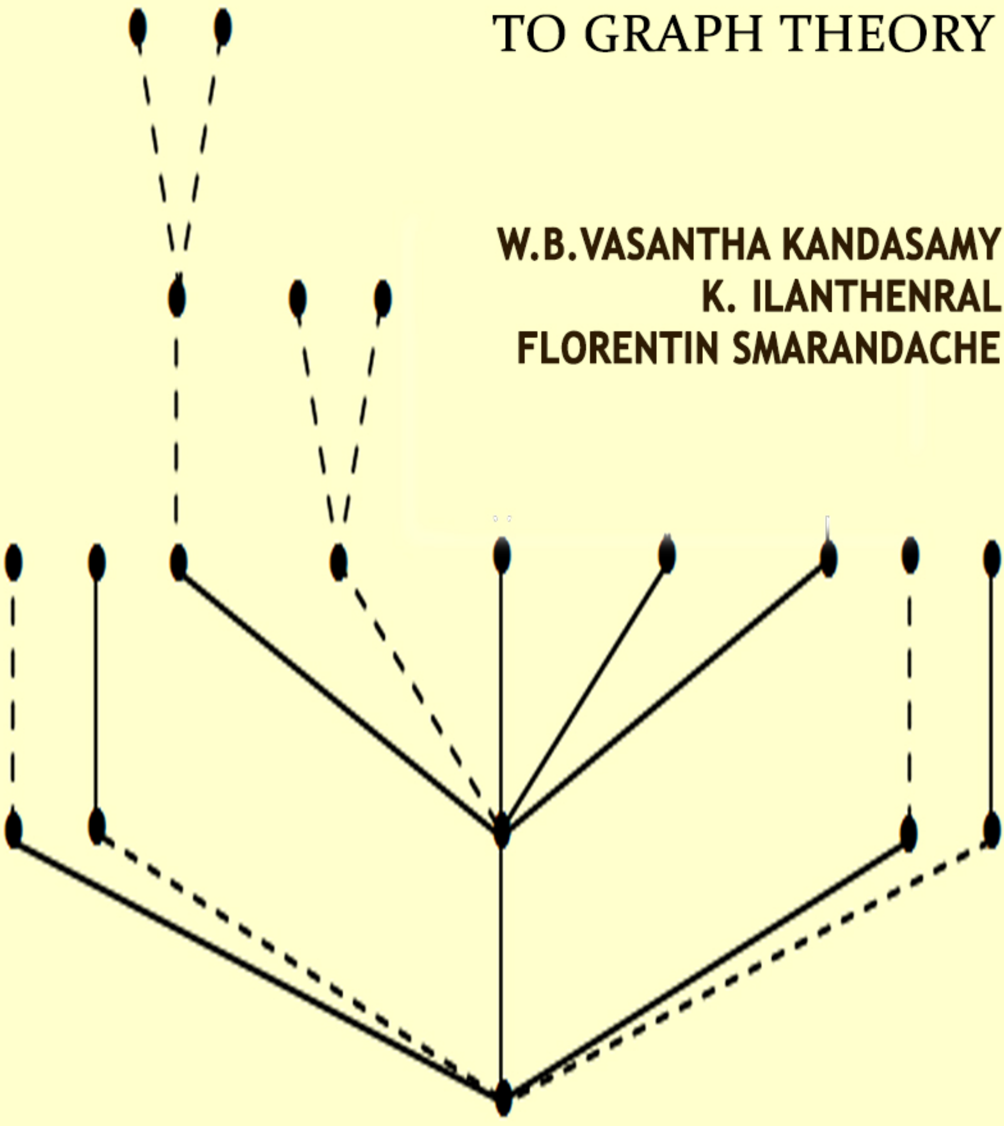
NEUTROSOPHIC GRAPHS:
A NEW DIMENSION TO GRAPH THEORY



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A New Dimension to Graph Theory

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PREFACE

In this book authors for the first time have made a through study of neutrosophic graphs. This study reveals that these neutrosophic graphs give a new dimension to graph theory. The important feature of this book is it contains over 200 neutrosophic graphs to provide better understanding of this concepts. Further these graphs happen to behave in a unique way inmost cases, for even the edge colouring problem is different from the classical one. Several directions and dimensions in graph theory are obtained from this study.

Finally certainly these new notions of neutrosophic graphs in general and in particular the bipartite neutrosophic graphs and neutrosophic trees follow special format distinctly different from the usual graphs.

Positively these can find applications in data mining and in other various engineering problems which has indeterminacy associated with it.

However these directed neutrosophic graphs have been applied by the authors in Neutrosophic Cognitive Maps (NCM) models, Neutrosophic Relational Maps (NRM) models and Neutrosophic Relational Equations (NRE).

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

W.B.VASANTHA KANDASAMY
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FLORENTIN SMARANDACHE

Chapter One

INTRODUCTION

In this chapter we just recall some basic definitions about neutrosophy. For basic concepts about graphs please refer [1].

Here we introduce the notion of neutrosophic logic created by Florentin Smarandache [2-4] which is an extension of the fuzzy logic in which indeterminancy is included. It has become very essential that the notion of neutrosophic logic play a vital role in several of the real valued problems like law, medicine, industry, finance, engineering IT, etc.

These neutrosophic cognitive maps models make use of neutrosophic graphs. Here the directed graphs of an FCMs (Fuzzy Cognitive Maps) or NCMs (Neutrosophic Cognitive Maps) or FRMs (Fuzzy Relational Maps) or NRMs (Neutrosophic Relational Maps) are nothing but the psychological inter relations or feelings of different nodes, where when we use NRMs and NCMs the concept of indeterminancy is also given a reasonable place.

We denote the indeterminancy by the letter I. I is such that $I^2 = I$, $I + I = 2I$, $I – I = 0$ and $I + I + \ldots + I$ (n times) $= nI$.

Neutrosophic algebraic structures like neutrosophic graphs, neutrosophic vector spaces etc; where introduced by the authors.

Further we have built analogous to Fuzzy Relational Equations (FRE) Neutrosophic Relational Equations (NRE) [7]. These models are also depicted by neutrosophic bipartite graphs. [5-8].

Thus we see the neutrosophic graphs happen to play a vital role in the building of neutrosophic models. Also these graphs can be used in networking, computer technology, communication, genetics, economics, sociology, linguistics, etc when the concept of indeterminancy is present.

Chapter Two

NEUTROSOPHIC GRAPHS

Here we proceed on to define the notion of neutrosophic graphs and their related matrices. If the edge values are from the set $\langle R \cup I\rangle$ or $\langle Q \cup I\rangle$ or $\langle Z_n \cup I\rangle$ or $\langle Z \cup I\rangle$ or $\langle C \cup I\rangle$ they will termed as neutrosophic graphs. If we take the edge values are taken from $\langle[0, 1] \cup [0, I]\rangle$ then we call such graphs of fuzzy neutrosophic graphs.

In most cases we will be using only fuzzy neutrosophic graphs in the fuzzy neutrosophic models used by us.

Example 2.1: Let us consider the graph this is a neutrosophic

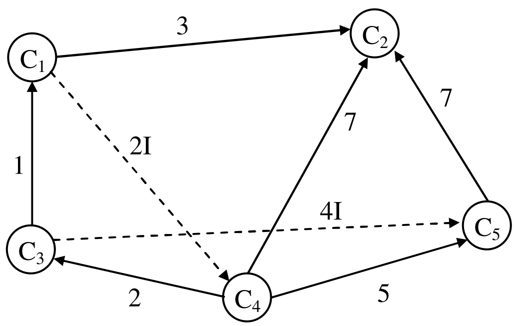


Figure 2.1

directed graph with 5 vertices and some edges are neutrosophic edges.

We denote the neutrosophic edges by dotted lines. The neutrosophic matrix associated with this graph is a 5×5 matrix M which is as follows:

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 2I & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 4I \\ 0 & 7 & 2 & 0 & 5 \\ 0 & 7 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Example 2.2: Let $\{V_1, V_2, \ldots, V_7\}$ be the vertices of a neutrosophic directed graph given in the following.

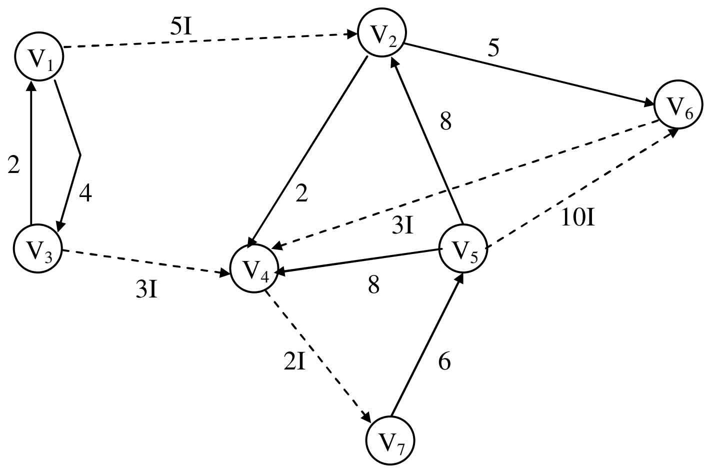


Figure 2.2

The matrix N associated with the neutrosophic directed graph is as follows:

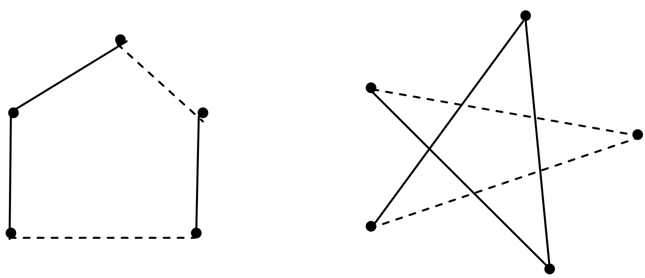
$$N = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{array} \!\!\! \begin{array}{c} \begin{array}{ccccccc} V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 \end{array} \\ \begin{bmatrix} 0 & 5I & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 5 & 0 \\ 2 & 0 & 0 & 3I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2I \\ 0 & 8 & 0 & 8 & 0 & 10I & 0 \\ 0 & 0 & 0 & 3I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 \end{bmatrix} \end{array}.$$

Clearly N is 7×7 neutrosophic matrix.

We see both the neutrosophic graphs are directed and are not complete neutrosophic graphs.

We now describe complete neutrosophic graphs.

Example 2.3: The following graph G is a neutrosophic graph.

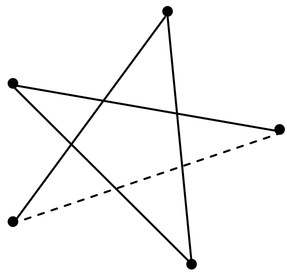


G **G′**

Figure 2.3

The above graph G′ is also a neutrosophic graph which is the complement of G.

Consider the neutrosophic graph G″.

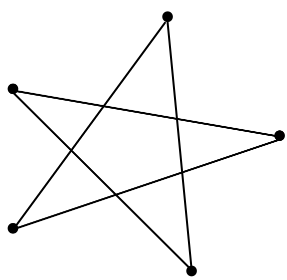


G″

Figure 2.4

Clearly G″ is not the complement of G.

Suppose H is the graph

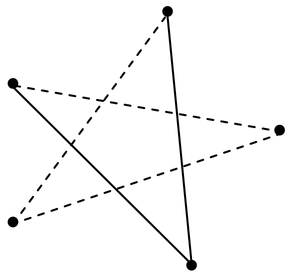


H

Figure 2.5

H is not the complement of G as H is not neutrosophic.

Consider H′



H′

Figure 2.6

H′ is the neutrosophic graph but H′ is not the complement of G. Further we see the neutrosophic graphs G and G′ are isomorphic.

However G and G″ are not isomorphic. Even G′ is not isomorphic with G″.

G, G′ and G″ are not isomorphic with H.

H′ is not isomorphic with G, G′, G″ and H. Thus in neutrosophic graphs we see there are several graphs which has same number of edges and same number of vertices but which are not isomorphic.

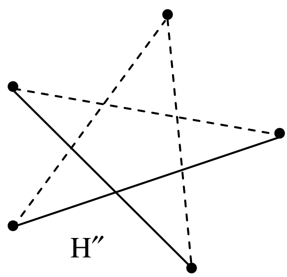


Figure 2.7

H″ is not isomorphic with any of these graphs. Further none of the graphs mentioned above are complements of H″. The only graph which is the complement of H″ is K″.

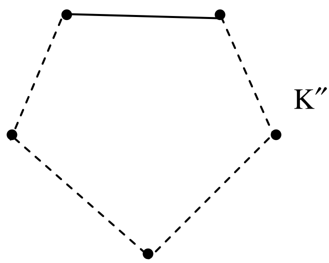


Figure 2.8

Further H″ and K″ are isomorphic. Thus we see neutrosophic graphs can be isomorphic if the number of vertices are the same, the number of edges are the same and the number of neutrosophic edges must be same in both graphs.

Thus if the number of edges are the same but the neutrosophic edges are not the same they are not isomorphic.

We say such graphs belong to the same class but not under isomorphic property.

Thus if the graph is a single point no change can be found. If G is a graph say

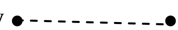
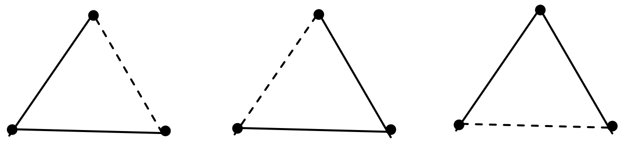
and G′ is the graph say  then

Figure 2.9 Figure 2.10

G is not isomorphic with G′.

Let G_1, G_2 and G_3 be three neutrosophic graphs



G_1 G_2 G_3

Figure 2.11

We say G_1, G_2 and G_3 are identical neutrosophic graphs.

Consider the neutrosophic graphs H_1 and H_2.

H_1 H_2

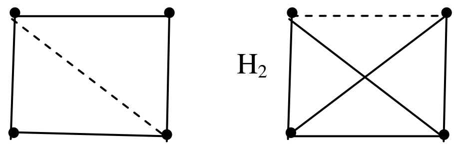
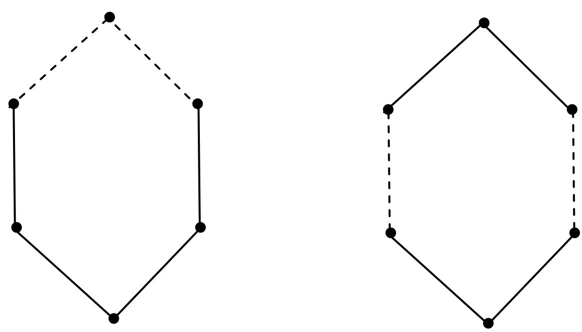


Figure 2.12

We see H_1 and H_2 are not isomorphic they are not identical either.

Consider the graphs P_1, P_2, P_3, …, P_6.



P_1 P_2

Figure 2.13

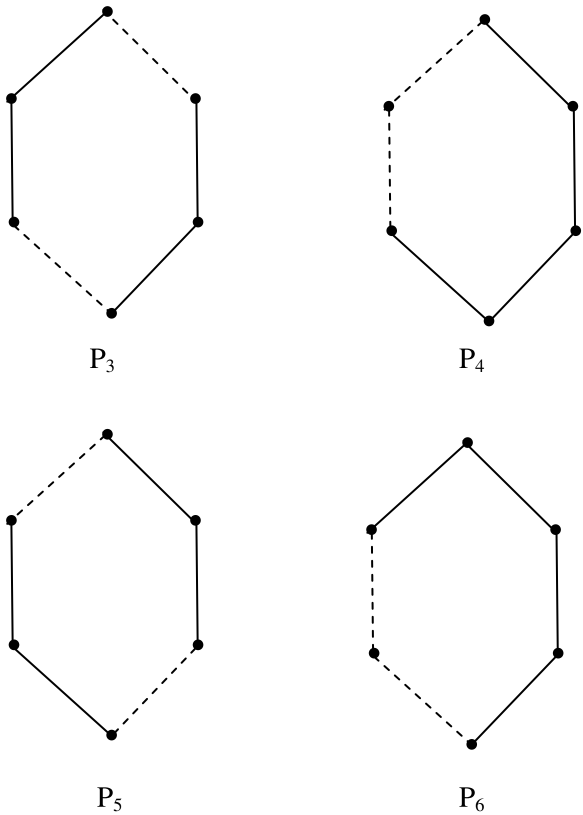


Figure 2.13

They are all identical neutrosophic graphs and are also isomorphic as graphs.

It is left as an open problem given p edges and n vertices of a neutrosophic graph ($n \geq 4$).

(i) Find the number of identical neutrosophic graphs.

(ii) Find the number of non identical neutrosophic graphs.

(iii) How many neutrosophic graphs can be constructed? (with n edges and n vertices).

We will just discuss this problem in case of three edges.

