NEUTROSOPHIC CUBIC (α, β)-IDEALS IN SEMIGROUPS

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Abstract. We generalize the concept of fuzzy point, intutionistic fuzzy point, cubic point by introducing the concept of neutrosophic cubic point. Based on neutrosophic cubic point we generalized the idea of (α, β)-fuzzy ideals, (α, β)-intutionistic fuzzy ideals, (α, β)-cubic ideals ideals by initiating the new concept of neutrosophic Cubic (α, β)-ideals. Particularly we give the idea of neutrosophic cubic $(\in, \in \vee q)$-ideals (resp., sub-semigroups, generalized bi ideals, bi-ideals, quasi ideals, interior ideals, prime and semiprime ideals).

1. Introduction

Fuzzy sets which were introduced by Zadeh [1], deal with possibilistic uncertainty, connected with imprecision of states, perceptions and preferences. Based on the (interval-valued) fuzzy sets, [2], cubic sets were introduced by Jun et al. [3] which are the generalization of Atanassov's, Intuitionistic fuzzy sets [18]. They introduced the notions of sub-algebras/ideals, cubic -sub-algebras and closed cubic ideals in BCK/BCI-algebras, and then they investigated several properties, see, [4, 5, 6, 7, 8]. Murali [13] defined the concept of belongingness of fuzzy point to a fuzzy subset under natural equivalence on fuzzy subset. The idea of quasi coincidence of fuzzy point to a fuzzy set played important role. Bhakat et al. [12], gave the concept of (α, β)-fuzzy subgroup by using the " belong to" relation $(\in)$ and "quasi-coincidence" with relation (q) between a fuzzy point and a fuzzy subgroup. Shabir et al. developed (α, β)-fuzzy ideals in semigroups [17]. Madad et al. generalized the concept of Jun's cubic sets in semigroups [19] by defining the concept of cubic point. The concept of neutrosophic set (NS) developed by Smarandache [14], is a more general platform which extends the concepts of the classic set and fuzzy set [15] and then neutrosophic set theory is applied in various directions. Jun et al. gave the idea of neutrosophic cubic sets and their different basic operations, [9, 10, 11].

In this paper we introduce the idea of neutrosophic cubic point, using the " belong to" relation $(\in)$ and "quasi-coincidence" with relation (q) between a neutrosophic cubic point and Neutrosophic cubic set. Based on neutrosophic cubic point we initiate the theory of neutrosophic Cubic (α, β)-ideals, neutrosophic Cubic (α, β)-sub-semigroups, Neutrosophic Cubic (α, β)-generalized bi-ideals, Neutrosophic Cubic (α, β)-bi-ideals and Neutrosophic Cubic (α, β)-interior ideals with examples. Particularly we present the idea of neutrosophic Cubic $(\in, \in \vee q)$-ideals, neutrosophic Cubic $(\in, \in \vee q)$-sub-semigroups, Neutrosophic Cubic $(\in, \in \vee q)$-generalized

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bi-ideals, Neutrosophic Cubic $(\in, \in \vee q)$-bi-ideals, Neutrosophic Cubic $(\in, \in \vee q)$-quasi ideals, Neutrosophic Cubic $(\in, \in \vee q)$-interior ideals and Neutrosophic Cubic $(\in, \in \vee q)$-semiprime (resp., prime) ideals.

2. Preliminaries

A non-empty set S together with an associative binary operation "$\cdot$" is called a semigroup. A non-empty subset A of a semigroup S is called a sub-semigroup if $AA \subseteq A$. A non-empty subset A of S is left(resp., right) ideal of S if $SA \subseteq A\,(AS \subseteq A)$. A sub-semigroup B of S is bi-ideal of S if $BSB \subseteq B$ and interior ideal of S if $SAS \subseteq A$.

Now we recall the concept of interval valued fuzzy sets**.** An interval number is $\widetilde{a} = [a^-, a^+]$, where $0 \leqslant a^- \leqslant a^+ \leqslant 1$. Let $D[0,1]$ denote the family of all closed subintervals of $[0,1]$, i.e.,

$$D[0,1] = \{\widetilde{a} = [a^-, a^+] : a^- \leqslant a^+, \text{ for } a^-, a^+ \in I\}.$$

We define the operations "$\succeq$", "$\preceq$", "$=$", "rmin" and "rmax" in case of two elements in $D[0,1]$. We consider two elements $\widetilde{a} = [a^-, a^+]$ and $\widetilde{b} = [b^-, b^+]$ in $D[0,1]$. Then (i) $\widetilde{a} \succeq \widetilde{b}$ if and only if $a^- \geq b^-$ and $a^+ \geq b^+$, (ii) $\widetilde{a} \preceq \widetilde{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$, (iii) $\widetilde{a} = \widetilde{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$, (iv) $\text{rmin}\{\widetilde{a}, \widetilde{b}\} = [\min\{a^-, b^-\}, \min\{a^+, b^+\}]$, (v) $\text{rmax}\{\widetilde{a}, \widetilde{b}\} = [\max\{a^-, b^-\}, \max\{a^+, b^+\}]$. It is obvious that $(D[0,1], \preceq, \vee, \wedge)$ is a complete lattice with $\widetilde{0} = [0,0]$ as its least element and $\widetilde{1} = [1,1]$ as its greatest element. Let $\widetilde{a_i} \in D[0,1]$ where $i \in \Lambda$. We define $\underset{i\in\Lambda}{\text{rinf}}\widetilde{a}_i = [\inf_{i\in\Lambda} a_i^-, \inf_{i\in\Lambda} a_i^+]$ and $\underset{i\in\Lambda}{\text{rsup}}\widetilde{a}_i = [\sup_{i\in\Lambda} a_i^-, \sup_{i\in\Lambda} a_i^+]$. An interval valued fuzzy set (briefly, IVF-set) $\tilde{\mu}_A$ on X is defined as $\tilde{\mu}_A = \{\langle x, [\mu_A^-(x), \mu_A^+(x)]\rangle : x \in X\}$, where $\mu_A^-(x) \leqslant \mu_A^+(x)$, for all $x \in X$. Then the ordinary fuzzy sets $\mu_A^- : X \to [0,1]$ and $\mu_A^+ : X \to [0,1]$ are called a lower fuzzy set and an upper fuzzy set of $\widetilde{\mu}$, respectively. Let $\tilde{\mu}_A(x) = [\mu_A^-(x), \mu_A^+(x)]$. Then $A = \left\{\left\langle x, \tilde{\mu}_A(x)\right\rangle : x \in X\right\}$, where $\tilde{\mu}_A : X \longrightarrow D[0,1]$.

Definition 1. [3] *Let X be a non-empty set. A cubic set in X is a structure of the form: $C = \{(x, \widetilde{\mu}(x), \lambda(x)) | x \in X\}$ where $\widetilde{\mu}$ is an interval-valued fuzzy set in X and λ is a fuzzy set in X.*

Definition 2. [14] *A neutrosophic set (NS) in X is a structure of the form: $\lambda = \{\langle \lambda_T(x), \lambda_I(x), \lambda_F(x)\rangle \,| x \in X\}$ where $\lambda_T : X \to [0,1]$ is a truth membership function, $\lambda_I : X \to [0,1]$ is an indeterminate membership function, and $\lambda_F : X \to [0,1]$ is a false membership function.*

Definition 3. [16] *Let X be a non-empty set. An interval neutrosophic set (INS) in X is a structure of the form: $\widetilde{\mu} = \{\langle \widetilde{\mu}_T(x), \widetilde{\mu}_I(x), \widetilde{\mu}_F(x)\rangle \,| x \in X\}$ where $\widetilde{\mu}_T, \widetilde{\mu}_I$ and $\widetilde{\mu}_F$ are interval-valued fuzzy set in X, which are called an interval truth membership function, an interval indeterminacy membership function and an interval falsity membership function, respectively.*

Definition 4. [9] *Let X be the space of points and A be a NCS, we define a neutrosophic cubic set $A(x) = \left\langle x | \left(\tilde{\mu}, \lambda\right)\right\rangle$ where $\tilde{\mu}, (x) = \langle x\,(\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F)\rangle$ and $\lambda(x) = \langle x\,(\lambda_T, \lambda_I, \lambda_F)\rangle$ with $\widetilde{\mu}_T : X \to D[0,1]$, $\widetilde{\mu}_I : X \to D[0,1]$, $\widetilde{\mu}_F : X \to D[0,1]$ and $\lambda_T : X \to [0,1]$, $\lambda_I : X \to [0,1]$, $\lambda_F : X \to [0,1]$. We will briefly denote by*

$A(x) = \langle x(\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)\rangle$, *where* $[0,0] \preceq \widetilde{\mu}_T + \widetilde{\mu}_I + \widetilde{\mu}_F \preceq [3,3]$ *and* $0 \leq \lambda_T + \lambda_I + \lambda_F \leq 3$.

3. Neutrosophic Cubic Point

Here in this section we generalize the concept of fuzzy point, intutionistic fuzzy point, cubic point by introducing the concept of neutrosophic cubic point.

Definition 5. *Let* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *and* $B = \langle x, (\widetilde{\upsilon}_T, \widetilde{\upsilon}_I, \widetilde{\upsilon}_F, \eta_T, \eta_I, \eta_F)(x)\rangle$ *be two NCS and* $x \in X$ *, we define*

$A \circ B = \langle x,\ (\widetilde{\mu}_T \circ \widetilde{\upsilon}_T,\ \widetilde{\mu}_I \circ \widetilde{\mu}_I,\ \widetilde{\mu}_F \circ \widetilde{\upsilon}_F,\ \lambda_T \circ \eta_T,\ \lambda_I \circ \eta_I,\ \lambda_F \circ \eta_F)(x)\rangle$, *where*

$$\widetilde{\mu}_T \circ \widetilde{\upsilon}_T(x) = \begin{cases} \bigvee\limits_{x=yz} \operatorname{rmin}\{\widetilde{\mu}_T(y), \widetilde{\upsilon}_T(z)\} \\ \widetilde{0} \quad \text{otherwise} \end{cases} \text{and } \lambda_T \circ \eta_T(x) = \begin{cases} \bigwedge\limits_{x=yz} \max\{\lambda_T(y), \eta_T(z)\} \\ 1 \quad \text{otherwise} \end{cases}$$

$$\widetilde{\mu}_I \circ \widetilde{\upsilon}_I(x) = \begin{cases} \bigvee\limits_{x=yz} \operatorname{rmin}\{\widetilde{\mu}_I(y), \widetilde{\upsilon}_I(z)\} \\ \widetilde{0} \quad \text{otherwise} \end{cases} \text{and } \lambda_T \circ \eta_T(x) = \begin{cases} \bigwedge\limits_{x=yz} \max\{\lambda_I(y), \eta_I(z)\} \\ 1 \quad \text{otherwise} \end{cases}$$

$$\widetilde{\mu}_F \circ \widetilde{\upsilon}_F(x) = \begin{cases} \bigvee\limits_{x=yz} \operatorname{rmin}\{\widetilde{\mu}_F(y), \widetilde{\upsilon}_F(z)\} \\ \widetilde{0} \quad \text{otherwise} \end{cases} \text{and } \lambda_F \circ \eta_F(x) = \begin{cases} \bigwedge\limits_{x=yz} \max\{\lambda_F(y), \eta_F(z)\} \\ 1 \quad \text{otherwise} \end{cases}$$

