VIKOR based MAGDM Strategy with Trapezoidal Neutrosophic Numbers

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Abstract. VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) is a popular strategy for multi- attribute decision making (MADM). We extend the VIKOR strategy for MAGDM problems in trapezoidal neutrosophic number environment. In decision making situation, single-valued trapezoidal neutrosophic numbers are employed to express the attribute values. Then we develop an extended VIKOR strategy to deal with MAGDM in single-valued trapezoidal neutrosophic number environment. The influence of decision-making mechanism coefficient is presented. To illustrate and validate the proposed VIKOR strategy, an illustrative numerical example of MAGDM problem is solved in trapezoidal neutrosophic number environment.

Keywords: Neutrosophic set, Trapezoidal neutrosophic fuzzy number, Multi-attribute decision making, VIKOR strategy..

1. Introduction:

Smarandache [1] poineered the neutrosophic set based on neutrosophy in 1998. In 2010, Wang et al. [2] proposed single valued neutrosophic set (SVNS). SVNS has been successfully applied to solve decision making problems [3-31], image processing [32-35], conflict resolution [36], educational problem [37, 38], social problem [39, 40], etc. Broumi et al. [41] presented an overview neutrosophic sets. Recently, Peng and Dai [42] presented a bibliometric analysis of neutrosophic sets for last two decades.
Single valued trapezoidal neutrosophic number (SVTrNN) [43] is an extension of SVNS. Every element of SVTrNN is a trapezoidal number with three membership degress namely, truth, indeterminacy and falsity membership degrees. Deli and Subhas [10] developed a ranking strategy of SVTrNNs. Biswas et al. [7] established value and ambiguity based ranking strategy for SVTrNN and employed the strategy to deal with MADM problem. Biswas et al. [44] developed TOPSIS strategy for MADM with trapezoidal neutrosophic numbers (TrNNs). Biswas et al. [45] presented distance measure based MADM strategy with interval trapezoidal neutrosophic numbers (ITrNNs). For simplicity, we call SVTrNN as TrNN.
In 1998, Opricovic [46] first introduced the VIKOR strategy in the literature to deal with multi criteria group decision making (MCGDM) with conflicting objectives[47,48].
Using TOPSIS and VIKOR, Pouresmaeil et al.[49] proposed an MAGDM strategy in SVNS environment. Bausys and Zavadskas[50] proposed the VIKOR strategy in interval neutrosophic set (INS) environment. Huang et al.[51] developed a VIKOR strategy in INS environment. Liu and Zhang[52] studied VIKOR strategy in neutrosophic hesitant fuzzy set environment. In 2017, Hu et al.[53] developed a projection based VIKOR strategy for doctor selection problem in INS environment. In 2017, Selvakumari et al. [54] studied VIKOR strategy using octagonal neutrosophic soft matrix. Pramanik et al. [55] developed a VIKOR based MAGDM strategy in bipolar neutrosophic set environment. In 2018, Pramanik et al. [56] introduced a VIKOR strategy for MAGDM in neutrosophic cubic set environment. Dalapati and Pramanik [57] further revisited VIKOR based MAGDM strategy [56] to make it more comprehensive.

VIKOR strategy in trapezoidal neutrosophic number (TrNN) environment is not studied in the literature.
To fill up this research gap, we propose a VIKOR strategy to deal with MAGDM problems in TrNN environment. Also, we solve an MAGDM problem based on VIKOR strategy in trapezoidal neutrosophic number.

The rest of the paper is develpoed as follows. In section 2, we briefly describe definitions of trapezoidal fuzzy number, TrNN, trapezoidal neutrosophic weighted arithmetic averaging (TrNWAA) operator, Hamming distance between two TrNNs. In section 3, we briefly describe extended VIKOR strategy. Thereafter in section 4, we present a VIKOR strategy in TrNN environment. In section 5, we solve an MAGDM problem using the proposed

VIKOR strategy. In section 6, we present the sensitivity analysis. We represent conclusion and scope of future research in section 7.

2. Preliminaries

We present some fundamental definitions of fuzzy sets, neutrosophic set, SVNS, and TrNN.

Definition 2.1 [58] Let $\ddot{Y}$ be a universal set . Then, a fuzzy set F is presented as:

$$F = \{< \dot{y}, \nu_F(\dot{y}) : \dot{y} \in \ddot{Y} >\} \quad (1)$$

where $\nu_F(\dot{y})$ is the degree of membership which maps $\ddot{Y}$ to [0,1] or we can express by $\nu_F : \ddot{Y} \to [0,1]$.

Definition 2.2[1]Let $\ddot{Y}$ be an universal set . A neutrosophic set N can be presented of the form:

$$N = \{< z : T_N(\dot{y}), I_N(\dot{y}), F_N(\dot{y}) > \dot{y} \in \ddot{Y}\} \quad (2)$$

where the functions $T, I, F : \ddot{Y} \to]^-0,1^+[$ define repectively the degree of truth membership, the degree of indeterminacy, and the degree of non-membership or falsity of the component $\dot{y} \in \ddot{Y}$ and satisfy the condition,

$$^-0 \le T_N(\dot{y}) + F_N(\dot{y}) + I_N(\dot{y}) \le 3^+ \quad (3)$$

Definition 2.3 [2] Let $\ddot{Y}$ be a universal set. An SVNS N in $\ddot{Y}$ is described by

$$N = \{< \dot{y} : T_N(\dot{y}), I_N(\dot{y}), F_N(\dot{y}) > \dot{y} \in \ddot{Y}\}$$

where $T_N(\dot{y}) : \ddot{Y} \to [0,1]$, $I_N(\dot{y}) : \ddot{Y} \to [0,1]$ and $F_N(\dot{y}) : \ddot{Y} \to [0,1]$ with the condition $0 \le T_N(\dot{y}) + F_N(\dot{y}) + I_N(\dot{y}) \le 3$ for all $\dot{y} \in \ddot{Y}$ .The functions $T_N(\dot{y}), I_N(\dot{y})$ and $F_N(\dot{y})$ are respectively, the truth membership function, the indeterminacy membership function and the falsity membership function of the element $\dot{y}$ to the set *N*.

