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Neutrosophic Crisp Sets & Neutrosophic Crisp Topological Spaces

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Abstract. In this paper, we generalize the crisp topological spaces to the notion of neutrosophic crisp topological space, and we construct the basic concepts of the neutrosophic crisp topology. In addition to these, we introduce the definitions of neutrosophic crisp continuous function and neutrosophic crisp compact spaces. Finally, some characterizations concerning neutrosophic crisp compact spaces are presented and one obtains several properties. Possible application to GIS topology rules are touched upon.

Keywords: *Neutrosophic Crisp Set; Neutrosophic Topology; Neutrosophic Crisp Topology.*

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their crisp and fuzzy counterparts, the most used one being the neutrosophic set theory [6, 7, 8]. After the introduction of the neutrosophic set concepts in [1, 2, 3, 4, 5, 9, 10, 11, 12] and after haven given the fundamental definitions of neutrosophic set operations, we generalize the crisp topological space to the notion of neutrosophic crisp set. Finally, we introduce the definitions of neutrosophic crisp continuous function and neutrosophic crisp compact space, and we obtain several properties and some characterizations concerning the neutrosophic crisp compact space.

2 Terminology

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [6, 7, 8, 12], and Salama et al. [1, 2, 3, 4, 5, 9, 10, 11, 12]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^-0, 1^+[$ is non-standard unit interval.

Hanafy and Salama et al. [10, 12] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations. We now improve some results by the following.

3 Neutrosophic Crisp Sets

3.1 Definition

Let X be a non-empty fixed set. A neutrosophic crisp set (NCS for short) A is an object having the form $A = \langle A_1, A_2, A_3 \rangle$ where A_1, A_2 and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$.

3.1 Remark

A neutrosophic crisp set $A = \langle A_1, A_2, A_3 \rangle$ can be identified as an ordered triple $\langle A_1, A_2, A_3 \rangle$, where A_1, A_2, A_3 are subsets on X, and one can define several relations and operations between NCSs.

Since our purpose is to construct the tools for developing neutrosophic crisp sets, we must introduce the types of NCSs ϕ_N, X_N in X as follows:

1) ϕ_N may be defined in many ways as a NCS, as follows:

i) $\phi_N = \langle \phi, \phi, X \rangle$, or

ii) $\phi_N = \langle \phi, X, X \rangle$, or

iii) $\phi_N = \langle \phi, X, \phi \rangle$, or

iv) $\phi_N = \langle \phi, \phi, \phi \rangle$

2) X_N may also be defined in many ways as a NCS:

i) $X_N = \langle X, \phi, \phi \rangle$,

ii) $X_N = \langle X, X, \phi \rangle$,

iii) $X_N = \langle X, X, \phi \rangle$,

Every crisp set A formed by three disjoint subsets of a non-empty set is obviously a NCS having the form $A = \langle A_1, A_2, A_3 \rangle$.

3.2 Definition

Let $A = \langle A_1, A_2, A_3 \rangle$ a NCS on , then the complement of the set A, (A^c for short may be defined in three different ways:

C_1 $A^c = \langle A_1{}^c, A_2{}^c, A_3{}^c \rangle$,

C_2 $A^c = \langle A_3, A_2, A_1 \rangle$

C_3 $A^c = \langle A_3, A_2{}^c, A_1 \rangle$

One can define several relations and operations between NCSs as follows:

3.3 Definition

Let X be a non-empty set, and the NCSs A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$, then we may consider two possible definitions for subsets $A \subseteq B$

$A \subseteq B$ may be defined in two ways:

1) $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$ and $A_3 \supseteq B_3$ or

2) $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$ and $A_3 \supseteq B_3$

3.1 Proposition

For any neutrosophic crisp set A the following hold:

i) $\phi_N \subseteq A$, $\phi_N \subseteq \phi_N$.

ii) $A \subseteq X_N$, $X_N \subseteq X_N$.

