



THE NEUTROSOPHIC STATISTICAL DISTRIBUTION, MORE PROBLEMS, MORE SOLUTIONS

S. K. Patro¹, F. Smarandache²

1. Khallikote University, Berhampur university, Ram Nagar 5th lane, Berhampur, Pin-760008, India, Email-ksantanupatro@gmail.com

2. University of New Mexico, Mathematics and Science Department, 705 Gurley Ave., Gallup, NM 87301, USA, Email-fs.gallup.unm.edu

Abstract: In this paper, the authors explore neutrosophic statistics, that was initiated by Florentin Smarandache in 1998 and developed in 2014, by presenting various examples of several statistical

distributions, from the work [1]. The paper is presented with more case studies, by means of which this neutrosophic version of statistical distribution becomes more pronounced.

Key words: Neutrosophy, Binomial & Normal distributions, Neutrosophic logic etc.

I. Introduction: Neutrosophy was first proposed by Prof. Florentin Smarandache in 1995. It is a new branch of philosophy, where one can study origin, nature and scope of neutralities. According to Prof. Dr. Huang, this gives advantages to break the mechanical understanding of human culture. For example, according to mechanical theory, existence and non-existence couldn't be simultaneously, due to some indeterminacy [2].

This theory considers every notion or idea $\langle A \rangle$ together with its opposite or negation $\langle \text{Anti-}A \rangle$. The $\langle \text{neut-}A \rangle$ and $\langle \text{Anti-}A \rangle$ ideas together called as a $\langle \text{non-}A \rangle$. Neutrosophic logic is a general framework for unification of many existing logics, intuitionistic logic, paraconsistent logic etc. The focal objective of neutrosophic logic is to characterize each logical statements in a 3D-neutrosophic space, where each dimension of space represents respectively the truth (T), falsehood (F) and indeterminacies of the statements under consideration. Where T, I, F are standard or non-standard real subset of $(-0, 1+)$ without necessary connection between them. [3]

The classical distribution is extended neutrosophically. That means that there is some indeterminacy related to the probabilistic experiment. Each experimental observation of each trial can result in an outcome of each trial can result in an outcome labelled failure (F) or some indeterminacy (I). Neutrosophic statistics is an extended form of classical statistics, dealing with crisp values. In this paper, we will discuss about one discrete random distribution such as Binomial distribution and a continuous one by approaching neutrosophically. Before focusing the light on this context, we should familiar with the following notions.

Neutrosophic statistical number 'N' has the form

$$N = d + I;$$

Where, d: Determinate part

I: Indeterminate part of N.

For example, $a = 5 + I$; where $I \in [0 , 0.4]$ is equivalent to $a \in [5 , 5.4]$. So for sure $a \geq 5$, where $I \in [0 , 0.4]$.

I.A. Preliminaries: In this context, we are going to discuss about the classical distributions[4] .

A). Binomial distribution,

B). Normal distribution.

I. A. a). Binomial distribution:

I. A.a.i. Definition: A random variable X is said to follow Binomial distribution, if it assumes only non-negative values and its probability mass function is given by,

$$p(X = x) = p(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

otherwise equal to zero .

I.A.a.ii. Physical conditions: We get Binomial distribution under the following conditions–

1. Each trials results in two exhaustive and mutually disjoint outcomes termed as success and failure.
2. The number of trials ‘n’ is finite.
3. The trials are independent on each other.
4. The probability of success ‘p’ is constant for each trial.

I.A.b. Normal Distribution:

I.A.b.i. Definition: A random variable is said to have a normal distribution with parameters μ and σ^2 , if its p.d.f is given by the probability law ,

$$f(x ; \mu ; \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$-\infty < x < \infty$ and $-\infty < \mu < \infty, \sigma_x > 0$.

A.I.b.ii. Chief characteristics of Normal Distribution and normal probability curve:

The normal probability curve is given by the equation

$$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}} ; -\infty < x < \infty$$

I.A.b.iii.Properties:

1.The point of inflexion of the curve are:

$$x = \mu_x \pm \sigma_x , f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-1/2}$$

2. The curve is symmetrical and bell shaped about the line $x = \mu$.

3. Mean, Median, Mode of distribution coincide.

4. X-axis is an asymptote to the curve.

5. Quartiles, $Q_1 = \mu - 0.6745\sigma$

$$Q_3 = \mu + 0.6745\sigma.$$

6. Q.D: M.D: S.D :: 10 : 12 : 15 implies Q.D : M.D: S.D :: 10 :12 : 15

II. Neutrosophic Statistical Distribution:

II.i. Neutrosophic Binomial Distribution: The neutrosophic binomial random variable ‘x’ is then defined as the number of success when we perform the experiment $n \geq 1$ times. The neutrosophic probability distribution of ‘x’ is also called neutrosophic binomial probability distribution.

