Neutrosophic Decision Making Model for Clay-Brick Selection in Construction Field Based on Grey Relational Analysis

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Abstract

The purpose of this paper is to present quality clay-brick selection approach based on multi-attribute decision-making with single valued neutrosophic grey relational analysis. Brick plays a significant role in construction field. So it is important to select quality clay-brick for construction based on suitable mathematical decision making tool. There are several selection methods in the literature. Among them decision making with neutrosophic set is very pragmatic and interesting. Neutrosophic set is one tool that can deal with indeterminacy and inconsistent data. In the proposed method, the rating of all alternatives is expressed with single-valued neutrosophic set which is characterized by truth-membership degree (acceptance), indeterminacy membership degree and falsity membership degree (rejection). Weight of each attribute is determined based on experts' opinions. Neutrosophic grey relational coefficient is used based on Hamming distance between each alternative to ideal neutrosophic estimates reliability solution and ideal neutrosophic estimates unreliability solution. Then neutrosophic relational degree is used to determine the ranking order of all alternatives (bricks). An illustrative numerical example for quality brick selection is solved to show the effectiveness of the proposed method.

Keywords: Single-valued neutrosophic set, Grey relational analysis; Neutrosophic relative relational degree, Multi-attribute decision making, Clay-brick selection

1. Introduction:

Operations research management science has been mostly studied with structured and well defined problems with crisply or fuzzily defined information. However, in realistic decision making situations, some information cannot be defined crisply or fuzzily where indeterminacy involves. In order to deal with this situation neutrosophic set studied by Smarandache [1] is very helpful. Several researchers studied decision making problems [2- 4] using single valued neutrosophic set proposed by Wang et al. [5]. Ye [6] proposed multi-criteria decision making problem using single valued neutrosophic sets. Biswas et al. [7] proposed multi attribute decision making (MADM) using neutrosophic grey relational analysis. Biswas et al. [8] also proposed a new method for MADM using entropy weight information based on neutrosophic grey relation analysis. Mondal and Pramanik [9] applied neutrosophic grey relational analysis based MADM to modeling school choice problem. Mondal and Pramanik [10] also applied single valued neutrosophic decision making concept for teacher recruitment in higher education. Mondal and Pramanik [11] also proposed a hybrid model namely rough neutrosophic multi-attribute decision making and applied in educational problem. So decision making in neutrosophic environment is an emergence area of research.

Brick selection is a special type of personnel selection problem. Pramanik and Mukhopadhyaya [12] studied grey relational analysis based intuitionistic fuzzy multi criteria group decision-making approach for teacher selection in higher education. Robertson and Smith [13] presented good reviews on personnel selection studies. They investigated the role of job analysis, contemporary models of work performance, and set of criteria employed in personnel selection process. Brick selection problem in intuitionistic fuzzy environment is studied by Mondal and Pramanik [14]. Brick selection problem in neutrosophic environment is yet to appear in the literature. In this paper brick selection in neutrosophic environment is studied.

Bricks are traditionally selected based on its color, size and total cost of brick, without considering the complexity of indeterminacy involved in characterizing the attributes of brick. In that case the building construction may have some problems regarding low rigidity, longevity, etc, which cause great threat for the construction. However, indeterminacy inherently involves in some of the attributes of bricks. So it is necessary to formulate new scientific based selection method which is capable of handling indeterminacy related information. In order to select the most suitable brick to construct a building, the following criteria of bricks obtained from experts' opinions considered by Mondal and Pramanik [14] are used in this paper. The criteria are namely, solidity, color, size and shape, strength of brick, cost of brick, and carrying cost.

A good quality brick is characterized by its regular shape and size, with smooth even sides and no cracks or defects. Poor quality bricks are generally produced as a result of employing poor techniques but these errors can often be easily corrected. If bricks are well-made and well fired, a metallic sound or ring is heard when they are knocked together. If the produced sound is a dull sound, it reflects that the bricks are either cracked or under fired [15, 16]. In the proposed approach, the information provided by the experts about the attribute values assumes the form of single valued neutrosophic set. In the proposed approach, the ideal neutrosophic estimates reliability solution and the ideal neutrosophic estimate un-reliable solution is used. Neutrosophic grey relational coefficient of each alternative is determined to rank the alternatives (bricks).