Definition 6. *Let* $A = \langle x, (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *and* $B = \langle x, (\widetilde{\upsilon}_T, \widetilde{\upsilon}_I, \widetilde{\upsilon}_F, \eta_T, \eta_I, \eta_F)(x)\rangle$ *be two NC sets, we define*

$$A \subseteq B, \text{ if } \widetilde{\mu}_T \succeq \widetilde{\upsilon}_T, \widetilde{\mu}_I \succeq \widetilde{\upsilon}_I, \widetilde{\mu}_F \succeq \widetilde{\upsilon}_F \text{ and } \lambda_T \leq \eta_T, \lambda_I \leq \eta_I, \lambda_F \leq \eta_F.$$

Definition 7. *Let* $\widetilde{\alpha} = \widetilde{t}, \widetilde{i}, \widetilde{f} \in D(0,1]$ *and* $\beta = s, o, g \in [0,1)$. *Then by neutrosophic cubic point (NCP) we mean* $x_{(\widetilde{\alpha},\beta)}(y) = \langle x_{\widetilde{\alpha}}(y), x_\beta(y)\rangle$ *where*

$$x_{\widetilde{\alpha}}(y) = \begin{cases} \widetilde{\alpha} & \text{if } x = y \\ \widetilde{0} & \text{otherwise} \end{cases} \quad \text{and } x_\beta(y) = \begin{cases} \beta & \text{if } x = y \\ 1 & \text{otherwise.} \end{cases}$$

- For any neutrosophic cubic set $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ and for a neutrosophic cubic point $x_{(\widetilde{\alpha},\beta)}$, then

  (i) $x_{(\widetilde{\alpha},\beta)} \in A$ if $\widetilde{\mu}_T(x) \succeq \widetilde{t}$, $\widetilde{\mu}_I(x) \succeq \widetilde{i}$, $\widetilde{\mu}_F(x) \succeq \widetilde{f}$ and $\lambda_T(x) \leq s$, $\lambda_I(x) \leq o$, $\lambda_F(x) \leq g$.

  (ii) $x_{(\widetilde{\alpha},\beta)} qA$ if $\widetilde{\mu}_T(x) + \widetilde{t} \succ \widetilde{1}$, $\widetilde{\mu}_I(x) + \widetilde{i} \succ \widetilde{1}$, $\widetilde{\mu}_F(x) + \widetilde{f} \succ \widetilde{1}$ and $\lambda_T(x) + s < 1$, $\lambda_I(x) + o < 1$, $\lambda_F(x) + g < 1$.

  (iii) $x_{(\widetilde{\alpha},\beta)} \in \vee qA$ if $x_{(\widetilde{\alpha},\beta)} \in A$ or $x_{(\widetilde{\alpha},\beta)} qA$.

  (iv) $x_{(\widetilde{\alpha},\beta)} \in \vee qA$ if $x_{(\widetilde{\alpha},\beta)} \in A$ and $x_{(\widetilde{\alpha},\beta)} qA$.

  (v) $x_{(\widetilde{\alpha},\beta)} q_k \Xi$ if $\widetilde{\mu}_T(x) + \widetilde{\alpha} + \widetilde{k}_1 \succ \widetilde{1}$, $\widetilde{\mu}_I(x) + \widetilde{\alpha} + \widetilde{k}_2 \succ \widetilde{1}$, $\widetilde{\mu}_F(x) + \widetilde{\alpha} + \widetilde{k}_3 \succ \widetilde{1}$ and $\lambda_T(x) + \beta + k_4 < 1$, $\lambda_I(x) + \beta + k_5 < 1$, $\lambda_F(x) + \beta + k_6 < 1$,where $\widetilde{k}_1, \widetilde{k}_2, \widetilde{k}_3 \in D(0,1]$ and $k_4, k_5, k_6 \in [0,1)$.
- Let $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ be a neutrosophic cubic set subset of S such that $\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F \preceq \widetilde{0.5}$ and $\lambda_T, \lambda_I, \lambda_F \geq 0.5$. Then $\widetilde{t} \in D(0,1], \widetilde{i} \in D(0,1], \widetilde{f} \in D(0,1]$ and $s \in [0,1), o \in [0,1), g \in [0,1)$ be such that $x_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)} \in \wedge qA$. Then $(\widetilde{\mu}_T(x) \succeq \widetilde{t},\ \widetilde{\mu}_I(x) \succeq \widetilde{i},\ \widetilde{\mu}_F(x) \succeq \widetilde{f}\ )$, $(\lambda_T(x) \leq s,\ \lambda_I(x) \leq o,\ \lambda_F(x) \leq g)$ and $(\widetilde{\mu}_T(x) + \widetilde{t} \succ \widetilde{1}, \widetilde{\mu}_I(x) + \widetilde{i} \succ \widetilde{1}, \widetilde{\mu}_F(x) + \widetilde{f} \succ \widetilde{1})$, $(\lambda_T(x) + s < 1, \lambda_I(x) + o < 1, \lambda_F(x) + g < 1)$. It

follows that $\widetilde{1} \prec \widetilde{\mu}_T(x) + \widetilde{\mu}_T(x) = 2\widetilde{\mu}_T(x), \widetilde{1} \prec \widetilde{\mu}_I(x) + \widetilde{\mu}_I(x) = 2\widetilde{\mu}_I(x), \widetilde{1} \prec \widetilde{\mu}_F(x) + \widetilde{\mu}_F(x) = 2\widetilde{\mu}_F(x)$ and $1 > \lambda_T(x) + \lambda_T(x) = 2\lambda_T(x)$, $1 > \lambda_I(x) + \lambda_I(x) = 2\lambda_I(x), 1 > \lambda_F(x) + \lambda_F(x) = 2\lambda_F(x)$. So $\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F \succ \widetilde{0.5}$ and $\lambda_T, \lambda_I, \lambda_F < 0.5$. This means that $x_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)} \in \wedge q = \phi$. So we will omit the case $\alpha = \in \wedge q$.

- Let $NCP(S)$ be the collection of all neutrosophic cubic point in a semigroup S. Then

$$\begin{aligned} & x_{(\widetilde{t_1},\widetilde{i_1},\widetilde{f_1},s_1,o_1,g_1)} \cdot y_{(\widetilde{t_2},\widetilde{i_2},\widetilde{f_2},s_2,o_2,g_2)} \\ = \; & (xy)_{(xy)_{(\mathrm{rmin}\{\widetilde{t_1},\widetilde{t_2}\},\mathrm{rmin}\{\widetilde{i_1},\widetilde{i_2}\},\mathrm{rmin}\{\widetilde{f_1},\widetilde{f_2}\},\max\{s_1,s_2\},\max\{o_1,o_2\},\max\{g,g_2\})}} \in NCP(S). \end{aligned}$$

Then $NCP(S)$ becomes a semigroup and it is a sub-semigroup of $\mathcal{NC}(\mathcal{S})$.

Definition 8. *Let S be a semigroup. Then the neutrosophic cubic characteristic function*

$$\chi_A = (\chi_{\widetilde{\mu}_T}, \chi_{\widetilde{\mu}_I}, \chi_{\widetilde{\mu}_F}, \chi_{\lambda_T}, \chi_{\lambda_I}, \chi_{\lambda_F})$$

of $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ is defined as

$$\begin{aligned}
\chi_{\widetilde{\mu}_T}(x) &= \begin{cases} [1,1] & \text{if } x \in A \\ [0,0] & \text{if } x \notin A \end{cases} \quad \text{and} \quad \chi_{\lambda_T}(x) = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{if } x \notin A \end{cases} \\
\chi_{\widetilde{\mu}_I}(x) &= \begin{cases} [1,1] & \text{if } x \in A \\ [0,0] & \text{if } x \notin A \end{cases} \quad \text{and} \quad \chi_{\lambda_I}(x) = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{if } x \notin A \end{cases} \\
\chi_{\widetilde{\mu}_F}(x) &= \begin{cases} [1,1] & \text{if } x \in A \\ [0,0] & \text{if } x \notin A \end{cases} \quad \text{and} \quad \chi_{\lambda_F}(x) = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{if } x \notin A \end{cases}
\end{aligned}$$

4. Neutrosophic Cubic (α, β)-sub-semigroups

Based on neutrosophic cubic point we generalized the idea of (α, β)-fuzzy ideals, (α, β)-intutionistic fuzzy ideals, (α, β)-cubic ideals ideals by initiating the new concept of neutrosophic Cubic (α, β)-sub-semigroups.

Definition 9. *A neutrosophic cubic subset $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ of a semigroup S is called the neutrosophic cubic (α, β)-sub-semigroup of S, where $\alpha, \beta \in \{\in, q, \in \vee q$ or $\in \wedge q\}$ but $\alpha \neq \in \wedge q$ if for all x, y in S and for all $\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, \widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2} \in D[0,1]$ and $s_1, o_1, g_1, s_2, o_2, g_2 \in [0,1)$ such that $x_{(\widetilde{t_1},\widetilde{i_1},\widetilde{f_1},s_1,o_1,g_1)} \alpha A$ and*

$$y_{(\widetilde{t_2},\widetilde{i_2},\widetilde{f_2},s_2,o_2,g_2)} \alpha A \Longrightarrow (xy)_{(\mathrm{rmin}\{\widetilde{t_1},\widetilde{t_2}\},\mathrm{rmin}\{\widetilde{i_1},\widetilde{i_2}\},\mathrm{rmin}\{\widetilde{f_1},\widetilde{f_2}\},\max\{s_1,s_2\},\max\{o_1,o_2\},\max\{g,g_2\})} \beta A.$$

Example 1. *Consider a semigroup $S = \{a, b, c\}$ with the following table,*

.	a	b	c
a	c	c	c
b	c	c	a
c	c	b	c

Define a neutrosophic cubic set $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ in S by

S	$\widetilde{\mu}_T$	$\widetilde{\mu}_I$	$\widetilde{\mu}_F$	λ_T	λ_I	λ_F
a	[0.3, 0.6]	[0.5, 0.7]	[0.6, 0.7]	0.4	0.6	0.8
b	[0.2, 0.4]	[0.3, 0.4]	[0.2, 0.5]	0.6	0.7	0.9
c	[0.7, 0.9]	[0.8, 0.9]	[0.7, 0.8]	0.2	0.3	0.4

where

$$\begin{aligned}\widetilde{t}_1 &= [0.2, 0.3], \widetilde{t}_2 = [0.1, 0.2], \widetilde{i}_1 = [0.1, 0.25], \widetilde{i}_2 = [0.2, 0.4], \widetilde{f}_1 = [0.01, 0.02], \widetilde{f}_2 = [0.1, 0.2] \\ \text{and } s_1 &= 0.7, s_2 = 0.8, o_1 = 0.7, o_2 = 0.9, g_1 = 0.9, g_2 = 0.95\end{aligned}$$

It is easy to verify that $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is a neutrosophic cubic* (α, β)*-sub-semigroup of* S.