II.i.a.Definitions:

1. **Neutrosophic Binomial Random Variable:** It is defined as the number of success when we perform the experiment $n \geq 1$ times, and is denoted as ‘x’.
2. **Neutrosophic Binomial Probability Distribution:** The neutrosophic probability distribution of ‘x’ is called n.p.d.
3. **Indeterminacy:** It is not confined to experimental results (either success or failures).
4. **Indeterminacy Threshold:** It is the number of trials whose outcome is indeterminate. Where

$$th \in \{0,1,2...n\}$$

Let P(S) = The chance of a particular trial results in a success.

P(F) = The chance of a particular trial results in a failure , for both S and f different from indeterminacy .

$P(I)$ = The chance of a particular trial results in an indeterminacy .

For example: for $x \in \{0,1,2,\dots,n\}$, $NP=(TX,IX,FX)$ with

TX : Chances of ‘x’ success and (n-x) failures and indeterminacy but such that the no. of indeterminacy is less than or equal to indeterminacy threshold.

FX : Chances of ‘y’ success , with $y \neq x$ and (n-y) failures and indeterminacy is less than the indeterminacy threshold.

IX : Chances of ‘z’ indeterminacy , where ‘z’ is strictly greater than the indeterminacy threshold.

$$TX + FX + IX = (P(S) + P(I) + P(F))^n$$

For complete probability, $P(S) + P(I) + P(F) = 1$;

For incomplete probability ,

$$0 \leq P(S) + P(I) + P(F) < 1 ;$$

For paraconsistent probability ,

$$1 < P(S) + P(I) + P(F) \leq 3 .$$

Now ,

$$\begin{aligned} Tx &= \frac{n!}{x!(n-x)!} [P(S)^x \sum_{k=0}^n \frac{k!}{(n-x)!(k-n+x)!} P(I)^k P(F)^{n-x-k}] \\ &= \frac{n!}{x!(n-x)!} P(S)^x \sum_{k=0}^n \frac{(n-x)!}{(n-x-k)!} P(I)^k P(F)^{n-x-k} \\ &= \frac{n!}{x!} P(S)^x \sum_{k=0}^n \frac{P(I)^k \cdot P(F)^{n-x-k}}{k!(n-x-k)!} \end{aligned}$$

$$\begin{aligned} Fx &= \sum_{y=0}^n T_y = \sum_{y=0, y \neq x}^n \frac{n!}{y!} P(S)^y \sum_{k=0}^n \frac{P(S)^k \cdot P(F)^{n-y-k}}{k!(n-y-k)!} \\ Ix &= \sum_{z=th+1}^n \frac{n!}{z!(n-z)!} P(I)^z \sum_{k=0}^{n-z} \frac{(n-z)!}{(n-z)!(n-z-k)!} P(S)^k \cdot P(F)^{n-z-k} \\ &= \sum_{z=th+1}^n \frac{n!}{z!} P(I)^z \sum_{k=0}^{n-z} \frac{P(S)^k \cdot P(F)^{n-z-k}}{k!(n-z-k)!} \end{aligned}$$

Where ,

$T_x, I_x, F_x, P(S), P(I), P(F)$ have their usual meaning. Now we are going to discuss several cases.

II.i.b.1. Case studies :

- Two friends Asish and Rajesh are going to throw 5 coins simultaneously. There are 60% of chance to get head and 30% of chance to get tail. Independent on the view of Asish ,Rajesh said that the probability of the result that are neither Head nor Tail is 20% . Then find the probability of getting 3 Heads when indeterminacy threshold is 2.