3.4 Definition

Let X is a non-empty set, and the NCSs A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$. Then:

1) $A \cap B$ may be defined in two ways:

i) $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ or

ii) $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$

2) $A \cup B$ may also be defined in two ways:

i) $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$ or

ii) $A \cup B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$

3) $[\ ]A = \langle A_1, A_2, A_1{}^c \rangle$.

4) $<> A = \langle A_3{}^c, A_2, A_3 \rangle$.

3.2 Proposition

For all two neutrosophic crisp sets A and B on X, then the followings are true:

1) $(A \cap B)^c = A^c \cup B^c$.

2) $(A \cup B)^c = A^c \cap B^c$.

We can easily generalize the operations of intersection and union in definition 3.2 to arbitrary family of neutrosophic crisp subsets as follows:

3.3 Proposition

Let $\{A_j : j \in J\}$ be arbitrary family of neutrosophic crisp subsets in X, then

1) $\cap A_j$ may be defined as the following types :

i) $\cap A_j = \langle \cap Aj_1, \cap A_{j_2}, \cup A_{j_3} \rangle$, or

ii) $\cap A_j = \langle \cap Aj_1, \cup A_{j_2}, \cup A_{j_3} \rangle$.

2) $\cup A_j$ may be defined as the following types :

i) $\cup A_j = \langle \cup Aj_1, \cap A_{j_2}, \cap A_{j_3} \rangle$ or

ii) $\cup A_j = \langle \cup Aj_1, \cup A_{j_2}, \cap A_{j_3} \rangle$.

3.5 Definition

The product of two neutrosophic crisp sets A and B is a neutrosophic crisp set $A \times B$ given by $A \times B = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \rangle$.

4 Neutrosophic Crisp Topological Spaces

Here we extend the concepts of topological space and intuitionistic topological space to the case of neutrosophic crisp sets.

4.1 Definition

A neutrosophic crisp topology (NCT for short) on a non-empty set is a family of neutrosophic crisp subsets in satisfying the following axioms

i) $\phi_N, X_N \in \Gamma$.

ii) $A_1 \cap A_2 \in \Gamma$ for any A_1 and $A_2 \in \Gamma$.

iii) $\cup A_j \in \Gamma \quad \forall \{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair (X,Γ) is called a neutrosophic crisp topological space ($NCTS$ for short) in X. The elements in Γ are called neutrosophic crisp open sets (NCOSs for short) in X. A neutrosophic crisp set F is closed if and only if its complement F^C is an open neutrosophic crisp set.

4.1 Remark

Neutrosophic crisp topological spaces are very natural generalizations of topological spaces and intuitionistic topological spaces, and they allow more general functions to be members of topology.

$$TS \rightarrow ITS \rightarrow NCTS$$

4.1 Example

Let $X=\{a,b,c,d\}$, ϕ_N, X_N be any types of the universal and empty subsets, and A, B two neutrosophic crisp subsets on X defined by $A=\langle\{a\},\{b,d\},\{c\}\rangle$, $B=\langle\{a\},\{b\},\{c\}\rangle$, then the family $\Gamma=\{\phi_N, X_N, A, B\}$ is a neutrosophic crisp topology on X.

4.2 Example

Let $(X,\tau_\circ)$ be a topological space such that $\tau_\circ$ is not indiscrete. Suppose $\{G_i : i\in J\}$ be a family and $\tau_\circ=\{X,\phi\}\cup\{G_i : i\in J\}$. Then we can construct the following topologies as follows

i) Two intuitionistic topologies

a) $\tau_1=\{\phi_I, X_I\}\cup\{\langle G_i,\phi\rangle, i\in J\}$.