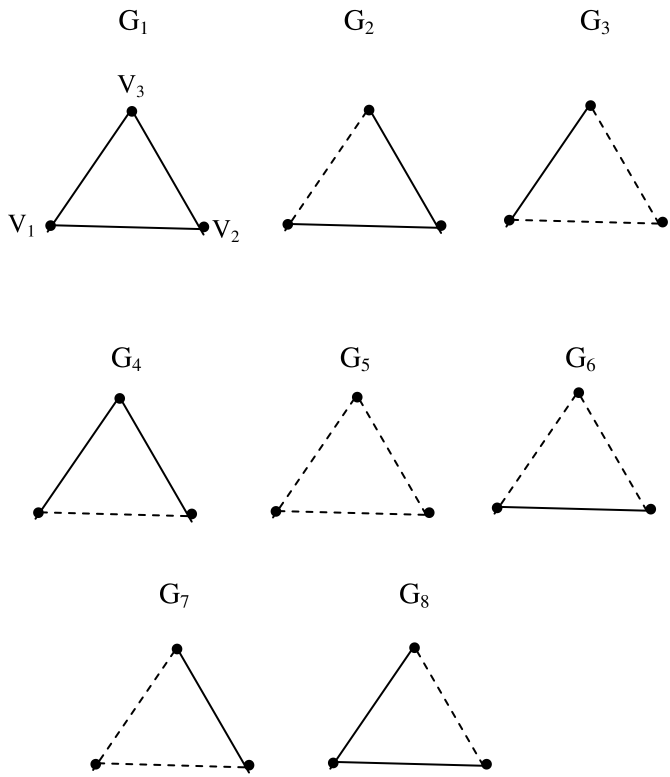


Figure 2.14

We see all the eight neutrosophic graphs are not identical or isomorphic. We see G_1 and G_5 remain unrelated with every other neutrosophic graphs.

However G_2, G_4 and G_8 are identical as graphs and G_3, G_7 and G_6 are identical as a graphs.

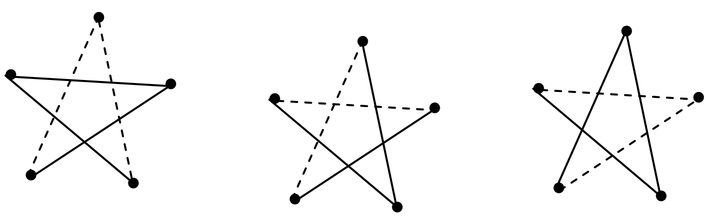
To this end we define some more concepts related with these neutrosophic graphs.

We in the first place say the base line in G_1, G_2, G_6 and G_8 are real and base line in the neutrosophic graphs G_3, G_4, G_5 and G_7 are neutrosophic edges.

Suppose we fix the vertices as V_1, V_2 and V_3 statement that V_2 V_3 is the base line and make the following subtle observations.

(i) We say, two neutrosophic graphs G and G′ are neutrosophically isomorphic if from a isomorphic vertex of G there are ‘t’ neutrosophic edges then it must be true for G′.

This is explained

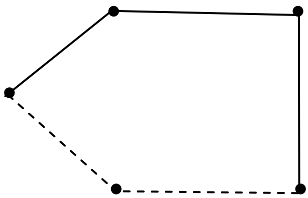


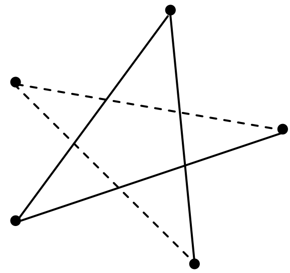
G G′ G″

Figure 2.15

Clearly G and G′ are not neutrosophically isomorphic. However G and G′ are isomorphic. We see G and G″ are neutrosophically isomorphic but they are also isomorphic as neutrosophic graphs.

Consider

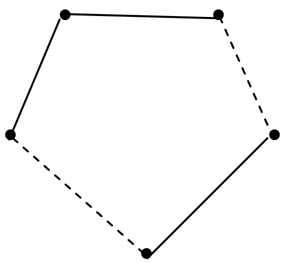




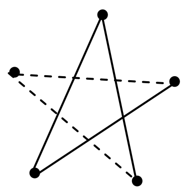
G H

Figure 2.16

G and H two neutrosophic graphs we see G and H both isomorphic as well as neutrosophically isomorphic



and



P_1 P_2

Figure 2.17

are isomorphic but not neutrosophically isomorphic.

THEOREM 2.1: *Two neutrosophic graphs which are neutrosophically isomorphic are isomorphic. But isomorphic neutrosophic graphs in general are not neutrosophically isomorphic.*

Proof: If two neutrosophic graphs G and H are neutrosophically isomorphic then we see the number of edges in

G is equal to number of edges in H and the number of vertices in H and G are equal. Further the number of neutrosophic edges in G is equal to the number of neutrosophic edges in H. Finally the number of neutrosophic edges emerging for any of the isomorphic vertices are the same, then the two neutrosophic graph are isomorphic.

If on the other hand two neutrosophic graphs are isomorphic they need not be neutrosophically isomorphic.

For consider the two neutrosophic graphs G and H where

G is

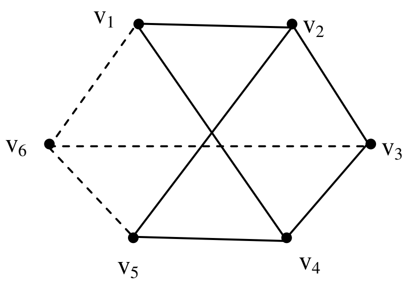


Figure 2.18

and

H =

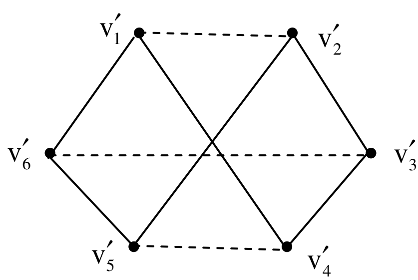


Figure 2.19

Clearly G and H are isomorphic but are not neutrosophically isomorphic. Hence the theorem.

As in case of usual graphs we can define in case of neutrosophic graphs the notion of union of graphs, intersection of graphs and difference of graphs.

We will only illustrate this situation by some examples.

Consider the neutrosophic graphs.

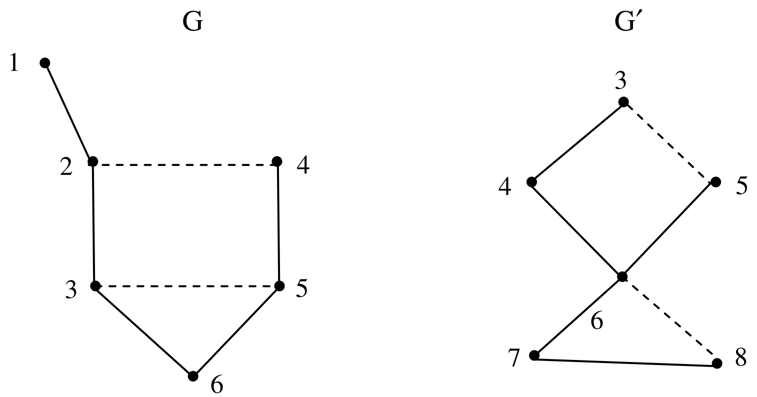


Figure 2.20

Now G ∪ G′

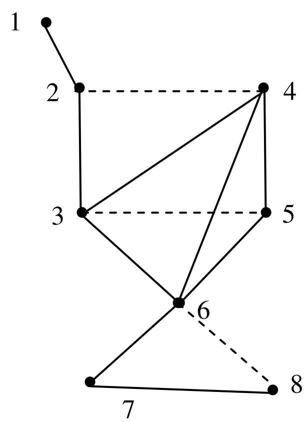


Figure 2.21

is again a neutrosophic graph.

G \ G′ =

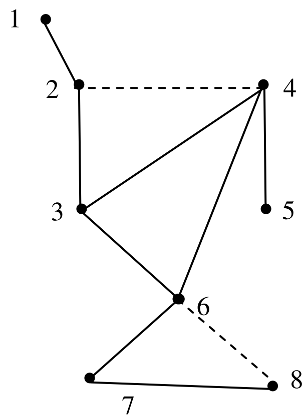


Figure 2.22

G – G′ is again a neutrosophic graph.

Consider G ∩ G′

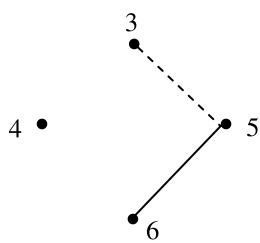


Figure 2.23

G ∩ G′ is again a neutrosophic graph.

However we can have two neutrosophic graphs whose difference and intersection, are not neutrosophic graphs.

To this end we give an example or two.

Consider the graph

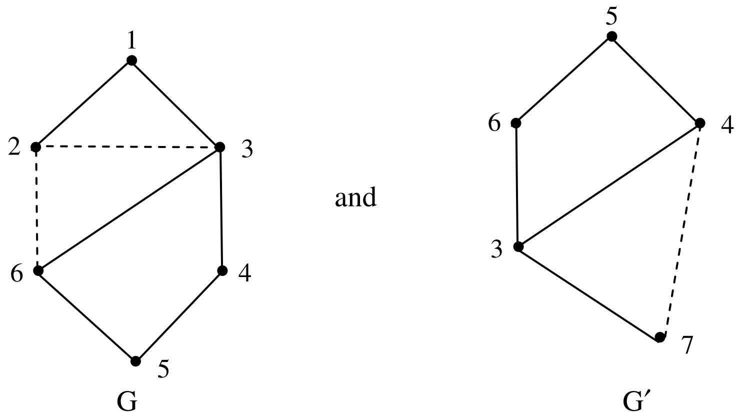


Figure 2.24

both G and G′ are neutrosophic graphs.

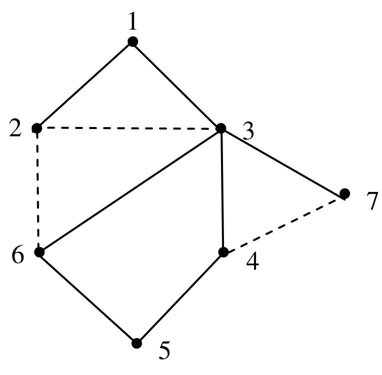


Figure 2.25

which is again a neutrosophic graph.

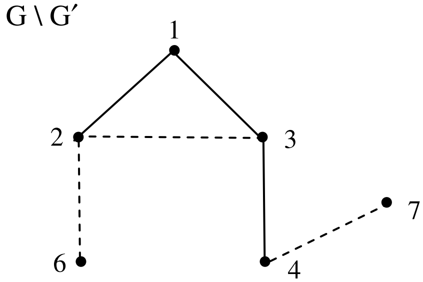


Figure 2.26

Consider G \ G′, G \ G′ is again a neutrosophic graph. Now G ∩ G′ is given by

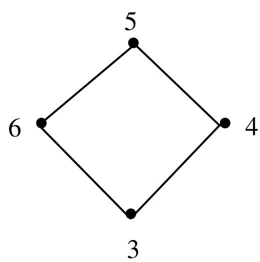


Figure 2.27

Clearly G ∩ G′ is not a neutrosophic graph.

Now if take the neutrosophic graphs G and G′;

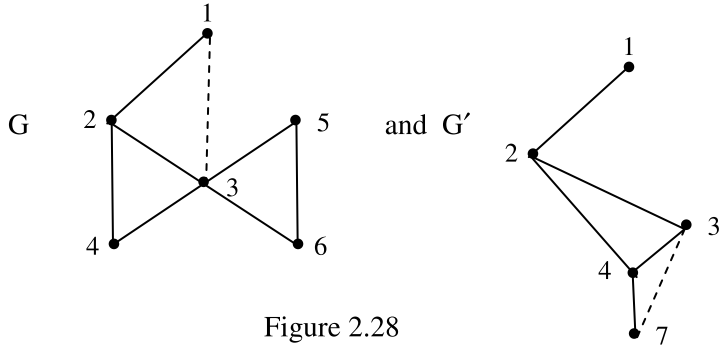


Figure 2.28

Clearly G ∪ G′ is as follows is again a neutrosophic graph.

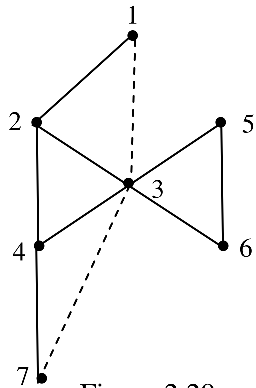


Figure 2.29

G ∩ G′ is only a graph which not neutrosophic.

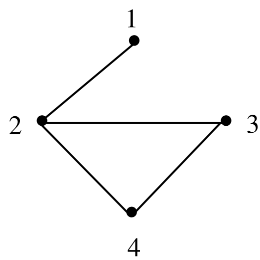


Figure 2.30

However G \ G′ is as follows.

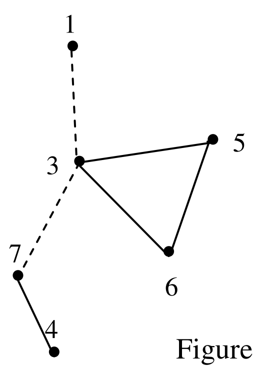


Figure 2.31

which is a neutrosophic graph.

Now we give an example where G \ G′ is not a neutrosophic graph. Let G and G′ be two neutrosophic graphs.

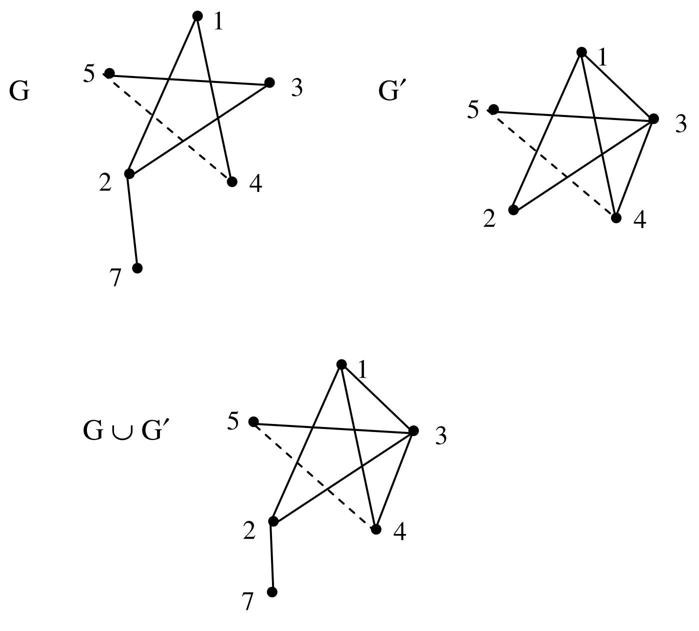


Figure 2.32

Clearly G ∪ G′ is a neutrosophic graph.

Consider G \ G′;

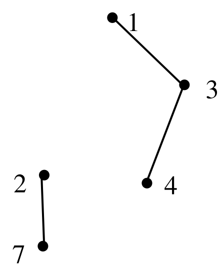


Figure 2.33

Clearly $G \setminus G'$ is not a neutrosophic graph.

Consider $G \cap G'$

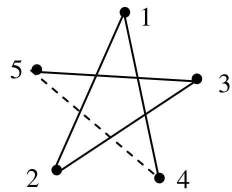


Figure 2.34

$G \cap G'$ is the neutrosophic graph.

In view of the above examples we have the following theorem.

THEOREM 2.2: *Let G and G′ be any two neutrosophic graphs.*

(i) $G \cup G'$ *is always a neutrosophic graph.*

(ii) $G \cap G'$ *in need not general be always a neutrosophic graph.*

(iii) $G \setminus G'$ *in general need not always be a neutrosophic graph.*

The proof of the above theorem is direct hence it is left as an exercise to the reader.

In the neutrosophic graph there is a neutrosophic path if there exist at least one edge $x_i\, x_{i+1}$ which is a neutrosophic edge.

If all the edges are neutrosophic edges then we call the path as the pure neutrosophic path.

We will just illustrate this by examples.

Example 2.4: Let $V = \{x_0, x_1, \ldots, x_8\}$ $E = \{x_0\, x_1, x_1\, x_2, \ldots, x_7\, x_8\}$

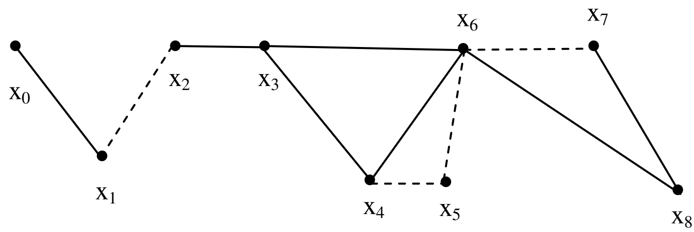


Figure 2.35

This path P is a neutrosophic path however P is not a pure neutrosophic path.

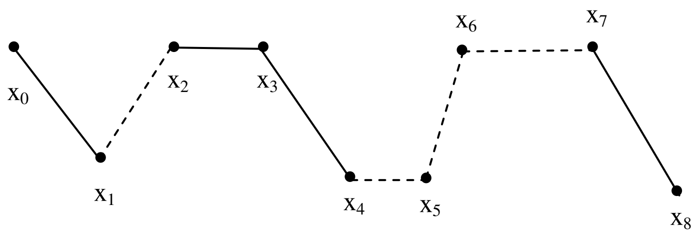


Figure 2.36

We give some more examples.