Solution:

$$\begin{aligned} Tx &= \frac{5!}{3!(5-3)!} [(0.6)^3 \sum_{k=0}^2 \frac{k!}{2!(k-2)!} (0.2)^k (0.3)^{2-k}] \\ &= \frac{5!}{3!(5-3)!} [(0.6)^3 \{ \frac{2!}{2!} (0.2)^2 \}] \\ &= 10[(0.6)^3 \{ (0.2)^2 \}] = 0.0864 \\ I_x &= \sum_{z=th+1}^n \frac{n!}{z!} P(I)^z \sum_{k=0}^{n-z} \frac{P(S)^k \cdot P(F)^{n-z-k}}{k!(n-z-k)!} \\ \therefore I_3 &= \sum_{z=3}^5 \frac{5!}{z!} (0.2)^z \sum_{k=0}^2 \frac{(0.6)^k (0.3)^{2-k}}{k!(2-k)!} \\ &= \sum_{z=3}^5 \frac{5!}{z!} (0.2)^z \{ \frac{(0.3)^2}{2!} + (0.6)(0.3) + \frac{(0.6)^2}{2!} \} \\ &= \sum_{z=3}^5 \frac{5!}{z!} (0.2)^z \{ \frac{(0.3)^2}{2!} + (0.6)(0.3) + \frac{(0.6)^2}{2!} \} \\ &= \{ 0.324 + 0.072 + 0.1008 \} = 0.496 \end{aligned}$$

$$\begin{aligned} F_x &= (P(S)+P(I)+P(F))^n - Tx - Fx \\ \therefore F_3 &= (0.6 + 0.3 + 0.2)^5 - 0.0864 - 0.496 \\ &= 1.02811 \end{aligned}$$

- Five coins are thrown simultaneously , the probability of success is 1/3 and the indeterminacy (the surface is very rough , so the coins may stand up) is 1/3 . Then find the probability of getting 3 Heads when the indeterminacy threshold is 2.

Solution:

Let x be no. of chances of getting heads in 5 trials .

$$T_x = \frac{n!}{x!} P(S)^x \sum_{k=0}^{th} \frac{P(I)^k P(F)^{n-x-k}}{k!(n-x-k)!}$$

$$\therefore T_3 = \frac{5!}{3!} (0.33)^3 \sum_{k=0}^2 \frac{(0.33)^k (0.33)^{2-k}}{k!(2-k)!}$$

$$= \frac{5!}{3!} (0.33)^3 \left\{ \frac{(0.33)^2}{2!} + (0.33)^2 + \frac{(0.33)^2}{2!} \right\}$$

$$= 40 \cdot (0.33)^5 = 0.15654$$

$$I_x = \sum_{z=th+1}^n \frac{n!}{z!} P(I)^z \sum_{k=0}^{n-z} \frac{P(S)^k P(F)^{n-z-k}}{k!(2-k)!}$$

$$\therefore I_3 = \sum_{z=3}^5 \frac{5!}{z!} (0.33)^z \sum_{k=0}^2 \frac{(0.33)^k (0.33)^{2-k}}{k!(2-k)!}$$

$$= (0.33)^5 [40 + (7.5)(0.33) + 1] = 0.17014$$

$$F_x = (P(S) + P(I) + P(F))^n - T_x - I_x$$

$$\therefore F_3 = (0.33 + 0.33 + 0.33)^5 - T_3 - I_3$$

$$so, (T_x, I_x, F_x) = (0.15654, 0.17014, 0.67332)$$

- Two friends Liza and Laxmi play a game in which their chance of winning is 2:3 . The chances of dismissing game is 30% . Then find the probability of Liza's chances of winning at least 3 games out of 5 games played when the indeterminacy threshold is 2.

solution:

x is the no. of chances of winning the game
Let $th = 2$

$$T_x = \frac{n!}{x!} P(S)^x \sum_{k=0}^{th} \frac{P(I)^k P(F)^{n-x-k}}{k!(n-x-k)!}$$

$$\therefore T_3 = \frac{5!}{3!} (0.4)^3 \sum_{k=0}^2 \frac{(0.3)^k (0.6)^{2-k}}{k!(2-k)!}$$

$$= 20(0.405)^4 = 0.53808$$

$$I_x = \sum_{z=th+1}^n \frac{n!}{z!} P(I)^z \sum_{k=0}^{n-z} \frac{P(S)^k P(F)^{n-z-k}}{k!(n-z-k)!}$$

$$I_3 = \sum_{z=3}^5 \frac{5!}{z!} (0.3)^z \sum_{k=0}^2 \frac{(0.4)^k (0.6)^{5-z-k}}{k!(5-z-k)!}$$

$$= 20(0.3)^3 (0.5) + 5(0.3)^4 (0.42)$$

$$= 0.28701$$

and $F_3 = 2.88785$, this is a paraconsistent probability which is ≤ 3 .