b) $\tau_2=\{\phi_I, X_I\}\cup\{\langle \phi, G_i^c\rangle, i\in J\}$

ii) Four neutrosophic crisp topologies

a) $\Gamma_1=\{\phi_N, X_N\}\cup\{\langle\phi,\phi,G_i^c\rangle, i\in J\}$

b) $\Gamma_2=\{\phi_N, X_N\}\cup\{\langle G_i,\phi,\phi\rangle, i\in J\}$

c) $\Gamma_3=\{\phi_N, X_N\}\cup\{\langle G_i,\phi,G_i^c\rangle, i\in J\}$,

d) $\Gamma_4=\{\phi_N, X_N\}\cup\{\langle G_i^c\ ,\phi,\phi\rangle, i\in J\}$

4.2 Definition

Let $(X,\Gamma_1),(X,\Gamma_2)$ be two neutrosophic crisp topological spaces on X. Then Γ_1 is said be contained in Γ_2 (in symbols $\Gamma_1\subseteq\Gamma_2$) if $G\in\Gamma_2$ for each $G\in\Gamma_1$. In this case, we also say that Γ_1 is coarser than Γ_2.

4.1 Proposition

Let $\{\Gamma_j : j\in J\}$ be a family of NCTs on X. Then $\cap\Gamma_j$ is a neutrosophic crisp topology on X. Furthermore, $\cap\Gamma_j$ is the coarsest NCT on X containing all topologies.

Proof

Obvious. Now, we define the neutrosophic crisp closure and neutrosophic crisp interior operations on neutrosophic crisp topological spaces:

4.3 Definition

Let (X,Γ) be NCTS and $A=\langle A_1,A_2,A_3\rangle$ be a NCS in X. Then the *neutrosophic crisp closure* of A ($NCCl(A)$ for short) and *neutrosophic interior crisp* ($NCInt(A)$ for short) of A are defined by

$$NCCl(A)=\cap\{K : K \text{ is an NCS in X and } A\subseteq K\}$$
$$NCInt(A)=\cup\{G : G \text{ is an NCOS in X and } G\subseteq A\},$$

where NCS is a neutrosophic crisp set, and NCOS is a neutrosophic crisp open set.

It can be also shown that $NCCl(A)$ is a NCCS (neutrosophic crisp closed set) and $NCInt(A)$ is a CNOS in X

a) A is in X if and only if $NCCl(A)\supseteq A$.

b) A is a NCCS in X if and only if $NCInt(A)=A$.

4.2 Proposition

For any neutrosophic crisp set A in (X,Γ) we have

(a) $NCCl(A^c)=(NCInt(A))^c$,

(b) $NCInt(A^c)=(NCCl(A))^c$.

Proof

a) Let $A=\langle A_1,A_2,A_3\rangle$ and suppose that the family of neutrosophic crisp subsets contained in A are indexed by the family if NCSs contained in A are indexed by the family $A=\{< A_{j_1}, A_{j_2}, A_{j3} >: i\in J\}$. Then we see that we have two types of $NCInt(A)=\{<\cup A_{j1},\cup A_{j_2},\cap A_{j3}>\}$ or $NCInt(A)=\{<\cup A_{j1},\cap A_{j_2},\cap A_{j3}>\}$ hence $(NCInt(A))^c=\{<\cap A_{j1},\cap A_{j_2},\cup A_{j3}>\}$ or $(NCInt(A))^c=\{<\cap A_{j1},\cup A_{j_2},\cup A_{j3}>\}$.

Hence $NCCl(A^c)=(NCInt(A))^c$, which is analogous to (a).

4.3 Proposition

Let (X,Γ) be a NCTS and A,B be two neutrosophic crisp sets in X. Then the following properties hold:

(a) $NCInt(A) \subseteq A,$
(b) $A \subseteq NCCl(A),$
(c) $A \subseteq B \Rightarrow NCInt(A) \subseteq NCInt(B),$
(d) $A \subseteq B \Rightarrow NCCl(A) \subseteq NCCl(B),$
(e) $NCInt(A \cap B) = NCInt(A) \cap NCInt(B),$
(f) $NCCl(A \cup B) = NCCl(A) \cup NCCl(B),$
(g) $NCInt(X_N) = X_N,$
(h) $NCCl(\phi_N) = \phi_N$

Proof. (a), (b) and (e) are obvious; (c) follows from (a) and from definitions.

