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Topological Manifold Space via Neutrosophic Crisp Set Theory

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Abstract. In this paper, we introduce and study a neutrosophic crisp manifold as a new topological structure of manifold via neutrosophic crisp set. Therefore, we study some new topological concepts and some metric distances on a neutrosophic crisp manifold.

Keywords: neutrosophic crisp manifold, neutrosophic crisp coordinate chart, neutrosophic crisp Haussdorff, neutrosophic crisp countable, neutrosophic crisp basis, neutrosophic crisp Homeomorphism, neutrosophic locally compact.

1 Introduction

Neutrosophics found their places into contemporary research; we have introduced the notions of neutrosophic crisp sets, neutrosophic crisp point and neutrosophic topology on crisp sets.

We presented some new topological concepts and properties on neutrosophic crisp topology. A manifold is a topological space that is locally Euclidean and around every point there is a neighborhood that is topologically the same as the open unit in R^n.

The aim of this paper is to build a new manifold topological structure called neutrosophic crisp manifold as a generalization of manifold topological space by neutrosophic crisp point and neutrosophic crisp topology and present some new topological concepts on a neutrosophic crisp manifold space.

Also, we study some metric distances on a neutrosophic crisp manifold.

The paper is structured as follows: in Section 2, we introduce preliminary definitions of the neutrosophic crisp point and neutrosophic crisp topology; in Section 3, some new topological concepts on neutrosophic crisp topology are presented and defined; in Section 4, we propose some topological concepts on neutrosophic crisp manifold space; Section 5 introduces some metric distances on a neutrosophic crisp manifold. Finally, our future work is presented in conclusion.

2 Terminologies [1, 2, 4]

We recollect some relevant basic preliminaries.

Definition 2.1:

Let $A =< A_1, A_2, A_3 >$ be a neutrosophic crisp set on a set X, then

$p =< \{p_1\}, \{p_2\}, \{p_3\} >$, $p_1 \neq p_2 \neq p_3 \in X$ is called a neutrosophic crisp point.

A NCP $p =< \{p_1\}, \{p_2\}, \{p_3\} >$ belongs to a neutrosophic crisp set

$A =< A_1, A_2, A_3 >$ of X denoted by $p \in A$ if it defined by: $\{p_1\} \subseteq A_1, \{p_2\} \subseteq A_2and\{p_3\} \subseteq A_3$.

Definition 2.2:

A neutrosophic crisp topology (NCT) on a non empty set X is a family of Γ of neutrosophic crisp subsets in X satisfying the following axioms:

i. $\phi_N, X_N \in \Gamma$
ii. $A_1 \cap A_2 \in \Gamma$ *for any* $A_1, A_2 \in \Gamma$
iii. $\cup A_j \in \Gamma \, \forall \{A_j\, j \in J\} \subseteq \Gamma$

Then (X, Γ) is called a neutrosophic crisp topological space (NCTS) in X and the elements in Γ are called neutrosophic crisp open sets (NCOSs).

3 Neutrosophic Crisp Topological Manifold Spaces [2, 5, 4, 7]

We present and study the following new topological concepts about the new neutrosophic crisp topological manifold Space.

Definition 3.1:

A neutrosophic crisp topological space (X, Γ) is a neutrosophic crisp Haussdorff (NCH) if for each two neutrosophic crisp points p $=<\{p_1\},\{p_2\},\{p_3\}>$ and q $=<\{q_1\},\{q_2\},\{q_3\}>$ in X such that $p \neq q$ there exist neutrosophic crisp open sets U $=<u_1,u_2,u_3>$ and V $=<v_1,v_2,v_3>$ such that p in U, q in V and $U \cap V = \phi_N$.

Definition 3.2:

β is collection of neutrosophic crisp open sets in (X, Γ) is said to be neutrosophic crisp base of neutrosophic crisp topology (NCT) if $\Gamma_{NC} = \cup$ β.

Definition 3.3:

Neutrosophic crisp topology (X, Γ) is countable if it has neutrosophic crisp countable basis for neutrosophic crisp topology, i.e. there exist a countable collection of neutrosophic crisp open set $\{U_\alpha\}_{\alpha \in N} =<u_{11},u_{12},u_{13}>,<u_{21},u_{22},u_{23}>,\ldots\ldots,<u_{n1},u_{n2},u_{n3}>$ such that for any neutrosophic crisp open set U containing a crisp neutrosophic point p in U, there exist a $\beta \in N$ such that $p \in U_\beta \subseteq U$.