Rest of the paper is constructed in the following manner. Section 2 presents preliminaries of neutrosophic sets. Section 3 describes the attributes of brick and their operational definitions. Section 4 is devoted to present multi-attribute decision making based on neutrosophic grey relational analysis for brick selection process. In section 5, illustrative example is provided for brick selection process. Section 6 describes the advantage of the proposed approach. Section 7 presents conclusion and future direction of research work.

2. Neutrosophic sets and Single-valued neutrosophic set (SVNS)

Neutrosophic set is derived from neutrosophy, a new branch of philosophy studied by Smarandache [1]. Neutrosophy is devoted to study the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

2.1 Definition of Neutrosophic set [1]

Definition 1: Let X be a space of points (objects) with generic element in X denoted by x. Then a neutrosophic set A in X is characterized by a truth membership function T_A an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A and F_A are real standard or non-standard subsets of $]^-0,1^+[$ that is T_A: $X \rightarrow]^-0,1^+[$; I_A: $X \rightarrow]^-0,1^+[$; F_A: $X \rightarrow]^-0,1^+[$ with the following relation

$$^-0 \le \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \le 3^+, \quad \forall x \in X$$

Definition 2: The complement [1] of a neutrosophic set A is denoted by A^c and is defined by

$$T_{A^c}(x) = \{1^+\} - T_A(x);\ I_{A^c}(x) = \{1^+\} - I_A(x);\ F_{A^c}(x) = \{1^+\} - F_A(x)$$

Definition 3: (Containment [1]): A neutrosophic set A is contained in the other neutrosophic set B, denoted by $A \subseteq B$ if and only if the following result holds.

$$\inf T_A(x) \le \inf T_B(x),\ \sup T_A(x) \le \sup T_B(x);$$

$$\inf I_A(x) \ge \inf I_B(x),\ \sup I_A(x) \ge \sup I_B(x);$$

$$\inf F_A(x) \ge \inf F_B(x),\ \sup F_A(x) \ge \sup F_B(x)$$

for all x in X.

Definition 4: (SVNS) [5]: Let X be a universal space of points (objects), with a generic element of X denoted by x. A SVNS set S is characterized by a true membership function $T_S(x)$, a falsity membership function $I_S(x)$, and an indeterminacy function $F_S(x)$, with $T_S(x)$, $I_S(x)$, $F_S(x) \in [0, 1]$.

$$S = \sum \langle T_S(x), F_S(x), I_S(x) \rangle / x, \quad \forall x \in X$$

It should be noted that for a SVNS S,

$$0 \leq \sup T_S(x) + \sup F_S(x) + \sup I_S(x) \leq 3, \quad \forall x \in X$$

For example, suppose ten members of a school managing committee will critically review a specific agenda. Six of them agree with this agenda, three of them disagree and rest of one member remain undecided. Then by neutrosophic notation it can be expressed as x (0.6, 0.3, 0.1).

Definition 5: The complement of a SVNS S is denoted by S^c and is defined by

$$T_S{}^c(x) = F_S(x);\ I_S{}^c(x) = 1 - I_S(x);\ F_S{}^c(x) = T_S(x)$$

Definition 6: A SVNS S_A is contained in the other single valued neutrosophic set S_B, denoted as $S_A \subseteq S_B$ iff

$$T_{S_A}(x) \leq T_{S_B}(x);\ I_{S_A}(x) \geq I_{S_B}(x);\ F_{S_A}(x) \geq F_{S_B}(x),\ \forall x \in X.$$

Definition 7: Two single valued neutrosophic sets S_A and S_B are equal, i.e, $S_A = S_B$, if and only if

$S_A \subseteq S_B$ and $S_A \supseteq S_B$

Definition 8 (Union): The union of two SVNSs S_A and S_B is a SVNS S_C, written as