Example 2.5: Let G be a

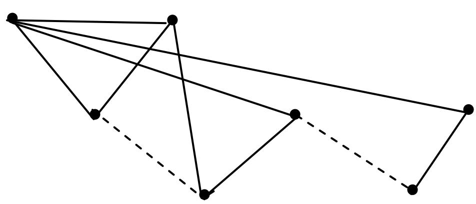


Figure 2.37

neutrosophic graph. The neutrosophic path of G is as follows

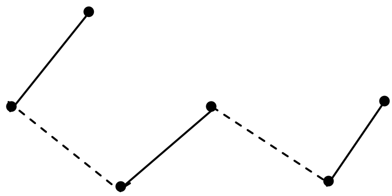


Figure 2.38

We see this path in G is not pure neutrosophic.

Example 2.6: Let G be a neutrosophic graph. Let P be the path.

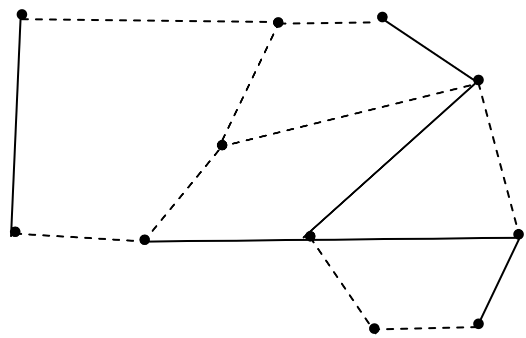


Figure 2.39

We have in this graph pure neutrosophic path also.

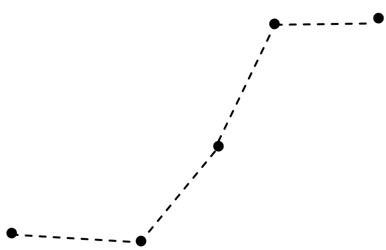


Figure 2.40

Now we proceed onto discuss about the degree or valency of a vertex v of a graph G. Degree of v can be purely neutrosophic or mixed neutrosophic or non neutrosophic.

We say degree of v is purely neutrosophic if all the edges at v are only neutrosophic edges and the number of such neutrosophic edges corresponds to pure neutrosophic degree.

If the number of edges at v is n and if $d \neq 0$ $(d < n)$ are neutrosophic and the rest $n - d$ are not neutrosophic we say the mixed degree of v is $n = d + (n-d)$.

If the number of edges at the vertex v are such that none of the edge is neutrosophic then we call the degree of v to be non neutrosophic degree. Thus in case of a neutrosophic graph the degree of the vertex can be three types.

To find effect we give some examples.

Example 2.7: Let G be a neutrosophic graph given in the following.

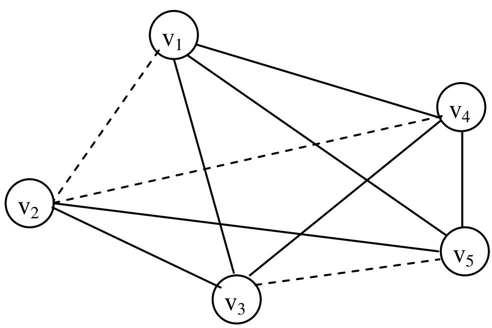


Figure 2.41

We see for all the vertices of G none of them have pure neutrosophic degree or non neutrosophic degree. All of them have a mixed neutrosophic degree. Mixed neutrosophic degree of v_1 is 4 with $d(v_1) = 1 + 3$ (1 neutrosophic edge rest non neutrosophic edges).

d (v_2) = 2 + 2 (2 neutrosophic edges and 2 non neutrosophic edge)

d (v_3) = 3 +1 (3 non neutrosophic edges and one neutrosophic edge)

d (v_4) = 1 + 3 (1 neutrosophic edges and 3 non neutrosophic edge) and

d (v_5) = 3 + 1 (3 non neutrosophic edges and one neutrosophic edge).

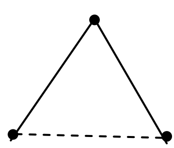
Likewise we can define three types of regularity in neutrosophic graphs.

If G be a neutrosophic graph and the degree of every vertex is the same and every vertex has a mixed neutrosophic edge or pure neutrosophic edge or non neutrosophic edge then we call G to be just mixed neutrosophic regular.

If each vertex has mixed neutrosophic k-edges say k = d + (k–d) with d neutrosophic edges and (k–d) non neutrosophic edges we call G to be uniformly mixed neutrosophic regular. If every vertex has k edges and all of them are pure neutrosophic we call them purely neutrosophic regular.

We see  this graph is purely neutrosophic regular.

Consider



G

Figure 2.42

the neutrosophic graph G is just neutrosophic mixed regular.

Let H

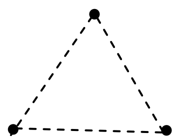


Figure 2.43

be a pure neutrosophic graph. H is purely neutrosophic 2 regular. Using 3 vertices it is impossible to get a uniformly mixed neutrosophic regular graph.

We have the following interesting theorem the proof of which is left to the reader.

THEOREM 2.3: *If a neutrosophic graph G is purely neutrosophic regular then G is a pure neutrosophic graph.*

We first give some examples of regular neutrosophic graphs of all the three types.

Example 2.8: Let G be a neutrosophic graph given in the following:

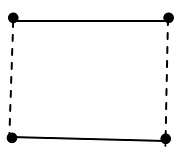


Figure 2.44

G is a uniformly mixed neutrosophic 2-regular graph.

Example 2.9: Let G be a neutrosophic graph

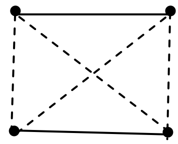


Figure 2.45

G is clearly a uniformly neutrosophic mixed 3-regular graph.

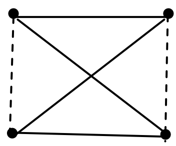
H = 

Figure 2.46

is again a uniformly mixed neutrosophic 3-regular graph.

It is pertinent to mention here that both G and H are uniformly mixed neutrosophic 3-regular graphs but they are different.

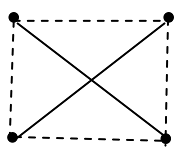
K is 

Figure 2.47

Neutrosophic graph with uniformly mixed neutrosophic 3-regular.

However K is different from H also K and G are not identical.

S is a purely neutrosophic graph

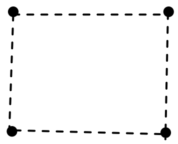


Figure 2.48

S is purely neutrosophic 2-regular

Take T a purely neutrosophic graph

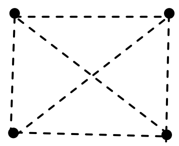


Figure 2.49

T is purely neutrosophic 3-regular.

Thus T and S are distinctly different. Consider B a neutrosophic graph.

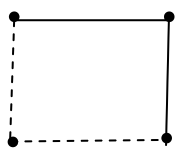


Figure 2.50

B is just neutrosophic mixed 2-regular graph.

Let N be a neutrosophic graph.

N = 

Figure 2.51

N is a just mixed neutrosophic 2-regular graph.

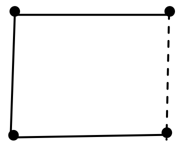
Let L be a neutrosophic graph L = 

Figure 2.52

L is a just mixed neutrosophic 2-regular graph.

Let D be a neutrosophic graph

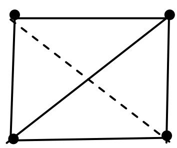


Figure 2.53

D is a just mixed neutrosophic 3-regular graph.

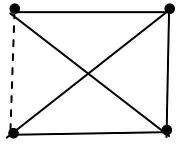
Let E be a neutrosophic graph E = 

Figure 2.54

E is a just mixed neutrosophic 3-regular graph.

Now we will find type of neutrosophic regular graphs with 5 and more vertices.

Let A be the neutrosophic graph.

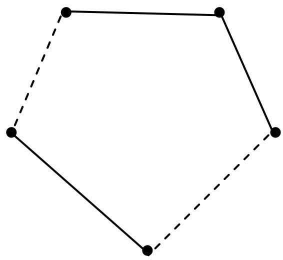


Figure 2.55

A is a just mixed neutrosophic 2-regular graph.

We cannot get a uniformly mixed neutrosophic regular graph with 5 vertices and 5 edges.

Let B be the pure neutrosophic graph

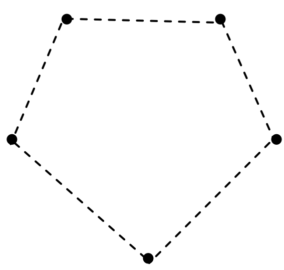


Figure 2.56

B is a purely neutrosophic 2-regular graph.

Consider H =

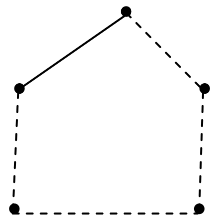


Figure 2.57

H is a neutrosophic graph which is just mixed neutrosophic 2 regular graph.

We see we have the following theorem.

THEOREM 2.4: *A neutrosophic graph with 5 vertices which is 2 regular cannot be uniformly mixed two regular.*

Proof is direct hence left as an exercise to the reader.

We wish to state if a neutrosophic graph G with odd number of vertices and if G is 2-regular; can G be uniformly mixed two regular?

Example 2.10: Let G be a neutrosophic graph with seven vertices.

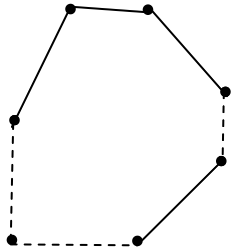
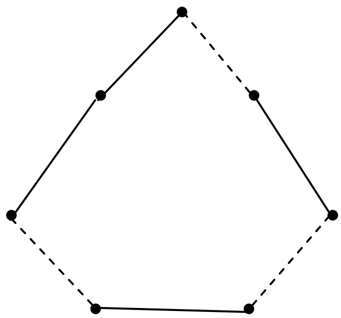


Figure 2.58

Clearly G is just mixed neutrosophic 2-regular.

But we see we cannot find a uniform neutrosophic mixed 2-regular.



H

Figure 2.59

The neutrosophic graph H is only just mixed neutrosophic 2-regular.

Clearly H is not uniformly neutrosophic 2-regular.

Let M be pure neutrosophic

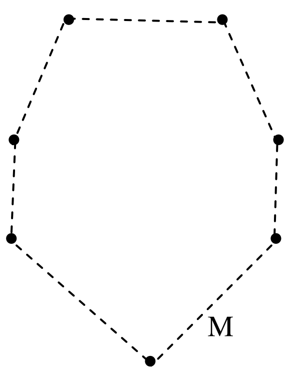


Figure 2.60

graph. Clearly M is a pure neutrosophic 2-regular.

We see we cannot get for a neutrosophic graph which is uniformly mixed neutrosophic 2-regular.

Can M be a uniformly mixed neutrosophic 3 regular with 7 vertices?

Is the above question true in case of 5 vertices

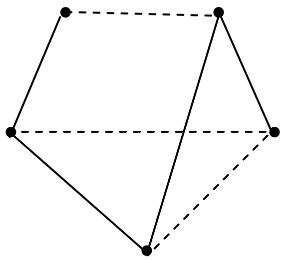


Figure 2.61

We leave the following as open problems.

Problem 2.1: Suppose G is a neutrosophic graph with five vertices.

(i) Can G be just neutrosophic mixed 3-regular?
(ii) Can G be uniformly neutrosophic mixed 3-regular?
(iii) Can G be pure neutrosophic 3-regular?

We just take G to be pure neutrosophic graph with five vertices.

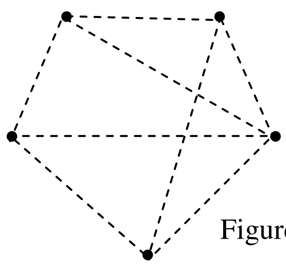


Figure 2.62

It is impossible to have 3-regular pure neutrosophic graph with five vertices.

However there cannot exist a 3-regular just mixed neutrosophic graph with 5 vertices. Likewise there cannot exist a 3-regular uniform mixed neutrosophic graph.

Can a neutrosophic graph with five vertices be 4-regular of any type?

G =

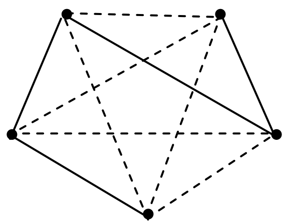


Figure 2.63

We see the neutrosophic graph G with 5 vertices. Clearly the graph G is uniformly mixed neutrosophic 4-regular we see degree of each of the vertices of the neutrosophic graph has two neutrosophic edges and two usual edges at each of the vertex of G.

Consider the pure neutrosophic graph G with five vertices given by

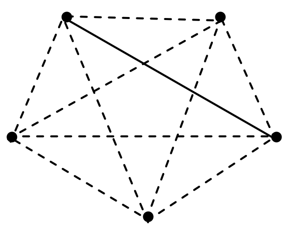


Figure 2.64

Clearly G is pure neutrosophic 4-regular graph.

Now we proceed onto study the neutrosophic graph G with seven vertices.

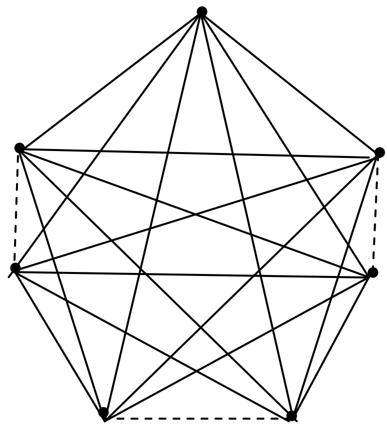


Figure 2.65

Clearly this neutrosophic graph G is just mixed 6-regular.

If we take pure neutrosophic graph H with seven vertices say H;

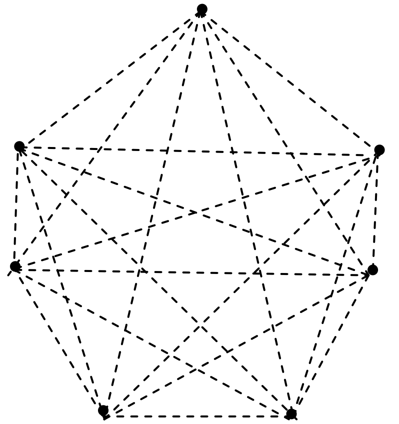


Figure 2.66

is a pure neutrosophic 6-regular graph.

In view of this we have the following theorem the proof of which is left to the reader.

THEOREM 2.5: *A complete pure neutrosophic graph with n vertices is a pure neutrosophic (n–1)-regular graph.*

Now we see only a few graphs of uniformly mixed neutrosophic regular graphs.

In view of this we leave the following as open problems.

Problem 2.2: Characterize those neutrosophic graphs which are uniformly mixed neutrosophic r-regular.

Problem 2.3: Characterize those neutrosophic graphs which are only just mixed neutrosophic k-regular.

A mixed neutrosophic walk in a neutrosophic graph G is a non empty alternating sequence $v_0e_0\ v_1e_1\ \ldots\ e_{k-1}\ v_k$ where at least one of the $e_i = \{v_i\ v_{i+1}\}$ is a neutrosophic edge.

If in the walk $v_0e_0\ v_1e_1\ \ldots\ e_{k-1}v_k$ in a neutrosophic graph G each e_i is only a neutrosophic edge then we define the walk to be a pure neutrosophic walk; $0 \leq i \leq k-1$.

If in a mixed neutrosophic walk $v_0e_0\ v_1e_1\ \ldots\ e_{k-1}\ v_k$ if e_i is a neutrosophic path than e_{i-1} and e_{i+1} are usual path, that is if e_0 is a neutrosophic path e_1 is a usual path, e_2 is a neutrosophic path and e_3 is the usual path and e_{k-1} is the usual path. Likewise if e_0 is the usual path, e_1 is a neutrosophic path so on the e_{k-1} is the neutrosophic path. Then we define the walk to be specially mixed alternating neutrosophic walk.

First we will supply with examples for all the three types of walk. The walk is a closed walk if $v_0 = v_{k-1}$.

Example 2.11: Let us consider the neutrosophic graph G

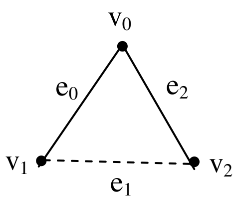


Figure 2.67

We see the walk is a closed walk and the walk is only a mixed neutrosophic closed walk.

Consider the neutrosophic graph H

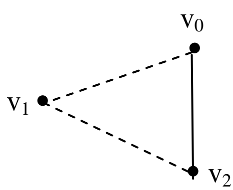


Figure 2.68

The walk is again only a mixed neutrosophic closed walk.

Consider the pure neutrosophic graph.

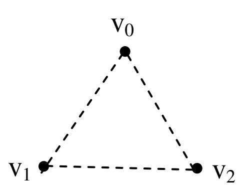


Figure 2.69

The walk is a pure neutrosophic closed walk.

However for the complete neutrosophic graph K^3 we cannot obtain a specially mixed alternating neutrosophic closed walk.

