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Weighted Neutrosophic Soft Sets

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Abstract. In this paper we study the concept of neutrosophic soft sets. Imposing some weights on the parameters considered we introduce here weighted neutrosophic soft sets. Some operations like union, intersection, complement, AND, OR etc. have been defined on this new concept. Some properties of these newly defined operations have also been verified .

Keywords: Soft sets, neutrosophic sets, neutrosophic soft sets, weighted neutrosophic soft sets.

1 Introduction

The soft set theory initiated by Molodtsov [1] has been proved as a generic mathematical tool to deal with problems involving uncertainties or imprecise data. So called traditional tools such as fuzzy sets [2], rough sets [3], vague sets [4], probability theory etc. can not be used successfully because of inadequecy present in the parametrization of the tools. Consequently, Molodtsov has shown that soft set theory has a potential to use in variety of many fields [1]. After its initiation a detailed theoretical construction has been introduced by Maji et al in [5] . Works on soft set theory is growing very rapidly with all its potentiality and is being used in different fields [6 – 11, 17,19]. In case of soft set the parametrization is done with the help of words, sentences, functions etc.. For different characteristics of the parameters present in soft set theory different hybridization viz. fuzzy soft sets [12], soft rough sets [13], intuitionistic fuzzy soft sets [14], vague soft sets [15], neutrosophic soft sets [16] etc. have been introduced. In [16] the parameters considered are neutrosophic in nature. Imposing the weights on the parameters (may be in a particular parameter also) we have introduced weighted neutrosophic soft sets in this paper. In section 2 of this paper we have a relevant recapitulation of some preliminaries for better understanding of the paper. In section 3 after defining weighted neutrosophic soft set we have defined some operations like union, intersection, AND, OR etc.. Some properties of these operations have also been verified in this section. Conclusions are there in the concluding section 4.

2 Preliminaries

In this section we recall some relevant definitions.

Definition 2.1 [18] A neutrosophic set A on the universe of discourse X is defined as A = {< x, $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ X}, where T, I, F : X →]− 0, 1+ [and $-0 \le T_A(x) + I_A(x) + F_A(x) \le 3+$.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of]− 0, 1+ [. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of]− 0, 1+ [. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].

Definition 2.2 [18] A neutrosophic set A is contained in another neutrosophic set B i.e. A ⊆ B if ∀x ∈ X, $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$.

Definition 2.3 [16] Let U be an initial universe set and E be a set of parameters.

Consider A ⊂ E. Let P(U) denotes the set of all neutrosophic sets of U.

The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by F : A → P(U).

Definition 2.4 [16] Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if A ⊂ B, and $T_{F(e)}(x) \le T_{G(e)}(x)$, $I_{F(e)}(x) \le I_{G(e)}(x)$, $F_{F(e)}(x) \ge T_{G(e)}(x)$,

∀e ∈ A, x ∈ U.
We denote it by (F, A) ⊆ (G, B).
(F, A) is said to be neutrosophic soft super set of (G, B) if (G, B) is a neutrosophic soft subset of (F, A). We denote it by (F, A) ⊇ (G, B).

Definition 2.5 [16] Equality of two neutrosophic soft sets.

Two NSSs (F, A) and (G, B) over the common universe U are said to be equal if (F, A) is neutrosophic soft subset of (G, B) and (G, B) is neutrosophic soft subset of (F, A). We denote it by (F, A) = (G, B).

Definition 2.6 [16] NOT set of a set of parameters.

Let E = {e_1, e_2 , · · · , e_n } be a set of parameters. The NOT set of E is denoted by ⌉E is defined by ⌉E = {⌉e_1, ⌉e_2 , · · · , ⌉e_n }, where ⌉e_i = not e_i ,∀i (it may be noted that ⌉and ⌉ are different operators).

Definition 2.7 [16] Complement of a neutrosophic soft set.
The complement of a neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c$, ⌉A), where F^c : ⌉A → P(U) is a mapping given by $F^c(\alpha)$ = neutrosophic soft complement with $T_{F^c(e)}(x) = F_{F(e)}(x)$, $I_{F^c(e)}(x) = I_{F(e)}(x)$ and $F_{F^c(e)}(x) = T_{F(e)}(x)$, ∀x ∈ U and ∀e ∈ ⌉A.

Definition 2.8 [16] Union of two neutrosophic soft sets.

Let (H, A) and (G, B) be two NSSs over the common universe U. Then the union of (H, A) and (G, B) is denoted by '(H, A) ∪ (G, B)' and is defined by (H, A) ∪ (G, B) = (K, C), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are as follows:

$T_{K(e)}(x) = T_{H(e)}(x)$, if e ∈ A − B,

$= T_{G(e)}(x)$, if e ∈ B − A,
$= \max (T_{H(e)}(x), T_{G(e)}(x))$, if e ∈ A ∩ B.