$S_C = S_A \cup S_B$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to those of S_A and S_B as follows:

$$T_{S_C}(x) = \max\left(T_{S_A}(x),\ T_{S_B}(x)\right);$$

$$I_{S_C}(x) = \min\left(I_{S_A}(x),\ I_{S_B}(x)\right);$$

$$F_{S_C}(x) = \min\left(F_{S_A}(x),\ F_{S_B}(x)\right)\ \forall x \in X.$$

Definition 9 (intersection): The intersection of two SVNSs S_A and S_B is a SVNS S_C written as $S_C = S_A \cap S_B$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of S_A and S_B as follows:

$$T_{S_C}(x) = \min\left(T_{S_A}(x),\ T_{S_B}(x)\right);\ I_{S_C}(x) = \max\left(I_{S_A}(x),\ I_{S_B}(x)\right);\ F_{S_C}(x) = \max\left(F_{S_A}(x),\ F_{S_B}(x)\right),\ \forall x \in X$$

Distance between two neutrosophic sets

The general SVNS has the following pattern:

$$S = \left\{\left(x/\left(T_S(x), I_S(x), F_S(x)\right)\right)\right\};\ x \in X$$

For finite SVNSs can be represented by the ordered tetrads:

S = $\{(x_1/(T_S(x_1), I_S(x_1), F_S(x_1))), \ldots, (x_m/(T_S(x_m), I_S(x_m), F_S(x_m)))\}$ for all $x \in X$

Definition 10: [17] Let

$S_A = \{(x_1/(T_{S_A}(x_1),\ I_{S_A}(x_1), F_{S_A}(x_1))),\ldots, \{(x_n/(T_{S_A}(x_n), I_{S_A}(x_n), F_{S_A}(x_n)))\}$

$S_B = \{(x_1/(T_{S_B}(x_1), I_{S_B}(x_1), F_{S_B}(x_1))),\ldots, \{(x_n/(T_{S_B}(x_n), I_{S_B}(x_n), F_{S_B}(x_n)))\}$

be two single-valued neutrosophic sets (SVNSs) in $x = \{x_1, x_2, x_3, \ldots, x_n\}$

Then the Hamming distance between two SVNSs S_A and S_B

is defined as follows:

$$d_S(S_A, S_B) = \sum_{i=1}^{n} \left\langle \left|T_{S_A}(x) - T_{S_B}(x)\right| + \left|I_{S_A}(x) - I_{S_B}(x)\right| + \left|F_{S_A}(x) - F_{S_B}(x)\right| \right\rangle \tag{1}$$

and normalized Hamming distance between (3) two SVNSs S_A and S_B is defined as follows:

$$^N d_S(S_A, S_B) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \left|T_{S_A}(x) - T_{S_B}(x)\right| + \left|I_{S_A}(x) - I_{S_B}(x)\right| + \left|F_{S_A}(x) - F_{S_B}(x)\right| \right\rangle \tag{2}$$

with the following two properties as follows:

$$0 \le d_S(S_A, S_B) \le 3n \tag{3}$$

$$0 \le {}^N d_S(S_A, S_B) \le 1 \tag{4}$$

Definition 11: Ideal neutrosophic reliability solution INERS [18]

$Q_S^+ = \left\langle q_{S_1}^+, q_{S_2}^+, \cdots, q_{S_n}^+ \right\rangle$ is a solution in which every component is presented by $q_{S_j}^+ = \left\langle T_j^+, I_j^+, F_j^+ \right\rangle$ where $T_j^+ = \max_i \{T_{ij}\}$, $I_j^+ = \min_i \{I_{ij}\}$ and $F_j^+ = \min_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$ for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

Definition 12: Ideal neutrosophic estimates un-reliability solution (INEURS) [18]

$Q_S^- = \left\langle q_{S_1}^-, q_{S_2}^-, \cdots, q_{S_n}^- \right\rangle$ is a solution in which every component is represented by $q_{S_j}^- = \langle T_j^-, I_j^-, F_j^- \rangle$ where $T_j^- = \min_i \{T_{ij}\}$, $I_j^- = \max_i \{I_{ij}\}$ and $F_j^- = \max_i \{F_{ij}\}$ in the neutrosophic decision matrix $D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$ for $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