The following lemma shows that every neutrosophic cubic sub-semigroup is neutrosophic cubic $(\in, \in)$-sub-semigroup of S.

Lemma 1. *A neutrosophic cubic* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of a semigroup* S *is neutrosophic cubic sub-semigroup if and only if, for all* $x, y \in S$ *and for all* $\widetilde{t}_1, \widetilde{i}_1, \widetilde{f}_1, \widetilde{t}_2, \widetilde{i}_2, \widetilde{f}_2 \in D[0,1]$ *and* $s_1, o_1, g_1, s_2, o_2, g_2 \in [0,1)$ *such that*

$$\begin{aligned}x_{(\widetilde{t}_1, \widetilde{i}_1, \widetilde{f}_1, s_1, o_1, g_1)} &\in A \text{ and } y_{(\widetilde{t}_2, \widetilde{i}_2, \widetilde{f}_2, s_2, o_2, g_2)} \in A \\ &\Longrightarrow (xy)_{(\mathrm{rmin}\{\widetilde{t}_1, \widetilde{t}_2\}, \mathrm{rmin}\{\widetilde{i}_1, \widetilde{i}_2\}, \mathrm{rmin}\{\widetilde{f}_1, \widetilde{f}_2\}, \max\{s_1, s_2\}, \max\{o_1, o_2\}, \max\{g_1, g_2\})} \in A\end{aligned}$$

Proof. Let $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ be a neutrosophic cubic sub-semigroup of S. Let $x, y \in S$ and $\widetilde{t}_1, \widetilde{i}_1, \widetilde{f}_1, \widetilde{t}_2, \widetilde{i}_2, \widetilde{f}_2 \in D(0,1]$ and $s_1, o_1, g_1, s_2, o_2, g_2 \in [0,1)$ be such that $x_{(\widetilde{t}_1, \widetilde{i}_1, \widetilde{f}_1, s_1, o_1, g_1)} \in A$ and $y_{(\widetilde{t}_2, \widetilde{i}_2, \widetilde{f}_2, s_2, o_2, g_2)} \in A$ then

$$\begin{aligned}\widetilde{\mu}_T(x) &\succeq \widetilde{t}_1, \widetilde{\mu}_I(x) \succeq \widetilde{i}_1, \widetilde{\mu}_F(x) \succeq \widetilde{f}_1, \widetilde{\mu}_T(y) \succeq \widetilde{t}_2, \widetilde{\mu}_I(y) \succeq \widetilde{i}_2, \widetilde{\mu}_F(y) \succeq \widetilde{f}_2, \\ \lambda_T(x) &\leq s_1, \lambda_I(x) \leq o_1, \lambda_F(x) \leq g_1 \ \lambda_T(y) \leq s_2, \lambda_I(y) \leq o_2, \lambda_F(y) \leq g_2\end{aligned}$$

So

$$\begin{aligned}\widetilde{\mu}_T(xy) &\succeq \mathrm{rmin}\{\widetilde{\mu}_T(x), \widetilde{\mu}_T(y)\} \succeq \mathrm{rmin}\{\widetilde{t}_1, \widetilde{t}_2\} \Rightarrow (xy)_{\mathrm{rmin}\{\widetilde{t}_1, \widetilde{t}_2\}} \in \widetilde{\mu}_T \\ \widetilde{\mu}_I(xy) &\succeq \mathrm{rmin}\{\widetilde{\mu}_I(x), \widetilde{\mu}_I(y)\} \succeq \mathrm{rmin}\{\widetilde{i}_1, \widetilde{i}_2\} \Rightarrow (xy)_{\mathrm{rmin}\{\widetilde{i}_1, \widetilde{i}_2\}} \in \widetilde{\mu}_I \\ \widetilde{\mu}_F(xy) &\succeq \mathrm{rmin}\{\widetilde{\mu}_F(x), \widetilde{\mu}_F(y)\} \succeq \mathrm{rmin}\{\widetilde{f}_1, \widetilde{f}_2\} \Rightarrow (xy)_{\mathrm{rmin}\{\widetilde{f}_1, \widetilde{f}_2\}} \in \widetilde{\mu}_F \\ \lambda_T(xy) &\leq \max\{\lambda_T(x), \lambda_T(y)\} \succeq \max\{s_1, s_2\} \Rightarrow (xy)_{\max\{s_1, s_2\}} \in \lambda_T \\ \lambda_I(xy) &\leq \max\{\lambda_I(x), \lambda_I(y)\} \succeq \max\{o_1, o_2\} \Rightarrow (xy)_{\max\{o_1, o_2\}} \in \lambda_I \\ \lambda_F(xy) &\leq \max\{\lambda_F(x), \lambda_F(y)\} \succeq \max\{g_1, g_2\} \Rightarrow (xy)_{\max\{g_1, g_2\}} \in \lambda_F\end{aligned}$$

Conversely, let $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ satisfies the given condition. We prove that $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ is neutrosophic cubic sub-semigroup. On contrary for $x, y \in S$ and $\widetilde{t}, \widetilde{s}, \widetilde{i} \in D(0,1]$, $s, o, g \in [0,1)$ such that

$$\begin{aligned}\widetilde{\mu}_T(xy) &\prec \widetilde{t} \preceq \mathrm{rmin}\{\widetilde{\mu}_T(x), \widetilde{\mu}_T(y)\} \Rightarrow x_{\widetilde{t}} \in \widetilde{\mu}_T \text{ and } \ y_{\widetilde{t}} \in \widetilde{\mu}_T \text{ but } \ (xy)_{\widetilde{t}} \notin \widetilde{\mu}_T \\ \widetilde{\mu}_I(xy) &\prec \widetilde{s} \preceq \mathrm{rmin}\{\widetilde{\mu}_I(x), \widetilde{\mu}_I(y)\} \Rightarrow x_{\widetilde{s}} \in \widetilde{\mu}_I \text{ and } \ y_{\widetilde{s}} \in \widetilde{\mu}_I \text{ but } \ (xy)_{\widetilde{s}} \notin \widetilde{\mu}_I \\ \widetilde{\mu}_F(xy) &\prec \widetilde{i} \preceq \mathrm{rmin}\{\widetilde{\mu}_F(x), \widetilde{\mu}_F(y)\} \Rightarrow x_{\widetilde{i}} \in \widetilde{\mu}_I \text{ and } \ y_{\widetilde{i}} \in \widetilde{\mu}_I \text{ but } \ (xy)_{\widetilde{i}} \notin \widetilde{\mu}_F \\ \lambda_T(xy) &> s \geq \max\{\lambda_T(x), \lambda_T(y)\} \Rightarrow x_s \in \lambda_T \text{ and } y_s \in \lambda_T \text{ but } \ (xy)_s \notin \lambda_T \\ \lambda_I(xy) &> o \geq \max\{\lambda_I(x), \lambda_I(y)\} \Rightarrow x_o \in \lambda_I \text{ and } y_o \in \lambda_I \text{ but } \ (xy)_o \notin \lambda_I \\ \lambda_F(xy) &> g \geq \max\{\lambda_F(x), \lambda_F(y)\} \Rightarrow x_g \in \lambda_F \text{ and } y_g \in \lambda_F \text{ but } \ (xy)_g \notin \lambda_F\end{aligned}$$

which is a contradiction. Hence $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ is neutrosophic cubic sub-semigroup. $\square$

Theorem 1. *Let* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic* (α, β)*-sub-semigroup of* S. *Then the set* $A_{(\widetilde{0},1)} = \{x \in s| \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F)(x) \succ \widetilde{0}$ *and* $(\lambda_T, \lambda_I, \lambda_F)(x) < 1\}$ *is sub-semigroup of* S.

Proof. Let $x, y \in A_{(\widetilde{0},1)}$. Then

$$\begin{aligned} \widetilde{\mu}_T(x) &\succ \widetilde{0}, \widetilde{\mu}_T(y) \succ \widetilde{0}, \widetilde{\mu}_I(x) \succ \widetilde{0}, \widetilde{\mu}_I(y) \succ \widetilde{0}, \widetilde{\mu}_F(x) \succ \widetilde{0}, \widetilde{\mu}_F(y) \succ \widetilde{0} \\ \lambda_T(x) &< 1, \lambda_T(y) < 1, \lambda_I(x) < 1, \lambda_I(y) < 1, \lambda_F(x) < 1, \lambda_F(y) < 1 \end{aligned}$$

Let $\widetilde{\mu}_T(xy) = \widetilde{0}, \widetilde{\mu}_I(xy) = \widetilde{0}, \widetilde{\mu}_F(xy) = \widetilde{0}$ and $\lambda_T(xy) = 1, \lambda_I(xy) = 1, \lambda_F(xy) = 1$. If $\alpha \in \{\in, \in \vee q\}$, then

$$x_{\widetilde{\mu}_T(x)}\alpha A, \ x_{\widetilde{\mu}_T(y)}\alpha A, x_{\widetilde{\mu}_I(x)}\alpha A, \ x_{\widetilde{\mu}_I(y)}\alpha A, x_{\widetilde{\mu}_F(x)}\alpha A, \ x_{\widetilde{\mu}_F(y)}\alpha A$$

and

$$x_{\lambda_T(x)}\alpha A, x_{\lambda_T(y)}\alpha A, x_{\lambda_I(x)}\alpha A, x_{\lambda_I(y)}\alpha A, x_{\lambda_F(x)}\alpha A, \ x_{\lambda_F(y)}\alpha A$$

but

$$\begin{aligned} \widetilde{\mu}_T(xy) &= \widetilde{0} \prec \text{rmin}\{\widetilde{\mu}_T(x), \widetilde{\mu}_T(y)\} \text{ and } \widetilde{\mu}_T(xy) + \text{rmin}\{\widetilde{\mu}_T(x), \widetilde{\mu}_T(y)\} \preceq \widetilde{0} + \widetilde{1} = \widetilde{1} \\ \widetilde{\mu}_I(xy) &= \widetilde{0} \prec \text{rmin}\{\widetilde{\mu}_I(x), \widetilde{\mu}_I(y)\} \text{ and } \widetilde{\mu}_I(xy) + \text{rmin}\{\widetilde{\mu}_I(x), \widetilde{\mu}_I(y)\} \preceq \widetilde{0} + \widetilde{1} = \widetilde{1} \\ \widetilde{\mu}_F(xy) &= \widetilde{0} \prec \text{rmin}\{\widetilde{\mu}_F(x), \widetilde{\mu}_F(y)\} \text{ and } \widetilde{\mu}_F(xy) + \text{rmin}\{\widetilde{\mu}_F(x), \widetilde{\mu}_F(y)\} \preceq \widetilde{0} + \widetilde{1} = \widetilde{1} \\ \lambda_T(xy) &= 1 > \max\{\lambda_I(x), \ \lambda_I(y)\} \text{ and } \lambda_T(xy) + \max\{\lambda_T(x), \lambda_T(y)\} \geq 1 \\ \lambda_I(xy) &= 1 > \max\{\lambda_I(x), \ \lambda_I(y)\} \text{ and } \lambda_I(xy) + \max\{\lambda_I(x), \lambda_I(y)\} \geq 1 \\ \lambda_F(xy) &= 1 > \max\{\lambda_F(x), \ \lambda_F(y)\} \text{ and } \lambda_F(xy) + \max\{\lambda_F(x), \lambda_F(y)\} \geq 1 \end{aligned}$$