$I_{K(e)}(x) = I_{H(e)}(x)$, if e ∈ A − B,

$= I_{G(e)}(x)$, if e ∈ B − A,
$= (I_{H(e)}(x) + I_{G(e)}(x))/2$, if e ∈ A ∩ B.

$F_{K(e)}(x) = F_{H(e)}(x)$, if e ∈ A − B,

$= F_{G(e)}(x)$, if e ∈ B − A,
$= \min (F_{H(e)}(x), F_{G(e)}(x))$, if e ∈ A ∩ B.

Definition 2.9 [16] Intersection of two neutrosophic soft sets.

Let (H, A) and (G, B) be two NSSs over the same universe U. Then the intersection of (H, A) and (G, B) is denoted by '(H, A) ∩ (G, B)' and is defined by (H, A) ∩ (G, B) = (K, C), where C = A ∩ B and the truth-membership, indeterminacy- membership and falsity-membership of (K, C) are as follows:
$T_{K(e)}(x) = \min (T_{H(e)}(x), T_{G(e)}(x))$, if e ∈ A ∩ B.
$I_{K(e)}(x) = (I_{H(e)}(x) + I_{G(e)}(x))/2$, if e ∈ A ∩ B.
$F_{K(e)}(x) = \max (F_{H(e)}(x), F_{G(e)}(x))$, if e ∈ A ∩ B.
Now we are in the position to define weighted neutrosophic soft sets.

3 Weighted Neutrosophic Soft Sets

Definition 3.1 A neutrosophic soft set is termed to be a weighted neutrosophic soft sets if a weight (w_i, a real positive number ≤ 1) be imposed on the parameter of it. The ijth entries of the weighted neutrosophic soft set,

$d_{ij} = w_{ij} \times c_{ij}$ where c_{ij} is the ij-th entry in the table of neutrosophic soft set.

The weighted neutrosophic soft sets (WNSS) for the neutrosophic soft sets (NSS) (F, A) with weights w associated with the parameter A is denoted by (F, A^w).

Example 3.1 For illustration we consider the **example** in [16]. Let U be the set of houses under consideration and E is the set of parameters which consist of neutrosophic words or phases with neutrosophic words. Consider E = { beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive }. Suppose that, there are five houses in the universe U given by, U = { h_1, h_2, h_3, h_4, h_5 } and the set of parameters A = {e_1, e_2, e_3, e_4 }, where e_1 stands for the parameter 'beautiful', e_2 stands for the parameter 'wooden', e_3 stands for the parameter 'costly' and the parameter e_4 stands for 'moderate'. Suppose that,
F(beautiful) = {< h_1, 0.5, 0.6, 0.3 >, < h_2 , 0.4, 0.7, 0.6 >, < h_3 , 0.6, 0.2, 0.3 >, < h_4 , 0.7, 0.3, 0.2 >, < h_5 , 0.8, 0.2, 0.3 >},
F(wooden) = {< h_1 , 0.6, 0.3, 0.5 >, < h_2, 0.7, 0.4, 0.3 >, < h_3, 0.8, 0.1, 0.2 >, < h_4 , 0.7, 0.1, 0.3 >, < h_5 , 0.8, 0.3, 0.6 >},
F(costly) = {< h_1, 0.7, 0.4, 0.3 >, < h_2, 0.6, 0.7, 0.2 >, < h_3, 0.7, 0.2, 0.5 >, < h_4, 0.5, 0.2, 0.6 >, < h_5, 0.7, 0.3, 0.4 >},
F(moderate) = {< h_1, 0.8, 0.6, 0.4 >, < h_2 , 0.7, 0.9, 0.6 >, < h_3 , 0.7, 0.6, 0.4 >, < h_4 , 0.7, 0.8, 0.6 >, < h_5 , 0.9, 0.5, 0.7 >}.
Then the neutrosophic soft set (F, A) describing the attractiveness of the houses given in the following tabular form.

U	beautiful	wooden	costly	moderate
h_1	(0.5, 0.6, 0.3)	(0.6, 0.3, 0.5)	(0.7,0.4, 0.3)	(0.8,0.6, 0.4)
h_2	(0.4, 0.7, 0.6)	(0.7, 0.4, 0.3)	(0.6,0.7, 0.2)	(0.7,0.9, 0.6)
h_3	(0.6, 0.2, 0.3)	(0.8, 0.1, 0.2)	(0.7,0.2, 0.5)	(0.7,0.6, 0.4)
h_4	(0.7, 0.3, 0.2)	(0.7, 0.1, 0.3)	(0.5,0.2, 0.6)	(0.7,0.8, 0.6)
h_5	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.6)	(0.7,0.3, 0.4)	(0.9,0.5, 0.7)

Table 1: The Neutrosophic Soft Sets (F, A).