3. Brick attributes

Six criteria [14] of bricks are considered, namely, solidity (C_1), color (C_2), size and shape (C_3), and strength of brick (C_4), brick cost (C_5), carrying cost (C_6). These six criteria's are explained as follows:

(i) Solid clay brick (C_1)**:** An ideal extended solid rigid body prepared by loam soil having fixed size and shape remains unaltered when fixed forces are applied. The distance between any two given points of the rigid body remains unchanged when external fixed forces applied on it. If we soap a solid brick in water and drop it from 3 or 4 feet heights [15, 16], it remains unbroken.

(ii) Color (C_2): Color of quality brick refers to reddish or light maroon.

(iii) Size and shape (C_3): All bricks are to be more or less same size and shape having same length, width and height. The size or dimensions of a brick are determined by how it is used in construction work. Standard size of a brick may vary. Size of a brick may be around $190\text{mm} \times 90\text{mm} \times 40\text{mm}$ [16].

Width

The width of a brick should be small enough to allow a bricklayer to lift the brick with one hand and place it on a bed of mortar. For the average person, the width should not be more than 115 mm.

Length

The length of a brick refers to twice its width plus 10 mm (for the mortar joint). A brick with this length will be easier to build with because it will provide and even surface on both sides of the wall. For example, if you follow the rule of the length being twice the width plus 10 mm, if you would like to have a brick x mm wide, then the ideal length would be (2x + 10) mm.

Height

The height of a brick is related to the length of the brick. The height of three bricks plus two 10 mm joints is equal to the length of a brick. This allows a bricklayer to lay bricks on end (called a soldier course) and join them into the wall without having to cut the bricks. The height of a brick is determined by subtracting 20 mm (the thickness of the two 10 mm mortar joints) from the length and dividing the result by three (this represents the three bricks).

Possible brick Sizes

In India the standard brick size is 190 mm x 90 mm x 40 mm while the British standard is 215 mm x 102.5 mm x 65 mm. To select your brick size, first contact the local public works department to see if your country has a standard size. If not, you will have to choose your own size. Possible brick sizes can be found in [14].

(iv) Well dried and burnt (strength of brick) (C_4) [15, 16]: Raw bricks are well dried in sunshine and then properly burnt. If bricks have been well- made and well-fired, a metallic sound is heard when they are knocked together. If knocking creates a dull sound, it reflects that they are either cracked or under-fired. A simple test for strength of a brick is to drop it from a height of 1.2 meters (shoulder height). A good brick will not break. This test should be repeated with a wet brick (a brick soaked in water for one week). If the soaked brick does not break when dropped, it reflects that the quality of the brick is good enough to build single storied structures.

v) Brick cost (C_5): Decision maker always tries to minimize purchasing cost. Reasonable price of quality brick is more acceptable.

vi) Carrying cost (C_6): The distance between brick field and construction site must be reasonable for maintaining minimum carrying cost.

4. GRA method for multiple attribute decision making problems with single valued neutrosophic information

Consider a multi-attribute decision making problem with m alternatives and n attributes. Let A_1, A_2, …, A_m and C_1, C_2 ,…, C_n represent the alternatives and attributes respectively. The rating reflects the performance of the alternative A_i against the attribute C_j. For MADM, weight vector $W = w_1, w_2,\ldots, w_n$ is fixed to the attributes. The weight $w_j > 0$, j = 1, 2, 3,…, n reflects the relative importance of attributes C_j , j = 1, 2, …, n to the decision making process. The weights of the attributes are usually determined on subjective basis. The values associated with the alternatives for MADM problems presented in the decision table 1.