So for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$

$$\begin{aligned} &(xy)_{\text{rmin}\{\widetilde{\mu}_T(x), \widetilde{\mu}_T(y)\}} \overline{\beta} A, (xy)_{\text{rmin}\{\widetilde{\mu}_I(x), \widetilde{\mu}_I(y)\}} \overline{\beta} A, (xy)_{\text{rmin}\{\widetilde{\mu}_F(x), \widetilde{\mu}_F(y)\}} \overline{\beta} A \\ &(xy)_{\max\{\lambda_T(x), \lambda_T(y)\}} \overline{\beta} A, (xy)_{\max\{\lambda_I(x), \lambda_I(y)\}} \overline{\beta} A, (xy)_{\max\{\lambda_F(x), \lambda_F(y)\}} \overline{\beta} A \end{aligned}$$

Hence $\widetilde{\mu}_T(xy) \succ \widetilde{0}, \widetilde{\mu}_I(xy) \succ \widetilde{0}, \widetilde{\mu}_F(xy) \succ \widetilde{0}$ and $\lambda_T(xy) < 1, \lambda_I(xy) < 1, \lambda_F(xy) < 1$. So $xy \in A_{(\widetilde{0},1)}$. Also for every $\beta \in \{\in, q, \in \vee q, \in \wedge q\}$

$$\begin{aligned} &x_1 q \widetilde{\mu}_T, \ x_1 q \widetilde{\mu}_I, x_1 q \widetilde{\mu}_F, \ y_1 q \widetilde{\mu}_T, \ y_1 q \widetilde{\mu}_I, y_1 q \widetilde{\mu}_F \\ &x_1 q \lambda_T, x_1 q \lambda_I, x_1 q \lambda_F, y_1 q \lambda_T, y_1 q \lambda_I, y_1 q \lambda_F \end{aligned}$$

but

$$(xy)_1 \overline{\beta} \widetilde{\mu}_T, (xy)_1 \overline{\beta} \widetilde{\mu}_I, (xy)_1 \overline{\beta} \widetilde{\mu}_F, (xy)_1 \overline{\beta} \lambda_T, (xy)_1 \overline{\beta} \lambda_I, (xy)_1 \overline{\beta} \lambda_F$$

So $A_{(\widetilde{0},1)} = \{x \in s| \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F)(x) \succ \widetilde{0}$ and $(\lambda_T, \lambda_I, \lambda_F)(x) < 1\}$ is sub-semigroup of S. $\square$

Theorem 2. *Let* B *be a sub-semigroup of* S *and let* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic subset of* S *such that*

$$\begin{aligned} \widetilde{\mu}_T(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - B \\ \succeq \widetilde{0.5} & x \in B \end{cases} \quad \text{and} \quad \lambda_T(x) = \begin{cases} 1 & \text{if } x \in S - B \\ \leq 0.5 & x \in B \end{cases} \\ \widetilde{\mu}_I(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - B \\ \succeq \widetilde{0.5} & x \in B \end{cases} \quad \text{and} \quad \lambda_I(x) = \begin{cases} 1 & \text{if } x \in S - B \\ \leq 0.5 & x \in B \end{cases} \\ \widetilde{\mu}_F(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - B \\ \succeq \widetilde{0.5} & x \in B \end{cases} \quad \text{and} \quad \lambda_F(x) = \begin{cases} 1 & \text{if } x \in S - B \\ \leq 0.5 & x \in B \end{cases} \end{aligned}$$

Then i) $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is the neutrosophic cubic* $(q, \in \vee q)$*-sub-semigroup of* S*.*

ii) $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is the neutrosophic cubic* $(\in, \in \vee q)$*-sub-semigroup of* S*.*

Proof. i) Let $x, y \in S$ and $\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, \widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2} \in D(0,1]$ and $s_1, o_1, g_1, s_2, o_2, g_2 \in [0,1)$ be such that $x_{(\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, s_1, o_1, g_1)} qA$ and $y_{(\widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2}, s_2, o_2, g_2)} qA$. Then

$$\begin{aligned}
\widetilde{\mu}_T(x) + \widetilde{t_1} &\succ \widetilde{1},\ \lambda_T(x) + s_1 < 1 \text{ and } \widetilde{\mu}_T(y) + \widetilde{t_2} \succ \widetilde{1},\ \lambda_T(y) + s_2 < 1 \\
\widetilde{\mu}_I(x) + \widetilde{i_1} &\succ \widetilde{1},\ \lambda_I(x) + o_1 < 1 \text{ and } \widetilde{\mu}_I(y) + \widetilde{i_2} \succ \widetilde{1},\ \lambda_I(y) + o_2 < 1 \\
\widetilde{\mu}_F(x) + \widetilde{f_1} &\succ \widetilde{1},\ \lambda_F(x) + g_1 < 1 \text{ and } \widetilde{\mu}_F(y) + \widetilde{f_2} \succ \widetilde{1},\ \lambda_F(y) + g_2 < 1
\end{aligned}$$

So $x, y \in L \Longrightarrow xy \in L$. Thus if

$$\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, \widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2} \preceq \widetilde{0.5} \text{ and } s_1, o_1, g_1, s_2, o_2, g_2 \geq 0.5$$

so

$$\begin{aligned}
\widetilde{\mu}_T(x) &\succeq \widetilde{0.5} \succeq \widetilde{t_1},\ \lambda_T(x) \leq 0.5 \leq s_1 \text{ and } \widetilde{\mu}_T(y) \succeq \widetilde{0.5} \succeq \widetilde{t_2},\ \lambda_T(y) \leq 0.5 \leq s_2 \\
\widetilde{\mu}_I(x) &\succeq \widetilde{0.5} \succeq \widetilde{i_1},\ \lambda_I(x) \leq 0.5 \leq o_1 \text{ and } \widetilde{\mu}_I(y) \succeq \widetilde{0.5} \succeq \widetilde{i_2},\ \lambda_I(y) \leq 0.5 \leq o_2 \\
\widetilde{\mu}_F(x) &\succeq \widetilde{0.5} \succeq \widetilde{f_1},\ \lambda_F(x) \leq 0.5 \leq g_1 \text{ and } \widetilde{\mu}_F(y) \succeq \widetilde{0.5} \succeq \widetilde{f_2},\ \lambda_F(y) \leq 0.5 \leq g_2
\end{aligned}$$

Hence $(xy)_{(\mathrm{rmin}\{\widetilde{t_1}, \widetilde{t_2}\}, \mathrm{rmin}\{\widetilde{i_1}, \widetilde{i_2}\}, \mathrm{rmin}\{\widetilde{f_1}, \widetilde{f_2}\}, \max\{s_1, s_2\}, \max\{o_1, o_2\}, \max\{g_1, g_2\})} \in A$. If

$$\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, \widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2} \succ \widetilde{0.5} \text{ and } s_1, o_1, g_1, s_2, o_2, g_2 < 0.5$$

so

$$\begin{aligned}
\widetilde{\mu}_T(x) + \widetilde{t_1} &\succeq \widetilde{0.5} + \widetilde{0.5} = \widetilde{1},\ \lambda_T(x) + s_1 \leq 0.5 + 0.5 = 1 \\
\widetilde{\mu}_T(y) + \widetilde{t_2} &\succeq \widetilde{0.5} + \widetilde{0.5} = \widetilde{1},\ \lambda_T(y) + s_2 \leq 0.5 + 0.5 = 1 \\
\widetilde{\mu}_I(x) + \widetilde{i_1} &\succeq \widetilde{0.5} + \widetilde{0.5} = \widetilde{1},\ \lambda_I(x) + o_1 \leq 0.5 + 0.5 = 1 \\
\widetilde{\mu}_I(y) + \widetilde{i_2} &\succeq \widetilde{0.5} + \widetilde{0.5} = \widetilde{1},\ \lambda_I(y) + s_2 \leq 0.5 + 0.5 = 1 \\
\widetilde{\mu}_F(x) + \widetilde{f_1} &\succeq \widetilde{0.5} + \widetilde{0.5} = \widetilde{1},\ \lambda_F(x) + g_1 \leq 0.5 + 0.5 = 1 \\
\widetilde{\mu}_F(y) + \widetilde{f_2} &\succeq \widetilde{0.5} + \widetilde{0.5} = \widetilde{1},\ \lambda_F(y) + g_2 \leq 0.5 + 0.5 = 1
\end{aligned}$$

Hence

$$(xy)_{(\mathrm{rmin}\{\widetilde{t_1}, \widetilde{t_2}\}, \mathrm{rmin}\{\widetilde{i_1}, \widetilde{i_2}\}, \mathrm{rmin}\{\widetilde{f_1}, \widetilde{f_2}\}, \max\{s_1, s_2\}, \max\{o_1, o_2\}, \max\{g_1, g_2\})}\, qA.$$

Thus

$$(xy)_{(\mathrm{rmin}\{\widetilde{t_1}, \widetilde{t_2}\}, \mathrm{rmin}\{\widetilde{i_1}, \widetilde{i_2}\}, \mathrm{rmin}\{\widetilde{f_1}, \widetilde{f_2}\}, \max\{s_1, s_2\}, \max\{o_1, o_2\}, \max\{g_1, g_2\})} \in \vee qA.$$

ii) Its proof is similar to the proof of (i). □

Theorem 3. *A neutrosophic cubic set* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of* S *is the neutrosophic cubic* $(\in, \in \vee q)$*-sub-semigroup of* S *if and only if*

$$\begin{aligned}
\widetilde{\mu}_T(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_T(x), \widetilde{\mu}_T(y), \widetilde{0.5}) \text{ and } \lambda_T(xy) \leq \max(\lambda_T(x), \lambda_T(y), 0.5) \\
\widetilde{\mu}_I(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_I(x), \widetilde{\mu}_I(y), \widetilde{0.5}) \text{ and } \lambda_I(xy) \leq \max(\lambda_I(x), \lambda_I(y), 0.5) \\
\widetilde{\mu}_F(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_F(x), \widetilde{\mu}_F(y), \widetilde{0.5}) \text{ and } \lambda_F(xy) \leq \max(\lambda_F(x), \lambda_F(y), 0.5)
\end{aligned}$$