Definition 2.4[59] A generalized trapezoidal fuzzy number $\dot{T}$ denoted by $\dot{T} = (\dot{b}_1, \dot{b}_2, \dot{b}_3, \dot{b}_4; v)$ is described as a fuzzy subset of the real number R with membership function $\underset{\cdot}{\kappa}$ which is defined by

$$\underset{\cdot}{\kappa}_{\dot{T}}(x) = \begin{cases} \dfrac{(x - \dot{b}_1)v}{(\dot{b}_2 - \dot{b}_1)}, \dot{b}_1 \le x < \dot{b}_2 \\ v, \dot{b}_2 \le x \le \dot{b}_3 \\ \dfrac{(\dot{b}_4 - x)v}{(\dot{b}_4 - \dot{b}_3)}, \dot{b}_3 < x \le \dot{b}_4 \\ 0, otherwise \end{cases} \quad (4)$$

where $\dot{b}_1, \dot{b}_2, \dot{b}_3, \dot{b}_4$ are real number satisfying $\dot{b}_1 \le \dot{b}_2 \le \dot{b}_3 \le \dot{b}_4$ and v is the membership degree.

Definition 2.5[43, 44] Let *x* be a TrNN. Then, its truth membership, indeterminacy membership, and falsity membership functions are presented respectively as:

$$T_x(z) = \begin{cases} \dfrac{(z - b_1')t_x}{(b_2' - b_1')}, b_1' \le z < b_2' \\ t_x, b_2' \le z \le b_3' \\ \dfrac{(b_4' - z)t_x}{(b_4' - b_3')}, b_3' \le z \le b_4' \\ 0, otherwise \end{cases} \quad (5)$$

$$I_x(z) = \begin{cases} \dfrac{(b_2' - z) + (z - b_1')i_x}{(b - a)}, b_1' \le z < b_2' \\ i_x, b_2' \le z \le b_3' \\ \dfrac{z - b_3' + (b_4' - z)i_x}{b_4' - b_3'}, b_3' < z \le b_4' \\ 0, otherwise \end{cases} \quad (6)$$

$$F_x(z)=\begin{cases}\dfrac{b_2'-z+(z-b_1')f_x}{b_2'-b_1'}, b_1'\le z<b_2'\\ f_x, b_2'\le z\le b_3'\\ \dfrac{z-b_3'+(b_4'-z)f_x}{b_4'-b_3'}, b_3'<z\le b_4'\\ 0, otherwise\end{cases} \tag{7}$$

Here $0\le T_x(z)\le 1, 0\le I_x(z)\le 1$ and $0\le F_x(z)\le 1$ and $0\le T_x(z)+I_x(z)+F_x(z)\le 3; b_1',b_2',b_3',b_4'\in R.$ Then $x=([b_1',b_2',b_3',b_4']:t_x,i_x,f_x)$ is called a TrNN.

Definition 2.6 [43] Let $m_i =<(b_{1i},b_{2i},b_{3i},b_{4i});T_{m_i},I_{m_i},F_{m_i}>$ (i = 1, 2, …., n') be a group of TrNNs, then a trapezoidal neutrosophic weighted arithmetic averaging (TrNWAA) operator is defined as follows:

$$\mathrm{TrNWAA}(m_1,m_2,...,m_n)=\sum_{i=1}^{n'}\dot{w}_i m_i \tag{8}$$

where, $\dot{w}_i$ is the weight of m_i (i = 1, 2, …, n) such that $\dot{w}_i$ >0 and $\sum_{i=1}^{n'}\dot{w}_i=1$.Specially, when $\dot{w}_i=1/n'$ for i=1,2,…, n' the TrNWAA operator transform into the trapezoidal neutrosophic arithmetic averaging (TrNAA) operator.

Definition 2.7[44] Let $m_1=([\ddot{p}_1,\ddot{q}_1,\ddot{r}_1,\ddot{s}_1];t_{m_1},i_{m_1},f_{m_1})$ and $m_2=([\ddot{p}_2,\ddot{q}_2,\ddot{r}_2,\ddot{s}_2];t_{m_2},i_{m_2}f_{m_2})$ be any two TrNNs. The normalized Hamming distance between m_1 and m_2 is defined as:

$$d(m_1,m_2)=\frac{1}{12}\begin{pmatrix}\left|\ddot{p}_1(2+t_{m_1}-i_{m_1}-f_{m_1})-\ddot{p}_2(2+t_{m_2}-i_{m_2}-f_{m_2})\right|+\left|\ddot{q}_1(2+t_{m_1}-i_{m_1}-f_{m_1})-\ddot{q}_2(2+t_{m_2}-i_{m_2}-f_{m_2})\right|\\ +\left|\ddot{r}_1(2+t_{m_1}-i_{m_1}-f_{m_1})-\ddot{r}_2(2+t_{m_2}-i_{m_2}-f_{m_2})\right|+\left|\ddot{s}_1(2+t_{m_1}-i_{m_1}-f_{m_1})-\ddot{s}_2(2+t_{m_2}-i_{m_2}-f_{m_2})\right|\end{pmatrix} \tag{9}$$

2.8. Standardize the decision matrix[44]

Let $D=(b_{ij})_{p\times n}$ be a neutrosophic matrix, where $b_{ij}=([b_{ij}^1,b_{ij}^2,b_{ij}^3,b_{ij}^4];t_{b_{ij}},i_{b_{ij}},f_{b_{ij}})$ is the rating value of the alternative x_i with respect to attribute y_i. To remove the effect of several physical dimensions, we standardize the decision matrix $(b_{ij})_{p\times n}$ for benefit type and cost type attributes.