Table1: Decision table of attribute values

$$D = \langle d_{ij} \rangle_{m\times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & d_{11} & d_{12} & \cdots & d_{1n} \\ A_2 & d_{21} & d_{22} & \cdots & d_{2n} \\ . & \cdots & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots & \cdots \\ A_m & d_{m1} & d_{m2} & \cdots & d_{mn} \end{array} \qquad (5)$$

GRA is one of the derived evaluation methods for MADM based on the concept of grey relational space. The main procedure of GRA method is firstly translating the performance of all alternatives into a comparability sequence. According to these sequences, a reference sequence (ideal target sequence) is defined. Then, the grey relational coefficient between all comparability sequences and the reference sequence for different values of distinguishing coefficient are calculated. Finally, based on these grey relational coefficients, the grey relational degree between the reference sequence and every comparability sequences is calculated. If an alternative gets the highest grey relational grade with the reference sequence, it means that the comparability sequence is the most similar to the reference sequence and that alternative would be the best choice (Fung [19]). The steps of improved GRA under SVNS are described below:

Step 1. Determination of the most important criteria

Generally, there exist many criteria or attributes in decision making problems where some of them are important and others may not be so important. So it is important to select the proper criteria or attributes for decision making situations. The most important criterion may be selected based on experts' opinions.

Step 2. Construction of the decision matrix with single valued neutrosophic sets (SVNSs)

The rating of alternatives A_i (i = 1, 2, . . . , m) with respect to the attribute C_j (j = 1, 2, . . ., n) is assumed as SVNS. It can be represented with the following forms:

$$A_i = \left[\frac{C_1}{\langle T_{i1}, I_{i1}, F_{i1} \rangle}, \frac{C_2}{\langle T_{i2}, I_{i2}, F_{i2} \rangle}, ..., \frac{C_n}{\langle T_{in}, I_{in}, F_{in} \rangle} \right]; \; C_j \in C$$

$$= \left[\frac{C_j}{\langle T_{ij}, I_{ij}, F_{ij} \rangle} : C_j \in C \right] \text{for j = 1, 2,..., n}$$

Here T_{ij}, I_{ij}, F_{ij} are the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i is satisfying the attribute Cj, respectively where

$0 \le T_{ij} \le 1, \; 0 \le I_{ij} \le 1, \quad 0 \le F_{ij} \le 1$ and $0 \le T_{ij} + I_{ij} + F_{ij} \le 3$

The decision matrix D_S is presented in the table 2.

Table2. Decision matrix D_S

$$D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n} = \begin{array}{c|cccc} & C_1 & C_2 & ... & C_n \\ \hline A_1 & \langle T_{11}, I_{11}, F_{11} \rangle & \langle T_{12}, I_{12}, F_{12} \rangle & ... & \langle T_{1n}, I_{1n}, F_{1n} \rangle \\ A_2 & \langle T_{21}, I_{21}, F_{21} \rangle & \langle T_{22}, I_{22}, F_{22} \rangle & ... & \langle T_{2n}, I_{2n}, F_{2n} \rangle \\ . & ... & ... & ... & ... \\ . & ... & ... & ... & ... \\ A_m & \langle T_{m1}, I_{m1}, F_{m1} \rangle & \langle T_{m2}, I_{m2}, F_{m2} \rangle & ... & \langle T_{mn}, I_{mn}, F_{mn} \rangle \end{array} \qquad (6)$$

Step 3. Determination of the weights of criteria

In the decision making process, decision maker may often encounter with unknown attribute weights. It may happen that the importance of the attributes is different. Therefore we need to determine reasonable attribute weight for making a proper decision.

Step 4. Determination of the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS) for neutrosophic decision matrix.

For a neutrosophic decision making matrix $D_S = [q_{S_{ij}}]_{m \times n} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m \times n}$, T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non membership of the alternative A_i of A satisfying the attribute C_j of C. The neutrosophic estimate reliability solution can be determined from the concept of SVNS cube proposed by Dezert [18].