Example 2.12: Let G be the neutrosophic graph;

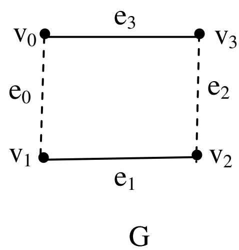


Figure 2.70

Clearly G has a closed specially mixed alternating neutrosophic walk.

Consider

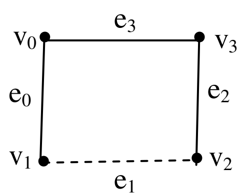


Figure 2.71

neutrosophic graph. The walk is only is a mixed neutrosophic closed walk.

Example 2.13: Let G be a pure neutrosophic graph

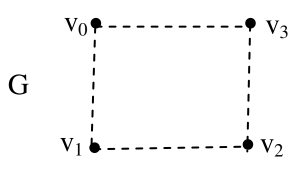


Figure 2.72

G has a pure neutrosophic closed walk.

Thus this graph with four vertices can have all the types of neutrosophic walks.

Example 2.14: Let G be the neutrosophic graph given by in the following

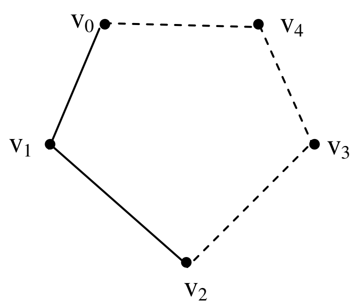


Figure 2.73

The walk of G is a closed mixed neutrosophic walk. Consider the neutrosophic graph

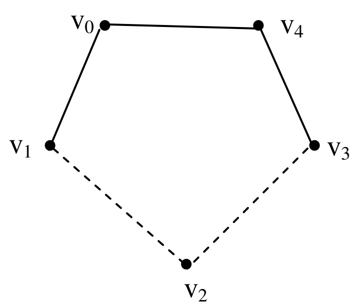


Figure 2.74

H is associated only a mixed neutrosophic closed walk.

Consider P

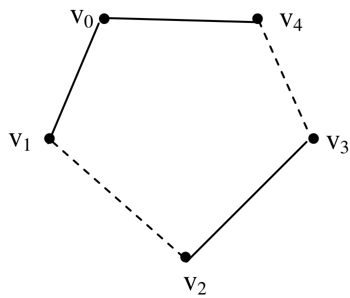


Figure 2.75

the neutrosophic graph. The walk associated with P is only a mixed neutrosophic closed walk.

However for the neutrosophic graph which has a closed walk with 5 vertices we can have only a mixed neutrosophic closed walk and never a specially mixed alternating neutrosophic closed walk.

In view of this we have the following theorem the proof of which is left as an exercise to the reader.

THEOREM 2.6: *If G is a neutrosophic graph with n vertices n odd and G is mixed neutrosophic two regular with a closed walk. Then the mixed neutrosophic closed walk can never be a specially mixed alternately neutrosophic closed walk.*

Corollary 2.1: If in the above theorem n is even there exist one specially alternative mixed neutrosophic closed walk.

This proof is also direct, hence is left as an exercise to the reader.

We will illustrate this special situation by some examples.

Example 2.15: Let G be the neutrosophic graph given in the following

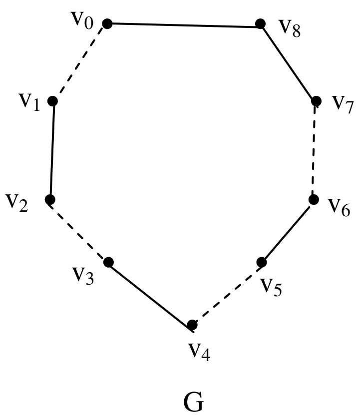


Figure 2.76

G can never have a specially alternatively mixed neutrosophic closed walk. Let H be a neutrosophic graph given in the following:

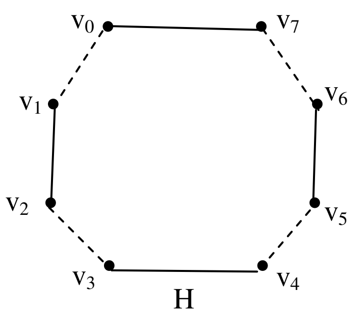


Figure 2.77

The neutrosophic graph H has a specially alternative mixed neutrosophic closed walk.

Example 2.16: Let G be a neutrosophic graph given in the following:

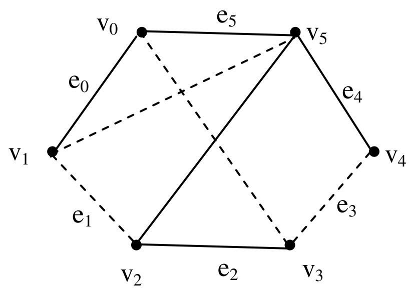


Figure 2.78

Consider the mixed neutrosophic walk from v_0e_0 v_1e_1 v_2e_2 v_3 e v_0 in G.

This is a closed specially alternative mixed neutrosophic walk.

However v_0e_0 v_1e_1 v_2e_2 … $v_5e_5v_0$ is a mixed neutrosophic closed walk which is not specially alternatively mixed neutrosophic closed walk.

Now we proceed onto define the notion of connectivity in neutrosophic graphs.

Let G be a neutrosophic graph which is non empty is called mixed neutrosophically connected if two of its vertices are linked by a mixed neutrosophic path in G; purely neutrosophically connected if two of its vertices are linked by a pure neutrosophic path.

We will give examples of them.

Example 2.17: Let G be a neutrosophic graph which is as follows.

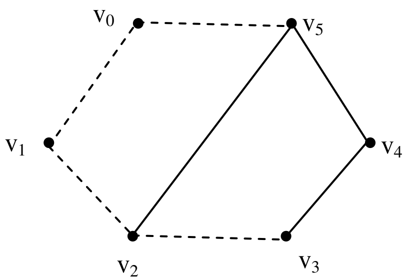


Figure 2.79

We see v_0 is neutrosophic connected to v_4 by a path which is a pure neutrosophic path.

The vertex v_3 is connected v_4 and the path is a mixed neutrosophic path.

Example 2.18: Let G be a neutrosophic graph which is as follows:

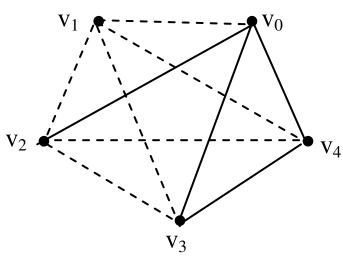


Figure 2.80

v_0 to v_4 has two paths. One is a usual path and the other is a mixed neutrosophic path.

Likewise v_0 to v_3 is the real path also v_0 to v_3 can be a mixed neutrosophic path.

We see in case of a neutrosophic graph the bridge can be neutrosophic edge or a usual edge.

We give examples of them.

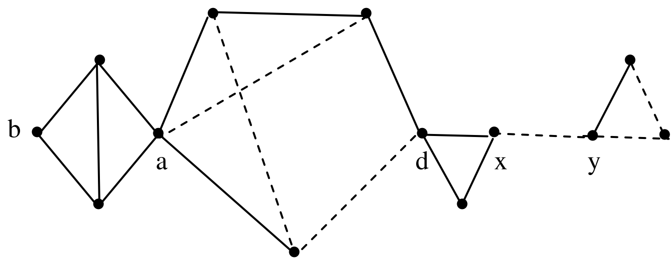


Figure 2.81

We see b, a, d, x and y are cut vertices and the bridge in this case is a neutrosophic edge.

We can have yet neutrosophic graphs whose edge is not a neutrosophic edge; we just illustrate this situation by an example.

Consider the neutrosophic graph

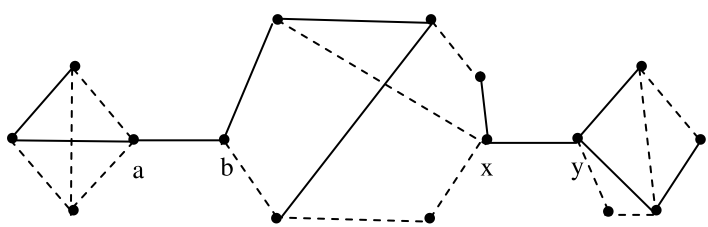


Figure 2.82

We see this neutrosophic graph has a, b, x and y to be cut vertices and $e_1 = ab$ and $e_2 = xy$ are the two bridges which are not neutrosophic edges.

Finally we can have neutrosophic graphs which can have both neutrosophic edge and usual edge.

Let G be a neutrosophic graph which is as follows:

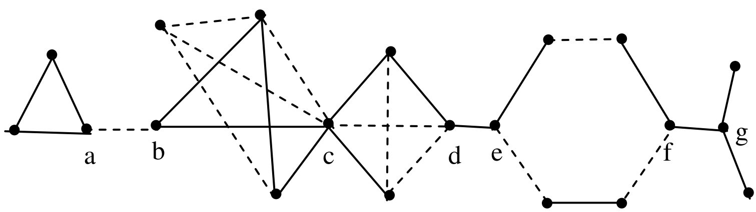


Figure 2.83

We see this neutrosophic graph G has a, b, c, d, e, f and g to be cut vertices. Further $e_1 = ab$, $e_2 = de$ and $e_3 = fg$ are the three bridges of G. We see the edges e_1 is neutrosophic where as e_2 and e_3 are usual.

Let G be a neutrosophic graph. Let P be a path say $x_0 \ldots x_{k-1}$, $k \geq 3$ of G; If P is a mixed neutrosophic path then $C = P + x_k, x_0$ is called the mixed neutrosophic cycle if P is a pure neutrosophic path then $C = P + x_{k-1}\ x_0$ is a pure neutrosophic cycle. If $x_{k-1}\ x_0$ is not neutrosophic edge we call the cycle to be a one mixed pure neutrosophic cycle.

The neutrosophic length of the cycle is the number of usual edges and neutrosophic edges. The minimum length of mixed neutrosophic cycle is called the mixed neutrosophic girth.

Maximum length of the neutrosophic cycle is the neutrosophic circumference. If the neutrosophic graph does not contain a mixed neutrosophic cycle we say the neutrosophic girth is infinite and the neutrosophic circumference is zero.

An edge which joins two vertices of a mixed neutrosophic cycle but is not itself an edge of a cycle is a neutrosophic chord of that mixed neutrosophic cycle if the edge is the neutrosophic edge, otherwise the call the chord as quasi neutrosophic chord.

The neutrosophic induced cycle in a neutrosophic graph G, a mixed neutrosophic cycle in forming an induced subgraph, is one that no neutrosophic chords or quasi neutrosophic chord.

Here it is pertinent to mention that for a neutrosophic graph G can have subgraphs which may be a neutrosophic subgraph or a usual subgraph.

The usual subgraph of a neutrosophic graph G will be defined as the quasi neutrosophic subgraph.

We will illustrate this situation by some examples.

Example 2.19: Let G be a neutrosophic graph given in the following.

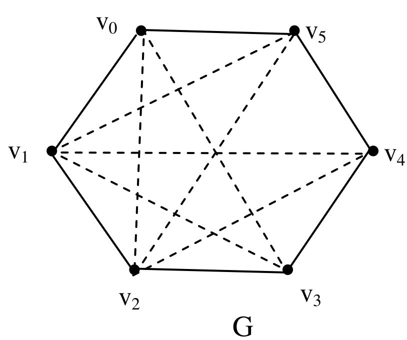
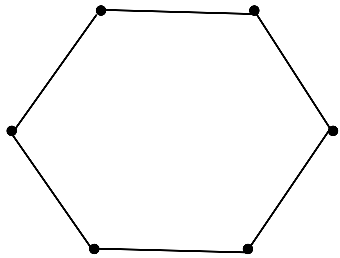


Figure 2.84

Consider the subgraph H of G



H

Figure 2.85

Clearly H is a quasi neutrosophic subgraph of G. Consider the subgraph P.

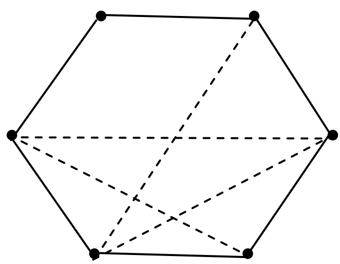


Figure 2.86

P is a neutrosophic subgraph. Consider the subgraph X of G

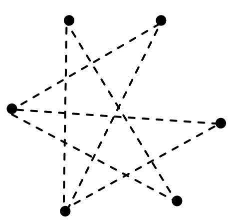
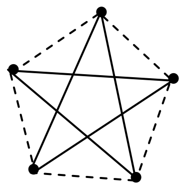


Figure 2.87

X is a pure neutrosophic subgraph which we define as the quasi pure neutrosophic subgraph.

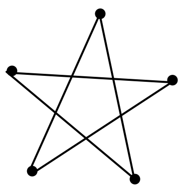
Example 2.20: Let G be a neutrosophic graph.



G

Figure 2.88

Consider the subgraph H of G

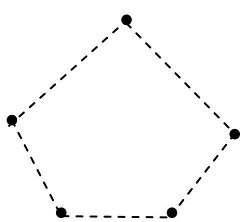


H

Figure 2.89

H is a quasi neutrosophic subgraph of G.

Consider the subgraph P of G

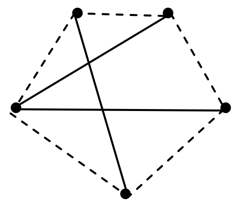


P

Figure 2.90

P is a quasi pure neutrosophic subgraph of G.

Let T be a subgraph given in the following.

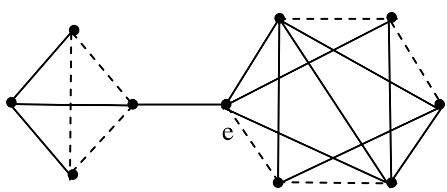


T

Figure 2.91

T is a neutrosophic subgraph of G.

Example 2.21: Let G be a neutrosophic given in the following.



G

Figure 2.92

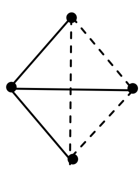
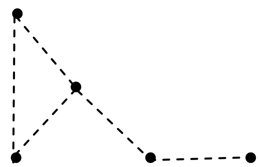
Let P  be a subgraph of G.

Figure 2.93

P is a neutrosophic subgraph of G.

Consider the subgraph H of G which is given by the following.

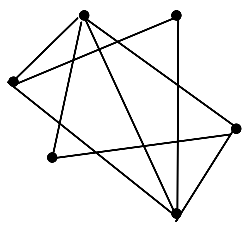


H

Figure 2.94

H is a quasi pure neutrosophic graph.

Let S be the subgraph which is as follows.



S

Figure 2.95

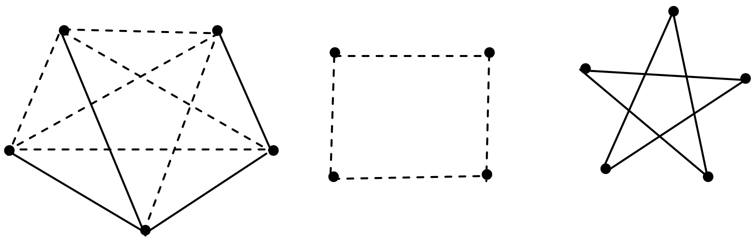
S is a quasi neutrosophic subgraph of G.

Thus a neutrosophic graph G may have a subgraph which is quasi pure neutrosophic, some subgraphs which are quasi neutrosophic and subgraphs which is just neutrosophic.

Now we see for a neutrosophic graph G all the components need not be neutrosophic subgraphs, some can be usual subgraphs and some pure neutrosophic.

We will illustrate this situation by an example.

Example 2.22: Let G be the neutrosophic graph which is as follows:

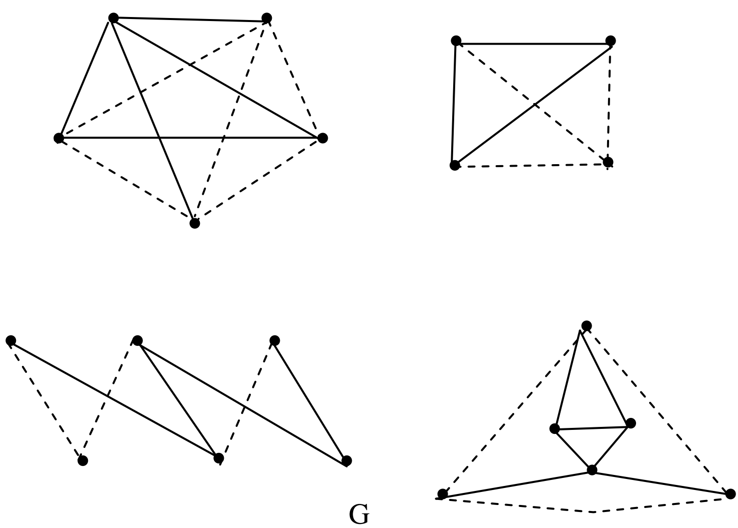


G

Figure 2.96

G has three components, one component subgraph is the usual graph, one a pure neutrosophic subgraph and another a neutrosophic graph which is not pure.