We denote the standardized decision matrix by $D^*=(s_{ij})_{p'\times n'}$

1. For benefit type attribute

$$b_{ij}^*=([\frac{b_{ij}^1}{v_j^+},\frac{b_{ij}^2}{v_j^+},\frac{b_{ij}^3}{v_j^+},\frac{b_{ij}^4}{v_j^+}];t_{b_{ij}},i_{b_{ij}},f_{b_{ij}}) \tag{10}$$

2.For cost type attribute:

$$b_{ij}^*=([\frac{v_j^-}{b_{ij}^4},\frac{v_j^-}{b_{ij}^3},\frac{v_j^-}{b_{ij}^2},\frac{v_j^-}{b_{ij}^1}];t_{b_{ij}},i_{b_{ij}},f_{b_{ij}}) \tag{11}$$

Here $v_j^+=\max\{b_{ij}^4: i=1, 2,..., p'\}$ and $v_j^-=\min\{b_{ij}^1: i=1,2, ..., p'\}$ for j = 1, 2, ..., n'

Hence, we obtain standardized matrix D^* as:

$$D^*=(s_{ij})_{p'\times n'}=\begin{pmatrix} & b_1 & b_2 & ... & b_{n'}\\ a_1 & s_{11} & s_{12} & ... & s_{1n'}\\ a_2 & s_{21} & s_{22} & ... & s_{2n'}\\ ... & ... & ... & ... & ...\\ a_{p'} & s_{p'1} & s_{p'2} & ... & s_{p'n'}\end{pmatrix} \tag{12}$$

3.VIKOR Strategy for MADM

Assume that $B_1,B_2,...,B_s$ are the s alternatives. For the alternative $B_{i'}$, assume that the rating of the j' th criterion is $h_{i'j'}$, i.e. $h_{i'j'}$ is the value of j' th criterion for the alternative B_i; the number of criteria is assumed to be r. Development of the extended VIKOR strategy is started with the following form of L_P - metric:

$$L_{i'q} = \{\sum_{i=1}^{r}[(h_{i'}^{+} - h_{i'j'}) / (\mathrm{h}_{i'}^{+} - h_{i'}^{-})]^{q}\}^{\frac{1}{q}}, 1 \le q \le \infty; i' = 1, 2, ..., \mathrm{s} \quad (13)$$

To formulate ranking measure, $L_{1,i'}$ (as $S_{i'}$) and $L_{\infty,i'}$ (as $R_{i'}$) are employed. The solution obtained by $\min S_{i'}$ reflects a maximum group utility (''majority'' rule), and the solution obtained by $\min R_{i'}$ reflects a minimum individual regret of the ''opponent''.

VIKOR stratgey is presented using the following steps:

(a) Evaluate the best $\mathrm{h}_{j'}^{+}$ and the worst $\mathrm{h}_{j'}^{-}$ values of all criteria $j' = 1, 2, ..., n'$.

$$h_{j'}^{+} = \max_{i'} h_{i'j'}, \mathrm{h}_{j'}^{-} = \min_{i'} h_{i'j'}, \text{for benefit criterion,} \quad (14)$$

$$h_{j'}^{+} = \min_{i'} h_{i'j'}, \mathrm{h}_{j'}^{-} = \max_{i'} h_{i'j'}, \text{for cost criterion} \quad (15)$$

(a) Calculate the values $F_{i'}$ and $G_{i'}$; i' = 1, 2, ..., m', by these relations:

$$F_{i'} = \sum_{j'=1}^{n'} \frac{w_{j'}(\mathrm{h}_{j'}^{+} - h_{i'j'})}{(\mathrm{h}_{j'}^{+} - h_{j'}^{-})} \quad (16)$$

$$G_{i'} = \max_{j'} \left(\frac{w_{j'}(\mathrm{h}_{j'}^{+} - h_{i'j'})}{(\mathrm{h}_{j'}^{+} - h_{j'}^{-})} \right) \quad (17)$$

where $w_{j'}$ (j' =1, 2, …, r) represent the weights of criteria.

(b) Evaluate the value $K_{i'}$; i' = 1, 2, ..., s, using following relation:

$$K_{i'} = \frac{v(\mathrm{F}_{i'} - F^{+})}{(\mathrm{F} - F^{+})} + \frac{(1-v)(\mathrm{G}_{i'} - G^{+})}{(\mathrm{G}^{-} - G^{+})} \quad (18)$$

where

$$F^{+} = \min_{i'} F_{i'}, \mathrm{F}^{-} = \max_{i'} F_{i'}$$

$$G^{+} = \min_{i'} G_{i'}, \mathrm{G}^{-} = \max_{i'} G_{i'}$$

Here, v indicates the weight of the technique of ''the majority of criteria'' (or ''the maximum group utility'') . Here we set v = 0.5.

(c) Sorting by the values F, G and K in decreasing order, we rank the alternatives.

(d) Propose the alternative B^1 as a compromise solution that is ranked the best by the measure K (minimal) subject to the conditions A1 and A2:

A1. Acceptable advantage:

$$\mathrm{K}(\mathrm{B}^2) - \mathrm{K}(\mathrm{B}^1) \ge \mathrm{DK}$$

where B^2 is second alternative in the ranking list by K; DK= 1/(s-1); s = the number of alternatives.

A2. Acceptable stability in decision making:

Using F or/and G, we must have alternative B^1 as the best ranked. We say the compromise solution as stable subject to

i. ''voting by majority rule'' (when v > 0.5 is needed),
ii. or ''by consensus'' v $\approx$ 0.5,
iii. or ''with veto'' (v < 0.5).

v refelcts the weight of the decision making strategy of ''the majority of criteria'' (or ''the maximum group utility'').

- B^1 and B^2 are compromise solutions if A2 is not satisfied, Or
- $B^1, B^2, ..., B^M$ are compromise solutions if A1 is not satisfied.