- In a precision bombing attack there is a 50% chance that any one bomb will strike the target . Two direct hits are required to destroy the target . If the chance of failure of mission is 30% , then find how many bombs

are required to give a 99% chance with $th=2$

Solution:

Let x is the no. of chances of hitting bomb

$$T_x = \frac{n!}{x!} P(S)^x \sum_{k=0}^{th} \frac{P(I)^k P(F)^{n-x-k}}{k!(n-x-k)!}$$

$$\therefore T_3 = \frac{5!}{3!} (0.5)^3 \sum_{k=0}^2 \frac{(0.3)^k (0.3)^{2-k}}{k!(2-k)!}$$

$$= 40(0.5)^5 = 0.0972$$

$$I_x = \sum_{z=th+1}^n \frac{n!}{z!} P(I)^z \sum_{k=0}^{n-z} \frac{P(S)^k P(F)^{n-z-k}}{k!(n-z-k)!}$$

$$\therefore I_3 = \sum_{z=3}^5 \frac{5!}{z!} (0.3)^z \sum_{k=0}^2 \frac{(0.5)^k (0.3)^{5-z-k}}{k!(5-z-k)!}$$

$$= 0.1728 + 0.0078975 = 0.18069$$

$$\text{therefore } F_3 = (P(S) + P(F) + P(I))^5 - T_3 - I_3$$

$$= (1.1)^5 - 0.27789 = 1.33262$$

so $(T_{x=3}, F_{x=3}, I_{x=3}) = (0.0972, 1.3326, 0.1806)$.

It is an example of paraconsistent probability.

- It is decided that a cricket player , Jagadiswar has to appear 4 times for a physical test . If the possibility of passing the test is 2/3 ; and one referee guess that the chance of dismiss of game is 30% , then what is the probability of that the player passes the test at least 3 times, provided $th=2$?

Solution:

Let x is no. of chances that the player passes the test

$$T_x = \frac{n!}{x!} P(S)^x \sum_{k=0}^{th} \frac{P(I)^k P(F)^{n-x-k}}{k!(n-x-k)!}$$

$$\therefore T_3 = \frac{4!}{3!} (0.66)^3 \sum_{k=0}^2 \frac{(0.3)^k (0.33)^{1-k}}{k!(1-k)!}$$

$$= 8(0.66)^3 (0.33) = 0.7589$$

$$I_x = \sum_{z=th+1}^n \frac{n!}{z!} P(I)^z \sum_{k=0}^{n-z} \frac{P(S)^k P(F)^{n-z-k}}{k!(n-z-k)!}$$

$$\therefore I_3 = \sum_{z=3}^4 4(0.33)^z \sum_{k=0}^1 \frac{\{(0.66)^k (0.33)^{1-k}\}}{k!(1-k)!}$$

$$= (0.33)^3 [3.96 + 0.33] = 0.15416$$

$$\text{therefore } F_3 = (0.66 + 0.33 + 0.3)^4 - T_3 - I_3$$

$$= 2.76922 - 0.91306 = 1.85616$$

$$SO (T_3, I_3, F_3) = (0.7589, 0.1541, 1.8561).$$

II.i.b.2.Exercises:

- In a B.Sc course , suppose that a student has to pass a minimum of 4 tests out of 8 conducted tests during the year to get promoted to next academic year . One student, Sarmistha says that his chance of winning is 80% , another student, Baisakhi

says that his chance of winning is 0.3 . Then find the probability of the promotion of Sarmistha , when the indeterminacy (either illegal paper correction or system error) is 20% , provided $th=2$.

2. If a car agency sells 40% of its inventory of a certain foreign cars equipped with air bags , the asst. manager says that the cars which are neither equipped with air bags nor a general one is 30% , then find the probability distribution of the 2 cars with airbags among the next 4 cars sold with $th=2$?
3. A question paper contain 5 questions and a candidate will be declared to have passed the exam. If he/she answered at least one question correctly, considering the uncertainty as 33% (may be improper paper correction or system error etc.). What is the probability that the candidate passes the examination?