Example 2.23: Let G be a neutrosophic graph given in the following:



G

Figure 2.97

G has four components and each of the component subgraph is a neutrosophic subgraph.

Next we proceed onto define neutrosophic tree and neutrosophic forest, pure neutrosophic trees and forest.

A acyclic neutrosophic graph not containing any mixed neutrosophic cycles or pure neutrosophic cycles or usual cycles is called a neutrosophic forest.

A connected neutrosophic forest is called a neutrosophic tree.

We will first illustrate this situation by some simple examples.

Example 2.24: A neutrosophic tree is as follows:

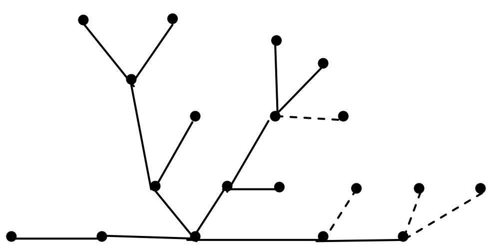


Figure 2.98

Example 2.25: A pure neutrosophic tree is as follows:

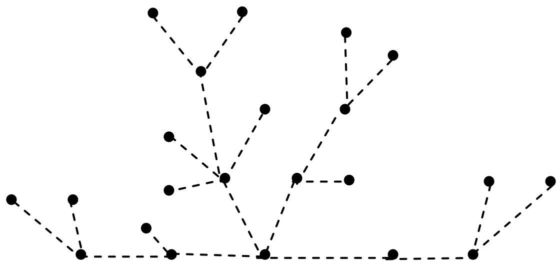
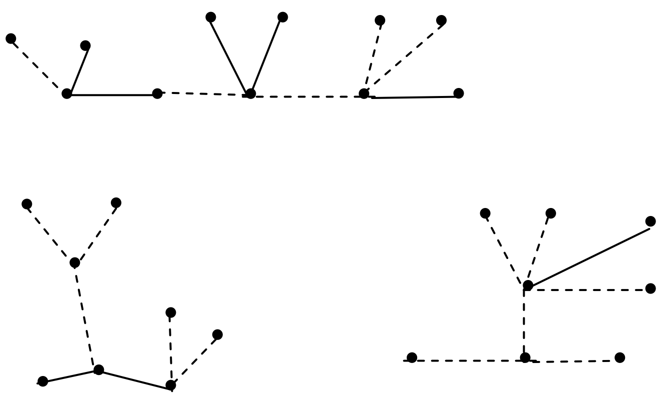


Figure 2.99

We just describe a neutrosophic forest and a pure neutrosophic forest.

Example 2.26: The following neutrosophic graph F.



F

Figure 2.100

is a neutrosophic forest.

Example 2.27: The following is a pure neutrosophic forest

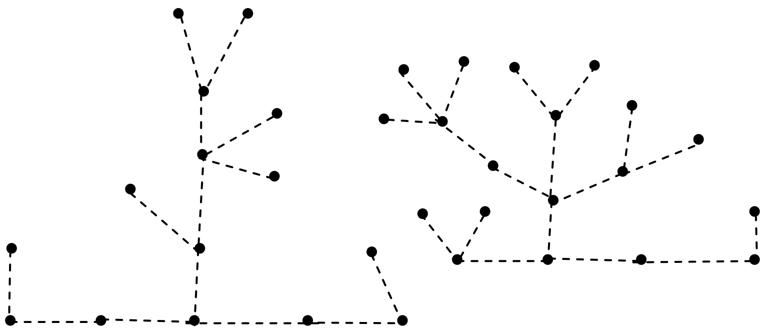


Figure 2.101

Now we have seen examples of neutrosophic trees and neutrosophic forests.

We see the modified result in case a neutrosophic tree.

THEOREM 2.7: *A connected neutrosophic graph with n vertices is a neutrosophic tree if and only if it has (n–1) edges and some of them are neutrosophic edges and some just edges.*

The proof is straight forward and hence is left as an exercise to the reader.

THEOREM 2.8: *A connected pure neutrosophic graph with n vertices is a pure neutrosophic tree if and only if it has (n–1) neutrosophic edges.*

This proof is also left as an exercise to the reader.

We will now illustrate these situations by some examples.

Example 2.28: Let us consider the neutrosophic tree T given in the following:

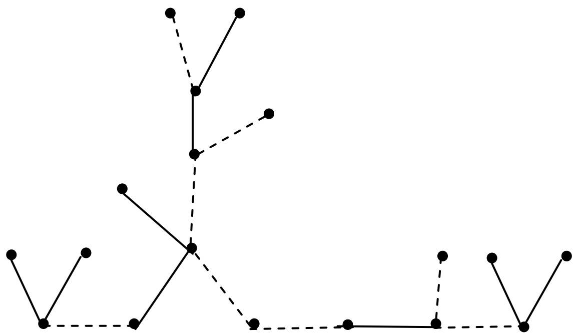


Figure 2.102

This tree has 18 vertices and 17 edges, of the 17 edges 9 are ordinary edges and 8 of the edges are neutrosophic.

Example 2.29: Let us consider the neutrosophic tree T given in the following:

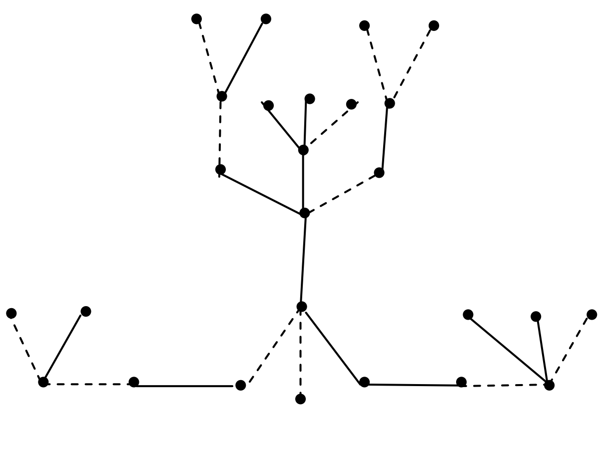


Figure 2.103

This neutrosophic tree T has 26 vertices and 25 edges of which 12 are neutrosophic edges and 13 are usual edges.

So the classical result about trees holds good in case of neutrosophic trees also.

We define neutrosophic trees with roots in the same way as that of usual trees.

The ordering of the vertices is also carried out in the same way.

Even in case of neutrosophic trees we define the notion of normal spanning trees or the depth first search trees, in a similar way they arise in computer searches on graphs.

We will describe this situation by an example or two.

Example 2.30: Let T be a neutrosophic tree with root r:

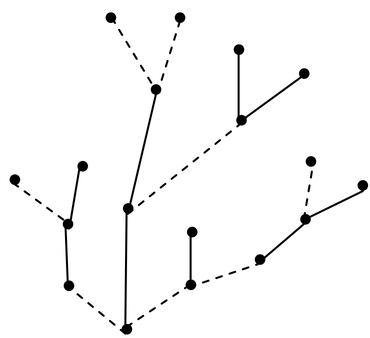


Figure 2.104

Example 2.31: Let T be a depth first search tree with root r:

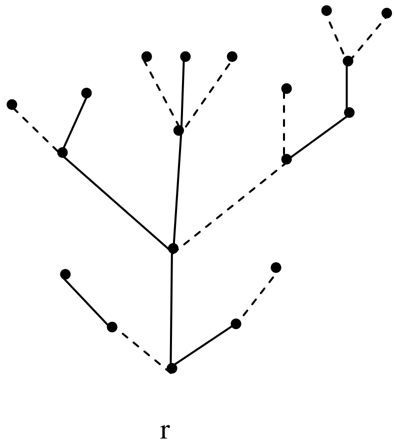


Figure 2.105

Example 2.32: Let T be a pure neutrosophic tree with root r:

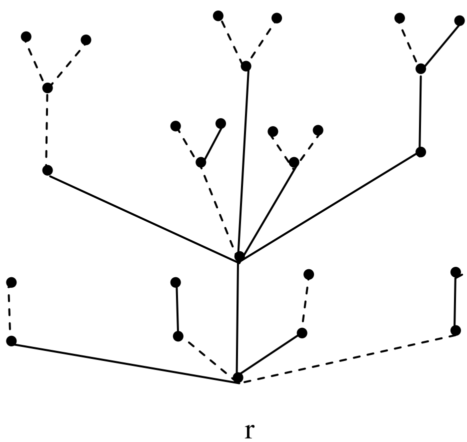


Figure 2.106

Example 2.33: Let T be a pure neutrosophic a depth first search and root r.

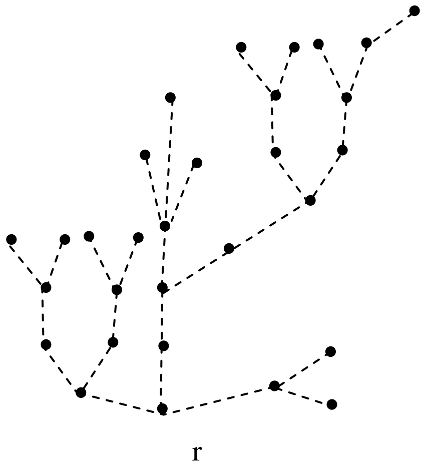


Figure 2.107

Chapter Three

NEUTROSOPHIC BIPARTITE GRAPHS

Now we proceed onto define neutrosophic bipartite graphs. Let G be a neutrosophic graphs if G admits a partition in two classes we call G to be a bipartite neutrosophic graph.

The neutrosophic graphs if they are bipartite we can have a partition depending on the neutrosophic graph G.

Let G be a neutrosophic graph if G is a partition into two graphs G_1 and G_2 such that $G_1 \cap G_2 = \phi$ and $G = G_1 \cup G_2$ is called 2-partite or bipartite, if G admits a partition into 2 classes such that every edge has its ends in different classes.

We will first illustrate this situation before we proceed onto describe and define r-partite neutrosophic graphs r > 2.

Example 3.1: Let G be a neutrosophic graph which is a bipartite graph given in the following.

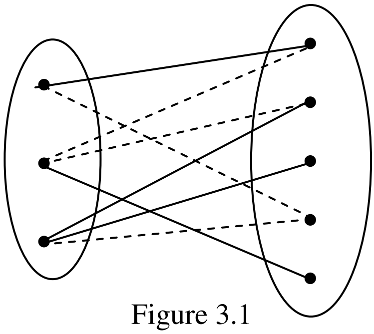


Figure 3.1

Example 3.2: Let G be a pure neutrosophic graph. G is a

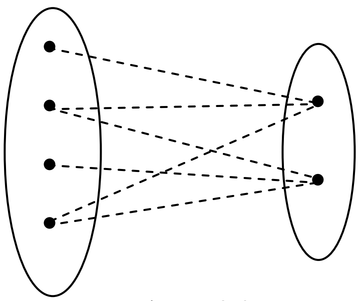


Figure 3.2

bipartite pure neutrosophic graph.

Example 3.3: Let G be a neutrosophic graph

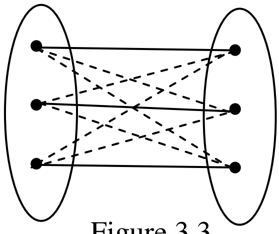


Figure 3.3

G is a neutrosophic bipartite graph.

We see the edges are both neutrosophic as well as non neutrosophic.

Example 3.4: Let G be a neutrosophic graph given in the following

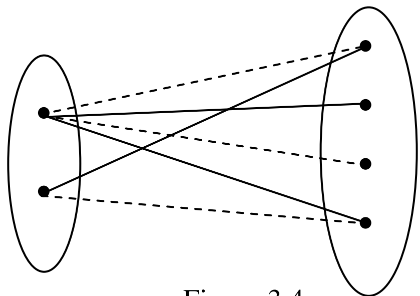


Figure 3.4

Now in case of neutrosophic graphs we have the following result.

THEOREM 3.1: *A neutrosophic graph is bipartite if and only if it contains no odd cycle.*

The proof is as in case of usual graphs, hence left as an exercise to the reader.

We define r-partite in case of a neutrosophic graph in an analogous way. However we provide a few examples.

Example 3.5: Let G be the neutrosophic graph given in the following.

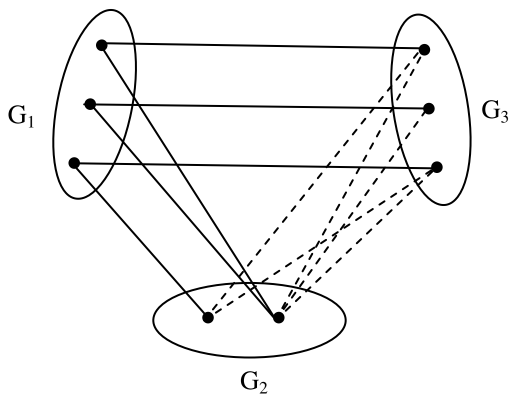


Figure 3.5

G is a 3-partite graph.

It is interesting to make the following observations.

Suppose $G = G_1 \cup G_2 \cup G_3$ we see $G_i \cap G_j = \phi$ if $i \neq j$.

Further it is important to note from the set G_1 the edges to both G_2 and G_3 are only real where as the edges from G_2 to G_3 are all neutrosophic.

This occurrence is very special.

Such 3-partite graphs we call as doubly 3-partite neutrosophic graphs.

Example 3.6: Let G be a neutrosophic graph.

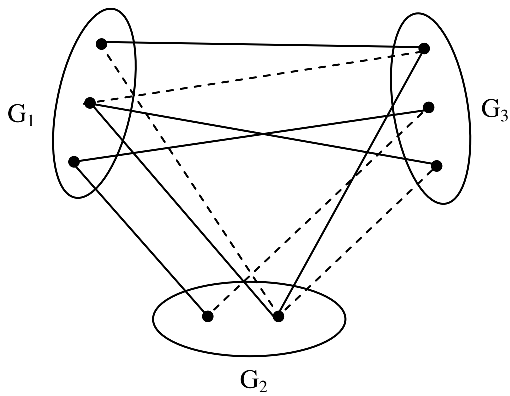


Figure 3.6

G is a 3 partite neutrosophic graph which is not doubly three partite.

Example 3.7: Let G be a neutrosophic graph.

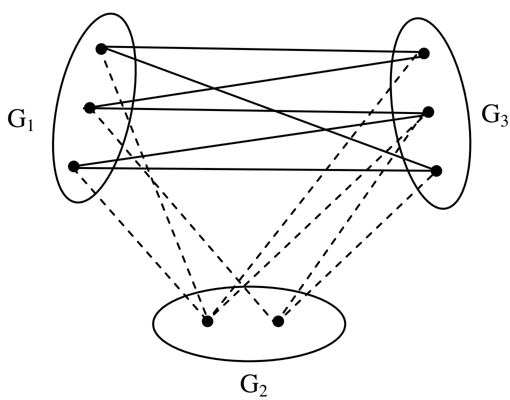


Figure 3.7

G is a doubly 3-partite neutrosophic graph.

THEOREM 3.2: *Every doubly 3-partite neutrosophic graph is a 3-partite neutrosophic graph but not conversely.*

The interested reader is requested to prove this theorem.

Example 3.8: Let G be a neutrosophic graph.

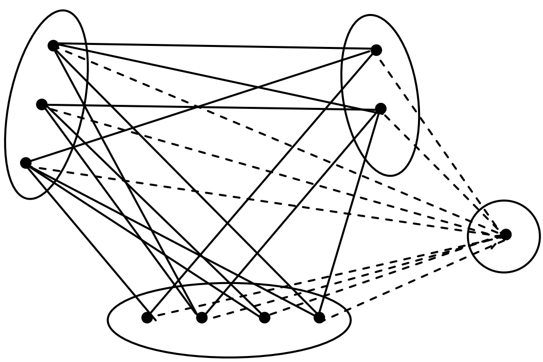


Figure 3.8

G is a doubly 4-partite neutrosophic graph.

Example 3.9: Let G be a pure neutrosophic graph which is as follows:

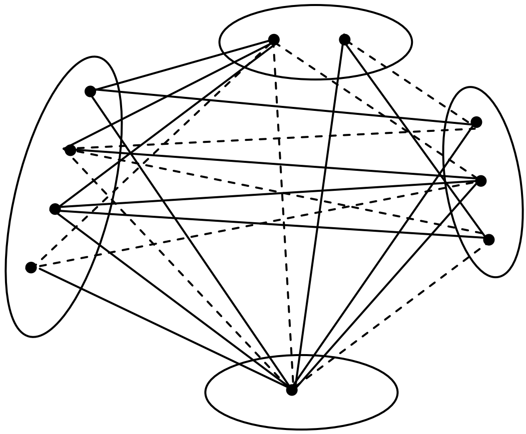


Figure 3.9

G is only a 4-partite neutrosophic graph.

Now we proceed onto define the notion of contraction and minors in case of neutrosophic graphs G.

Let G be a neutrosophic graph. Let e = xy be an edge which is neutrosophic or otherwise of the graph G = (V, E).

By G/e we denote the neutrosophic graph obtained from G by contracting the edge e into a vertex v_e which becomes adjacent to all the former neighbours of x and of y.