  Evaluate $\mathrm{K}(\mathrm{B}^2) - \mathrm{K}(\mathrm{B}^1) \ge \mathrm{DK}$ to determine B^{M} for maxium M (the positions of these alternatives are ''in closeness'').

The minimal value of K determines the best alternative.

4.VIKOR strategy for solving MAGDM problem in TrNN environment:

Consider an MADGM problem consisitng of r alternatives and t attributes. The alternatives and attributes are presented by $\alpha' = \{\alpha'_1, \alpha'_2, ..., \alpha'_r\}$ and $\beta' = \{\beta'_1, \beta'_2, ..., \beta'_t\}$ respectively. Assume that $\tau = \{\tau_1, \tau_2, ..., \tau_t\}$ is the set of weights of the attributes, where $\tau_{i'} \geq 0$ and $\sum_{i'=1}^{t} \tau_{i'} = 1$. Assume that $B = \{B_1, B_2, ..., B_K\}$ be the set of K decision makers and $\dot{\sigma} = \{\dot{\sigma}_1, \dot{\sigma}_2, ..., \dot{\sigma}_K\}$ be the set of weights of the decision makers, where $\dot{\sigma}_K \geq 0$ and $\sum_{i'=1}^{K} \dot{\sigma}_{i'} = 1$. The rating values offfered by the experts are presented in terms of Trnn

The MAGDM strategy is described as follows:

Step-1: Let $D = (p_{i'j'}^{N'})$ (N' = 1, 2, ..., s) be the N'-th decision matrix where $\alpha'_{i'}$ is alternative with respect to attribute $\beta'_{i'}$. The N'-th decision matrix denoted by $D^{N'}$ is presented as:

$$D^M = \begin{pmatrix} & \beta'_1 & \beta'_2 & \cdots & \beta'_t \\ \alpha'_1 & p_{11}^{N'} & p_{12}^{N'} & \cdots & p_{1t}^{N'} \\ \alpha'_2 & p_{21}^{N'} & p_{22}^{N'} & \cdots & p_{2t}^{N'} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha'_r & p_{r1}^{N'} & p_{r2}^{N'} & \cdots & p_{rt}^{N'} \end{pmatrix} \tag{19}$$

where N' = 1, 2, ..., s; i' = 1,2, ..., r; j' = 1, 2, ..., t.

Step-2: To standardize the benefit criterion, we use the equation (10) and for cost criterion, we use (11). After standardizing, the decision matrix reduces to

$$D^M = \begin{pmatrix} & \beta'_1 & \beta'_2 & \cdots & \beta'_t \\ \alpha'_1 & p_{11}^{*N'} & p_{12}^{*N'} & \cdots & p_{1t}^{*N'} \\ \alpha'_2 & p_{21}^{*N'} & p_{22}^{*N'} & \cdots & p_{2t}^{*N'} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha'_r & p_{r1}^{*N'} & p_{r2}^{*N'} & \cdots & p_{rt}^{*N'} \end{pmatrix}$$

N'=1, 2, ..., s; i'=1, 2, ..., r; j' = 1, 2, ..., t..

Step-3:To obtain aggregate decision matrix, we use trapezoidal neutrosophic weighted arithmetic operator(TrNWAA) which is presented below:

$$p_{ij} = \text{TrNWAA}(p_{ij}^1, p_{ij}^2, ..., p_{ij}^M)$$
$$= \sum_{q=1}^{M} \sigma_q p_{ij}^q \tag{20}$$

Therefore, we obtain the aggregated decision matrix as:

$$D^{N'} = \begin{pmatrix} & \beta'_1 & \beta'_2 & \cdots & \beta'_t \\ \alpha'_1 & \ddot{p}_{11} & \ddot{p}_{12} & \cdots & \ddot{p}_{1t} \\ \alpha'_2 & \ddot{p}_{21} & \ddot{p}_{22} & \cdots & \ddot{p}_{2t} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha'_r & \ddot{p}_{r1} & \ddot{p}_{r2} & \cdots & \ddot{p}_{rt} \end{pmatrix}$$

Step -4: Define the positive ideal solution (PIS) S^+ and negative ideal solution (NIS) S^-

$$S^+ = ([\dot{b}_1^+, \dot{b}_2^+, \dot{b}_3^+, \dot{b}_4^+]; t_{\dot{b}}^+, i_{\dot{b}}^-, f_{\dot{b}}^-) = ([\dot{b}_1^+, \dot{b}_2^+, \dot{b}_3^+, \dot{b}_4^+]; \max t_{\dot{b}}, \min i_{\dot{b}}, \min f_{\dot{b}}) \tag{21}$$

$$S^- = ([\dot{b}_1^-, \dot{b}_2^-, \dot{b}_3^-, \dot{b}_4^-]; t_{\dot{b}}^-, i_{\dot{b}}^+, f_{\dot{b}}^+) = ([\dot{b}_1^-, \dot{b}_2^-, \dot{b}_3^-, \dot{b}_4^-]; \max t_{\dot{b}}, \min i_{\dot{b}}, \min f_{\dot{b}}) \tag{22}$$