Proof. Let $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ is the neutrosophic cubic $(\in, \in \vee q)$-sub-semigroup of S, and assume on contrary that there exist $x, y \in S$ and $\widetilde{t}, \widetilde{i}, \widetilde{f} \in D[0,1]$ *and* $s, o, g \in [0,1)$ such that

$$
\begin{aligned}
\widetilde{\mu}_T(xy) &\prec \widetilde{t} \preceq \text{rmin}\left\{\widetilde{\mu}_T(x), \widetilde{\mu}_T(y), \widetilde{0.5}\right\} \Rightarrow x_{\widetilde{t}} \in \widetilde{\mu}_T \text{ and } y_{\widetilde{t}} \in \widetilde{\mu}_T \text{ but } (xy)_{\widetilde{t}} \overline{\in \vee q} \widetilde{\mu}_T \\
\widetilde{\mu}_I(xy) &\prec \widetilde{s} \preceq \text{rmin}\left\{\widetilde{\mu}_I(x), \widetilde{\mu}_I(y), \widetilde{0.5}\right\} \Rightarrow x_{\widetilde{s}} \in \widetilde{\mu}_I \text{ and } y_{\widetilde{s}} \in \widetilde{\mu}_I \text{ but } (xy)_{\widetilde{s}} \overline{\in \vee q} \widetilde{\mu}_I \\
\widetilde{\mu}_F(xy) &\prec \widetilde{i} \preceq \text{rmin}\left\{\widetilde{\mu}_F(x), \widetilde{\mu}_F(y), \widetilde{0.5}\right\} \Rightarrow x_{\widetilde{i}} \in \widetilde{\mu}_I \text{ and } y_{\widetilde{i}} \in \widetilde{\mu}_I \text{ but } (xy)_{\widetilde{i}} \overline{\in \vee q} \widetilde{\mu}_F \\
\lambda_T(xy) &> s \geq \max\{\lambda_T(x), \lambda_T(y), 0.5\} \Rightarrow x_s \in \lambda_T \text{ and } y_s \in \lambda_T \text{ but } (xy)_s \overline{\in \vee q} \lambda_T \\
\lambda_I(xy) &> o \geq \max\{\lambda_I(x), \lambda_I(y), 0.5\} \Rightarrow x_o \in \lambda_I \text{ and } y_o \in \lambda_I \text{ but } (xy)_o \overline{\in \vee q} \lambda_I \\
\lambda_F(xy) &> g \geq \max\{\lambda_F(x), \lambda_F(y), 0.5\} \Rightarrow x_g \in \lambda_F \text{ and } y_g \in \lambda_F \text{ but } (xy)_g \overline{\in \vee q} \lambda_F
\end{aligned}
$$

which is a contradiction. Hence $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ satisfies the given conditions. Conversely, let $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ satisfies the given conditions. Let $x, y \in S$ and $\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, \widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2} \in D(0,1]$ and $s_1, o_1, g_1, s_2, o_2, g_2 \in [0,1)$ be such that $x_{\left(\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, s_1, o_1, g_1\right)} \in A$ and $y_{\left(\widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2}, s_2, o_2, g_2\right)} \in A$ then

$$
\begin{aligned}
\widetilde{\mu}_T(x) &\succeq \widetilde{t_1}, \widetilde{\mu}_I(x) \succeq \widetilde{i_1}, \widetilde{\mu}_F(x) \succeq \widetilde{f_1}, \widetilde{\mu}_T(y) \succeq \widetilde{t_2}, \widetilde{\mu}_I(y) \succeq \widetilde{i_2}, \widetilde{\mu}_F(y) \succeq \widetilde{f_2}, \\
\lambda_T(x) &\leq s_1, \lambda_I(x) \leq o_1, \lambda_F(x) \leq g_1 \ \lambda_T(y) \leq s_2, \lambda_I(y) \leq o_2, \lambda_F(y) \leq g_2
\end{aligned}
$$

so

$$
\begin{aligned}
\widetilde{\mu}_T(xy) &\succeq \text{rmin}\{\widetilde{\mu}_T(x), \widetilde{\mu}_T(y), \widetilde{0.5}\} \succeq \text{rmin}\{\widetilde{t_1}, \widetilde{t_2}, \widetilde{0.5}\} \\
\widetilde{\mu}_I(xy) &\succeq \text{rmin}\{\widetilde{\mu}_I(x), \widetilde{\mu}_I(y), \widetilde{0.5}\} \succeq \text{rmin}\{\widetilde{i_1}, \widetilde{i_2}, \widetilde{0.5}\} \\
\widetilde{\mu}_F(xy) &\succeq \text{rmin}\{\widetilde{\mu}_F(x), \widetilde{\mu}_F(y), \widetilde{0.5}\} \succeq \text{rmin}\{\widetilde{f_1}, \widetilde{f_2}, \widetilde{0.5}\} \\
\lambda_T(xy) &\leq \max\{\lambda_T(x), \lambda_T(y), 0.5\} \succeq \max\{s_1, s_2, 0.5\} \\
\lambda_I(xy) &\leq \max\{\lambda_I(x), \lambda_I(y), 0.5\} \succeq \max\{o_1, o_2, 0.5\} \\
\lambda_F(xy) &\leq \max\{\lambda_F(x), \lambda_F(y), 0.5\} \succeq \max\{g_1, g_2, 0.5\}
\end{aligned}
$$

Now if,

$$\text{rmin}\{\widetilde{t_1}, \widetilde{t_2}\}, \text{rmin}\{\widetilde{i_1}, \widetilde{i_2}\}, \text{rmin}\{\widetilde{f_1}, \widetilde{f_2}\} \preceq \widetilde{0.5} \text{ and } \max\{s_1, s_2\}, \max\{o_1, o_2\}, \max\{g_1, g_2\} \geq 0.5,$$

then

$$
\begin{aligned}
\widetilde{\mu}_T(xy) &\succeq \text{rmin}\left\{\widetilde{\mu}_T(x), \widetilde{\mu}_T(y), \widetilde{0.5}\right\} \succeq \text{rmin}\{\widetilde{t_1}, \widetilde{t_2}\} \Rightarrow (xy)_{\text{rmin}\left\{\widetilde{t_1}, \widetilde{t_2}\right\}} \in \widetilde{\mu}_T \\
\widetilde{\mu}_I(xy) &\succeq \text{rmin}\left\{\widetilde{\mu}_I(x), \widetilde{\mu}_I(y), \widetilde{0.5}\right\} \succeq \text{rmin}\{\widetilde{i_1}, \widetilde{i_2}\} \Rightarrow (xy)_{\text{rmin}\{\widetilde{i_1}, \widetilde{i_2}\}} \in \widetilde{\mu}_I \\
\widetilde{\mu}_F(xy) &\succeq \text{rmin}\left\{\widetilde{\mu}_F(x), \widetilde{\mu}_F(y), \widetilde{0.5}\right\} \succeq \text{rmin}\{\widetilde{f_1}, \widetilde{f_2}\} \Rightarrow (xy)_{\text{rmin}\left\{\widetilde{f_1}, \widetilde{f_2}\right\}} \in \widetilde{\mu}_F \\
\lambda_T(xy) &\leq \max\{\lambda_T(x), \lambda_T(y), 0.5\} \succeq \max\{s_1, s_2\} \Rightarrow (xy)_{\max\{s_1, s_2\}} \in \lambda_T \\
\lambda_I(xy) &\leq \max\{\lambda_I(x), \lambda_I(y), 0.5\} \succeq \max\{o_1, o_2\} \Rightarrow (xy)_{\max\{o_1, o_2\}} \in \lambda_I \\
\lambda_F(xy) &\leq \max\{\lambda_F(x), \lambda_F(y), 0.5\} \succeq \max\{g_1, g_2\} \Rightarrow (xy)_{\max\{g_1, g_2\}} \in \lambda_F
\end{aligned}
$$

Now if,

$$\text{rmin}\{\widetilde{t_1}, \widetilde{t_2}\}, \text{rmin}\{\widetilde{i_1}, \widetilde{i_2}\}, \text{rmin}\{\widetilde{f_1}, \widetilde{f_2}\} \succ \widetilde{0.5} \text{ and } \max\{s_1, s_2\}, \max\{o_1, o_2\}, \max\{g_1, g_2\} < 0.5,$$

so

$$\begin{aligned}
\widetilde{\mu}_T(xy)+\text{rmin}\left\{\widetilde{\mu}_T(x),\widetilde{\mu}_T(y)\right\} &\succ \widetilde{0.5}+\widetilde{0.5}\Rightarrow (xy)_{\text{rmin}\left\{\widetilde{t_1},\widetilde{t_2}\right\}}\in\vee q\widetilde{\mu}_T\\
\widetilde{\mu}_I(xy)+\text{rmin}\left\{\widetilde{\mu}_I(x),\widetilde{\mu}_I(y)\right\} &\succ \widetilde{0.5}+\widetilde{0.5}\Rightarrow (xy)_{\text{rmin}\left\{\widetilde{i_1},\widetilde{i_2}\right\}}\in\vee q\widetilde{\mu}_I\\
\widetilde{\mu}_F(xy)+\text{rmin}\left\{\widetilde{\mu}_F(x),\widetilde{\mu}_F(y)\right\} &\succ \widetilde{0.5}+\widetilde{0.5}\Rightarrow (xy)_{\text{rmin}\left\{\widetilde{f_1},\widetilde{f_2}\right\}}\in\vee q\widetilde{\mu}_F\\
\lambda_T(xy)+\max\left\{\lambda_T(x),\lambda_T(y)\right\} &< 0.5+0.5=1\Rightarrow (xy)_{\max\{s_1,s_2\}}\in\vee q\lambda_T\\
\lambda_I(xy)+\max\left\{\lambda_I(x),\lambda_I(y)\right\} &< 0.5+0.5=1\Rightarrow (xy)_{\max\{o_1,o_2\}}\in\vee q\lambda_I\\
\lambda_F(xy)+\max\left\{\lambda_F(x),\lambda_F(y)\right\} &< 0.5+0.5=1\Rightarrow (xy)_{\max\{g_1,g_2\}}\in\vee q\lambda_F
\end{aligned}$$

Thus $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ is the neutrosophic cubic $(\in,\in\vee q)$-sub-semigroup of S. □

Theorem 4. *A non-empty subset A of a semigroup S is sub-semigroup of S if and only if the neutrosophic cubic characteristic function $\chi_A=(\chi_{\widetilde{\mu}_T},\chi_{\widetilde{\mu}_I},\chi_{\widetilde{\mu}_F},\chi_{\lambda_T},\chi_{\lambda_I},\chi_{\lambda_F})$ is an neutrosophic cubic $(\in,\in\vee q)$-sub-semigroup of S.*

5. Neutrosophic Cubic (α,β)-left (resp., right) Ideals

In this section we focus at Neutrosophic Cubic (α,β)-left (resp., right) Ideals with different results.

Definition 10. *A neutrosophic cubic subset $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ of a semigroup S is called the neutrosophic cubic (α,β)-left(resp., right) ideal of S, where $\alpha,\beta\in\{\in,q,\in\vee q$ or $\in\wedge q\}$ but $\alpha\neq\in\wedge q$ if $x\in S$ and $\widetilde{t},\widetilde{i},\widetilde{f}\in D[0,1]$ and $s,o,g\in[0,1)$ such that $y_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\alpha A\Longrightarrow(xy)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\beta A$(resp., $(yx)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\beta A$) for all $x,y\in S$.*

A neutrosophic cubic subset $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ is neutrosophic cubic (α,β)-ideal of S, if it is both neutrosophic cubic (α,β)-left and neutrosophic cubic (α,β)-right ideal of S.