Imposing the weights w_1 = 0.3, w_2 = 0.6, w_3= 0.4, w_4= 0.7 respectively for the parameters 'beautiful', 'wooden', 'costly' and 'moderate' the weighted neutrosophic soft sets (WNSS) corresponding to the neutrosophic soft sets (F, A) denoted by (F, A^w) and is given in the following tabular form:

U	beautiful, w_1 = 0.3	wooden, w_2 = 0.6,	costly, w_3= 0.4,	moderate, w_4= 0.7
h_1	(0.15, 0.18,0.09)	(0.36, 0.18,0.30)	(0.28,0.16, 0.12)	(0.56,0.42, 0.28)

h_2	(0.12, 0.21,0.18)	(0.42, 0.24, 0.18)	(0.24,0.28, 0.08)	(0.49,0.63, 0.42)
h_3	(0.18, 0.06,0.18)	(0.48, 0.06,0.12)	(0.28,0.08, 0.20)	(0.49,0.42, 0.28)
h_4	(0.21, 0.09,0.06)	(0.42, 0.06,0.18)	(0.20,0.08, 0.24)	(0.49,0.56, 0.42)
h_5	(0.24, 0.06,0.09)	(0.48, 0.18,0.36)	(0.28,0.12, 0.16)	(0.63,0.35, 0.49)

Table 2: The Weighted Neutrosophic Soft Sets (F, A^w).

Definition 3.2 Subset of weighted NSS

Let (F, A^w) and (G, B^w) be two weighted neutrosophic soft sets over the common universe U. (F, A^w) is said to be weighted neutrosophic soft subset of (G, B^w) if A ⊂ B, and $T_{F(e)}(x) \le T_{G(e)}(x)$, $I_{F(e)}(x) \le I_{G(e)}(x)$, $F_{F(e)}(x) \ge T_{G(e)}(x)$, ∀e ∈ A, x ∈ U.

We denote it by (F, A^w) ⊆ (G, B^w).

(F, A^w) is said to be neutrosophic soft super set of (G, B^w) if (G, B^w) is a neutrosophic soft subset of (F, A^w). We denote it by (F, A) ⊇ (G, B). It is to be noted that the weights w for A and B may not be same.

Definition 3.3 Equality of two weighted neutrosophic soft sets.

Two WNSSs (F, A^w) and (G, B^w) over the common universe U are said to be equal if (F, A^w) is neutrosophic soft subset of (G, B^w) and (G, B^w) is neutrosophic soft subset of (F, A^w). We denote it by (F, A^w) = (G, B^w).

Definition 3.4 NOT set of a set of parameters.

Let E = {e_1, e_2 , · · · , e_n } be a set of parameters. The NOT set of E is denoted by ⌉E is defined by E = {⌉e_1, ⌉e_2 , · · · , ⌉e_n }, where ⌉e_i = not e_i ,∀i (it may be noted that ⌉ and ⌉ are different operators).

Definition 3.5 Complement of a weighted neutrosophic soft set.

The complement of a weighted neutrosophic soft set (F, A^w) denoted by $(F, A^w)^c$ and is defined as

$(F, A^w)^c = (F^c, A^w)$, where F^c :⌉A $^\omega$ → P(U) is a mapping given by $F^c(e)$ = neutrosophic soft complement with $T_{F^c(e^w)}(x) = F_{F(e^w)}(x)$, $I_{F^c(e^w)}(x) = I_{F(e^w)}(x)$ and $F_{F^c(e^w)}(x) = T_{F(e^w)}(x)$.

Example 3.2 Consider the WNSS (F, A^w) as in **example 3.1** above.

The tabular representation of the complement of $(F, A^w)^c$ is as below:

U	not beautiful, w_1 = 0.3	not wooden, w_2 = 0.6,	not costly, w_3= 0.4,	not moderate, w_4= 0.7
h_1	(0.09, 0.18,0.15)	(0.30,0.18, 0.36)	(0.12,0.16, 0.28)	(0.28,0.42, 0.56)
h_2	(0.18, 0.21,0.12)	(0.18, 0.24, 0.42)	(0.08,0.28, 0.24)	(0.42,0.63, 0.49)
h_3	(0.18, 0.06,0.18)	(0.12 0.06, 0.48)	(0.20,0.08, 0.28)	(0.28,0.42, 0.49)
h_4	(0.06,0.09,0.21)	(0.18,0.06, 0.42)	(0.24,0.08, 0.20)	(0.42,0.56, 0.49)
h_5	(0.09, 0.06, 0.24)	(0.36,0.18, 0.48)	(0.16,0.12, 0.28)	(0.49,0.35, 0.63)

Table 3: The Weighted Neutrosophic Soft Sets $(F, A^w)^c$.

