Reliability and Importance Discounting of Neutrosophic Masses

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Abstract. In this paper, we introduce for the first time the discounting of a neutrosophic mass in terms of reliability and respectively the importance of the source.

We show that reliability and importance discounts commute when dealing with classical masses.

1. **Introduction.** Let $\Phi = \{\Phi_1, \Phi_2, \dots, \Phi_n\}$ be the frame of discernment, where $n \geq 2$, and the set of **focal elements**:

$$F = \{A_1, A_2, \dots, A_m\}, \text{ for } m \geq 1, F \subset G^{\Phi}. \ (1)$$

Let $G^{\Phi} = (\Phi, \cup, \cap, \mathcal{C})$ be the **fusion space**.

A **neutrosophic mass** is defined as follows:

$$m_n \colon G \to [0, 1]^3$$

for any $x \in G$, $m_n(x) = (t(x), i(x), f(x))$, (2)

where $t(x)$ = believe that x will occur (truth);

$i(x)$ = indeterminacy about occurence;

and $f(x)$ = believe that x will not occur (falsity).

Simply, we say in neutrosophic logic:

$t(x) =$ believe in x;

$i(x) =$ believe in neut(x)
[the neutral of x, i.e. neither x nor anti(x)];

and $f(x) =$ believe in anti(x) [the opposite of x].

Of course, $t(x),\ i(x),\ f(x) \in [0, 1]$, and

$$\sum_{x \in G}[t(x) + i(x) + f(x)] = 1, (3)$$

while

$$m_n(\phi) = (0, 0, 0). \ (4)$$

It is possible that according to some parameters (or data) a source is able to predict the believe in a hypothesis x to occur, while according to other parameters (or other data) the same source may be able to find the believe in x not occuring, and upon a third category of parameters (or data) the source may find some indeterminacy (ambiguity) about hypothesis occurence.

An element $x \in G$ is called **focal** if

$$n_m(x) \neq (0, 0, 0), (5)$$

i.e. $t(x) > 0$ or $i(x) > 0$ or $f(x) > 0$.

Any **classical mass**:

$$m : G^{\phi} \rightarrow [0, 1] \ (6)$$

can be simply written as a neutrosophic mass as:

$$m(A) = (m(A), 0, 0). (7)$$

2. Discounting a Neutrosophic Mass due to Reliability of the Source.

Let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ be the reliability coefficient of the source, $\alpha \in [0,1]^3$.

Then, for any $x \in G^\theta \setminus \{\theta, I_t\}$,

where $\theta =$ the empty set

and $I_t =$ total ignorance,

$$m_n(x)_\alpha = \big(\alpha_1 t(x), \alpha_2 i(x), \alpha_3 f(x)\big), \ (8)$$

and

$$m_n(I_t)_\alpha = \left(t(I_t) + (1-\alpha_1) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} t(x) , \right.$$
$$\left. i(I_t) + (1-\alpha_2) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} i(x), f(I_t) + (1-\alpha_3) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} f(x) \right)$$

(9),

and, of course,

$$m_n(\phi)_\alpha = (0,0,0).$$

The missing mass of each element x, for $x \neq \phi, x \neq I_t$, is transferred to the mass of the total ignorance in the following way:

$t(x) - \alpha_1 t(x) = (1-\alpha_1) \cdot t(x)$ is transferred to $t(I_t)$, (10)

$i(x) - \alpha_2 i(x) = (1-\alpha_2) \cdot i(x)$ is transferred to $i(I_t)$, (11) and $f(x) - \alpha_3 f(x) = (1 - \alpha_3) \cdot f(x)$ is transferred to $f(I_t)$. (12)

3. Discounting a Neutrosophic Mass due to the Importance of the Source.

Let $\beta \in [0, 1]$ be the importance coefficient of the source. This discounting can be done in several ways.