Example 2. *Consider a semigroup $S=\{a,b,c\}$ with the following table,*

.	a	b	c
a	a	a	a
b	a	a	a
c	a	a	c

Define a neutrosophic cubic set $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ in S by

S	$\widetilde{\mu}_T$	$\widetilde{\mu}_I$	$\widetilde{\mu}_F$	λ_T	λ_I	λ_F
a	$[0.7,0.9]$	$[0.8,0.9]$	$[0.7,0.8]$	0.2	0.3	0.4
b	$[0.2,0.4]$	$[0.3,0.4]$	$[0.2,0.5]$	0.6	0.7	0.9
c	$[0.1,0.3]$	$[0.5,0.6]$	$[0.2,0.4]$	0.7	0.5	0.8

where

$$\widetilde{t}=[0.01,0.2],\widetilde{i}=[0.4,0.5],\widetilde{f}=[0.1,0.2]\ \text{ and } s=0.8,o=0.7,g=0.85$$

It is easy to verify that $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ is a neutrosophic cubic (α,β)-left ideal of S.

The following lemma shows that every neutrosophic cubic left (resp., right) ideal is a neutrosophic cubic $(\in,\in)$-left (resp., right) ideal of S.

Lemma 2. *A neutrosophic cubic subset* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of a semigroup* S *is neutrosophic cubic left (resp., right) ideal if and only if, for all* $x, y \in S$ *and for all* $\widetilde{t}, \widetilde{i}, \widetilde{f} \in D[0,1]$ *and* $s, o, g \in [0,1)$ *such that*

$$y_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)} \in A \Longrightarrow (xy)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)} \in A \text{ (resp., } (yx)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)} \in A \text{)}$$

Theorem 5. *Let* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic* (α, β)*-left (resp., right) ideal of* S*. Then the set* $A_{(\widetilde{0},1)} = \{x \in s| \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F)(x) \succ \widetilde{0}$ *and* $(\lambda_T, \lambda_I, \lambda_F)(x) < 1\}$ *is left (resp., right) ideal of* S*.*

Theorem 6. *Let* L *be a left (resp., right) ideal of* S *and let* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic subset of* S *such that*

$$\begin{aligned}
\widetilde{\mu}_T(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - L \\ \succeq \widetilde{0.5} & x \in L \end{cases} \quad \text{and} \quad \lambda_T(x) = \begin{cases} 1 & \text{if } x \in S - L \\ \leq 0.5 & x \in L \end{cases} \\
\widetilde{\mu}_I(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - L \\ \succeq \widetilde{0.5} & x \in L \end{cases} \quad \text{and} \quad \lambda_I(x) = \begin{cases} 1 & \text{if } x \in S - L \\ \leq 0.5 & x \in L \end{cases} \\
\widetilde{\mu}_F(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - L \\ \succeq \widetilde{0.5} & x \in L \end{cases} \quad \text{and} \quad \lambda_F(x) = \begin{cases} 1 & \text{if } x \in S - L \\ \leq 0.5 & x \in L \end{cases}
\end{aligned}$$

Then i) $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is the neutrosophic cubic* $(q, \in \vee q)$*-left (resp., right) ideal of* S*.*

ii) $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is the neutrosophic cubic* $(\in, \in \vee q)$*-left (resp., right) ideal of* S*.*

Theorem 7. *A neutrosophic cubic set* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of* S *is the neutrosophic cubic* $(\in, \in \vee q)$*-left(resp., right) ideal of* S *if and only if*

$$\begin{aligned}
\widetilde{\mu}_T(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_T(x), \widetilde{0.5}) \text{ and } \lambda_T(xy) \leq \max(\lambda_T(x), 0.5) \\
\widetilde{\mu}_I(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_I(x), \widetilde{0.5}) \text{ and } \lambda_I(xy) \leq \max(\lambda_I(x), 0.5) \\
\widetilde{\mu}_F(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_F(x), \widetilde{0.5}) \text{ and } \lambda_F(xy) \leq \max(\lambda_F(x), 0.5)
\end{aligned}$$

Corollary 1. *A neutrosophic cubic set* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of* S *is the neutrosophic cubic* $(\in, \in \vee q)$*-ideal of* S *if and only if*

$$\begin{aligned}
\widetilde{\mu}_T(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_T(x), \widetilde{0.5}) \text{ and } \lambda_T(xy) \leq \max(\lambda_T(x), 0.5) \\
\widetilde{\mu}_T(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_T(y), \widetilde{0.5}) \text{ and } \lambda_T(xy) \leq \max(\lambda_T(y), 0.5) \\
\widetilde{\mu}_I(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_I(x), \widetilde{0.5}) \text{ and } \lambda_I(xy) \leq \max(\lambda_I(x), 0.5) \\
\widetilde{\mu}_I(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_I(y), \widetilde{0.5}) \text{ and } \lambda_I(xy) \leq \max(\lambda_I(y), 0.5) \\
\widetilde{\mu}_F(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_F(x), \widetilde{0.5}) \text{ and } \lambda_F(xy) \leq \max(\lambda_F(x), 0.5) \\
\widetilde{\mu}_F(xy) &\succeq \mathrm{rmin}(\widetilde{\mu}_F(y), \widetilde{0.5}) \text{ and } \lambda_F(xy) \leq \max(\lambda_F(y), 0.5)
\end{aligned}$$

Theorem 8. *Let* $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic* $(\in, \in \vee q)$*-left ideal and* $B = \langle x, (\widetilde{\upsilon}_T, \widetilde{\upsilon}_I, \widetilde{\upsilon}_F, \eta_T, \eta_I, \eta_F)(x)\rangle$ *be a neutrosophic cubic* $(\in, \in \vee q)$*-right ideal of* S*. Then* $A \circ B$ *is neutrosophic cubic* $(\in, \in \vee q)$*-two sided ideal of* S*.*

Lemma 3. *Intersection of neutrosophic cubic* $(\in, \in \vee q)$*-two sided ideals is a neutrosophic cubic* $(\in, \in \vee q)$*-two sided ideal of* S*.*

Remark 1. *Let* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *and* $B = \langle x, (\widetilde{\upsilon}_T, \widetilde{\upsilon}_I, \widetilde{\upsilon}_F, \eta_T, \eta_I, \eta_F)(x)\rangle$ *be neutrosophic cubic* $(\in, \in \vee q)$*- ideals of* S*. Then* $A \circ B \subsetneq A \cap B$.

Theorem 9. *A non-empty subset* A *of a semigroup* S *is two sided ideals of* S *if and only if the neutrosophic cubic characteristic function* $\chi_A = (\chi_{\widetilde{\mu}_T}, \chi_{\widetilde{\mu}_I}, \chi_{\widetilde{\mu}_F}, \chi_{\lambda_T}, \chi_{\lambda_I}, \chi_{\lambda_F})$ *is an neutrosophic cubic* $(\in, \in \vee q)$*-two sided ideals of* S.

6. Neutrosophic Cubic (α, β)-generalized bi (resp., bi)-ideals

Here in this section we study Neutrosophic Cubic (α, β)-generalized bi (resp., bi)-ideals in semigroups.

Definition 11. *A neutrosophic cubic subset* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of a semigroup* S *is called the neutrosophic cubic* (α, β)*-generalized bi-ideal of* S*, where* $\alpha, \beta \in \{\in, q, \in \vee q$ *or* $\in \wedge q\}$ *but* $\alpha \neq \in \wedge q$ *if for all* $x, y, z \in S$ *and for all* $\widetilde{t_1}, \widetilde{i}_1, \widetilde{f}_1, \widetilde{t_2}, \widetilde{i}_2, \widetilde{f}_2 \in D[0,1]$ *and* $s_1, o_1, g_1, s_2, o_2, g_2 \in [0, 1)$ *such that* $x_{\left(\widetilde{t_1}, \widetilde{i}_1, \widetilde{f}_1, s_1, o_1, g_1\right)} \alpha A$ *and*

$$z_{\left(\widetilde{t_2}, \widetilde{i}_2, \widetilde{f}_2, s_2, o_2, g_2\right)} \alpha A \Longrightarrow (xyz)_{\left(\mathrm{rmin}\left\{\widetilde{t_1}, \widetilde{t_2}\right\}, \mathrm{rmin}\left\{\widetilde{i_1}, \widetilde{i_2}\right\}, \mathrm{rmin}\left\{\widetilde{f_1}, \widetilde{f_2}\right\}, \max\{s_1, s_2\}, \max\{o_1, o_2\}, \max\{g_1, g_2\}\right)} \beta A.$$

Definition 12. *A neutrosophic cubic* (α, β)*-sub-semigroup of* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of a semigroup* S *is called the neutrosophic cubic* (α, β)*-bi-ideal of* S*, where* $\alpha, \beta \in \{\in, q, \in \vee q$ *or* $\in \wedge q\}$ *but* $\alpha \neq \in \wedge q$ *if for all* $x, y, z \in S$ *and for all* $\widetilde{t_1}, \widetilde{i}_1, \widetilde{f}_1, \widetilde{t_2}, \widetilde{i}_2, \widetilde{f}_2 \in D[0,1]$ *and* $s_1, o_1, g_1, s_2, o_2, g_2 \in [0, 1)$ *such that* $x_{\left(\widetilde{t_1}, \widetilde{i}_1, \widetilde{f}_1, s_1, o_1, g_1\right)} \alpha A$ *and*

$$z_{\left(\widetilde{t_2}, \widetilde{i}_2, \widetilde{f}_2, s_2, o_2, g_2\right)} \alpha A \Longrightarrow (xyz)_{\left(\mathrm{rmin}\left\{\widetilde{t_1}, \widetilde{t_2}\right\}, \mathrm{rmin}\left\{\widetilde{i_1}, \widetilde{i_2}\right\}, \mathrm{rmin}\left\{\widetilde{f_1}, \widetilde{f_2}\right\}, \max\{s_1, s_2\}, \max\{o_1, o_2\}, \max\{g_1, g_2\}\right)} \beta A.$$

Remark 2. *Every neutrosophic cubic* (α, β)*-bi-ideal is an neutrosophic cubic* (α, β)*-generalized bi-ideal of* S*, but converse is not true.*

Example 3. *Consider the semigroup* $S = \{a, b, c, d\}$.

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

We define neutrosophic cubic set $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *by*

A	$\widetilde{\mu}_T(x)$	$\widetilde{\mu}_I(x)$	$\widetilde{\mu}_F(x)$	$\lambda_T(x)$	$\lambda_I(x)$	$\lambda_F(x)$
a	$[0.5, 0.8]$	$[0.4, 0.7]$	$[0.5, 0.7]$	0.1	0.3	0.2
b	$[0.3, 0.6]$	$[0.2, 0.5]$	$[0.1, 0.5]$	0.6	0.5	0.6
c	$[0.4, 0.7]$	$[0.3, 0.6]$	$[0.3, 0.6]$	0.2	0.4	0.4
d	$[0.1, 0.4]$	$[0.2, 0.6]$	$[0.2, 0.5]$	0.5	0.7	0.5

Then $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is neutrosophic cubic* $(\in, \in \vee q)$*-generalized bi-ideal of* S *but not a neutrosophic cubic* $(\in, \in \vee q)$*-bi-ideal of* S.

The following lemma shows that every neutrosophic cubic generalized bi (resp., bi) ideal is a neutrosophic cubic $(\in, \in)$-generalized bi (resp., bi) ideal of S.