Step 5:Compute

$$\Gamma_m = \sum_{j=1}^{t} \frac{\tau_m \times d(([\dot{b}_1^+, \dot{b}_2^+, \dot{b}_3^+, \dot{b}_4^+]; t_{\dot{b}}^+, i_{\dot{b}}^-, f_{\dot{b}}^-), ([\dot{b}_{j1}, \dot{b}_{j2}, \dot{b}_{j3}, \dot{b}_{j4}]; t_{nj}, i_{nj}, f_{nj}))}{d(([\dot{b}_1^+, \dot{b}_2^+, \dot{b}_3^+, \dot{b}_4^+]; t_{\dot{b}}^+, i_{\dot{b}}^-, f_{\dot{b}}^-), ([\dot{b}_1^-, \dot{b}_2^-, \dot{b}_3^-, \dot{b}_4^-]; t_{\dot{b}}^-, i_{\dot{b}}^+, f_{\dot{b}}^+))} \tag{23}$$

$$Z_m = \max \sum_{j=1}^{t} \frac{\tau_m \times d(([\dot{b}_1^+, \dot{b}_2^+, \dot{b}_3^+, \dot{b}_4^+]; t_{\dot{b}}^+, i_{\dot{b}}^-, f_{\dot{b}}^-), ([\dot{b}_{j1}, \dot{b}_{j2}, \dot{b}_{j3}, \dot{b}_{j4}]; t_{nj}, i_{nj}, f_{nj}))}{d(([\dot{b}_1^+, \dot{b}_2^+, \dot{b}_3^+, \dot{b}_4^+]; t_{\dot{b}}^+, i_{\dot{b}}^-, f_{\dot{b}}^-), ([\dot{b}_1^-, \dot{b}_2^-, \dot{b}_3^-, \dot{b}_4^-]; t_{\dot{b}}^-, i_{\dot{b}}^+, f_{\dot{b}}^+))} \tag{24}$$

where τ_m is the weight of β'_m.

Using equation (9), we obtain

$$d(([\dot{b}_1^+, \dot{b}_2^+, \dot{b}_3^+, \dot{b}_4^+]; t_{\dot{b}}^+, i_{\dot{b}}^-, f_{\dot{b}}^-), ([\dot{b}_{j1}, \dot{b}_{j2}, \dot{b}_{j3}, \dot{b}_{j4}]; t_{nj}, i_{nj}, f_{nj})$$

$$= \frac{1}{12}\left(\begin{array}{l} \left|\dot{b}_1^+(2 + t_{\dot{b}}^+ - i_{\dot{b}}^- - f_{\dot{b}}^-) - \dot{b}_{j1}(2 + t_{nj} - i_{nj} - f_{nj})\right| + \left|\dot{b}_2^+(2 + t_{\dot{b}}^+ - i_{\dot{b}}^- - f_{\dot{b}}^-) - \dot{b}_{j2}(2 + t_{nj} - i_{nj} - f_{nj})\right| \\ + \left|\dot{b}_3^+(2 + t_{\dot{b}}^+ - i_{\dot{b}}^- - f_{\dot{b}}^-) - \dot{b}_{j3}(2 + t_{nj} - i_{nj} - f_{nj})\right| + \left|\dot{b}_4^+(2 + t_{\dot{b}}^+ - i_{\dot{b}}^- - f_{\dot{b}}^-) - \dot{b}_{j4}(2 + t_{nj} - i_{nj} - f_{nj})\right| \end{array} \right)$$

and

$$d(([\dot{b}_1^+, \dot{b}_2^+, \dot{b}_3^+, \dot{b}_4^+]; t_{\dot{b}}^+, i_{\dot{b}}^-, f_{\dot{b}}^-), ([\dot{b}_1^-, \dot{b}_2^-, \dot{b}_3^-, \dot{b}_4^-]; t_{\dot{b}}^-, i_{\dot{b}}^+, f_{\dot{b}}^+))$$

$$= \frac{1}{12}\left(\begin{array}{l} \left|\dot{b}_1^+(2 + t_{\dot{b}}^+ - i_{\dot{b}}^- - f_{\dot{b}}^-) - \dot{b}_1^-(2 + t_{\dot{b}}^- - i_{\dot{b}}^+ - f_{\dot{b}}^+))\right| + \left|\dot{b}_2^+(2 + t_{\dot{b}}^+ - i_{\dot{b}}^- - f_{\dot{b}}^-) - \dot{b}_2^-(2 + t_{\dot{b}}^- - i_{\dot{b}}^+ - f_{\dot{b}}^+))\right| \\ + \left|\dot{b}_3^+(2 + t_{\dot{b}}^+ - i_{\dot{b}}^- - f_{\dot{b}}^-) - \dot{b}_3^-(2 + t_{\dot{b}}^- - i_{\dot{b}}^+ - f_{\dot{b}}^+))\right| + \left|\dot{b}_4^+(2 + t_{\dot{b}}^+ - i_{\dot{b}}^- - f_{\dot{b}}^-) - \dot{b}_4^-(2 + t_{\dot{b}}^- - i_{\dot{b}}^+ - f_{\dot{b}}^+))\right| \end{array} \right)$$

Step 6: Compute the Θ by the following formula:

$$\Theta_m = \Psi \frac{(\Gamma_m - \Gamma''_m)}{(\Gamma_m^+ - \Gamma''_m)} + (1 - \Psi)\frac{(Z_m - Z''_m)}{(Z_m^+ - Z''_m)} \tag{25}$$

where $\Gamma''_m = \min_m \Gamma_m$, $\Gamma_m^+ = \max_m \Gamma_m$ (26)

$$Z''_m = \min_m Z_m \ , \ \ Z_m^+ = \max_m Z_m \tag{27}$$

Here, Ψ denotes "decision-making mechanism coefficient".

i. Θ is the minimal if $\Psi \le 0.5$

ii. Θ is the "maximum group utility" if $\Psi \ge 0.5$,

iii. Θ is both the minimal and the "maximum group utility" if $\Psi = 0.5$.

Step-7: Ranking the alternative by Γ_m, Z_m, and Θ_m.

Step-8: Determine the compromise solution

Obtain alternative α'^1 as a compromise solution, that is ranked as the best by the measure α' (minimal) if the A1 and A2 are satisfied:

A1. Acceptable stability:

$$\Theta(\alpha'^2) - \Theta(\alpha'^1) \ge \frac{1}{r-1} \tag{28}$$

where α'^1, α'^2 are the alternatives with 1st and 2nd positions in the ranking by Θ **;** r = the number of alternatives.

A2. Acceptable stability in decision making:

Alternative α'^1 must also be the best ranked by Γ or/and Z. This compromise solution is stable within whole decision making process.