Step 5. Calculation of the neutrosophic grey relational coefficient of alternative from INERS

Grey relational coefficient of each alternative from INERS is as follows:

$$g_{ij}^{+} = \frac{\min_i \min_j \Delta_{ij}^{+} + \rho \max_i \max_j \Delta_{ij}^{+}}{\Delta_{ij}^{+} + \rho \max_i \max_j \Delta_{ij}^{+}} \text{, where}$$

$\Delta_{ij}^{+} = d\,(q_{S_j}^{+}, q_{S_{ij}})$, i = 1, 2, …, m and j = 1, 2, …., n (7)

Step 6. Calculation of the neutrosophic grey relational coefficient of alternative from INEURS

Grey relational coefficient of each alternative from INEURS is as follows:

$$g_{ij}^{-} = \frac{\min_i \min_j \Delta_{ij}^{-} + \rho \max_i \max_j \Delta_{ij}^{-}}{\Delta_{ij}^{-} + \rho \max_i \max_j \Delta_{ij}^{-}}, \quad (8)$$

Here $\Delta_{ij}^{-} = d(q_{S_{ij}}, q_{S_j}^{-})$, i = 1, 2, …, m and j = 1, 2, …., n

$\rho \in [0,1]$ is the distinguishable coefficient or the identification coefficient used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When $\rho = 1$, the comparison environment is unaltered; when $\rho = 0$, the comparison environment disappears. Smaller value of distinguishing coefficient will yield in large range of grey relational coefficient. Generally, $\rho = 0.5$ is considered for decision making.

Step 7. Calculation of the neutrosophic grey relational coefficient

Calculate the degree of neutrosophic grey relational coefficient of each alternative from INERS and INEURS using the following equation respectively:

$g_i^{+} = \sum_{j=1}^{n} w_j\, g_{ij}^{+}$ for i = 1, 2, …, m (9)

$g_i^{-} = \sum_{j=1}^{n} w_j\, g_{ij}^{-}$ for i = 1, 2, …, m (10)

Step 8. Calculation the neutrosophic relative relational degree

We calculate the neutrosophic relative relational degree of each alternative from indeterminacy truthfulness falsity positive ideal solution (ITFPIS) with the help of following equations:

$R_i = \dfrac{g_i^{+}}{g_i^{-} + g_i^{+}}$, for i = 1, 2, …, m (11)

Step 9. Ranking the alternatives

According to the relative relational degree, the ranking order of all alternatives can be determined. The highest value of R_i represents the most important alternative.

Step10. End

5. Example of brick selection

The steps of brick selection procedure using the proposed approach are arranged as follows:

Step 1: Determination of the most important criteria

The most important criterion of brick is selected based on experts' opinions are namely, solidity, color, size and shape, strength of brick, cost of brick, and carrying cost.

Step 2: Construction of the decision matrix with single valued neutrosophic sets (SVNSs)

Here the most important criterion of brick is chosen based on experts' opinions. When the four possible alternatives with respect to the six criteria are evaluated by the expert, we can obtain the following single-valued neutrosophic decision matrix:

$D_S = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{4\times 6} =$

$$\begin{array}{c|cccccc} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \hline A_1 & \langle 0.7, 0.2, 0.1 \rangle & \langle 0.8, 0.1, 0.1 \rangle & \langle 0.7, 0.1, 0.2 \rangle & \langle 0.6, 0.2, 0.2 \rangle & \langle 0.7, 0.0, 0.0 \rangle & \langle 0.5, 0.3, 0.3 \rangle \\ A_2 & \langle 0.7, 0.1, 0.0 \rangle & \langle 0.7, 0.2, 0.1 \rangle & \langle 0.8, 0.1, 0.1 \rangle & \langle 0.6, 0.1, 0.1 \rangle & \langle 0.8, 0.1, 0.1 \rangle & \langle 0.5, 0.4, 0.1 \rangle \\ A_3 & \langle 0.8, 0.1, 0.1 \rangle & \langle 0.8, 0.0, 0.1 \rangle & \langle 0.7, 0.2, 0.1 \rangle & \langle 0.6, 0.2, 0.1 \rangle & \langle 0.7, 0.1, 0.1 \rangle & \langle 0.6, 0.4, 0.2 \rangle \\ A_4 & \langle 0.7, 0.0, 0.1 \rangle & \langle 0.9, 0.2, 0.0 \rangle & \langle 0.7, 0.3, 0.1 \rangle & \langle 0.8, 0.2, 0.1 \rangle & \langle 0.6, 0.2, 0.3 \rangle & \langle 0.7, 0.3, 0.2 \rangle \end{array} \quad (12)$$