Lemma 4. *A neutrosophic cubic subset* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of a semigroup* S *is neutrosophic cubic generalized bi-ideal if and only if for all* $x, y, z \in S$ *and for all* $\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, \widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2} \in D[0,1]$ *and* $s_1, o_1, g_1, s_2, o_2, g_2 \in [0,1)$ *such that*

$$\begin{aligned} x_{(\widetilde{t_1},\widetilde{i_1},\widetilde{f_1},s_1,o_1,g_1)} &\in A \text{ and } z_{(\widetilde{t_2},\widetilde{i_2},\widetilde{f_2},s_2,o_2,g_2)} \in A \\ &\Longrightarrow (xyz)_{(\mathrm{rmin}\{\widetilde{t_1},\widetilde{t_2}\},\mathrm{rmin}\{\widetilde{i_1},\widetilde{i_2}\},\mathrm{rmin}\{\widetilde{f_1},\widetilde{f_2}\},\max\{s_1,s_2\},\max\{o_1,o_2\},\max\{g_1,g_2\})} \in A \end{aligned}$$

Corollary 2. *A neutrosophic cubic* $(\in, \in \vee q)$*-sub-semigroup* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of a semigroup* S *is neutrosophic cubic bi-ideal if and only if for all* $x, y, z \in S$ *and for all* $\widetilde{t_1}, \widetilde{i_1}, \widetilde{f_1}, \widetilde{t_2}, \widetilde{i_2}, \widetilde{f_2} \in D[0,1]$ *and* $s_1, o_1, g_1, s_2, o_2, g_2 \in [0,1)$ *such that*

$$\begin{aligned} x_{(\widetilde{t_1},\widetilde{i_1},\widetilde{f_1},s_1,o_1,g_1)} &\in A \text{ and } z_{(\widetilde{t_2},\widetilde{i_2},\widetilde{f_2},s_2,o_2,g_2)} \in A \\ &\Longrightarrow (xyz)_{(\mathrm{rmin}\{\widetilde{t_1},\widetilde{t_2}\},\mathrm{rmin}\{\widetilde{i_1},\widetilde{i_2}\},\mathrm{rmin}\{\widetilde{f_1},\widetilde{f_2}\},\max\{s_1,s_2\},\max\{o_1,o_2\},\max\{g_1,g_2\})} \in A \end{aligned}$$

Theorem 10. *Let* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic* (α, β)*-generalized bi (resp., bi) ideal of* S*. Then the set* $A_{(\widetilde{0},1)} = \{x \in s |$ $(\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F)(x) \succ \widetilde{0}$ *and* $(\lambda_T, \lambda_I, \lambda_F)(x) < 1\}$ *is generalized bi (resp., bi) ideal of* S*.*

Theorem 11. *Let* B *be a generalized bi (resp., bi) ideal of* S *and let* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic subset of* S *such that*

$$\begin{aligned} \widetilde{\mu}_T(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - B \\ \succeq \widetilde{0.5} & x \in B \end{cases} \quad \text{and} \quad \lambda_T(x) = \begin{cases} 1 & \text{if } x \in S - B \\ \leq 0.5 & x \in B \end{cases} \\ \widetilde{\mu}_I(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - B \\ \succeq \widetilde{0.5} & x \in B \end{cases} \quad \text{and} \quad \lambda_I(x) = \begin{cases} 1 & \text{if } x \in S - B \\ \leq 0.5 & x \in B \end{cases} \\ \widetilde{\mu}_F(x) &= \begin{cases} \widetilde{0} & \text{if } x \in S - B \\ \succeq \widetilde{0.5} & x \in B \end{cases} \quad \text{and} \quad \lambda_F(x) = \begin{cases} 1 & \text{if } x \in S - B \\ \leq 0.5 & x \in B \end{cases} \end{aligned}$$

Then i) $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is the neutrosophic cubic* $(q, \in \vee q)$*-generalized bi (resp., bi) ideal of* S*.*

ii) $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is the neutrosophic cubic* $(\in, \in \vee q)$*-generalized bi (resp., bi) ideal of* S*.*

Theorem 12. *A neutrosophic cubic set* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of* S *is the neutrosophic cubic* $(\in, \in \vee q)$*-generalized bi (resp., bi) ideal of* S *if and only if*

$$\begin{aligned} \widetilde{\mu}_T(xyz) &\succeq \mathrm{rmin}(\widetilde{\mu}_T(x), \widetilde{\mu}_T(z), \widetilde{0.5}) \text{ and } \lambda_T(xyz) \leq \max(\lambda_T(x), \lambda_T(y), 0.5) \\ \widetilde{\mu}_I(xyz) &\succeq \mathrm{rmin}(\widetilde{\mu}_I(x), \widetilde{\mu}_I(z), \widetilde{0.5}) \text{ and } \lambda_I(xyz) \leq \max(\lambda_I(x), \lambda_T(y), 0.5) \\ \widetilde{\mu}_F(xyz) &\succeq \mathrm{rmin}(\widetilde{\mu}_F(x), \widetilde{\mu}_F(z), \widetilde{0.5}) \text{ and } \lambda_F(xyz) \leq \max(\lambda_F(x), \lambda_T(y), 0.5) \end{aligned}$$

Theorem 13. *A non-empty subset* A *of a semigroup* S *is generalized bi (resp., bi) ideal of* S *if and only if the neutrosophic cubic characteristic function* $\chi_A = (\chi_{\widetilde{\mu}_T}, \chi_{\widetilde{\mu}_I}, \chi_{\widetilde{\mu}_F}, \chi_{\lambda_T}, \chi_{\lambda_I}, \chi_{\lambda_F})$ *is an neutrosophic cubic* $(\in, \in \vee q)$*-generalized bi (resp., bi) ideal of* S*.*

7. Neutrosophic Cubic (α, β)-quasi ideals

In this section we give the concept of Neutrosophic Cubic (α, β)-quasi ideals particularly Neutrosophic Cubic $(\in, \in \vee q)$-quasi ideals. We also discuss the relation between Neutrosophic Cubic $(\in, \in \vee q)$-quasi ideals and other Neutrosophic Cubic (α, β)-ideals.

Definition 13. *A neutrosophic cubic set $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ of S is the neutrosophic cubic $(\in, \in \vee q)$-quasi ideal of S if it satisfies*

$$\begin{aligned}
\widetilde{\mu}_T(x) &\succeq \text{rmin}\{(\widetilde{1}_T \circ \widetilde{\mu}_T)(x), (\widetilde{\mu}_T \circ \widetilde{1}_T)(x), \widetilde{0.5}\} \\
\lambda_T(x) &\leq \max\{(0_T \circ \lambda_T)(x), (\lambda_T \circ 0_T)(x), 0.5\} \\
\widetilde{\mu}_I(x) &\succeq \text{rmin}\{(\widetilde{1}_I \circ \widetilde{\mu}_I)(x), (\widetilde{\mu}_I \circ \widetilde{1}_I)(x), \widetilde{0.5}\} \\
\lambda_I(x) &\leq \max\{(0_I \circ \lambda_I)(x), (\lambda_I \circ 0_I)(x), 0.5\} \\
\widetilde{\mu}_F(x) &\succeq \text{rmin}\{(\widetilde{1}_F \circ \widetilde{\mu}_F)(x), (\widetilde{\mu}_F \circ \widetilde{1}_F)(x), \widetilde{0.5}\} \\
\lambda_F(x) &\leq \max\{(0_F \circ \lambda_F)(x), (\lambda_F \circ 0_F)(x), 0.5\}
\end{aligned}$$

where

$$\widetilde{1}_T = [1\ 1],\ \widetilde{1}_I = [1\ 1],\ \widetilde{1}_F = [1\ 1],\ \textit{and}\ 0_T = 0,\ 0_I = 0,\ 0_F = 0\ .$$

Theorem 14. *Let $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ be a neutrosophic cubic $(\in, \in \vee q)$-quasi ideal of S. Then the set $A_{(\widetilde{0},1)} = \{x \in s | (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F)(x) \succ \widetilde{0}$ and $(\lambda_T, \lambda_I, \lambda_F)(x) < 1\}$ is quasi ideal of S.*

Remark 3. *Every neutrosophic cubic quasi ideal of S is neutrosophic cubic $(\in, \in \vee q)$-quasi ideal of S.*

Lemma 5. *Every neutrosophic cubic $(\in, \in \vee q)$-left (resp., right) ideal of S is neutrosophic cubic $(\in, \in \vee q)$-quasi ideal of S.*

Lemma 6. *Every neutrosophic cubic $(\in, \in \vee q)$-quasi ideal of S is neutrosophic cubic $(\in, \in \vee q)$-bi ideal of S.*

Theorem 15. *A non-empty subset A of a semigroup S is quasi ideal of S if and only if the neutrosophic cubic characteristic function $\chi_A = (\chi_{\widetilde{\mu}_T}, \chi_{\widetilde{\mu}_I}, \chi_{\widetilde{\mu}_F}, \chi_{\lambda_T}, \chi_{\lambda_I}, \chi_{\lambda_F})$ is an neutrosophic cubic $(\in, \in \vee q)$-quasi ideal of S.*

8. Neutrosophic Cubic (α, β)-interior ideals

Here we study Neutrosophic Cubic (α, β)-interior ideals with different results.

Definition 14. *A neutrosophic cubic subset $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ of a semigroup S is called the neutrosophic cubic (α, β)-interior ideal of S, where $\alpha, \beta \in \{\in, q, \in \vee q$ or $\in \wedge q\}$ but $\alpha \neq \in \wedge q$ if for all $x, a, z \in S$ and for all $\widetilde{t}, \widetilde{i}, \widetilde{f} \in D[0,1]$ and $s, o, g \in [0,1)$ such that*

$$a_{(\widetilde{t}, \widetilde{i}, \widetilde{f}, s, o, g)} \alpha A \ \textit{and} \implies (xaz)_{(\widetilde{t}, \widetilde{i}, \widetilde{f}, s, o, g)} \beta A.$$

Example 4. *Consider the semigroup $S = \{a, b, c, d\}$.*

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	b
d	a	a	b	c

We define neutrosophic cubic subset $A = \langle x, \ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ by

A	$\widetilde{\mu}_T(x)$	$\widetilde{\mu}_I(x)$	$\widetilde{\mu}_F(x)$	$\lambda_T(x)$	$\lambda_I(x)$	$\lambda_F(x)$
a	$[0.7, 0.9]$	$[0.6, 0.8]$	$[0.7, 0.8]$	0.1	0.2	0.3
b	$[0.4, 0.5]$	$[0.2, 0.4]$	$[0.3, 0.5]$	0.4	0.5	0.6
c	$[05, 0.6]$	$[0.3, 0.5]$	$[0.5, 0.6]$	0.3	0.4	0.5
d	$[0.4, 0.5]$	$[0.2, 0.3]$	$[0.4, 0.5]$	0.4	0.5	0.6

Then $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is neutrosophic cubic* $(\in \vee q)$*-interior ideal of* S.

The following lemma shows that every neutrosophic cubic interior ideal is a neutrosophic cubic $(\in, \in)$-interior ideal of S.