5 Neutrosophic Crisp Continuity

Here come the basic definitions first

5.1 Definition

(a) If $B = \langle B_1, B_2, B_3 \rangle$ is a NCS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B) = \langle f^{-1}(B_1), f^{-1}(B_2), f^{-1}(B_3) \rangle$.
(b) If $A = \langle A_1, A_2, A_3 \rangle$ is a NCS in X, then the image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A) = \langle f(A_1), f(A_2), f(A_3)^c) \rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following sections .

5.1 Corollary

Let A, $\{A_i : i \in J\}$, be NCSs in X, and B, $\{B_j : j \in K\}$ NCS in Y, and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2),$
$B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2),$
(b) $A \subseteq f^{-1}(f(A))$ and if f is injective, then $A = f^{-1}(f(A))$.
(c) $f^{-1}(f(B)) \subseteq B$ and if f is surjective, then $f^{-1}(f(B)) = B,$.
(d) $f^{-1}(\cup B_i) = \cup f^{-1}(B_i),\ f^{-1}(\cap B_i) = \cap f^{-1}(B_i),$
(e) $f(\cup A_i) = \cup f(A_i);\ f(\cap A_i) \subseteq \cap f(A_i);$ and if f is injective, then $f(\cap A_i) = \cap f(A_i);$
(f) $f^{-1}(Y_N) = X_N,\ f^{-1}(\phi_N) = \phi_N$.
(g) $f(\phi_N) = \phi_N,\ f(X_N) = Y_N,$ if f is subjective.

Proof

Obvious.

5.2 Definition

Let (X,Γ_1) and (Y,Γ_2) be two NCTSs, and let $f : X \to Y$ be a function. Then f is said to be continuous iff the preimage of each NCS in Γ_2 is a NCS in Γ_1.

5.3 Definition

Let (X,Γ_1) and (Y,Γ_2) be two NCTSs and let $f : X \to Y$ be a function. Then f is said to be open iff the image of each NCS in Γ_1 is a NCS in Γ_2.

5.1 Example

Let (X,Γ_o) and (Y,ψ_o) be two NCTSs

(a) If $f : X \to Y$ is continuous in the usual sense, then in this case, f is continuous in the sense of Definition 5.1 too. Here we consider the NCTs on X and Y, respectively, as follows :
$\Gamma_1 = \{\langle G, \phi, G^c \rangle : G \in \Gamma_o\}$ and
$\Gamma_2 = \{\langle H, \phi, H^c \rangle : H \in \Psi_o\}$,
In this case we have, for each $\langle H, \phi, H^c \rangle \in \Gamma_2$, $H \in \Psi_o$,
$f^{-1}\langle H, \phi, H^c \rangle = \langle f^{-1}(H), f^{-1}(\phi), f^{-1}(H^c) \rangle$
$= \langle f^{-1}H, f(\phi), (f(H))^c \rangle \in \Gamma_1$.

(b) If $f : X \to Y$ is open in the usual sense, then in this case, f is open in the sense of Definition 3.2. Now we obtain some characterizations of continuity:

5.1 Proposition

Let $f : (X,\Gamma_1) \to (Y,\Gamma_2)$.

f is continuous if the preimage of each CNCS (crisp neutrosophic closed set) in Γ_2 is a CNCS in Γ_2 **.**

5.2 Proposition

The following are equivalent to each other:

(a) $f : (X,\Gamma_1) \to (Y,\Gamma_2)$ is continuous.
(b) $f^{-1}(CNInt(B) \subseteq CNInt(f^{-1}(B))$ for each CNS B in Y.
(c) $CNCl(f^{-1}(B)) \subseteq f^{-1}(CNCl(B))$ for each CNC B in Y.

5.2 Example

Let (Y,Γ_2) be a NCTS and $f: X \to Y$ be a function. In this case $\Gamma_1 = \{f^{-1}(H): H \in \Gamma_2\}$ is a NCT on X. Indeed, it is the coarsest NCT on X which makes the function $f: X \to Y$ continuous. One may call it the initial neutrosophic crisp topology with respect to f.