Definition 3.6 Empty or Null neutrosophic soft set with respect to a parameter.

A weighted neutrosophic soft set (H, A^w) over the universe U is termed to be empty or weighted null neutrosophic soft set with respect to the parameter A if $T_{H(e^w)}(x) = 0$,

$I_{H(e^w)}(x) = 0$ and $F_{H(e^w)}(x) = 0$, ∀x ∈ U, ∀e ∈ A.

In this case the weighted null neutrosophic soft set (WNNSS) is denoted by $\Phi_A{}^w$.

Example 3.3 Let U = { h_1, h_2, h_3, h_4, h_5 } the set of five houses be considered as the universal set and A = { beautiful, wooden, in the green surroundings } be the set of parameters that characterizes the houses. Consider the neutrosophic soft set (H, A^w) which describes the attractiveness of the houses and

H(beautiful, w_1= 0.4)={< h_1, 0,0,0 >, < h_2, 0,0,0 >, < h_3, 0,0,0 >, < h_4, 0,0,0 >, <h_5, 0,0,0> },

H(wooden, w_2= 0.8)={< h_1, 0,0,0 >, < h_2, 0,0,0 >, < h_3, 0,0,0 >, < h_4, 0,0,0 >, <h_5, 0,0,0> },

H(in the green surroundings, w_3= 0.6)={< h_1, 0,0,0 >, <h_2, 0,0,0 >,< h_3, 0,0,0 >, <h_4, 0,0,0 >, <h_5, 0,0,0> }.

Here the (H, A^w) is the weighted null neutrosophic soft set.

Definition 3.7 Union of two weighted neutrosophic soft sets.

Let (F, A^w) and (G, B^w) be two WNSSs over the common universe U. Then the union of (F, A^w) and (G, B^w) is denoted by '(F, A^w) ⊔ (G, B^w)' and is defined by (F, A^w) ⊔ (G, B^w) = (K, C^w), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C^w) are as follows:

$T_{K(e^w)}(x) = T_{F(e^w)}(x)$, if e ∈ A − B,

$= T_{G(e^w)}(x)$, if e ∈ B − A,

$= \max.(w_1,w_2).\max.(T_{F(e^w)}(x),T_{G(e^w)}(x))$, if e ∈ A∩B,

$I_{K(e^w)}(x) = I_{F(e^w)}(x)$, if e ∈ A− B,

$= I_{G(e^w)}(x)$, if e ∈ B − A,

$= (I_{F(e^w)}(x) + I_{G(e^w)}(x))/2$, if e ∈ A∩B,

$F_{K(e^w)}(x) = F_{F(e^w)}(x)$, if e ∈ A − B,

$= F_{G(e^w)}(x)$, if e ∈ B − A,

$= \min.(w_1,w_2).\min.(F_{F(e^w)}(x),F_{G(e^w)}(x))$, if e ∈ A∩B,

Example 3.4 Let (F, A^w) and (G, B^w) be two WNSSs over the common universe U = { h_1, h_2, h_3, h_4, h_5 } and their tabular representations are given below:

	U	beautiful	wooden	moderate
(F, A)	h_1	(0.6.,0.3,0.7)	(0.7,0.3,0.5)	(0.6,0.4,0.5)
	h_2	(0.5,0.4,0.5)	(0.6,0.7,0.3)	(0.6,0.5,0.4)
	h_3	(o.7,0.4,0.3)	(0.7,0.3,0.5)	(0.7,0.4,0.5)
	h_4	(0.8,0.4,0.7)	(0.6,0.3,0.6)	(0.7,0.5,0.6)
	h_5	(0.6,0.7,0.2)	(0.7,0.3,0.4)	(0.8,0.6,0.5)
weight		**w_1 = 0.4**	**w_2 = 0.3**	**w_3 = 0.6**
(F, A^w)	h_1	(0.24,0.12,0.28)	(0.21,0.09,0.15)	(0.36,0.24,0.30)
	h_2	(0.20,0.16,0.20)	(0.18,0.21,0.09)	(0.36,0.30,0.24)
	h_3	(0.28,0.16,0.12)	(0.21,0.09,0.15)	(0.42,0.24,0.30)
	h_4	(0.32,0.16,0.28)	(0.18,0.09,0.18)	(0.42,0.30,0.36)
	h_5	(0.24,0.28,0.08)	(0.21,0.09,0.12)	(0.48,0.36,0.30)

Table 4: The Weighted Neutrosophic Soft Sets (F, A^w).