- α'^1 and α'^2 are compromise solutions if A2 is not satisfied, or
- $\alpha'^1, \alpha'^2, \ldots, \alpha'^r$ are compromise solutions if A1 is not satisfied and α'^r is decided by constraint $\Theta(\alpha'^2) - \Theta(\alpha'^1) \le \frac{1}{r-1}$ for maximum r.

The minimal value of Θ determines the best alternative.

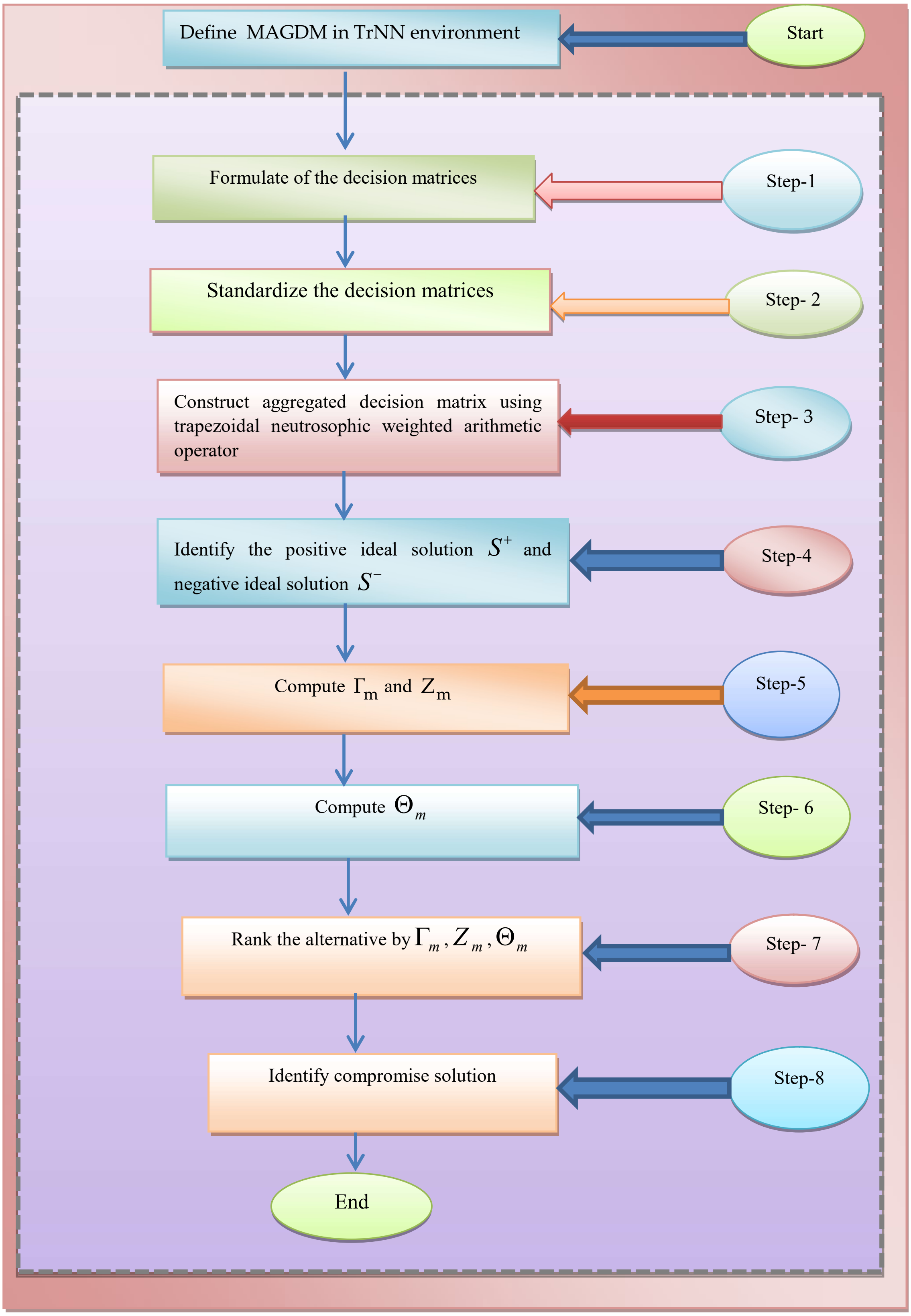

Figure 1. VIKOR based MAGDM strategy in trapezoidal neutrosophic number environment

5. Numerical example

To illustrate the developed VIKOR strategy, we consider an MAGDM problem adapted from [57]. The considered MAGDM problem is described as follows:

An investment company constitutes a decision making board with three experts to invest certain amount of money in the best alternative. The experts evaluate the four alternatives and three attributes which are described below :

Alteranatives:

1. Car company(α_1')
2. Food company(α_2')
3. Computer company(α_3')
4. Arms company(α_4')

Attributes:

1. Risk factor(β_1')
2. Growth factor(β_2')
3. Environment impact(β_3')

Suppose, $\tau = (0.30, 0.42, 0.28)$ be the set of weigts of the decision makers and $\sigma = (0.33, 0.39.0.28)$ be the set of weights of the attributes.