6 Neutrosophic Crisp Compact Space (NCCS)

First we present the basic concepts:

6.1 Definition

Let (X,Γ) be an NCTS.

(a) If a family $\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$ of NCOSs in X satisfies the condition $\cup\{\langle X, G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\} = X_N$, then it is called an neutrosophic open cover of X.

(b) A finite subfamily of an open cover $\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$ on X, which is also a neutrosophic open cover of X , is called a neutrosophic finite subcover $\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$.

(c) A family $\{\langle K_{i_1}, K_{i_2}, K_{i_3}\rangle : i \in J\}$ of NCCSs in X satisfies the finite intersection property (FIP for short) iff every finite subfamily $\{\langle K_{i_1}, K_{i_2}, K_{i_3}\rangle : i = 1,2,\ldots,n\}$ of the family satisfies the condition $\cap\{\langle K_{i_1}, K_{i_2}, K_{i_3}\rangle : i \in J\} \neq \phi_N$.

6.2 Definition

A NCTS (X,Γ) is called neutrosophic crisp compact iff each crisp neutrosophic open cover of X has a finite subcover.

6.1 Example

a) Let $X = N$ and let's consider the NCSs (neutrosophic crisp sets) given below:

$$A_1 = \langle\{2,3,4,\ldots\}, \phi, \phi\rangle,\quad A_2 = \langle\{3,4,\ldots\}, \phi, \{1\}\rangle,$$
$$A_3 = \langle\{4,5,6,\ldots\}, \phi, \{1,2\}\rangle, \ldots$$

$$A_n = \langle\{n+1, n+2, n+3, \ldots\}, \phi, \{1,2,3,\ldots n-1\}\rangle.$$

Then $\Gamma = \{\phi_N, X_N\} \cup \{A_n : n = 3,4,5,\ldots\}$ is a NCT on X and (X,Γ) is **a** neutrosophic crisp compact.

b) Let $X = (0,1)$ and let's take the NCSs

$$A_n = \left\langle X, \left(\frac{1}{n}, \frac{n-1}{n}\right), \phi, \left(0, \frac{1}{n}\right)\right\rangle, n = 3,4,5,\ldots \text{in X.}$$

In this case $\Gamma = \{\phi_N, X_N\} \cup \{A_n : n = 3,4,5,\ldots\}$ is an NCT on X, which is not a neutrosophic crisp compact.

6.1 Corollary

A NCTS (X,Γ) is a neutrosophic crisp compact iff every family $\{\langle X, G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$ of NCCSs in X having the FIP has nonempty intersection.

6.2 Corollary

Let (X,Γ_1), (Y,Γ_2) be NCTSs and $f: X \to Y$ be a continuous surjection. If (X,Γ_1) is a neutrosophic crisp compact, then so is (Y,Γ_2)

6.3 Definition

(a) If a family $\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$ of NCCSs in X satisfies the condition $A \subseteq \cup\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$, then it is called a neutrosophic crisp open cover of A.

(b) Let's consider a finite subfamily of a neutrosophic crisp open subcover of $\{\langle G_{i_1}, G_{i_2}, G_{i_3}\rangle : i \in J\}$.

A neutrosophic crisp set $A = \langle A_1, A_2, A_3\rangle$ in a NCTS (X,Γ) is called neutrosophic crisp compact iff every neutrosophic crisp open cover of A has a finite neutrosophic crisp open subcover.

6.3 Corollary

Let (X,Γ_1), (Y,Γ_2) be NCTSs and $f: X \to Y$ be a continuous surjection. If A is a neutrosophic crisp compact in (X,Γ_1), then so is $f(A)$ in (Y,Γ_2).

7 Conclusion

In this paper we introduce both the neutrosophic crisp topology and the neutrosophic crisp compact space, and we present properties related to them.

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