a. For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_1} = \big(\beta \cdot t(x), i(x), f(x) + (1 - \beta) \cdot t(x)\big), (13)$$

which means that $t(x)$, the believe in x, is diminished to $\beta \cdot t(x)$, and the missing mass, $t(x) - \beta \cdot t(x) = (1 - \beta) \cdot t(x)$, is transferred to the believe in $anti(x)$.

b. Another way:

For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_2} = \big(\beta \cdot t(x), i(x) + (1 - \beta) \cdot t(x), f(x)\big), (14)$$

which means that $t(x)$, the believe in x, is similarly diminished to $\beta \cdot t(x)$, and the missing mass $(1 - \beta) \cdot t(x)$ is now transferred to the believe in $neut(x)$.

c. The third way is the most general, putting together the first and second ways.

For any $x \in G^\theta \setminus \{\phi\}$,

$$m_n(x)_{\beta_3} = \big(\beta \cdot t(x), i(x) + (1 - \beta) \cdot t(x) \cdot \gamma, f(x) + (1 - \beta) \cdot t(x) \cdot (1 - \gamma)\big), (15)$$

where $\gamma \in [0, 1]$ is a parameter that splits the missing mass $(1-\beta) \cdot t(x)$ a part to $i(x)$ and the other part to $f(x)$.

For $\gamma = 0$, one gets the first way of distribution, and when $\gamma = 1$, one gets the second way of distribution.

4. Discounting of Reliability and Importance of Sources in General Do Not Commute.

a. Reliability first, Importance second.

For any $x \in G^{\theta} \setminus \{\phi, I_t\}$, one has after reliability α discounting, where

$$\alpha = (\alpha_1, \alpha_2, \alpha_3):$$

$$m_n(x)_{\alpha} = \big(\alpha_1 \cdot t(x), \alpha_2 \cdot t(x), \alpha_3 \cdot f(x)\big), (16)$$

and

$$m_n(I_t)_{\alpha} = \left(t(I_t) + (1-\alpha_1) \cdot \sum_{x \in G^{\theta} \setminus \{\phi, I_t\}} t(x), i(I_t) + (1-\alpha_2) \cdot \sum_{x \in G^{\theta} \setminus \{\phi, I_t\}} i(x), f(I_t) + (1-\alpha_3) \cdot \sum_{x \in G^{\theta} \setminus \{\phi, I_t\}} f(x) \right) \stackrel{\text{def}}{=} \left(T_{I_t}, I_{I_t}, F_{I_t}\right). \tag{17}$$

Now we do the importance β discounting method, the third importance discounting way which is the most general:

$$m_n(x)_{\alpha\beta_3} = \big(\beta\alpha_1 t(x), \alpha_2 i(x) + (1-\beta)\alpha_1 t(x)\gamma, \alpha_3 f(x) + (1-\beta)\alpha_1 t(x)(1-\gamma)\big)$$

(18)

and

$$m_n(I_t)_{\alpha\beta_3} = \left(\beta \cdot T_{I_t}, I_{I_t} + (1-\beta)T_{I_t} \cdot \gamma, F_{I_t} + (1-\beta)T_{I_t}(1-\gamma)\right). \quad (19)$$

b. Importance first, Reliability second.

For any $x \in G^{\theta} \setminus \{\phi, I_t\}$, one has after importance β discounting (third way):

$$m_n(x)_{\beta_3} = \left(\beta \cdot t(x), i(x) + (1-\beta)t(x)\gamma, f(x) + (1-\beta)t(x)(1-\gamma)\right) \quad (20)$$

and

$$m_n(I_t)_{\beta_3} = \left(\beta \cdot t\left(I_{I_t}\right), i(I_{I_t}) + (1-\beta)t(I_t)\gamma,\ f(I_t) + (1-\beta)t(I_t)(1-\gamma)\right). \quad (21)$$

Now we do the reliability $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ discounting, and one gets:

$$m_n(x)_{\beta_3\alpha} = \left(\alpha_1 \cdot \beta \cdot t(x), \alpha_2 \cdot i(x) + \alpha_2(1-\beta)t(x)\gamma, \alpha_3 \cdot f(x) + \alpha_3 \cdot (1-\beta)t(x)(1-\gamma)\right) \quad (22)$$

and

$$m_n(I_t)_{\beta_3\alpha} = \left(\alpha_1 \cdot \beta \cdot t(I_t), \alpha_2 \cdot i(I_t) + \alpha_2(1-\beta)t(I_t)\gamma, \alpha_3 \cdot f(I_t) + \alpha_3(1-\beta)t(I_t)(1-\gamma)\right). \quad (23)$$

Remark.