Step 3. Determination weights of the criteria

In the decision making situation, decision makers recognize that all the criteria of bricks are not equal importance. Here the importance of the criteria is obtained from expert opinion through questionnaire method i.e. the weights of the criteria are previously determined such that the sum of the weights of the criteria is equal to unity. Data was collected from fifteen constructional engineers, ten construction labors of Nadia district from twelve brick fields of surrounding areas. After extended interviews and discussions with the experts, the criteria of brick were found the same as found in [14] namely, solidity, color, size and shape, strength of brick, brick cost, and carrying cost. We have the weight of each criterion w_j, j = 1, 2, 3, 4, 5, 6 as follows:

$w_1 = 0.275$, $w_2 = 0.175$, $w_3 = 0.2$, $w_4 = 0.1$, $w_5 = 0.05$, $w_6 = 0.2$ such that $\sum_{j=1}^{6} w_j = 1$

Step 4. Determine the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS)

$$Q_S^+ = \left[q_{S_1}^+, q_{S_2}^+, q_{S_3}^+, q_{S_4}^+, q_{S_5}^+, q_{S_6}^+\right] =$$

$$\left[\begin{array}{l} \left\langle \max_i \{T_{i1}\}, \min_i \{I_{i1}\}, \min_i \{F_{i1}\} \right\rangle, \left\langle \max_i \{T_{i2}\}, \min_i \{I_{i2}\}, \min_i \{F_{i2}\} \right\rangle, \left\langle \max_i \{T_{i3}\}, \min_i \{I_{i3}\}, \min_i \{F_{i3}\} \right\rangle, \\ \left\langle \max_i \{T_{i4}\}, \min_i \{I_{i4}\}, \min_i \{F_{i4}\} \right\rangle, \left\langle \max_i \{T_{i5}\}, \min_i \{I_{i5}\}, \min_i \{F_{i5}\} \right\rangle, \left\langle \max_i \{T_{i6}\}, \min_i \{I_{i6}\}, \min_i \{F_{i6}\} \right\rangle \end{array}\right]$$

$$= \left[\langle 0.8, 0.0, 0.0 \rangle, \langle 0.9, 0.0, 0.0 \rangle, \langle 0.8, 0.1, 0.1 \rangle, \langle 0.6, 0.1, 0.1 \rangle, \langle 0.8, 0.0, 0.0 \rangle, \langle 0.7, 0.3, 0.1 \rangle\right]$$

$$Q_S^- = \left[q_{S_1}^-, q_{S_2}^-, q_{S_3}^-, q_{S_4}^-, q_{S5}^-, q_{S_6}^-\right] =$$

$$\left[\begin{array}{l}\left\langle \min_i \{T_{i1}\}, \max_i \{I_{i1}\}, \max_i \{F_{i1}\}\right\rangle, \left\langle \min_i \{T_{i2}\}, \max_i \{I_{i2}\}, \max_i \{F_{i2}\}\right\rangle, \left\langle \min_i \{T_{i3}\}, \max_i \{I_{i3}\}, \max_i \{F_{i3}\}\right\rangle, \\ \left\langle \min_i \{T_{i4}\}, \max_i \{I_{i4}\}, \max_i \{F_{i4}\}\right\rangle, \left\langle \min_i \{T_{i5}\}, \max_i \{I_{i5}\}, \max_i \{F_{i5}\}\right\rangle, \left\langle \min_i \{T_{i6}\}, \max_i \{I_{i6}\}, \max_i \{F_{i6}\}\right\rangle\end{array}\right]$$

$$= \left[\langle 0.7, 0.2, 0.1\rangle, \langle 0.7, 0.2, 0.1\rangle, \langle 0.7, 0.3, 0.2\rangle, \langle 0.6, 0.2, 0.2\rangle, \langle 0.6, 0.2, 0.3\rangle, \langle 0.5, 0.4, 0.3\rangle\right]$$