	U	costly	moderate
(G, B)	h_1	(0.7,0. 6.,0.6)	(0.7,0.8,0.6)
	h_2	(0.8,0.4,0.5)	(0.8,0.8,0.3)
	h_3	(0.7,0.4,0.6)	(0.5,0.6,0.7)
	h_4	(0.6,0.3,0.5)	(0.8,0.5,0.6)
	h_5	(0.8,0.5,0.4)	(0.6,0.3,0.5)
weight		**w_1 = 0.3**	**w_3 = 0.4**
(G, B^w)	h_1	(0.21,0.18,0.18)	(0.28,0.32,0.24)
	h_2	(0.24,0.12,0.15)	(0.32,0.32,0.12)
	h_3	(0.21,0.12,0.18)	(0.20,0.24,0.28)
	h_4	(0.18,0.09,0.15)	(0.32,0.20,0.24)
	h_5	(0.24,0.15,0.12)	(0.24,0.12,0.20)

Table 5: The Weighted Neutrosophic Soft Sets (G, B^w).

Then the tabular representation of their union (K, C^w) = (F, A^w) ⊔ (G, B^w) is as below:

U	beautiful	wooden	costly	moderate
h_1	(0.24,0.12,0.28)	(0.21,0.09,0.15)	(0.21,0.18,0.18)	(0.42,0.28,0.20)
h_2	(0.20,0.16,0.20)	(0.18,0.21,0.09)	(0.24,0.12,0.15)	(0.48,0.31,0.12)
h_3	(0.28,0.16,0.12)	(0.21,0.09,0.15)	(0.21,0.12,0.18)	(0.42,0.24,0.20)
h_4	(0.32,0.16,0.28)	(0.18,0.09,0.18)	(0.18,0.09,0.15)	(0.36,0.25,0.24)
h_5	(0.24,0.28,0.08)	(0.21,0.09,0.12)	(0.24,0.15,0.12)	(0.48,0.24,0.20)

Table 6: The Weighted Neutrosophic Soft Sets (K, C^w).

Definition 3.8 Intersection of two weighted neutrosophic soft sets.
Let (F, A^w) and (G, B^w) be two WNSSs over the common universe U. Then the intersection of (F, A^w) and (G, B^w) is denoted by '(F, A^w) ⊓ (G, B^w)' and is defined by (F, A^w) ⊓ (G, B^w) = (K, C^w), where C = A ∪ B and the truth-membership, indeterminacy-membership and falsity-membership of (K, C^w) are as follows:

$T_{K(e^w)}(x) = T_{F(e^w)}(x)$, if $e \in A - B$,
$= T_{G(e^w)}(x)$, if $e \in B - A$,
$=$ min. (w_1,w_2). min. $(T_{F(e^w)}(x), T_{G(e^w)}(x))$, if $e \in A \cap B$,
$I_{K(e^w)}(x) = I_{F(e^w)}(x)$, if $e \in A - B$,
$= I_{G(e^w)}(x)$, if $e \in B - A$,
$= (I_{F(e^w)}(x) + I_{G(e^w)}(x))/2$, if $e \in A \cap B$,
$F_{K(e^w)}(x) = F_{F(e^w)}(x)$, if $e \in A - B$,
$= F_{G(e^w)}(x)$, if $e \in B - A$,
$=$ max. (w_1,w_2). max. $(F_{F(e^w)}(x), F_{G(e^w)}(x))$, if $e \in A \cap B$,

Example 3.5 Consider the WNSSs (F, A^w) and (G, B^w) as in **example 3.4,** then their intersection is given in the following tabular form:

U	beautiful	wooden	costly	moderate
h_1	(0.24,0.12,0.28)	(0.21,0.09,0.15)	(0.21,0.18,0.18)	(0.24,0.28,0.36)
h_2	(0.20,0.16,0.20)	(0.18,0.21,0.09)	(0.24,0.12,0.15)	(0.24,0.31,0.24)
h_3	(0.28,0.16,0.12)	(0.21,0.09,0.15)	(0.21,0.12,0.18)	(0.20,0.24,0.42)
h_4	(0.32,0.16,0.28)	(0.18,0.09,0.18)	(0.18,0.09,0.15)	(0.28,0.25,0.36)
h_5	(0.24,0.28,0.08)	(0.21,0.09,0.12)	(0.24,0.15,0.12)	(0.24,0.24,0.30)

Table 7: The Weighted Neutrosophic Soft Sets (F, A^w) ⊓ (G, B^w)

Consider (F, A^w), (G, B^w) and (K, C^w) be three WNSSs over the common universe U. Based on the definitions of union and intersections of them we have the following Propositions:

Proposition: 3.1

i. $(F, A^w) \sqcup (F, A^w) = (F, A^w)$.
ii. $(F, A^w) \sqcup (G, B^w) = (G, B^w) \sqcup (F, A^w)$.
iii. $(F, A^w) \sqcap (F, A^w) = (F, A^w)$.
iv. $(F, A^w) \sqcap (G, B^w) = (G, B^w) \sqcap (F, A^w)$.