Formally G/e is a neutrosophic graph (V′, E′) (not a neutrosophic graph if e is the only neutrosophic edge of G) with vertex set $V' = (V \setminus \{x, y\}) \cup \{v_e\}$ (where v_e is the new vertex $v_e \notin V \cup E$) and the edge set

$E' = \{vw \in E / \{v, w\} \cap \{x, y\} = \phi\} \cup \{v_e\, w / x\, w \in E \setminus \{e\} \text{ or } yw \in E \setminus \{e\}\}$.

We will illustrate this situation by some examples.

Example 3.10: Let G be the neutrosophic graph.

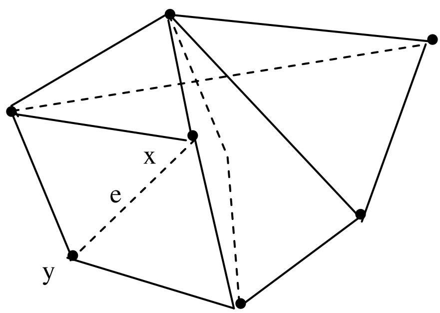


Figure 3.10

Contracting the edge e = xy we get G/e which is again a neutrosophic graph.

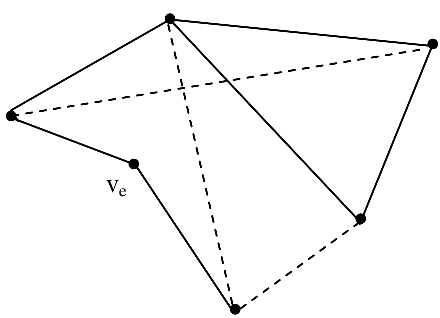


Figure 3.11

G/e is also a neutrosophic graph in this case.

Example 3.11: Let G be a neutrosophic graph.

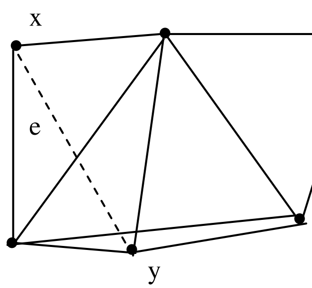


Figure 3.12

Contracting the edge e = xy we get the graph G/e.

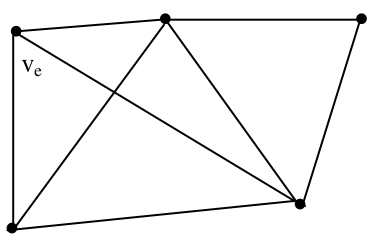


Figure 3.13

Clearly G/e is not a neutrosophic graph as G/e has no neutrosophic edges.

Example 3.12: let G be a neutrosophic graph given in the following.

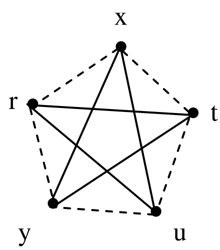


Figure 3.14

Contracting the edge e = xy we get

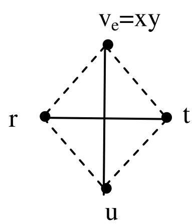


Figure 3.15

We get in this case we face some problems.

We see the edge $v_e t$ is neutrosophic after contraction, but the edge xt is also neutrosophic but the edge yt is not neutrosophic so $v_e t$ should it be neutrosophic or otherwise we say these neutrosophic graphs are not contracted in the usual way.

In view of this we have the following theorem.

THEOREM 3.3: *A neutrosophic graph in general need not always be contracted at all the edges.*

We will illustrate this by some examples.

Example 3.13: Let G be a neutrosophic graph.

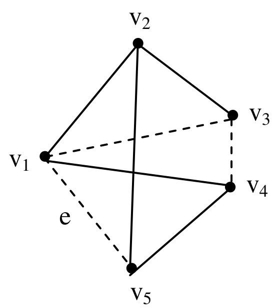


Figure 3.16

Suppose e = v_1 v_5. Contracting the edge e we get G/e the neutrosophic graph.

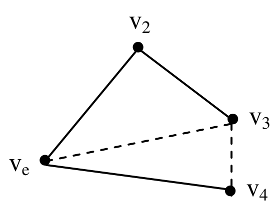


Figure 3.17

Example 3.14: Let G be a neutrosophic graph.

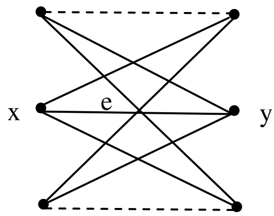


Figure 3.18

Let xy = e.

By contracting the edge xy we get

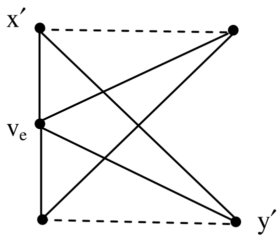


Figure 3.19

G/e = H.

Let e′ = x′ y′

By contracting the edge e′ we get H/e′

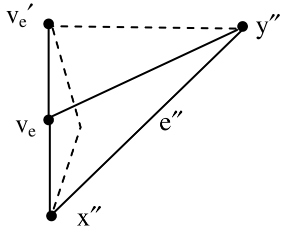


Figure 3.20

We see H/e' is a neutrosophic graph.

Let $H/e' = P$.

$e'' = x'' y''$.

By contracting the edge P/e'' we get a neutrosophic graph

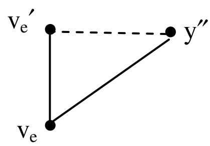
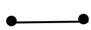


Figure 3.21

Another interesting feature we wish to study about neutrosophic graphs is that if we have a neutrosophic graph G with $v_1, \ldots, v_n$ as its n vertices and has $e_1, \ldots, e_p$ as edges.

Suppose $e_t = v_i v_j$ is contracted and suppose we get the contracted graph $G/e_t = H_1$ with $(n-1)$ vertices $\{v_1, \ldots, \breve{v}_i, \ldots, \breve{v}_j, \ldots, v_n\} \cup \{v_{e_t}\}$ and say q edges.

Now suppose $e_m = v_{e_t} v_k$ an edge in $G/e_t = H_1$ after contraction of the edge e_m let $H_1/e_m = H_2$ be the new neutrosophic graph $v_{e_m} \in H_2$ now suppose $v_{e_m} v_i = e_r$ an edge in H_2 after contraction of the edge e_r we get H_2/e_r, is again a graph and so on.

When will we reach a graph with two vertices of the form  ?

This study is interesting we illustrate this for some special type of graphs.

Let us consider the neutrosophic graph, G

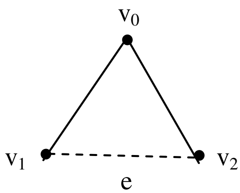


Figure 3.22

Suppose we contract the graph G with edge $e = v_1 v_2$ we get G/e as

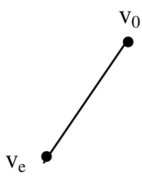


Figure 3.23

Suppose we want contract the graph G with edge $e_1 = v_0 v_2$ we get G/e_1 for we get v_1v_2 a pure neutrosophic graph G/e_1

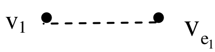


Figure 3.24

Suppose we contract the edge $v_0 v_1 = e_2$ we get again a pure neutrosophic graph

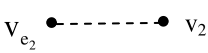


Figure 3.25

One may feel should v_1 v_{e_1} be a neutrosophic edge or usual edge. If the original structure of the graph is essentially to be maintained the edge is a neutrosophic edge.

If one wishes to accept the changed edge then it is the usual edge. If one does not agree upon the contraction for it is ambiguous one can say cannot be contracted.

All the three cases are accepted for while using it in problems flexibility leads to more true solutions.

Now we study the graph with four vertices and four edges.

Consider the neutrosophic graph G.

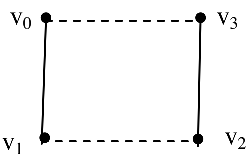


Figure 3.26

Suppose $e = v_1v_2$ is contracted in G. We find the new graph $H = G/e$ with vertices v_0, v_e v_3 which is as follows.

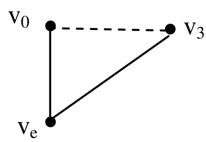
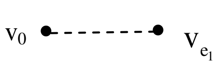


Figure 3.27

Now we can in the graph $H = G/e$ contract the edge $e_1 = v_e$ v_3 we get the resultant graph H/e_1 which is as follows:



is the H/e_1.

Figure 3.28

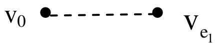
We see after two contraction we arrive at the graph of the form 

Figure 3.29

Now we study the same problem with five vertices and five edges. Let G be a neutrosophic graph which is as follows:

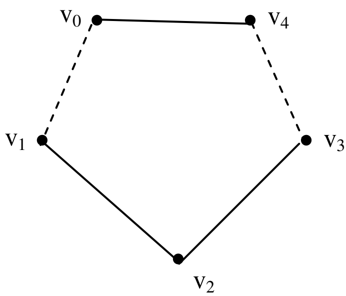


Figure 3.30

Let $e = v_2\, v_3$ by contracting G the edge e we get $G/e = H$ which is as follows:

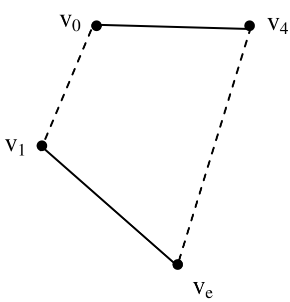


Figure 3.31

Now we contract the neutrosophic graph H by the edge $e_1 = v_e\ v_4$ which is as follows:

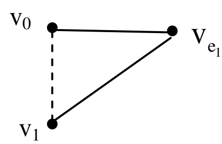


Figure 3.32

H/e_1 = P is the neutrosophic graph. Now contracting P by the edge $e_2 = v_1\ v_{e_1}$ we get S

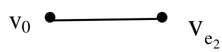


Figure 3.33

Thus the final graph S is got after three stages.

We see get one more example before we pose a problem.

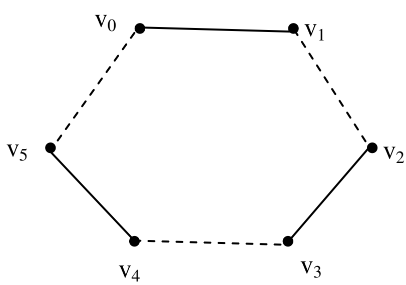


Figure 3.34

Let G be a neutrosophic graph with $\{v_0, v_1, v_2, v_3, v_4, v_5\} = V$ as its vertices.

Let e = v_5 v_4 be the edge using which we contract the neutrosophic graph G. We get G/e = H which is as follows with vertices $V_1 = \{v_0\ v_1\ v_2, v_3, v_e\}$.

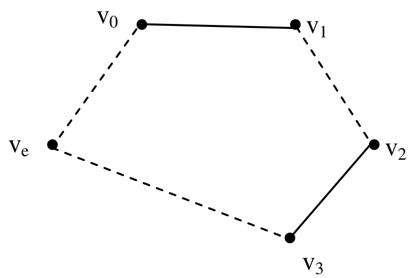


Figure 3.35

Now for this neutrosophic graph H we obtain the contracted graph by contracting the edge $e_1 = v_e v_3$. Let P_1 have vertices $V_2 = \{v_0, v_1, v_2, v_{e_1}\}$. Here $P_1 = H/e_1$.

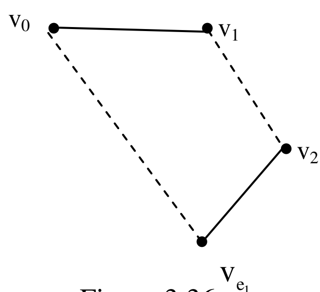


Figure 3.36

Now we find the graph $S_1 = P_1 / e_2$ where $e_2 = v_{e_1} v_2$ is the edge which is contracted to the vertex v_{e_2}.

The vertices of S_1 are $V_{S_1} = \{v_0, v_1, v_{e_2}\}$.

The graph with vertices V_{S_1} is as follows:

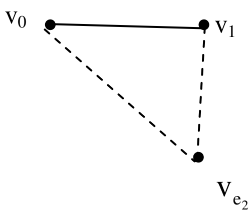


Figure 3.37

We see if S_1 is contracted for this pure neutrosophic graph with V_{S_1} vertices by the edge $v_1 v_{e_2} = e_3$ we get the pure neutrosophic graph

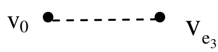


Figure 3.38

With these we pose the following problem.

Problem 3.1: Let G be a neutrosophic graph of the form with n vertices and n edges given by the following:

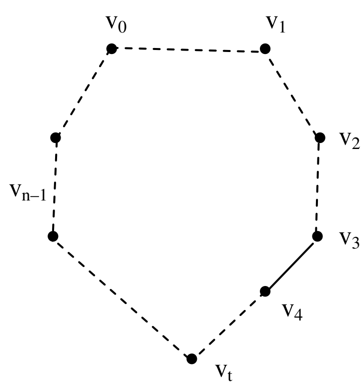


Figure 3.39

That is $V = \{v_0, v_1, \ldots, v_{n-1}\}$ and G has n vertices and n edges.

Will (n–2) contractions, by contracting first $v_{n-1}\ v_{n-2} = e_0$ as $G/e_0 = P_1$ with vertices $V_1 = \{v_0, v_1, \ldots, v_{n-3}, v_{e_0}\}$ and the second contraction is P_1 / e_1 got by contracting the edge $e_1 = v_e\ v_{n-3}$ and so on, so that P_1/e_1 has vertices $V_2 = \{v_0, \ldots, v_{n-4}, v_{e_1}\}$. Thus the ith contraction will be P_{i-1} / e_{i-1} and has vertices $v_i = \{v_0, \ldots, v_{n-(i+2)}, v_{e_{i-1}}\}$; for i = 1, 2, …, n–2 lead a to a graph of the form

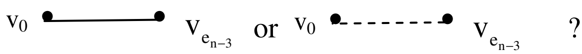


Figure 3.40

Now we see about other type of graphs.

Example 3.15: Suppose G is a neutrosophic graph with six edges and four vertices.

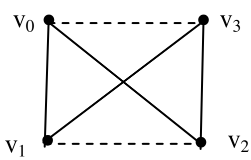


Figure 3.41

We see how many contractions are needed to make this G into

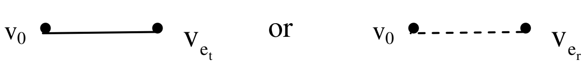


Figure 3.42

Consider the edge e = v_2 v_3 by contracting the edge e of G we get the graph G/e which is as follows:

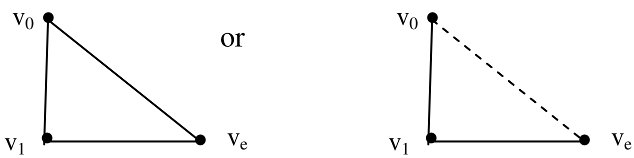


Figure 3.43

We see in the next stage we get

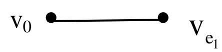


Figure 3.44

Suppose we contract the edge e = v_1 v_3; the G/e is as follows:

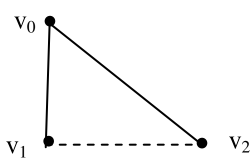


Figure 3.45

Next we consider the neutrosophic graph G with 5 vertices V = {v_0, v_1, v_2, v_3, v_4} which is as follows:

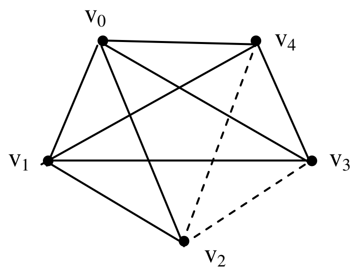


Figure 3.46

Suppose we take the edge e = v_3 v_4 we find the contracted graph G/e which is as follows:

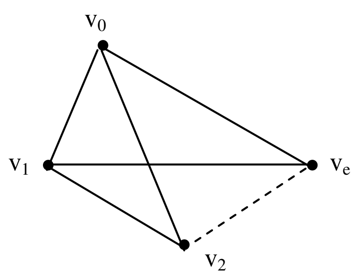


Figure 3.47

Now let H = G/e take the edge e_1 = v_2 v_e we contract H by the edge e_1; H/e_1 is as follows:

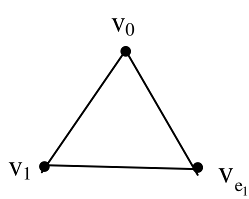


Figure 3.48

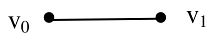
Contracting by the edge $v_1 \; v_{e_1}$ we get 

Figure 3.49

Suppose we contract the graph G by the edge e = $v_4 \; v_2$ we get G/e to be the contracted graph which is as follows:

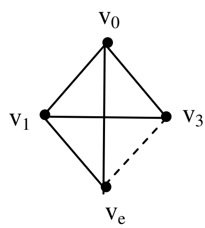


Figure 3.50

Once again by contracting $v_e \; v_1$ or $v_e \; v_3$ we get a three vertex complete graph next stage of contraction leads to

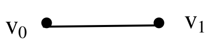


Figure 3.51

Finally let us consider the neutrosophic graph G with six vertices (v_0, v_1, v_2, v_3, v_4, v_5) and more than six edges.