We see that (a) and (b) are in general different, so reliability of sources does not commute with the importance of sources.

5. Particular Case when Reliability and Importance Discounting of Masses Commute.

Let's consider a classical mass

$$m: G^{\theta} \rightarrow [0,1] \ (24)$$

and the focal set $F \subset G^{\theta}$,

$$F = \{A_1, A_2, \dots, A_m\}, m \geq 1, (25)$$

and of course $m(A_i) > 0$, for $1 \leq i \leq m$.

Suppose $m(A_i) = a_i \in (0,1]$. (26)

a. Reliability first, Importance second.

Let $\alpha \in [0,1]$ be the reliability coefficient of $m\ (\cdot)$.

For $x \in G^{\theta} \setminus \{\phi, I_t\}$, one has

$$m(x)_{\alpha} = \alpha \cdot m(x), (27)$$

$$\text{and } m(I_t) = \alpha \cdot m(I_t) + 1 - \alpha. (28)$$

Let $\beta \in [0,1]$ be the importance coefficient of $m\ (\cdot)$.

Then, for $x \in G^{\theta} \setminus \{\phi, I_t\}$,

$$m(x)_{\alpha\beta} = \big(\beta\alpha m(x), \alpha m(x) - \beta\alpha m(x)\big) = \alpha \cdot m(x) \cdot (\beta, 1-\beta), (29)$$

considering only two components: believe that x occurs and, respectively, believe that x does not occur.

Further on,

$$m(I_t)_{\alpha\beta} = (\beta\alpha m(I_t) + \beta - \beta\alpha, \alpha m(I_t) + 1 - \alpha - \beta\alpha m(I_t) - \beta + \beta\alpha) =$$
$$[\alpha m(I_t) + 1 - \alpha] \cdot (\beta, 1 - \beta). \text{ (30)}$$

b. Importance first, Reliability second.

For $x \in G^\theta \setminus \{\phi, I_t\}$, one has

$$m(x)_\beta = \left(\beta \cdot m(x), m(x) - \beta \cdot m(x)\right) = m(x) \cdot (\beta, 1 - \beta), \text{ (31)}$$

and $m(I_t)_\beta = \left(\beta m(I_t), m(I_t) - \beta m(I_t)\right) = m(I_t) \cdot (\beta, 1 - \beta)$. (32)

Then, for the reliability discounting scaler α one has:

$$m(x)_{\beta\alpha} = \alpha m(x)(\beta, 1 - \beta) = \left(\alpha m(x)\beta, \alpha m(x) - \alpha\beta m(m)\right) \text{ (33)}$$

and $m(I_t)_{\beta\alpha} = \alpha \cdot m(I_t)(\beta, 1 - \beta) + (1 - \alpha)(\beta, 1 - \beta) = [\alpha m(I_t) + 1 - \alpha] \cdot (\beta, 1 - \beta) = (\alpha m(I_t)\beta, \alpha m(I_t) - \alpha m(I_t)\beta) + (\beta - \alpha\beta, 1 - \alpha - \beta + \alpha\beta) = (\alpha\beta m(I_t) + \beta - \alpha\beta, \alpha m(I_t) - \alpha\beta m(I_t) + 1 - \alpha - \beta - \alpha\beta)$. (34)

Hence (a) and (b) are equal in this case.