Proof: Proofs being straightforward are not given.

Proposition: 3.2

i. $(F, A^w) \sqcup [(G, B^w) \sqcup (K, C^w)] = [(F, A^w) \sqcup (G, B^w)] \sqcup (K, C^w)$.
ii. $(F, A^w) \sqcap [(G, B^w) \sqcap (K, C^w)] = [(F, A^w) \sqcap (G, B^w)] \sqcap (K, C^w)$.
iii. $(F, A^w) \sqcup [(G, B^w) \sqcap (K, C^w)] = [(F, A^w) \sqcup (G, B^w)] \sqcap [(F, A^w) \sqcap (K, C^w)]$.
iv. $(F, A^w) \sqcap [(G, B^w) \sqcup (K, C^w)] = [(F, A^w) \sqcap (G, B^w)] \sqcup [(F, A^w) \sqcap (K, C^w)]$.

Proofs: Proofs being straightforward are not given.

We can verify the De Morgan's laws in case of union and intersection of two WNSSs.

Proposition: 3.3

i. $[(F, A^w) \sqcap (G, B^w)]^c = (F, A^w)^c \sqcup (G, B^w)^c$.
ii. $[(F, A^w) \sqcup (G, B^w)]^c = (F, A^w)^c \sqcap (G, B^w)^c$.

Proof: (i). Let $(K, D^w) = (F, A^w) \sqcap (G, B^w)$. Therfore

$T_{K(e^w)}(x) = T_{F(e^w)}(x)$, if $e \in A - B$,
$= T_{G(e^w)}(x)$, if $e \in B - A$,
$= \min.(w_1,w_2).\ \min.(T_{F(e^w)}(x), T_{G(e^w)}(x))$, if $e \in A \cap B$,
$I_{K(e^w)}(x) = I_{F(e^w)}(x)$, if $e \in A - B$,
$= I_{G(e^w)}(x)$, if $e \in B - A$,
$= (I_{F(e^w)}(x) + I_{G(e^w)}(x))/2$, if $e \in A \cap B$,
$F_{K(e^w)}(x) = F_{F(e^w)}(x)$, if $e \in A - B$,
$= F_{G(e^w)}(x)$, if $e \in B - A$,
$= \max.(w_1,w_2).\ \max.(F_{F(e^w)}(x), F_{G(e^w)}(x))$, if $e \in A \cap B$,

So,

$T_{K^c(e^w)}(x) = F_{F(e^w)}(x)$, if $e \in A - B$,
$= F_{G(e^w)}(x)$, if $e \in B - A$,
$= \max.(w_1,w_2).\ \max.(F_{F(e^w)}(x), F_{G(e^w)}(x))$, if $e \in A \cap B$,
$I_{K^c(e^w)}(x) = I_{F(e^w)}(x)$, if $e \in A - B$,
$= I_{G(e^w)}(x)$, if $e \in B - A$,
$= (I_{F(e^w)}(x) + I_{G(e^w)}(x))/2$, if $e \in A \cap B$,
$F_{K^c(e^w)}(x) = T_{F(e^w)}(x)$, if $e \in A - B$,
$= T_{G(e^w)}(x)$, if $e \in B - A$,
$= \min.(w_1,w_2).\ \min.(T_{F(e^w)}(x), T_{G(e^w)}(x))$, if $e \in A \cap B$.

Again for $(F, A^w)^c \sqcup (G, B^w)^c$, let $(P, D^w) = (H, A^w)^c$, $(Q, E^w) = (G, B^w)^c$ and $(R, S^w) = (P, D^w)^c \sqcup (Q, E^w)$, where $S = D \cup E$.

Therefore,

$T_{R(e^w)}(x) = T_{p^c(e^w)}(x) = F_{H(e^w)}(x)$, if $e \in A - B$,
$= T_{Q^c(e^w)}(x) = F_{G(e^w)}(x)$, if $e \in B - A$,
$= \max.(w_1,w_2).\ \max.(T_{p^c(e^w)}(x), T_{q^c(e^w)}(x)) = \max.(w_1,w_2).\ \max(F_{H(e^w)}(x), F_{G(e^w)}(x))$, if $e \in A \cap B$.

$I_{R(e^w)}(x) = (I_{P^c(e^w)}(x) + I_{q^c(e^w)}(x))/2 = (I_{H(e^w)}(x) + I_{G(e^w)}(x))/2$, if $e \in A \cap B$,
$= I_{p^c(e^w)}(x) = I_{H(e^w)}(x)$, if $e \in A - B$,
$= I_{Q^c(e^w)}(x) = I_{G(e^w)}(x)$, if $e \in B - A$,

$F_{R(e^w)}(x) = F_{p^c(e^w)}(x) = T_{H(e^w)}(x)$, if $e \in A - B$,
$= F_{Q^c(e^w)}(x) = T_{G(e^w)}(x)$, if $e \in B - A$,
$= \min.(w_1,w_2).\ \min.(F_{p^c(e^w)}(x), F_{q^c(e^w)}(x)) = \min.(w_1,w_2).\ \min(T_{H(e^w)}(x), T_{G(e^w)}(x))$, if $e \in A \cap B$.

