the best one is K_2 and the worst one is K_1 or K_4 in the TQA process of the teachers.

It is obvious that the sorting orders of the five teachers obtained based on the proposed TT-SvNNWA and TT-SvNNWG operators in the SvNS setting reveal some differences, which show that different aggregation algorithms may affect the sorting order.

6.2. Related Comparison

To reveal the efficiency and suitability of the established MADM technique for the TQA problem of teachers, this part compares the established MADM technique with the related techniques in the environment of SvNSs.

Using Eqs. (1)–(6), the evaluation results of the five teachers are given by existing MADM techniques [7, 14]. Then, all the decision results based on the established MADM technique and the existing MADM techniques [7, 14] are shown in Table 1.

Table 1. Decision results of various MADM techniques

MADM technique	Score value	Sorting order	The best one	The worst one
Existing MADM technique using Eq. (1) [7]	0.7425, 0.8058, 0.7768, 0.7499, 0.7658	$K_2 > K_3 > K_5 > K_4 > K_1$	K_2	K_1
Existing MADM technique using Eq. (2) [7]	0.7251, 0.7708, 0.7516, 0.6989, 0.7264	$K_2 > K_3 > K_5 > K_1 > K_4$	K_2	K_4
Existing MADM technique using Eq. (5) [14]	0.9418, 0.9719, 0.9650, 0.9582, 0.9638	$K_2 > K_3 > K_5 > K_4 > K_1$	K_2	K_1
Existing MADM technique using Eq. (6) [14]	0.9324, 0.9534, 0.9516, 0.9309, 0.9429	$K_2 > K_3 > K_5 > K_1 > K_4$	K_2	K_4
Established MADM technique using Eq. (11)	0.6633, 0.7370, 0.7022, 0.6746, 0.6909	$K_2 > K_3 > K_5 > K_4 > K_1$	K_2	K_1
Established MADM technique using Eq. (12)	0.6441, 0.6973, 0.6738, 0.6191, 0.6475	$K_2 > K_3 > K_5 > K_1 > K_4$	K_2	K_4

In Table 1, the sorting orders given by the established MADM technique using Eqs. (11) and (12) and the existing MADM techniques using Eqs. (1) and (2) and Eqs. (5) and (6) are the same. In the meantime, the best one is K_2 and the worst one is K_1 or K_4 in the MADM problem. In the existing MADM techniques [7, 14] and the established MADM technique, different aggregation operators will affect the sorting order of the five teachers. In general, the weighted average aggregation operators mainly tend to group opinions, while the weighted geometric aggregation operators mainly tend to individual opinions. However, one of the aggregation operators is selected depending on decision makers' preference and some actual requirements in the TQA process.

7. Conclusions

Since the TQA of teachers shows its importance and necessity to improve the teaching quality in colleges/universities, it is critical to establish a suitable assessment mechanism in the teaching process. Then, the evaluation information for the teaching quality of teachers implies incompleteness, indeterminacy, and inconsistency due to the indeterminacy and inconsistency of human cognition and judgements to the evaluated objects. In this case, this research proposed an MADM technique for the TQA of teachers in the SvNN situation. To perform this task, we proposed the operational laws of TT-SvNNs and the TT-SvNNWA and TT-SvNNWG operators, and then established an MADM technique using the TT-SvNNWA and TT-SvNNWG operators in the SvNS setting. Consequently, the established MADM technique was applied in an actual MADM example about the TQA problem of teachers and compared with existing related MADM techniques. The comparative results revealed the efficiency and suitability of the established MADM technique in the SvNS setting.

However, the established MADM technique is another complement to existing MADM techniques. Since the tangent trigonometric function shows the main superiority of its periodicity and symmetry about the origin, fitting the preference of decision makers for multiple time phase parameters, this new technique will also be extended to new aggregation operations and applied to the areas of slope stability/risk assessment and medical diagnosis in the environment of simplified NSs (SvNSs and INSs).

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1 Smarandache, F. *Neutrosophy: neutrosophic probability, set, and logic*. American Research Press, Rehoboth, USA, **1998**.

2 Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. *Interval neutrosophic sets and logic: Theory and applications in computing*. Hexis, Phoenix, AZ, **2005**.

3 Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single valued neutrosophic sets. *Multispace Multistructure* **2010**, 4, 410–413.

4 Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent & Fuzzy Systems* **2014**, 26, 2459-2466.

5 Ye, J. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *International Journal of General Systems* **2013**, 42, 386–394.

6 Liu, P.D.; Wang, Y.M. Multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean. *Neural Computing and Applications* **2014**, 25(7-8), 2001–2010.

7 Peng, J.J.; Wang, J.Q.; Wang, J.; Zhang, H.Y.; Chen, X.H. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *International Journal of Systems Science* **2016**, 47(10), 2342–2358.

8 Zhou, L.P.; Dong, J.Y.; Wan, S.P. Two new approaches for multi-attribute group decision-making with interval-valued neutrosophic Frank aggregation operators and incomplete weights. *IEEE Access* **2019**, 7, 102727–102750.

9 Tang, N.; Li, B.; Elhoseny, M. Assessment of English teaching systems using a single-valued neutrosophic MACROS method. *Neutrosophic Sets and Systems* **2021**, 46, 87–110.

10 Zhao, M.; Ye, J. MCGDM approach using the weighted hyperbolic sine similarity measure of neutrosophic (indeterminate fuzzy) multivalued sets for the teaching quality assessment of teachers.

Neutrosophic Sets and Systems **2022**, 48, 1-8.

11 Do, D.A.; Pham, T.M.; Hang, D.T. et al. Evaluation of lecturers' performance using a novel hierarchical multi-criteria model based on an interval complex neutrosophic set. *Decision Science Letters* **2020**, 9(2), 119–144.

12 Garg, H. New logarithmic operational laws and their applications to multiattribute decision-making for single-valued neutrosophic numbers. *Cognitive Systems Research* **2018**, 52, 931–946.

13 Ashraf, S.; Abdullah, S.; Smarandache, F. Logarithmic hybrid aggregation operators based on single valued neutrosophic sets and their applications in decision support systems. *Symmetry* **2019**, 11, 364.

14 Ashraf, S.; Abdullah, S.; Zeng, S. et al. Fuzzy decision support modeling for hydrogen power plant selection based on single valued neutrosophic sine trigonometric aggregation operators. *Symmetry* **2020**, 12, 298.

Received: Feb 8, 2022. Accepted: Jun 4, 2022