Step-1: In this step,we construct the decision matrix in TrNNs form

Decision matrix D^1

$$\left(\begin{array}{cccc} & \beta_1' & \beta_2' & \beta_3' \\ \alpha_1' & <(0.5,0.6,0.7,0.8);0.1,0.4,0.7> & <(0.1,0.1,0.2,0.3);0.6,0.7,0.5> & <(0.1,0.2,0.2,0.3);0.7,0.2,0.4> \\ \alpha_2' & <(0.3,0.4,0.5,0.5);0.4,0.5,0.2> & <(0.1,0.2,0.2,0.4);0.1,0.4,0.3> & <(0.1,0.1,0.2,0.3);0.5,0.3,0.5> \\ \alpha_3' & <(0.3,0.3,0.30.3);0.1,0.2,0.3> & <(0.2,0.3,0.4,0.4);0.8,0.2,0.5> & <(0.6,0.7,0.8,0.9);0.4,0.3,0.1> \\ \alpha_4' & <(0.7,0.8,0.8,0.9);0.3,0.3,0.2> & <(0.1,0.2,0.3,0.3);0.6,0.5,0.2> & <(0.2,0.2,0.2,0.2);0.5,0.2,0.2> \end{array}\right) \tag{29}$$

Decision matrix D^2

$$\left(\begin{array}{cccc} & \beta_1' & \beta_2' & \beta_3' \\ \alpha_1' & <(0.1,0.1,0.2,0.3);0.2,0.5,0.1> & <(0.2,0.2,0.3,0.4);0.2,0.5,0.1> & <(0.4,0.5,0.6,0.7);0.5,0.7,0.2> \\ \alpha_2' & <(0.2,0.3,0.4,0.5);0.3,0.3,0.2> & <(0.1,0.1,0.2,0.3);0.3,0.3,0.4> & <(0.2,0.2,0.3,0.3);0.4,0.5,0.2> \\ \alpha_3' & <(0.1,0.2,0.2,0.3);0.2,0.5,0.6> & <(0.2,0.3,0.3,0.4);0.2,0.2,0.1> & <(0.4,0.5,0.6,0.6);0.8,0.1,0.1> \\ \alpha_4' & <(0.5,0.6,0.7,0.7);0.5,0.2,0.1> & <(0.2,0.2,0.2,0.2);0.3,0.4,0.5> & <(0.1,0.1,0.2,0.2);0.3,0.7,0.4> \end{array}\right) \tag{30}$$

Decision matrix D^3

$$\left(\begin{array}{cccc} & \beta_1' & \beta_2' & \beta_3' \\ \alpha_1' & <(0.3,0.4,0.4,0.5);0.5,0.1,0.1> & <(0.1,0.2,0.2,0.3);0.5,0.1,0.1> & <(0.2,0.2,0.3,0.4);0.6,0.2,0.1> \\ \alpha_2' & <(0.2,0.2,0.2,0.2);0.3,0.2,0.7> & <(0.1,0.1,0.1,0.1);0.4,0.4,0.1> & <(0.6,0.7,0.8,0.8);0.4,0.1,0.1> \\ \alpha_3' & <(0.2,0.3,0.4,0.5);0.4,0.5,0.3> & <(0.2,0.3,0.3,0.4);0.5,0.4,0.3> & <(0.3,0.4,0.4,0.5);0.5,0.2,0.3)> \\ \alpha_4' & <(0.3,0.4,0.4,0.5);0.5,0.2,0.1> & <(0.1,0.2,0.2,0.3);0.5,0.1,0.1> & <(0.1,0.2,0.3,0.4);0.2,0.2,0.5)> \end{array}\right) \tag{31}$$

Step-2: We do not need to standardize the defining matrix as all the criteria are profit type.

Step-3:Using TrNWAA operator of equation (20),we get aggregate decision matrix of (29), (30), and (31) which is presented below:

$$\left(\begin{array}{cccc} & \beta_1' & \beta_2' & \beta_3' \\ \alpha_1' & <(0.276,0.334,0.406,0.506);0.273,0.298,0.179> & <(0.142,0.17,0.242,0.342);.431,0.352,0.162> & <(0.254,0.326,0.396,0.496);0.633,0.294,0.203> \\ \alpha_2' & <(0.23,0.302,0.374,0.416);0.332,0.312,0.284> & <(0.1,0.13,0.172,0.274);0.277,0.354,0.249> & <(0.282,0.31,0.41,0.44);0.564,0.242,0.217> \\ \alpha_3' & <(0.188,0.258,0.286,0.356);0.235,0.380,0.401> & <(0.2,0.3,0.33,0.372);0.537,0.242,0.162> & <(0.432,0.532,0.604,0.662);0.640,0.169,0.136> \\ \alpha_4' & <(0.504,0.604,0.646,0.704);0.447,0.226,0.123> & <(0.142,0.2,0.23,0.258);0.461,0.290,0.242> & <(0.13,0.158,0.228,0.256);0.343,0.338,0.346> \end{array}\right)$$

step-4:Here we define positive ideal soluton and negative solution by employing equations (21) and (22)

The positive ideal solution R^+ is presented as:

$$\begin{array}{ccc} \beta_1' & \beta_2' & \beta_3' \\ <(0.504,0.604,0.646,0.704);0.447,0.226,0.123> & <(0.2,0.3,0.33,0.372);0.537,0.242,0.162> & <(0.432,0.532,0.604,0.662);0.640,0.169,0.136> \end{array}$$

The negative ideal solution R^- is presented as:

$$\begin{array}{ccc} \beta_1' & \beta_2' & \beta_3' \\ <(0.188,0.258,0.286,0.356);0.235,0.380,0.401> & <(0.1,0.13,0.172,0.274);0.277,0.354,0.249> & <(0.13,0.158,0.228,0.256);0.343,0.338,0.346> \end{array}$$

Step-5:Using equations (23) and (24), we compute Γ_m and Z_m which are presented as:

$$\Gamma_1 = \left(\frac{0.33\times 0.202}{0.294}\right)+\left(\frac{0.39\times 0.070}{0.121}\right)+\left(\frac{0.28\times 0.172}{0.327}\right)=0.601,$$
$$\Gamma_2 = \left(\frac{0.33\times 0.238}{0.294}\right)+\left(\frac{0.39\times 0.119}{0.121}\right)+\left(\frac{0.28\times 0.181}{0.327}\right)=0.805,$$
$$\Gamma_3 = \left(\frac{0.33\times 0.298}{0.294}\right)+\left(\frac{0.39\times 0}{0.121}\right)+\left(\frac{0.28\times 0}{0.327}\right)=0.334,$$
$$\Gamma_4 = \left(\frac{0.33\times 0}{0.294}\right)+\left(\frac{0.39\times 0.080}{0.121}\right)+\left(\frac{0.28\times 0.284}{0.327}\right)=0.501.$$