Thus the result is proved.

Proof (ii). The proof is similar to the proof of (i).

Definition 3.9 AND operations of two WNSSs.

Let (F, A^w) and (G, B^w) be two WNSSs over the common universe U. Then the 'AND' operation of (F, A^w) and (G, B^w) is denoted by '$(F,A^w) \wedge (G,B^w)$' and is defined by $(F,A^w) \wedge (G,B^w) = (K, C^w)$, where $C = A \times B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C^w) are as follows:

$T_{K(\alpha^w,\beta^w)}(x) = \min.(w_1,w_2).\ \min.(T_{F(\alpha)}(x), T_{G(\beta)}(x))$, $\forall\alpha \in A$, $\forall\beta \in B$,
$I_{K(\alpha^w,\beta^w)}(x) = (T_{F(\alpha^w)}(x) + T_{G(\beta^w)}(x))/2$, $\forall\alpha \in A$, $\forall\beta \in B$,
$F_{K(\alpha^w,\beta^w)}(x) = \max.(w_1,w_2).\ \max.(F_{F(\alpha)}(x), F_{G(\beta)}(x))$, $\forall\alpha \in A$, $\forall\beta \in B$.

Example 3.6 Consider the **example 3.5** above. The tabular representation of the WNSS $(F,A^w) \wedge (G,B^w)$ is given below:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)	(wooden, moderate)	(moderate, costly)	(moderate, moderate)
h_1	(0.18,0.15, 0.28)	(0.24,0.22,0 .28)	(0.21,0.135, 0.18)	(0.21,0.205, 0.24)	(0.18,0.21,0 .36)	(0.24,0.28,0 .36)
h_2	(0.15,0.14, 0.20)	(0.20,0.24,0 .20)	(0.18,0.165, 0.15)	(0.18,0.165, 0.12)	(0.18,0.21,0 .30)	(0.24,0.31,0 .24)
h_3	(0.21,0.14, 0.24)	(0.20,0.20,0 .28)	(0.21,0.105, 0.18)	(0.15,0.165, 0.28)	(0.21,0.18,0 .36)	(0.26,0.24,0 .42)
h_4	(0.18,0.125 ,0.28)	(0.32,0.185, 0.28)	(0.18,0.09,0 .18)	(0.18,0.145, 0.24)	(0.18,0.195, 0.36)	(0.28,0.25,0 .36)
h_5	(0.18,0.215 ,0.16)	(0.24,0.20,0 .20)	(0.21,0.12,0 .12)	(0.18,0.105, 0.20)	(0.24,0.255, 0.30)	(0.24,0.24,0 .30)

Table 8: The Weighted Neutrosophic Soft Sets $(F,A^w) \wedge (G,B^w)$

Definition 3.10. OR operations of two WNSSs.

If (F, A^w) and (G, B^w) be two WNSSs over the common universe U then '(F, A^w) **OR** (G, B^w)' denoted by

$(F,A^w)\vee(G,B^w)$ is defined by $(F,A^w)\vee(G,B^w)$ = (O, C^w), where C = A × B and the truth-membership, indeterminacy-membership and falsity-membership of (O, C^w) are given as follows:

$T_{O(\alpha^w,\beta^w)}(x)$ = max. (w_1,w_2). max. $(T_{F(\alpha)}(x),T_{G(\beta)}(x))$, $\forall\alpha \in A,\ \forall\beta \in B$,

$I_{O(\alpha^w,\beta^w)}(x)= (I_{F(\alpha^w)}(x) + I_{G(\beta^w)}(x))/2$, $\forall\alpha \in A$, $\forall\beta \in B$,

$F_{O(\alpha^w,\beta^w)}(x)$ = min. (w_1,w_2). min. $(F_{F(\alpha)}(x),F_{G(\beta)}(x))$, $\forall\alpha \in A,\ \forall\beta \in B$.