6. Examples.

1. Classical mass.

The following classical is given on $\theta = \{A, B\}$:

	A	B	AUB
m	0.4	0.5	0.1

(35)

Let $\alpha = 0.8$ be the reliability coefficient and $\beta = 0.7$ be the importance coefficient.

a. Reliability first, Importance second.

	A	B	AUB
m_α	0.32	0.40	0.28
$m_{\alpha\beta}$	(0.224, 0.096)	(0.280, 0.120)	(0.196, 0.084)

(36)

We have computed in the following way:

$$m_\alpha(A) = 0.8m(A) = 0.8(0.4) = 0.32, (37)$$

$$m_\alpha(B) = 0.8m(B) = 0.8(0.5) = 0.40, (38)$$

$$m_\alpha(AUB) = 0.8(\text{AUB}) + 1 - 0.8 = 0.8(0.1) + 0.2 = 0.28, (39)$$

and

$$m_{\alpha\beta}(B) = \big(0.7m_\alpha(A), m_\alpha(A) - 0.7m_\alpha(A)\big) = \big(0.7(0.32), 0.32 - 0.7(0.32)\big) = (0.224, 0.096), (40)$$

$$m_{\alpha\beta}(B) = \left(0.7m_\alpha(B), m_\alpha(B) - 0.7m_\alpha(B)\right) = \left(0.7(0.40), 0.40 - 0.7(0.40)\right) = (0.280, 0.120), \text{(41)}$$

$$m_{\alpha\beta}(AUB) = \left(0.7m_\alpha(AUB), m_\alpha(AUB) - 0.7m_\alpha(AUB)\right) = \left(0.7(0.28), 0.28 - 0.7(0.28)\right) = (0.196, 0.084). \text{(42)}$$

b. Importance first, Reliability second.

	A	B	AUB
m	0.4	0.5	0.1
m_β	(0.28, 0.12)	(0.35, 0.15)	(0.07, 0.03)
$m_{\beta\alpha}$	(0.224, 0.096	(0.280, 0.120)	(0.196, 0.084)

(43)

We computed in the following way:

$$m_\beta(A) = \left(\beta m(A), (1-\beta)m(A)\right) = \left(0.7(0.4), (1-0.7)(0.4)\right) = (0.280, 0.120), \text{(44)}$$

$$m_\beta(B) = \left(\beta m(B), (1-\beta)m(B)\right) = \left(0.7(0.5), (1-0.7)(0.5)\right) = (0.35, 0.15), \text{(45)}$$

$$m_\beta(AUB) = \left(\beta m(AUB), (1-\beta)m(AUB)\right) = \left(0.7(0.1), (1-0.1)(0.1)\right) = (0.07, 0.03), \text{(46)}$$

and $m_{\beta\alpha}(A) = \alpha m_\beta(A) = 0.8(0.28, 0.12) = \left(0.8(0.28), 0.8(0.12)\right) = (0.224, 0.096), \text{(47)}$

$$m_{\beta\alpha}(B) = \alpha m_\beta(B) = 0.8(0.35, 0.15) = \left(0.8(0.35), 0.8(0.15)\right) = (0.280, 0.120), \text{(48)}$$

$$m_{\beta\alpha}(AUB) = \alpha m(AUB)(\beta, 1-\beta) + (1-\alpha)(\beta, 1-\beta) = 0.8(0.1)(0.7, 1-0.7) + (1-0.8)(0.7, 1-0.7) = 0.08(0.7, 0.3) + 0.2(0.7, 0.3) = (0.056, 0.024) + (0.140, 0.060) = (0.056+0.140, 0.024+0.060) = (0.196, 0.084). \quad (49)$$

Therefore reliability discount commutes with importance discount of sources when one has classical masses.

The result is interpreted this way: believe in A is 0.224 and believe in $nonA$ is 0.096, believe in B is 0.280 and believe in $nonB$ is 0.120, and believe in total ignorance AUB is 0.196, and believe in non-ignorance is 0.084.