Definition 3.4:

Neutrosophic crisp homeomorphism is a bijective mapping f of NCTs (X, Γ_1) onto NCTs (Y, Γ_2) is called a neutrosophic crisp homeomorphism if it is neutrosophic crisp continuous and neutrosophic crisp open.

Definition 3.5:

Neutrosophic crisp topology is neutrosophic crisp Locally Euclidean of dimension n if for each neutrosophic crisp point p $=<\{p_1\},\{p_2\},\{p_3\}>$ in X, there exist a neutrosophic crisp open set U $=<u_1,u_2,u_3>$ and a map $\phi: U \to R^n$ such that $\phi: U \to \phi(U)$ *which is* $\phi(U) =<\phi(u_1),\phi(u_2),\phi(u_3)>$ is a homeomorphism; in particular $\phi(U)$ is neutrosophic crisp open set of R^n .

We define a neutrosophic crisp topological manifold (NCM) as follows:

Definition 3.6:

(NCM) is a neutrosophic crisp topological manifold space if the following conditions together satisfied

1. (NCM) is satisfying neutrosophic crisp topology axioms.
2. (NCM) is neutrosophic crisp Haussdorff.
3. (NCM) is countable neutrosophic crisp topology.
4. (NCM) is neutrosophic crisp Locally Euclidean of dimension n.

We give the terminology $(M_{NC})^n$ to mean that it is a neutrosophic crisp manifold of dimension n.

The following graph represents the neutrosophic crisp topological manifold space as a generalization of topological manifold space:

Topological Manifold
↙ ↓ ↘
Haussdorff   Second Countable   Locally Euclidean
↓   ↓   ↓
Neutrosophic crisp Haussdorff   Neutrosophic crisp Countable
Neutrosophic crisp Locally Euclidean
↘ ↓ ↙
Neutrosophic Crisp Topological Manifold

Figure 3.1 A graph of generalization of topological manifold space

4 Some New Topological Concepts on NCM Space [2, 3, 4, 6, 8]

The neutrosophic crisp set U and map $\phi(U)$ in the Definition 3.5 of neutrosophic crisp Locally Euclidean is called a neutrosophic crisp coordinate chart.

Definition 4.1:

A neutrosophic crisp coordinate chart on $(M_{NC})^n$ is a pair $(U, \phi(U))$where U in $(M_{NC})^n$ is open and $\phi: U \to \phi(U) \subseteq R^n$ is a neutrosophic crisp homeomorphism, and then the neutrosophic crisp set U is called a neutrosophic crisp coordinate domain or a neutrosophic crisp coordinate neighborhood.

A neutrosophic crisp coordinate chart $(U, \phi(U))$ is centered at p if
$\phi(p) = 0$ *where*
a neutrosophic crisp *coordinate ball* $\phi(U)$
is a ball in R^n.

Definition 4.1.1:

A Ball in neutrosophic crisp topology is an open ball (r, ϵ, p) , r is radius

$$0 \leq r \leq 1\ ,\ 0 < \epsilon < r \text{ and } p \text{ is } NCP.$$

Theorem 4.1:

Every NCM has a countable basis of *coordinate ball*.

Theorem 4.2:

In $(M_{NC})^n$ every neutrosophic crisp point $p = (<\{p_1\},\{p_2\},\{p_3\}>) \in (M_{NC})^n$ is contained in neutrosophic coordinate ball centered at p if:

$(\phi^{-1}(\phi(p)), \phi(\phi^{-1}(\phi(p))))$

and then if we compose ϕ with a translating we must get $p = \phi(p) = 0$.

Proof: Since $(M_{NC})^n$ neutrosophic crisp Locally Euclidean, p must be contained in a coordinate chart$(U, \phi(U))$. Since $\phi(U)$ is a neutrosophic crisp open set containing$\phi(p)$, by the NCT of R^n there must be an open ball B containing $\phi(p)$ and contained in$\phi(U)$. The appropriate coordinate ball is $(\phi^{-1}(\phi(p)), \phi(\phi^{-1}(\phi(p))))$. Compose ϕ with a translation taking $\phi(p)$ to 0 , then $p = \phi(p) = 0$, we have completed the proof.