Lemma 7. *A neutrosophic cubic subset* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of a semigroup* S *is neutrosophic cubic interior ideal if and only if for all* $x, a, z \in S$ *and for all* $\widetilde{t}, \widetilde{i}, \widetilde{f} \in D[0,1]$ *and* $s, o, g \in [0,1)$ *such that*

$$a_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)} \in A \text{ and } \Longrightarrow (xaz)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)} \in A.$$

Theorem 16. *Let* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic* (α, β)*-interior ideal of* S. *Then the set* $A_{(\widetilde{0},1)} = \{x \in s \mid (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F)(x) \succ \widetilde{0}$ *and* $(\lambda_T, \lambda_I, \lambda_F)(x) < 1\}$ *is interior ideal of* S.

Theorem 17. *Let* I *be a interior ideal of* S *and let* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *be a neutrosophic cubic subset of* S *such that*

$$\widetilde{\mu}_T(x) = \begin{cases} \widetilde{0} & \text{if } x \in S - I \\ \succeq \widetilde{0.5} & x \in I \end{cases} \quad \text{and} \quad \lambda_T(x) = \begin{cases} 1 & \text{if } x \in S - I \\ \leq 0.5 & x \in I \end{cases}$$

$$\widetilde{\mu}_I(x) = \begin{cases} \widetilde{0} & \text{if } x \in S - I \\ \succeq \widetilde{0.5} & x \in I \end{cases} \quad \text{and} \quad \lambda_I(x) = \begin{cases} 1 & \text{if } x \in S - I \\ \leq 0.5 & x \in I \end{cases}$$

$$\widetilde{\mu}_F(x) = \begin{cases} \widetilde{0} & \text{if } x \in S - I \\ \succeq \widetilde{0.5} & x \in I \end{cases} \quad \text{and} \quad \lambda_F(x) = \begin{cases} 1 & \text{if } x \in S - I \\ \leq 0.5 & x \in I \end{cases}$$

Then i) $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is the neutrosophic cubic* $(q, \in \vee q)$*-interior ideal of* S.

ii) $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *is the neutrosophic cubic* $(\in, \in \vee q)$*-interior ideal of* S.

Theorem 18. *A neutrosophic cubic set* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of* S *is the neutrosophic cubic* $(\in, \in \vee q)$*-interior ideal of* S *if and only if*

$$\begin{aligned} \widetilde{\mu}_T(xaz) &\succeq \operatorname{rmin}(\widetilde{\mu}_T(a), \widetilde{0.5}) \text{ and } \lambda_T(xaz) \leq \max(\lambda_T(a), 0.5) \\ \widetilde{\mu}_I(xaz) &\succeq \operatorname{rmin}(\widetilde{\mu}_I(a), \widetilde{0.5}) \text{ and } \lambda_I(xaz) \leq \max(\lambda_I(a), 0.5) \\ \widetilde{\mu}_F(xaz) &\succeq \operatorname{rmin}(\widetilde{\mu}_F(a), \widetilde{0.5}) \text{ and } \lambda_F(xaz) \leq \max(\lambda_F(a), 0.5) \end{aligned}$$

Corollary 3. *Every neutrosophic cubic* $(\in, \in \vee q)$*-ideal is an neutrosophic cubic* $(\in, \in \vee q)$*-interior ideal of* S *but converse is not true.*

Example 5. *In Example 4,the neutrosophic cubic subset* $A = \langle x,\ (\widetilde{\mu}_T, \widetilde{\mu}_I, \widetilde{\mu}_F, \lambda_T, \lambda_I, \lambda_F)(x)\rangle$ *of* S *is neutrosophic cubic* $(\in \vee q)$*-interior ideal of* S *but it is not neutrosophic cubic* $(\in \vee q)$*-ideal of* S. *Since???*

Theorem 19. *A non-empty subset* A *of a semigroup* S *is interior ideal of* S *if and only if the neutrosophic cubic characteristic function* $\chi_A = (\chi_{\widetilde{\mu}_T}, \chi_{\widetilde{\mu}_I}, \chi_{\widetilde{\mu}_F}, \chi_{\lambda_T}, \chi_{\lambda_I}, \chi_{\lambda_F})$ *is an neutrosophic cubic* $(\in, \in \vee q)$*-interior ideal of* S.

9. Neutrosophic Cubic (α,β) semiprime/prime ideals

In this last section we provide some results on Neutrosophic Cubic (α,β) semiprime/prime ideals of semigroups.

Definition 15. *A neutrosophic cubic subset $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ of a semigroup S is called the neutrosophic cubic (α,β)-semiprime ideal of S, where $\alpha,\beta\in\{\in,q,\in\vee q$ or $\in\wedge q\}$ but $\alpha\neq\in\wedge q$ if for all $x\in S$ and for all $\widetilde{t},\widetilde{i},\widetilde{f}\in D[0,1]$ and $s,o,g\in[0,1)$ such that*

$$x^2_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\alpha A\Longrightarrow(x)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\beta A.$$

Definition 16. *A neutrosophic cubic subset $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ of a semigroup S is called the neutrosophic cubic (α,β)-prime ideal of S, where $\alpha,\beta\in\{\in,q,\in\vee q$ or $\in\wedge q\}$ but $\alpha\neq\in\wedge q$ if for all $x,y\in S$ and for all $\widetilde{t},\widetilde{i},\widetilde{f}\in D[0,1]$ and $s,o,g\in[0,1)$ such that*

$$(xy)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\alpha A\Longrightarrow(x)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\beta A\ \text{or}\ (y)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\beta A.$$

The following lemma shows that every neutrosophic cubic semiprime (resp., prime) ideal is a neutrosophic cubic $(\in,\in)$-semiprime (resp., prime) ideal of S.

Lemma 8. *A neutrosophic cubic subset $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ of a semigroup S is neutrosophic cubic semiprime (resp., prime) ideal if and only if for all $x\in S$ and for all $\widetilde{t},\widetilde{i},\widetilde{f}\in D[0,1]$ and $s,o,g\in[0,1)$ such that*

$$x^2_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\in A\Longrightarrow(x)_{(\widetilde{t},\widetilde{i},\widetilde{f},s,o,g)}\in A.$$

Theorem 20. *Let $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ be a neutrosophic cubic (α,β)-semiprime (resp., prime) ideal of S. Then the set $A_{(\widetilde{0},1)}=\{x\in s|$ $(\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F)(x)\succ\widetilde{0}$ and $(\lambda_T,\lambda_I,\lambda_F)(x)<1\}$ is semiprime (resp., prime) ideal of S.*

Theorem 21. *Let P be a semiprime (resp., prime) ideal of S and let $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ be a neutrosophic cubic subset of S such that*

$$\begin{aligned}
\widetilde{\mu}_T(x)&=\begin{cases}\widetilde{0} & \text{if } x\in S-P\\ \succeq\widetilde{0.5} & x\in P\end{cases}\quad\text{and}\quad\lambda_T(x)=\begin{cases}1 & \text{if } x\in S-P\\ \leq 0.5 & x\in P\end{cases}\\
\widetilde{\mu}_I(x)&=\begin{cases}\widetilde{0} & \text{if } x\in S-P\\ \succeq\widetilde{0.5} & x\in P\end{cases}\quad\text{and}\quad\lambda_I(x)=\begin{cases}1 & \text{if } x\in S-P\\ \leq 0.5 & x\in P\end{cases}\\
\widetilde{\mu}_F(x)&=\begin{cases}\widetilde{0} & \text{if } x\in S-P\\ \succeq\widetilde{0.5} & x\in P\end{cases}\quad\text{and}\quad\lambda_F(x)=\begin{cases}1 & \text{if } x\in S-P\\ \leq 0.5 & x\in P\end{cases}
\end{aligned}$$

Then i) $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ is the neutrosophic cubic $(q,\in\vee q)$-semiprime (resp., prime) ideal of S.

ii) $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ is the neutrosophic cubic $(\in,\in\vee q)$-semiprime (resp., prime) ideal of S.

Theorem 22. *A neutrosophic cubic set $A=\langle x,\ (\widetilde{\mu}_T,\widetilde{\mu}_I,\widetilde{\mu}_F,\lambda_T,\lambda_I,\lambda_F)(x)\rangle$ of S is the neutrosophic cubic $(\in,\in\vee q)$-semiprime (resp., prime) ideal of S if and only* *if*

$$\begin{aligned}
\widetilde{\mu}_T\left(x^2\right) &\succeq \operatorname{rmin}(\widetilde{\mu}_T\left(x\right),\widetilde{0.5}) \text{ and } \lambda_T\left(x^2\right) \leq \max\left(\lambda_T\left(x\right),0.5\right) \\
\widetilde{\mu}_I\left(x^2\right) &\succeq \operatorname{rmin}(\widetilde{\mu}_I\left(x\right),\widetilde{0.5}) \text{ and } \lambda_I\left(x^2\right) \leq \max\left(\lambda_I\left(x\right),0.5\right) \\
\widetilde{\mu}_F\left(x^2\right) &\succeq \operatorname{rmin}(\widetilde{\mu}_F\left(x\right),\widetilde{0.5}) \text{ and } \lambda_F\left(x^2\right) \leq \max\left(\lambda_F\left(x\right),0.5\right)
\end{aligned}$$

Corollary 4. *Every neutrosophic cubic $(\in, \in \vee q)$-prime is an neutrosophic cubic $(\in, \in \vee q)$-semiprime ideal of S but converse is not true.*

Theorem 23. *A non-empty subset A of a semigroup S is $(\in, \in \vee q)$-semiprime (resp., prime) ideal of S if and only if the neutrosophic cubic characteristic function $\chi_A = (\chi_{\widetilde{\mu}_T}, \chi_{\widetilde{\mu}_I}, \chi_{\widetilde{\mu}_F}, \chi_{\lambda_T}, \chi_{\lambda_I}, \chi_{\lambda_F})$ is an neutrosophic cubic $(\in, \in \vee q)$- $(\in, \in \vee q)$-semiprime (resp., prime) ideal of S.*

Conclusion 1. *In this paper we study different types of neutrosophic cubic (α, β)-ideals of semigroups. In our future study we are focusing at the following points.*

(i) To study neutrosophic cubic $(\in, \in \vee q_k)$-ideals of semigroups.

(ii) To study neutrosophic cubic $(\in_\gamma, \in_\gamma \vee q_\delta)$-ideals of semigroups.

(iii) To define the upper and lower parts of neutrosophic cubic $(\in, \in \vee q)$-ideals, neutrosophic cubic $(\in, \in \vee q_k)$-ideals and neutrosophic cubic $(\in_\gamma, \in_\gamma \vee q_\delta)$-ideals of semigroups.

(iv) To study the regular and intra-regular semigroups in terms of neutrosophic cubic $(\in, \in \vee q)$-ideals, neutrosophic cubic $(\in, \in \vee q_k)$-ideals and neutrosophic cubic $(\in_\gamma, \in_\gamma \vee q_\delta)$-ideals.

(v) To study neutrosophic cubic $(\in, \in \vee q)$-relations, neutrosophic cubic $(\in, \in \vee q_k)$-relations and neutrosophic cubic $(\in_\gamma, \in_\gamma \vee q_\delta)$-relations.

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