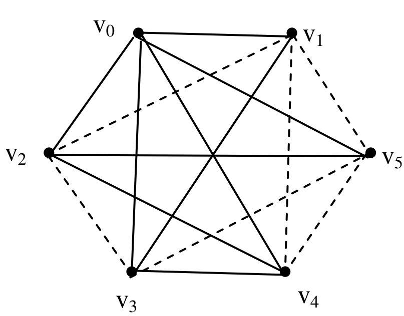


Figure 3.52

Let e = v_4 v_5 by contracting G by the edge e we get G/e which is as follows:

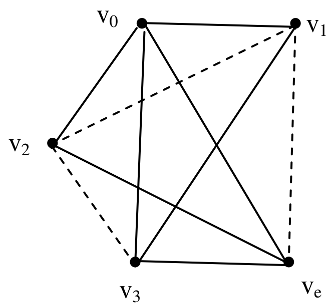


Figure 3.53

Now we see contracting any edge say $v_3v_e = e_1$ leads to H/e_1 (H = G/e) which is as follows:

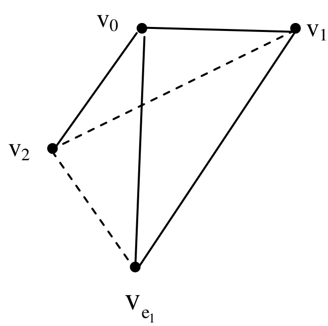


Figure 3.54

and so on.

Thus with the following observations we propose the following problem.

Problem 3.2: Let G be a neutrosophic graph with n vertices and each vertex is of degree n–1, regular we will contract say stage by stage also lead to graphs of same type such that G/e will have n–1 vertices with degree of each vertex n–2, … and so on?

Recall a minor of a graph or neutrosophic graph G obtained from G by contracting edges, deleting edges and deleting isolated vertices, a proper minor of G is any minor other than G itself.

We give examples of minors got by contracting a edge.

Example 3.16: Let G be a neutrosophic graph given by the following :

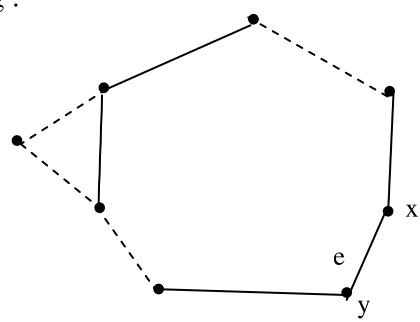


Figure 3.55

Let e = xy. To find H = G/e

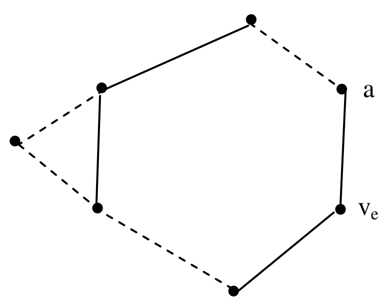


Figure 3.56

G/e is also a neutrosophic graph a minor of G.

Consider H / e_1 where $e_1 = av_e$ by contracting using the edge $e_1 = av_e$ we get P = H/e_1 which is as follows:

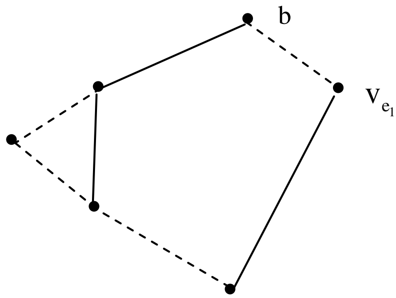


Figure 3.57

P is a minor of H as well as minor of G.

Let $e_2 = b\,v_{e_1}$ to find the contracted graph of by contracting the edge e_2.

P/e_2 is a graph given by

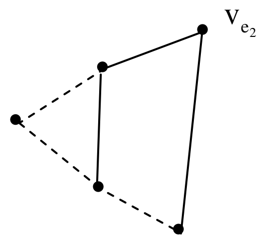


Figure 3.58

and so on.

P/e_2 is a minor of G, P and H.

We just recall the definition of subdivision of a graph.

Let G be a neutrosophic graph.

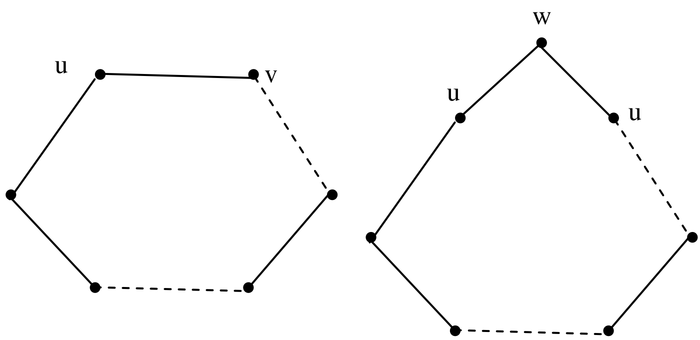


Figure 3.59

A subdivision of the edge e = uv of a neutrosophic graph G is the replacement of the edge e by a new vertex w and two new edges uw and wv.

The operation is also called an elementary subdivision of G.

If G be the neutrosophic edge and G is as follows:

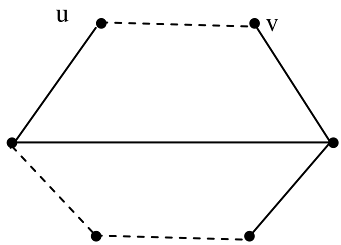


Figure 3.60

the subdivision of edge uv of a graph G is the replacement of the neutrosophic edge e by a new vertex w and two new neutrosophic edges uw and wv.

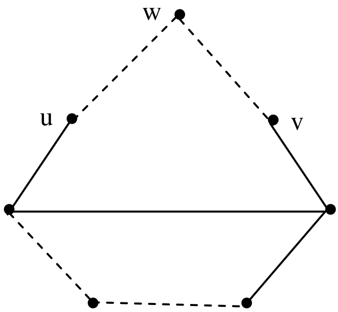


Figure 3.61

We can have subdivision of a complete graph.

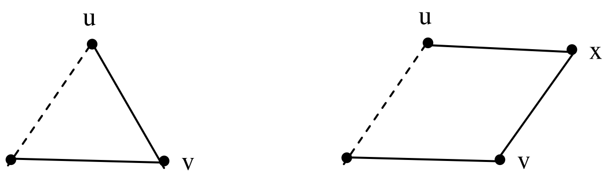


Figure 3.62

Let G be a neutrosophic complete graph.

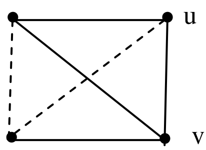


Figure 3.63

The subdivision of uv leads to H

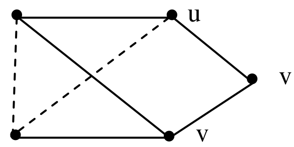


Figure 3.64

Clearly H is not a complete neutrosophic graph.

Let G be a neutrosophic graph.

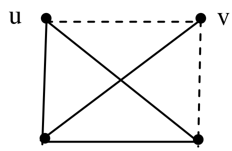


Figure 3.65

but subdivision of the edge uv we get the neutrosophic graph W which is as follows:

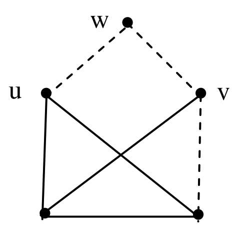


Figure 3.66

W is not a complete graph.

Consider the neutrosophic graph G which is as follows:

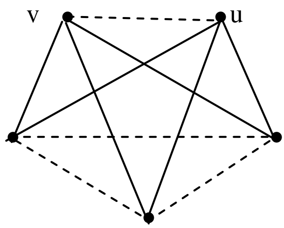


Figure 3.67

The subdivision of the edge uv gives the new neutrosophic graphs which is as follows:

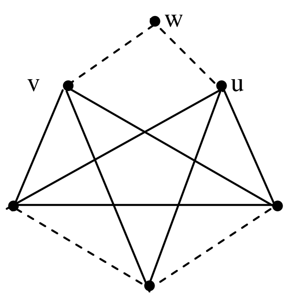


Figure 3.68

Study in the direction is both interesting and innovative.

Chapter Four

APPLICATIONS OF NEUTROSOPHIC GRAPHS

To the best of our knowledge these neutrosophic graphs find applications in fuzzy models [5-8]. Further we feel that when we have net work in which some of the edges cannot be predicted we can use these neutrosophic graphs.

They can be helpful in that case. Also with the advent of neutrosophic graphs the colouring of edges will have an impact for the same colour can be used for the edges if one edge is usual and other is neutrosophic.

So it is interesting to redefine the edge colouring problems in case of neutrosophic graphs.

Proper colouring of the graph remains the same in case of neutrosophic graphs.

Now for neutrosophic graph the edge colouring problem reduces the number of colours for we need to colour differently to adjacent edges only both the edges happen to be usual or both are neutrosophic edge.

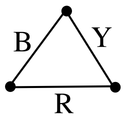


Figure 4.1

To colour this graph we need minimum 3 colours say red yellow and black.

The same is true in case of pure neutrosophic graph.

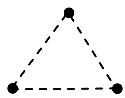


Figure 4.2

Now consider the neutrosophic graph.

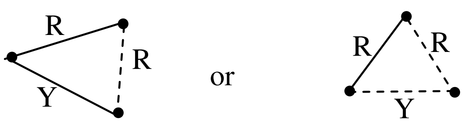


Figure 4.3

Two colour are enough for only two usual adjacent edges or two neutrosophic edges.

Consider the graph G

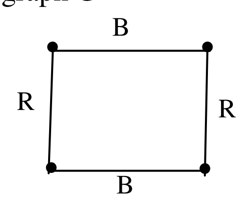


Figure 4.4

Minimum two colours are need to colour G.

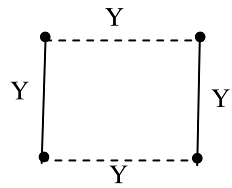


Figure 4.5

We see one colour is enough to edge colour this graph.

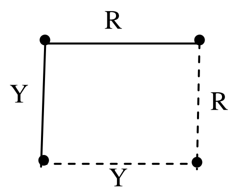


Figure 4.6

This graph needs atleast two colours.

So one can treat following as the open problem.

Characterize those neutrosophic graphs which need same number to colour the edges as that of the usual graphs.

Example 4.1: Let G be the graph.

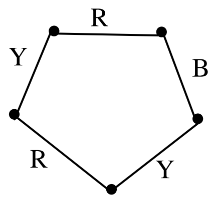


Figure 4.7

We need minimum three colour to edge colour the graph G.

Consider the neutrosophic graph H.

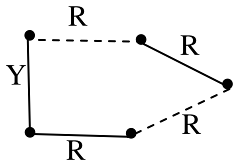


Figure 4.8

Two colours are sufficient to colour H′

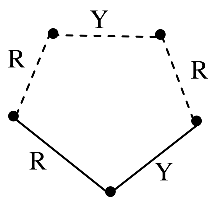


Figure 4.9

Two colours are sufficient to edge colour H′.

Example 4.2: Consider the usual graph G.

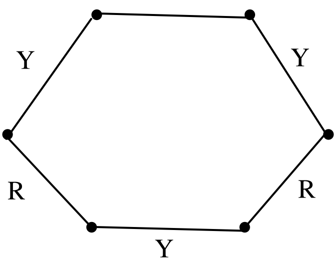


Figure 4.10

Two colour are enough to edge colour G.

Consider the neutrosophic graph H.

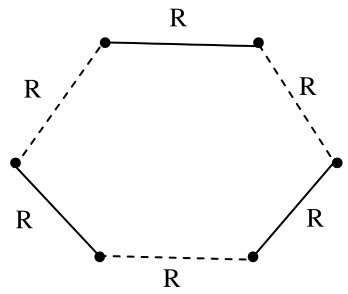


Figure 4.11

One colour is enough to edge colour the neutrosophic graph H.

Consider the neutrosophic graph K.

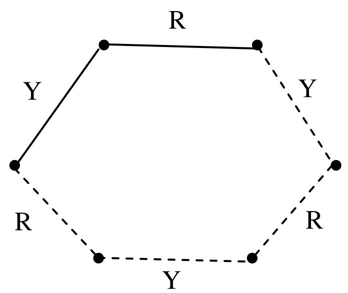


Figure 4.12

Two colour are needed to edge colour K.

Consider the neutrosophic graph H_1

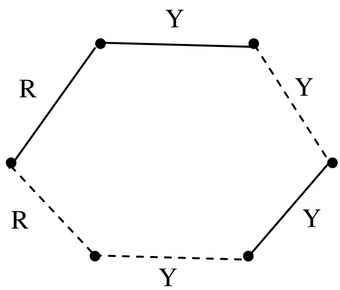


Figure 4.13

We need two colours to edge colour H_1.

Example 4.3: Let G be a graph

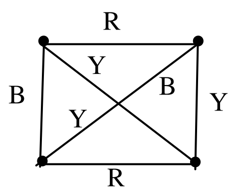


Figure 4.14

Three colours are needed to edge colour G.

If H is a pure neutrosophic graph.

We need 3 colours to edge colour H.

Chapter Five

SUGGESTED PROBLEMS

In this chapter we suggest a few problems some of which are very difficult, some are at research level. In this juncture authors wish to keep on record neutrosophic graphs behave in a very unique manner.

Several factors easily found in case of usual graphs are in fact very difficult or at time impossible to arrive at a conclusion. So the notion of indeterminacy's vital role is seen in neutrosophic graphs in an explicit way.

1. Find some interesting features enjoyed by neutrosophic graphs.

2. Find the number of neutrosophic graphs with 5 vertices.

3. Find the number of neutrosophic graphs with 5 edges which is connected.

4. Find all the subgraphs S(G) of the neutrosophic graph G.

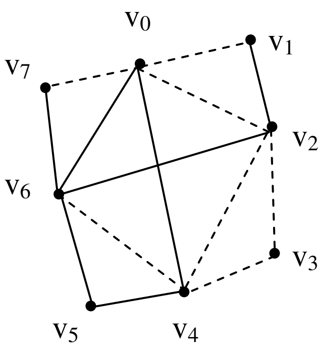


Figure 5.1

5. Give an example of a neutrosophic graph which is non planar with five vertices.

6. Let G be the neutrosophic graph.

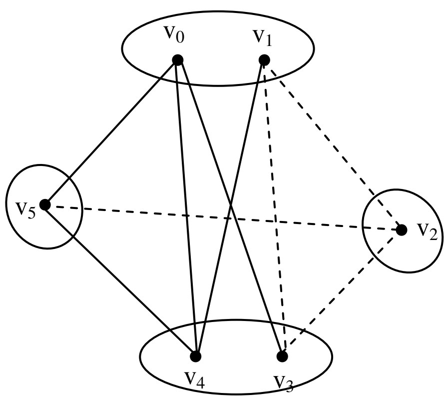


Figure 5.2

Find S(G). What is the cardinality of S(G).

7. Let G be a neutrosophic graph given in the following

8. Let G be a neutrosophic graph. S(G) = {Collection of all subgroups of G}.

   Find some interesting properties enjoyed by S(G).

9. Let G be the neutrosophic given in the following:

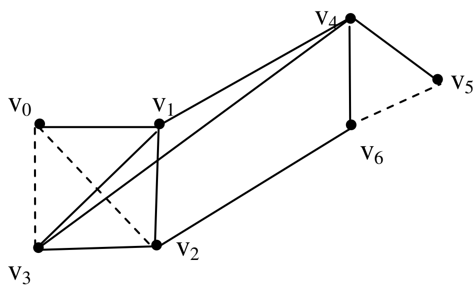


Figure 5.3

   i. Find all subgraphs of G.
   ii. Find cardinality of S(G).
   iii. Find the neutrosophic adjacency matrix of G.
   iv. How many subgraphs of G are not neutrosophic?

10. Let G be the graph given in problem 9.

   i. Find the neutrosophic adjacency matrix A of G.
   ii. Find A^2 and verify the diagonal element of A^2 denotes the number of edges at the vertices $v_0, v_1, \ldots, v_6$.

11. For the neutrosophic graph given in problem 9 find A(G) the neutrosophic incidence matrix of G.

12. Let G be the neutrosophic graph given in the following:

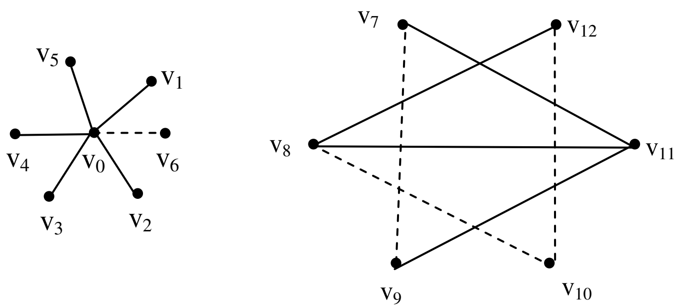


Figure 5.4

i. Find the adjacency matrix A of G.
ii. Show the neutrosophic matrix Y = A + A + … + A has zeros.
iii. Find A(G) the neutrosophic incidence matrix associated with G.