7. Same Example with Different Redistribution of Masses Related to Importance of Sources.

Let's consider the third way of redistribution of masses related to importance coefficient of sources. $\beta = 0.7$, but $\gamma = 0.4$, which means that 40% of β is redistributed to $i(x)$ and 60% of β is redistributed to $f(x)$ for each $x \in G^{\theta} \setminus \{\phi\}$; and $\alpha = 0.8$.

a. Reliability first, Importance second.

	A	B	AUB
m	0.4	0.5	0.1
m_{α}	0.32	0.40	0.28
$m_{\alpha\beta}$	(0.2240, 0.0384, 0.0576)	(0.2800, 0.0480, 0.0720)	(0.1960, 0.0336, 0.0504).

(50)

We computed m_α in the same way.

But:

$$m_{\alpha\beta}(A) = \big(\beta \cdot m_\alpha(A), i_\alpha(A) + (1-\beta)m_\alpha(A)\cdot\gamma, f_\alpha(A) + (1-\beta)m_\alpha(A)(1-\gamma)\big) = \big(0.7(0.32), 0 + (1-0.7)(0.32)(0.4), 0 + (1-0.7)(0.32)(1-0.4)\big) = (0.2240, 0.0384, 0.0576). \quad (51)$$

Similarly for $m_{\alpha\beta}(B)$ and $m_{\alpha\beta}(AUB)$.

b. *Importance first, Reliability second.*

	A	B	AUB
m	0.4	0.5	0.1
m_β	(0.280, 0.048, 0.072)	(0.350, 0.060, 0.090)	(0.070, 0.012, 0.018)
$m_\beta\alpha$	(0.2240, 0.0384, 0.0576)	(0.2800, 0.0480, 0.0720)	(0.1960, 0.0336, 0.0504).

(52)

We computed $m_\beta(\cdot)$ in the following way:

$$m_\beta(A) = \big(\beta \cdot t(A), i(A) + (1-\beta)t(A)\cdot\gamma, f(A) + (1-\beta)t(A)(1-\gamma)\big) = \big(0.7(0.4), 0 + (1-0.7)(0.4)(0.4), 0 + (1-0.7)0.4(1-0.4)\big) = (0.280, 0.048, 0.072). \quad (53)$$

Similarly for $m_\beta(B)$ and $m_\beta(AUB)$.

To compute $m_{\beta\alpha}(\cdot)$, we take $\alpha_1 = \alpha_2 = \alpha_3 = 0.8$, (54)

in formulas (8) and (9).

$$m_{\beta\alpha}(A) = \alpha \cdot m_\beta(A) = 0.8(0.280, 0.048, 0.072)$$
$$= \big(0.8(0.280), 0.8(0.048), 0.8(0.072)\big)$$
$$= (0.2240, 0.0384, 0.0576). \ (55)$$

Similarly $m_{\beta\alpha}(B) = 0.8(0.350, 0.060, 0.090) = (0.2800, 0.0480, 0.0720)$. (56)

For $m_{\beta\alpha}(AUB)$ we use formula (9):

$$m_{\beta\alpha}(AUB) = \big(t_\beta(AUB) + (1-\alpha)\big[t_\beta(A) + t_\beta(B)\big],\ i_\beta(AUB)$$
$$+ (1-\alpha)\big[i_\beta(A) + i_\beta(B)\big],$$
$$f_\beta(AUB) + (1-\alpha)\big[f_\beta(A) + f_\beta(B)\big]\big)$$
$$= (0.070 + (1-0.8)[0.280 + 0.350], 0.012$$
$$+ (1-0.8)[0.048 + 0.060], 0.018 + (1-0.8)[0.072 + 0.090])$$
$$= (0.1960, 0.0336, 0.0504).$$

Again, the reliability discount and importance discount commute.

8. Conclusion.

In this paper we have defined a new way of discounting a classical and neutrosophic mass with respect to its importance. We have also defined the discounting of a neutrosophic source with respect to its reliability.

In general, the reliability discount and importance discount do not commute. But if one uses classical masses, they commute (as in Examples 1 and 2).

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