Step 5. Calculation of the neutrosophic grey relational coefficient of each alternative from INERS

Using Equation (7), the neutrosophic grey relational coefficient of each alternative from INERS can be obtained as follows:

$$g_{ij}^{+} = \begin{bmatrix} 0.3333 & 0.4641 & 0.4641 & 0.4641 & 1.0000 & 0.4545 \\ 0.4641 & 0.2899 & 1.0000 & 1.0000 & 0.7143 & 0.5556 \\ 0.4641 & 0.4641 & 0.4641 & 0.5505 & 0.5556 & 0.5556 \\ 0.4641 & 0.3797 & 0.3539 & 0.3539 & 0.5556 & 1.0000 \end{bmatrix} \tag{13}$$

Step 6. Calculation of the neutrosophic grey relational coefficient of each alternative from INEURS

Similarly, from Equation (8) the neutrosophic grey relational coefficient of each alternative from INEURS can be obtained as follows:

$$g_{ij}^{-} = \begin{bmatrix} 1.0000 & 0.4641 & 0.3797 & 1.0000 & 0.3333 & 1.0000 \\ 0.4641 & 1.0000 & 0.3333 & 0.4641 & 0.3750 & 0.7500 \\ 0.4641 & 0.3539 & 0.4641 & 0.5505 & 0.4286 & 0.7500 \\ 0.3797 & 0.3539 & 0.5505 & 0.3539 & 1.0000 & 0.5000 \end{bmatrix} \tag{14}$$

Step 7. Determination of the degree of neutrosophic grey relational co-efficient of each alternative from INERS and INEURS

The required neutrosophic grey relational co-efficient corresponding to INERS is obtained using equation (9) as follows:

$$g_1^{+} = 0.63635,\; g_2^{+} = 0.62520,\quad g_3^{+} = 0.49562\;,\quad g_4^{+} = 0.52720 \tag{15}$$

and corresponding to INEURS is obtained with the help of equation (10) as follows:

$$g_1^{-} = 0.74882,\; g_2^{-} = 0.58445,\quad g_3^{-} = 0.50886\;,\quad g_4^{-} = 0.46184 \tag{16}$$

Step 8. Calculation of neutrosophic relative relational degree -

Thus neutrosophic relative degree of each alternative from INERS can be obtained with the help of equation (11) as follows:

$R_1 = 0.459402$; $R_2 = 0.516844$; $R_3 = 0.493410$; R (17)

Step 9. Ranking the alternatives

The ranking order of all alternatives can be determined according the value of neutrosophic relational degree i.e

$R_4 \succ R_2 \succ R_3 \succ R_1$

It is seen that the highest value of neutrosophic relational degree is R_4. Therefore the best alternative brick is identified as A_4.

Step10. End

6. Advantages of the proposed approach

The proposed approach is very flexible as it uses the realistic nature of attributes i.e. the degree of indeterminacy as well as degree of rejection and acceptance simultaneously. In this paper, we showed how the proposed approach could provide a well-structured, practical, and scientific selection. New criteria are easily incorporated in the formulation of the proposed approach.

7. Conclusion

In this study, the concept of single valued neutrosophic set proposed by Wang et al. [5] with grey relational analysis [20] is used to deal with realistic brick selection process. Neutrosophic decision making based on grey relational analysis approach is a practical, versatile and powerful tool that identifies the criteria and offers a consistent structure and process for selecting bricks by employing the concept of acceptance, indeterminacy and rejection of single valued neutrosophic sets simultaneously. In this study, we demonstrated how the proposed approach could provide a well-structured, rational, and scientific selection practice.

Therefore, in future, the proposed approach can be used for dealing with multi-attribute decision-making problems such as project evaluation, supplier selection, manufacturing system, data mining, medical diagnosis and many other areas of management decision making. Neutrosophic sets, degree of rejection (non membership), degree of acceptance (membership) and degree of indeterminacy (hesitancy) are independent to each other. In this sense, the concept of single valued neutrosophic set applied in this paper is a realistic application of brick selection process. This selection process can be extended in the environment dealing with interval single valued neutrosophic set [21].

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