Theorem 4. 3:

The neutrosophic crisp graph $G(f)$ of a continuous function$f: U \to R^k$,
where U *is neutrosophic crisp set in* R^n , is NCM.

$$G(f) = \{(p, f(p)) \; in \; R^n \times R^k : p \;\; NCPin \; U\}$$

Proof: Obvious.

Example: Spheres are NCM. An n-sphere is defined as:
$S^n = \{p \; NCP \; in \; R^{n+1} : |p|^2 = \sqrt[2]{p_1{}^2 + p_2{}^2 + p_3{}^2} = 1\}$.

Definition 4.2:

Every neutrosophic crisp point p has a neutrosophic crisp neighborhood point p_{NCbd} contained in an open ball B.

Definition 4.3:

Here come the basic definitions first.
Let (X, Γ) be a NCTS.

a) If a family$\{< G_{i1}, G_{i2}, G_{i3} >: i \in J\}$ of NCOSs in X satisfies the condition $\cup \{< G_{i1}, G_{i2}, G_{i3} >: i \in J\} = X_N$ then it is called a neutrosophic open cover of X.
b) A finite subfamily of an open cover $\{< G_{i1}, G_{i2}, G_{i3} >: i \in J\}$ on X, which is also a neutrosophic open cover of X is called a neutrosophic finite subcover $\{ < G_{i1}, G_{i2}, G_{i3} >: i \in J\}$.
c) A family $\{< K_{i1}, K_{i2}, K_{i3} >: i \in J\}$ of NCOSs in X satisfies the finite intersection property [FIP] iff every finite subfamily $\{< K_{i1}, K_{i2}, K_{i3} >: i = 1, 2, \dots\dots, n\}$ of the family satisfies the condition: $\cap \{< K_{i1}, K_{i2}, K_{i3} >: i \in J\} \neq \phi_N$.
d) A NCTS (X, Γ) is called a neutrosophic crisp compact iff each crisp neutrosophic open cover of X has a finite subcover.

Corollary:

A NCTS (X, Γ) is a neutrosophic crisp compact iff every family $\{ < G_{i1}, G_{i2}, G_{i3} >: i \in J\}$ of NCCS in X having the FIP has non-empty intersection.

Definition 4.4:

Every neutrosophic point has a neutrosophic neighborhood contained in a neutrosophic compact set is called neutrosophic locally compact set.

Corollary:

Every NCM is neutrosophic locally compact set.

5 Some Metric Distances on a Neutrosophic Crisp Manifold [10, 9]

5.1. Haussdorff Distance between Two Neutrosophic Crisp Sets on NCM:
Let $A =< A_1, A_2, A_3 >$ and $B =< B_1, B_2, B_3 >$ two neutrosophic crisp sets on NCM then the Haussdorff distance between A and B is

$$d_H(A, B) = sup(d(A_i, B_j), d(B_j, A_i))$$

$$d(A_i, B_j) = inf|A_i - B_j| \;\; , \forall \; i, j \in J$$

5.2. Modified Haussdorff Distance between Two Neutrosophic Crisp Sets on NCM:
Let $A =< A_1, A_2, A_3 >$ and $B =< B_1, B_2, B_3 >$ two neutrosophic crisp sets on NCM then the Haussdorff distance between A and B is

$$d_H(A, B) = \frac{1}{n}[sup(d(A_i, B_j), d(B_j, A_i))], n \; is \; number \; of \; NCPs$$

$$d(A_i, B_j) = inf|A_i - B_j| \;\; , \forall \; i, j \in J.$$

Conclusion and Future Work

In this paper, we introduced and studied the neutrosophic crisp manifold as a new topological structure of manifold via neutrosophic crisp set, and some new topological concepts on a neutrosophic crisp manifold space via neutrosophic crisp set, and also some metric distances on a neutrosophic crisp manifold. Future work will approach neutrosophic fuzzy manifold, a new topological structure of manifold via neutrosophic fuzzy set, and some new topological concepts on a neutrosophic fuzzy manifold space via neutrosophic fuzzy set.

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