II.ii.Neutrosophic Normal Distribution:

Neutrosophic normal distribution of a continuous variable X is a classical normal distribution of X , but such that its mean μ or its standard deviation σ or variance σ^2 or both are imprecise. For example , μ or σ or both can be set with two or more elements . The most common such distribution are when μ , σ or both are intervals .

$$X_N \sim N_N(\mu_N, \sigma_N) = \frac{1}{\sigma_N \sqrt{2\pi}} e^{-\frac{(X-\mu_N)^2}{2\sigma_N^2}}$$

N_N : Normal distribution may be neutrosophic

X_N : X may be neutrosophic

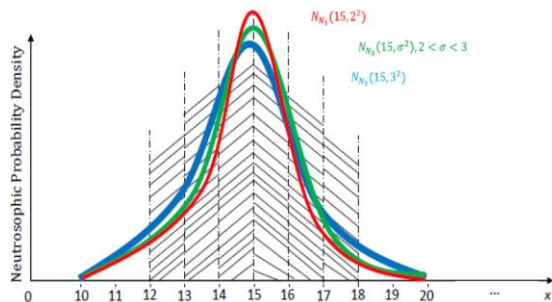


FIG-1: CREDIT TO FLORENTIN SMARANDACHE IN NEUTROSOPHIC STATISTICS

II.ii.a. Case studies:

1. In a college examination of a particular year, 60% of the Student failed when the mean of marks was 50% and the standard deviation is 5% with uncertainty $I \in [0,0.4]$.The college decided to relax the condition of passing by lowering the passing marks to show its result as 80% passed , find the minimum marks to be kept for passing when marks are distributed normally .

Solution : Let $\mu = 50$, $\sigma = 5$ with indeterminacy $I \in [0,0.4]$,so $\sigma = 5 + [0,0.4] = [5,5.4]$. therefore, $\mu \pm \sigma = 50 \pm [5,5.4] = [50-5.4, 50+5] = [44.6 ,55]$. Thus, 66.04 % of values lies in $[44.6,55]$.

$$\begin{aligned} 0.8 &= P(X_N \geq a_N) \\ &= 1 - P(X_N \leq a_N) \\ &= 1 - P\left(\frac{X_N - \mu_N}{\sigma_N} \leq \frac{a_N - \mu_N}{\sigma_N}\right) = 1 - P(Z_N \leq \frac{a_N - 50}{[5,5.4]}) \\ \therefore P(Z_N \leq \frac{a_N - 50}{[5,5.4]}) &= 0.2, \text{ clearly } \frac{a_N - 50}{[5,5.4]} < 0, \text{ Let so, } P(Z_N \leq Z_{0.2}) = 0.8 \\ Z_{0.2} &= -\left(\frac{a_N - 50}{[5,5.4]}\right) = 48.45\% \text{ approx.} \end{aligned}$$

2. If the monthly machine repair and maintenance cost X in a certain factory is known to be neutrosophically normal with mean 1000 and standard deviation 10000 , find the followings-

$$\mu \pm \sigma, \mu \pm 2\sigma, \text{ when } I \in [0,0.3].$$

Solution: Let $\mu=10000$, $\sigma=1000+[0,0.3]$, then $\mu \pm \sigma = 10000 \pm [1000, 1000.03]$. Thus 66.06% of values lies in $[9000.03, 11000]$. And $\mu \pm 2\sigma = 10000 \pm 2[1000, 1000.03] = [7999.97, 12000]$. Thus 75.04% of values lies in $[7999.97, 12000]$.

II.ii.b. Exercises:

1. A machine fills boxes weighting B kg with A kg of salt , where A and B are neutrosophically normal with mean 200kg and 10kg respectively and standard deviation of 2kg and 1kg respectively , what percentage of filled boxes weighting between 110kg an 120kg are to be expected when $I \in [0,0.5]$.
2. The average life of a bulb is 2000 hours and the standard deviation is 400 hours .If X_N is the life period of a bulb which is distributed

normally in a neutrosophic plane. Find the probability that a randomly picked bulb will lasts ≤ 600 hrs. , considering the distribution is neutrosophically normal with indeterminacy $I \in [0, 0.2]$.

Till now, we have discussed various types of practical cases in statistical approach. Now we review the general formula for fusioning classical subjective probability provided by 2 sources.