Example 3.7 Consider the **example 3.5** above. The tabular representation of the WNSS $(F,A^w)\vee(G,B^w)$ is given below:

U	(beautiful, costly)	(beautiful, moderate)	(wooden, costly)	(wooden, moderate)	(moderate, costly)	(moderate, moderate)
h_1	(0.28,0.15, 0.18)	(0.28,0.22,0.24)	(0.21,0.135, 0.15)	(0.28,0.205, 0.15)	(0.42,0.21,0.15)	(0.42,0.28,0.20)
h_2	(0.32,0.14, 0.15)	(0.32,0.24,0.12)	(0.24,0.165, 0.09)	(0.32,0.165, 0.09)	(0.48,0.21,0.12)	(0.48,0.31,0.12)
h_3	(0.32,0.14, 0.09)	(0.28,0.20,0.12)	(0.21,0.105, 0.15)	(0.28,0.165, 0.15)	(0.42,0.18,0.15)	(0.42,0.24,0.20)
h_4	(0.32,0.125,0.15)	(0.32,0.185, 0.24)	(0.18,0.09,0.15)	(0.32,0.145, 0.18)	(0.42,0.195, 0.15)	(0.48,0.25,0.24)
h_5	(0.32,0.215,0.06)	(0.24,0.20,0.08)	(0.24,0.12,0.12)	(0.28,0.105, 0.12)	(0.48,0.255, 0.12)	(0.48,0.24,0.20)

Table 9 : The Weighted Neutrosophic Soft Sets $(F,A^w)\vee(G,B^w)$

It is to be noted that for either AND or OR operations on two WNSSs the set of parameter is a subset of E × E whereas for three WNSSs the associated parameters are subset of E × E × E.

Conclusion

In this paper we introduce the concept of weighted neutrosophic soft sets which is a hybridization of soft sets and weighted parameter of neutrosophic soft sets. We have also introduced some operations like union, intersection, AND, OR etc. on this newly defined concept. Some properties of these operations have also been investigated.

References

[1] D. Molodtsov, Soft Set Theory-First Results, Comput. Math. Appl., 37 (1999), 19 - 31.

[2] L. A. Zadeh, Fuzzy Sets, Inform. Control, 8 (1965), 338 - 353.

[3] Z. Pawlak, Rough Sets, Int. J. Inform. Comput. Sci., 11 (1982), 341 - 356.

[4] W. L. Gau and D. J. Buehrer, Vague Sets, IEEE Trans. Sys. Man Cybernet. 23 (2) (1993), 610 – 614.

[5] P. K. Maji, R. Biswas, and A.R. Roy, Soft Set Theory, Comput. Math. Appl., 45 (2003), 555 - 562.

[6] T. Herawan and M. M. deris, A Soft Set Approach for Association Rules Mining, Knowledge-Based Sys., 24 (2011), 186 - 195.

[7] S. J. Kalayathankal and G.S. Singh, A fuzzy Soft Flood Alarm Model, Math. Comput. Simulat., 80 (2010), 887 - 893.

[8] Z. Xiao, K. Gong and Y. Zou, A Combined Forecasting Approach Based on Fuzzy Soft Sets, J. Comput. Appl. Maths. 228 (1) (2009), 326 - 333.

[9] D. Chen, E. C. C. Tsang, D. S. Yeung and X. Wang, The Parametrization Reduction of Soft sets and its Applications, Comput. Math. Appl. 49 (5 - 6) (2005), 757 - 763.

[10] P. K. Maji, R. Biswas and A. R. Roy, An Application of Soft Set in a Decision Making Problem, Comput. Math. Appl., 44 (2002), 1077 - 1083.

[11] A. R. Roy and P. K. Maji, A Fuzzy Soft Set Theoretic in a Decision Making Problem, Comput. Appl. Math., 203 (2007), 412 - 418.

[12] P. K. Maji, R. Biswas, and A.R. Roy, Fuzzy Soft Sets, The Journal of Fuzzy Mathematics, 9 (2001), 589 - 602.

[13] F. Feng, X. Liu, V. Leoreanu-Foeta and Y. B. Jun, Soft Sets and Soft Rough Sets, Inform. Sc. 181 (2011), 1125 - 1137.

[14] P. K. Maji, R. Biswas, and A.R. Roy, Intuitionistic Fuzzy Soft Sets, The Journal of Fuzzy Mathematics, 9 (3)(2001), 677 - 692.

[15] W. Xu., J. Ma, S. Wang and G. Hao, Vague Soft Sets and their Properties, Comput. Math. Appl. 59 (2010), 787 - 794.

[16] P. K. Maji, Neutrosophic Soft Sets, Annals of Fuzzy Maths. Inform. Vol.-5, No.1, (2013), 157-168.

[17] P. K. Maji, A Neutrosophic Soft Set Approach to a Decision Making Problem, Annals of Fuzzy Maths. Inform. Vol.-3, No.2, (2012), 313-319.

[18] F. Smarandache, Neutrosophic Set, a Generalisation of the Intuitionistic Fuzzy Sets, Int. J. Pure Appl. Math. 24 (2005), 287 – 297.

[19] P. K. Maji, Weighted Neutrosophic Soft Sets in Decision Making Problem, communicated.

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