Here we use Hamming distance to measure the distantance between two TrNN .
And

$$Z_1 = \max\left\{\left(\frac{0.33\times 0.202}{0.294}\right)+\left(\frac{0.39\times 0.070}{0.121}\right)+\left(\frac{0.28\times 0.172}{0.327}\right)\right\}=0.227,$$
$$Z_2 = \max\left\{\left(\frac{0.33\times 0.238}{0.294}\right)+\left(\frac{0.39\times 0.119}{0.121}\right)+\left(\frac{0.28\times 0.181}{0.327}\right)\right\}=0.383,$$
$$Z_3 = \max\left\{\left(\frac{0.33\times 0.298}{0.294}\right)+\left(\frac{0.39\times 0}{0.121}\right)+\left(\frac{0.28\times 0}{0.327}\right)\right\}=0.334,$$
$$Z_4 = \max\left\{\left(\frac{0.33\times 0}{0.294}\right)+\left(\frac{0.39\times 0.080}{0.121}\right)+\left(\frac{0.28\times 0.284}{0.327}\right)\right\}=0.258.$$

Step-6:Using (25),(26), and (27) we calculate Θ_i

$\Theta_1 = 0.283, \Theta_2 = 1, \Theta_3 = 0.342, \Theta_4 = 0.274$

Step -7:The ranking order of alternatives is

$\Theta_4 \le \Theta_1 \le \Theta_3 \le \Theta_2$

Table 1. Preference ranking order and compromise solution based on Γ , Z and Θ

	α'_1	α'_2	α'_3	α'_4	Ranking	Compromise solution
Γ	0.6	0.805	0.334	0.501	$\alpha'_3 < \alpha'_4 < \alpha'_1 < \alpha'_2$	α'_3
Z	0.228	0.383	0.334	0.258	$\alpha'_1 < \alpha'_3 < \alpha'_3 < \alpha'_2$	α'_1
$\Theta(\Psi = 0.5)$	0.282	1	0.342	0.274	$\alpha'_4 < \alpha'_1 < \alpha'_3 < \alpha'_2$	α'_4

Step 8: Determine the compromise solution

If we rank Θ in decreasing order, the best position alternative is α'_4 with Θ (α'_4) =0.274, and the 2nd best position α'^1 with Θ (α'^1)=0.283. Therefore, Θ (α'_1)- Θ (α'_4)=O.008<0.33(since r = 4;1/(r-1)=0.33) , which does not satisfy the condition 1(Θ (α'^2)- Θ (α'^1) $\ge \frac{1}{r-1}$ **).**

Here α'_4 is ranked best by Γ and Z and satisfies the condition 2.

So, the compromise solution as follows:

Θ (α'_1)- Θ (α'_4)=0.008<0.33,

Θ (α'_2)- Θ (α'_4)=0.726>0.33,

$\Theta(\alpha_3')$- $\Theta(\alpha_4')$=0.05<0.33,

Therefore, α_1', α_3' and α_4' are compromise solutions.

6.1 The impact of parameter Ψ

Table 2 demonstrates how the different values of Ψ impact the ranking order of the alternatives $\alpha_{i'}'$.

Table 2. For different values of Ψ, ranking the value of $\alpha_{i'}'$ (i' = 1, 2, 3, 4) .

Values of Ψ	Values of $\alpha_{i'}'$	Preference order
$\Psi = 0.1$	α_1' =0.057, α_2' =1, α_3' =0.615 , α_4' =0.209	$\alpha_1' < \alpha_4' < \alpha_3' < \alpha_2'$
$\Psi = 0.2$	α_1' =0.113 , α_2' =1, α_3' =0.547, α_4' =0.225	$\alpha_1' < \alpha_4' < \alpha_3' < \alpha_2'$
$\Psi = 0.3$	α_1' =0.170 , α_2' =1, α_3' =0.479, α_4' =0.241	$\alpha_1' < \alpha_4' < \alpha_3' < \alpha_2'$
$\Psi = 0.4$	α_1' =0.227 , α_2' =1, α_3' =0.410 , α_4' =0.257	$\alpha_1' < \alpha_4' < \alpha_3' < \alpha_2'$
$\Psi = 0.5$	α_1' =0.282 , α_2' =1, α_3' =0.342 , α_4' =0.274	$\alpha_4' < \alpha_1' < \alpha_3' < \alpha_2'$
$\Psi = 0.6$	α_1' =0.340 , α_2' =1, α_3' =0.274, α_4' =0.290	$\alpha_3' < \alpha_4' < \alpha_1' < \alpha_2'$
$\Psi = 0.7$	α_1' =0.370, α_2' =1, α_3' =0.205, α_4' =0.306	$\alpha_3' < \alpha_4' < \alpha_1' < \alpha_2'$
$\Psi = 0.8$	α_1' =0.454 , α_2' =1, α_3' =0.137 , α_4' =0.399	$\alpha_3' < \alpha_4' < \alpha_1' < \alpha_2'$
$\Psi = 0.9$	α_1' =0.510 , α_2' =1, α_3' =0.068 , α_4' =0.338	$\alpha_3' < \alpha_4' < \alpha_1' < \alpha_2'$

7. Conclusions

Extended VIKOR strategy for MAGDM in trapezoidal neutrosophic number environment is presented in the paper. TrNWAA operator and Hamming distance are employed to develop the VIKOR strategy for MAGDM. Finally, an MAGDM problem is solved to demonstrate the proposed VIKOR strategy. Here, a sensitivity analysis is performed to demonstrate the impact of different values of the "decision making mechanism coefficient'' on ranking system. The proposed extended VIKOR strategy for MAGDM problems can be used to deal with decision making problems such as brick selection [60, 61], logistics center selection [62], teacher selection [63], weaver selection [64], etc.

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