The principle of redistributing the conflicting chances for ex. t and I are same as in PCR5 rule for the DSMT used in information fusion if 2 sources of information S_1 and S_2 give the subjective probability P_1 and P_2 about 't' to combining by PCR5 rule ,[5]

$$(P_1 \wedge P_2)E = P_1(E)P_2(E) + \sum_{x \in E \wedge I \cap E = \phi} \left[\frac{P_1(E)^2 P_2(X)}{P_1(E) + P_2(X)} + \frac{P_2(E)^2 P_1(X)}{P_2(E) + P_1(X)} \right]$$

It helps to the generalization of classical probability theory, fuzzy set, fuzzy logic to their respective domains. They are useful in artificial intelligence, neutrosophic dynamic system, quantum mechanics [6].

This theory can be used for topical communication study [7]. It may also be applied to neutrosophic cognitive map study [8].

Thus we have presented our discussion with certain essential area of neutrosophy in a synchronized manner. Now we are going to explore some open challenges as follows.

Which are designed for inquiring minds.

Open Problems:

1. Can this Neutrosophic Statistics be applied to Industrial Management study?
2. Can we apply it with the study of Digital Signal Processing?
3. Can we merge the Representation theory [9]with Neutrosophy for a new theory ?
4. Is the uncertain theory, K-theory [10]solve the recent intriguing statistical problems by the power of this Neutrosophic logic ?
5. Can we construct a special master-space by the fusion of manifold concepts [11], soft topology [12], Ergodic theory [13],with Neutrosophic distribution ?

6. Is it possible for the construction of Neutrosophic manifold?
7. Is it possible for the construction of neutrosophic algebraic geometry[14] ?

III. Conclusion:

The actual motto of this short paper is to present the theory of Neutrosophic probability distribution in a more lucid and clear-cut way .The author presents various solved and unsolved problems, which are existed in reference to Neutrosophic 3D- space .Various practical situations are described and were tried to solve by Neutrosophic logic. The spectra of this theory may be applied to Quantum physics [15]and M-theory [16]. It may be said that it can also be applied to Human psychology as well as Behavioral study. I hope that the more extended version (with large no. of case studies) with the area of application of this theory will see the light of the day in recent future. Here we limited our discussion of problem analysis to some extent due to limited scope of presentation. And lastly but important that if some unmatched/contradicted idea will occur in this paper, then it is surely unintentional. Finally I hope that the idea on the advanced version of this theory,which is already raised in my brain, will change their abstractskeleton into a paper, in coming future.

Acknowledgement:

The first author owe special debt of gratitude towards the journal Neutrosophic Sets and System (NSS), also to Dr. DillipSenapati, Dr. S. N Kund, Meenati panda for their constant encouragement. And finally I thankful to my whole family for their full-term cooperation.

References:

- [1].Smarandache .F, Neutrosophicalstatistics,Sitech& Education publishing , 2014.
- [2].Smarandache.F, Neutrosophy and its application (collected papers),sitech publisher, 2014.
- [3]. Dr. Huang ,cheng-gui, A note on neutrosophy, amritahcg@263.net
- [4]. Gupta.A.C,Kapoor.V.K , Mathematical statistics , s chandpublisher,india , 2009.
- [5]. Smarandache F, Introduction to Neutrosophic Measure, Integral, Probability, Sitech Education publisher.

- [6]. Smarandache.F , A unifying field of logic:Neutrosophic logic , American Research press, 2000.
- [7]. Vladutesa.S ,Smarandache.F, Topical communication uncertainty , Zip publisher, 2014.
- [8].Khandasami.V.B, Smarandache .F, Fuzzy Cognitive Maps& Neutrosophic Cognitive Maps, Xiquan ,2003
- [9].Kwalski.E, Representation theory , ETH Zurich.
- [10].Max karoubi , Lectures on K-theory,2002.
- [11].Loring W.Tu, An introduction to manifold, Springer publishing, 2010.
- [12].Zorlutuna.I, M.Akdag , et al. Remarks on soft topological space , Annals of FMI, www.afmi.or.kr
- [13].Charles walkden, Ergodic theory,2015.
- [14].Milne.J.S, Algebric geometry,2015 .
- [15].Phillips.A.C, Introduction to quantum mechanics ,wiley publisher.
- [16].Jchwarz.J et al., S and M theory, Cambridge, 2006 .

Received: May 08, 2016. Accepted: July 08, 2016.