13. Let G be the neutrosophic graph which is as follows.

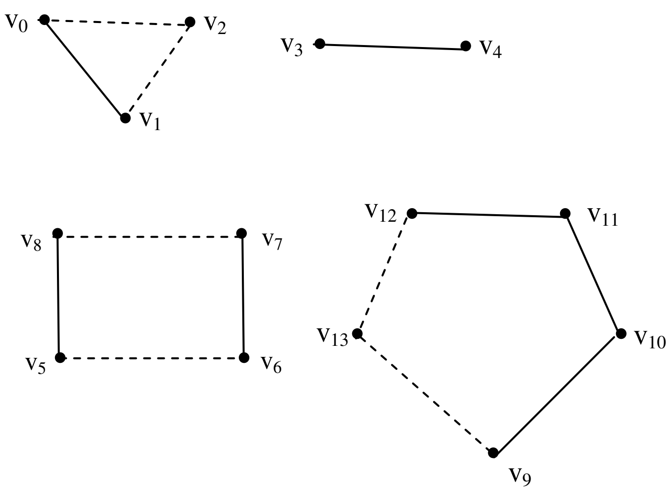


Figure 5.5

i. Find the neutrosophic adjacency matrix A of G.

ii. Prove G is disjoint using $Y = A + A^2 + \ldots + A^{13}$.
iii. Find A(G) the incidence matrix of G.
iv. Prove A is a diagonal symmetric super neutrosophic square matrix.
v. Hence or otherwise prove Y is a diagonal super symmetric neutrosophic square matrix.

14. Find all the subgraphs of G given in problem 13.

15. Find the incidence matrix A(G) of the neutrosophic graph given in problem 13.

16. Let G be a pure neutrosophic graph which is as follows.

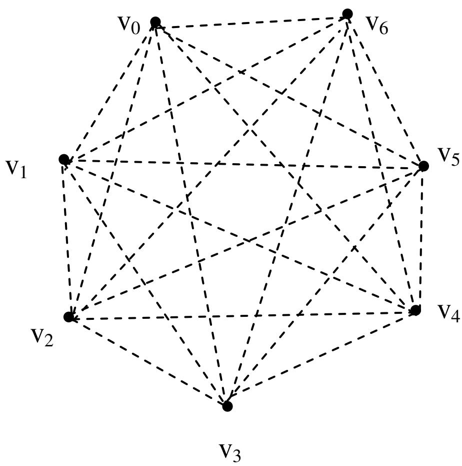


Figure 5.6

i. Find A the neutrosophic adjacency matrix of G.
ii. Find all subgraphs of G.

17. Let G be a graph which is neutrosophic planar.

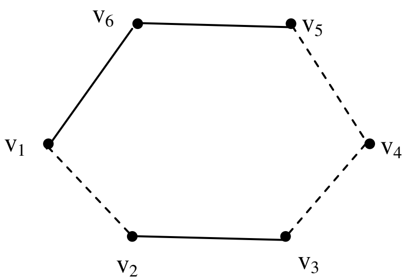


Figure 5.7

   i. How many distinct neutrosophic planar complete graphs with six vertices exist? (including pure neutrosophic planar graph with 6 vertices).

18. Let G be a neutrosophic graph which is as follows:

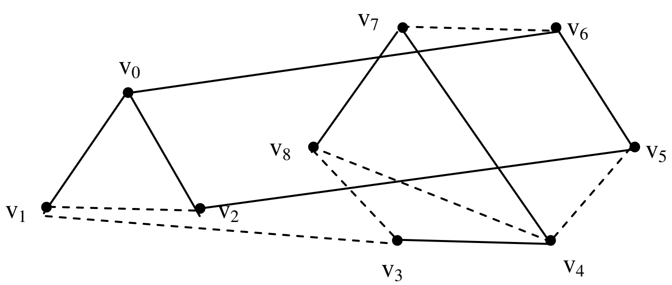


Figure 5.8

   i. Find the complement of this neutrosophic graph.
   ii. How many subgraphs of G are pure neutrosophic?
   iii. How many subgraphs of G are neutrosophic?
   iv. How many subgraphs of G are not neutrosophic?

19. Let G be the neutrosophic graph

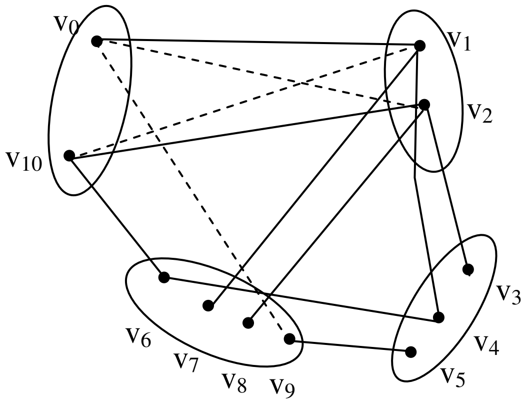


Figure 5.9

i. Find the adjacency neutrosophic matrix A associated with G.

20. Let G be a neutrosophic graph given in the following.

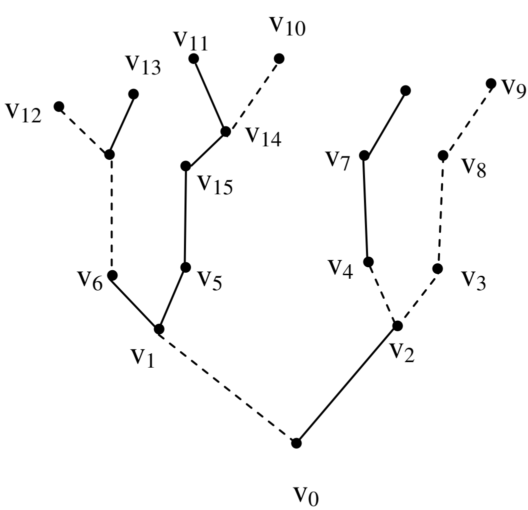


Figure 5.10

i. Find A the adjacency neutrosophic matrix of G.

21. For graph given in problem 20.

i. Find A^2; hence or otherwise state the number of edges which passes through each v_i $0 \leq i \leq 15$
ii. Find at least 3 pure neutrosophic subgraphs of G.
iii. Find 5 neutrosophic subgraphs of G.
iv. Find six usual subgraphs of G.

22. Let G be a neutrosophic graph which is as follows:

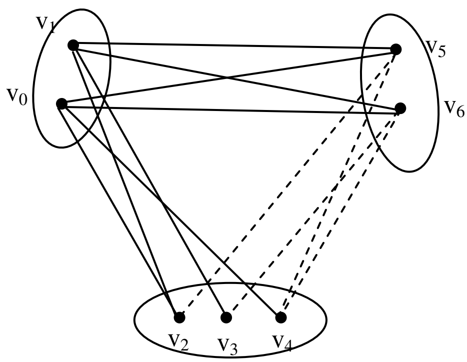


Figure 5.11

i. Find all neutrosophic subgraphs of G.
ii. Find pure neutrosophic subgraphs of G.
iii. Find usual subgraphs of G.

23. Let G be a neutrosophic graph given in the following.

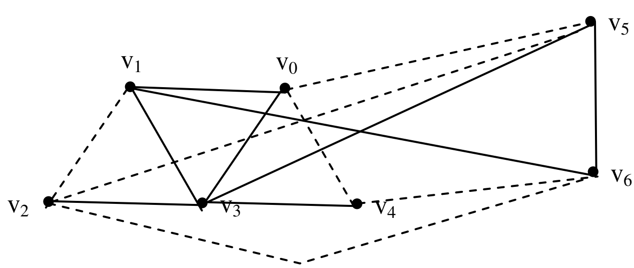


Figure 5.12

i. Find the neutrosophic adjacency matrix A associated with G.

24. Let G be the neutrosophic graph which is as follows:

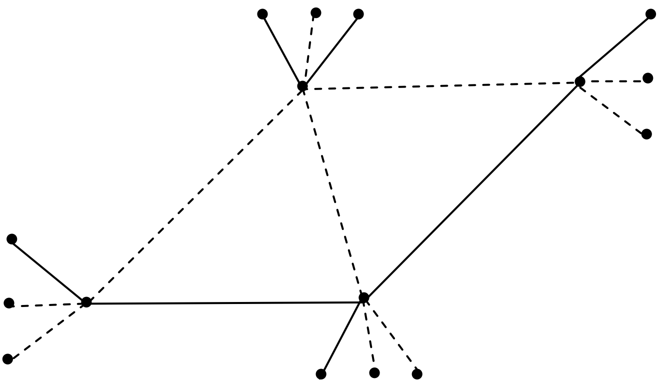


Figure 5.13

i. Find A the adjacency neutrosophic matrix of G.
ii. Find 3 pure neutrosophic subgraphs of G.
iii. Find 5 neutrosophic subgraphs of G.

25. Let G be the neutrosophic graph which is as follows:

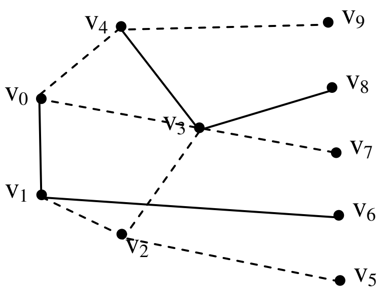


Figure 5.14

i. Find the largest neutrosophic subgraphs of G. Is it connected?
ii. Find the largest usual subgraphs fo G. Is it connected?

26. Find the adjacency matrix of the graph G given in problem 25.

27. Let G be neutrosophic graph which is as follows:

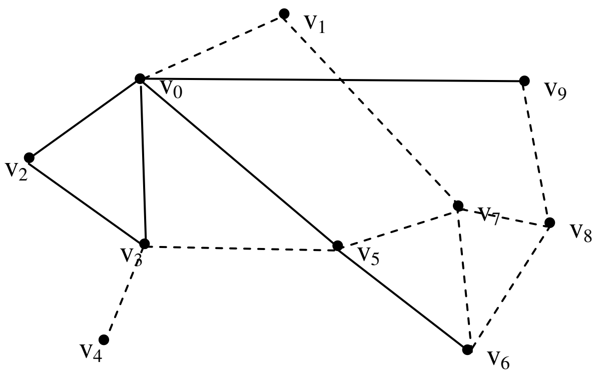


Figure 5.15

i. Find the largest pure neutrosophic subgraph of G and its incidence neutrosophic matrix.

28. Let G be the neutrosophic graph given below

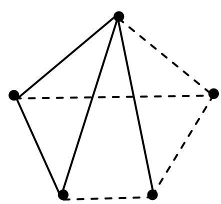


Figure 5.16

i. Find the minimum number of colors required to edge color G.

29. Let G be the neutrosophic graph.

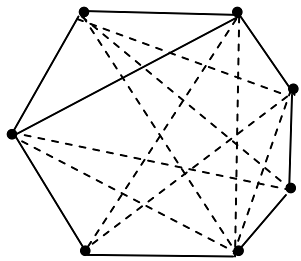


Figure 5.17

Find the minimum number of colors required to edge color G.

30. Find the largest pure neutrosophic subgraph of G given in problems 29 and 30.

31. Let G be a neutrosophic graph which is as follows:

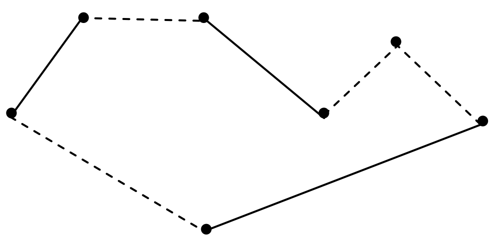


Figure 5.18

Find the minimum number of color required to edge color G.

32. Let G be a neutrosophic graph.

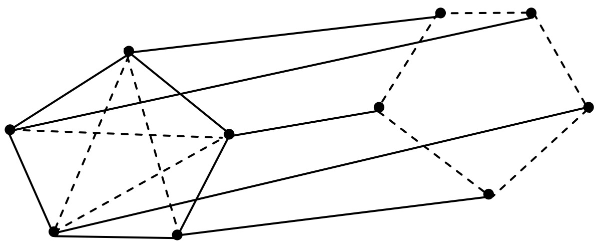


Figure 5.19

Find all pure neutrosophic subgraphs of G.

33. Let G be the neutrosophic graph

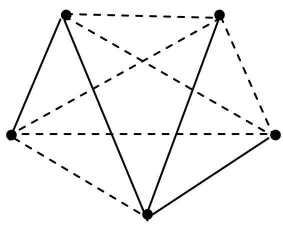


Figure 5.20

and H be another neutrosophic graph.

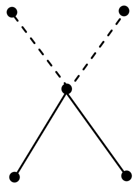


Figure 5.21

Compare the subgraphs of G and H.

34. Let P be the neutrosophic graph which is as follows:

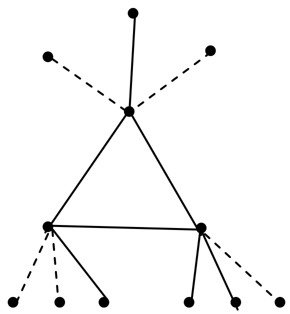


Figure 5.22

Find S(P). What is the o(S(P))?

35. Let G be the non planar graph

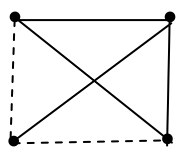


Figure 5.23

and H be the planar graph.

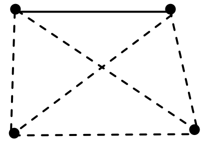


Figure 5.24

Compare G and H.

36. Find some nice applications of neutrosophic graphs other than in fuzzy neutrosophic models.

37. Let G be a neutrosophic graph with 5 vertices.
    Find how many neutrosophic graphs can be constructed with 5 vertices?

38. Find if G has n vertices; how many neutrosophic graphs can be constructed using n vertices.

39. Suppose G be a neutrosophic graph which is as follows:

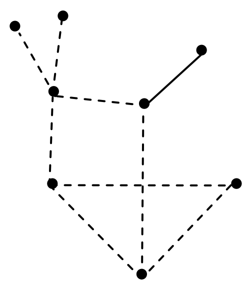


Figure 5.25

    i. Find the maximum number of neutrosophic subgraphs using G.

40. Let G be a neutrosophic graph which is as follows:

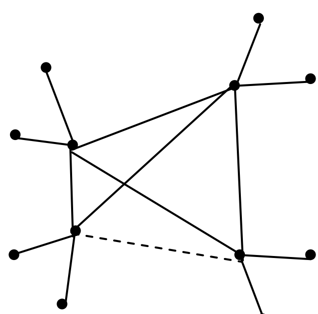


Figure 5.26

i. Find the maximum number of neutrosophic subgraphs.
ii. Find the total number of usual subgraphs of G.

41. Let G and G′ be the neutrosophic graphs which are as follows:

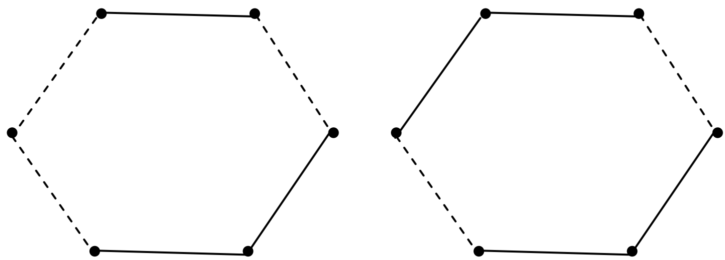


Figure 5.27

Are G and G′ neutrosophically isomorphic?

42. Suppose G and G′ are two neutrosophic graphs which have same number of edges and vertices.

If they have also same number of neutrosophic edges will they be neutrosophically isomorphic?

43. Find the neutrosophic path in the neutrosophic graph G.

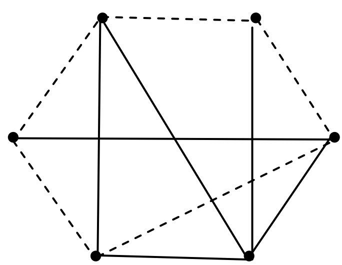


Figure 5.28

44. Will every neutrosophic graph have a neutrosophic path? Justify your claim.

45. Can the neutrosophic graph G which is as follows have a neutrosophic path?

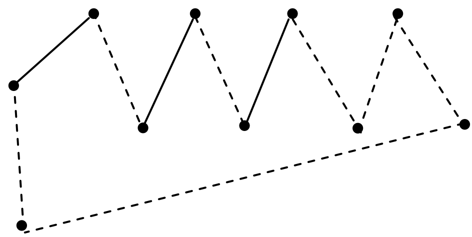


Figure 5.29

46. Can a complete neutrosophic graph have always a neutrosophic path?

47. Give an example of a neutrosophic walk in a neutrosophic graph.

48. Will every neutrosophic graph have a neutrosophic walk?

49. Will every neutrosophic graph have a usual walk?

50. Give an example of a neutrosophic forest.

51. Will every neutrosophic graph be a neutrosophic forest?

52. Obtain some interesting properties about neutrosophic graphs which are not enjoyed by usual graphs.

53. Show k-regular neutrosophic graph need not be regular usually.

54. Find the number colors needed to edge color the neutrosophic graph.

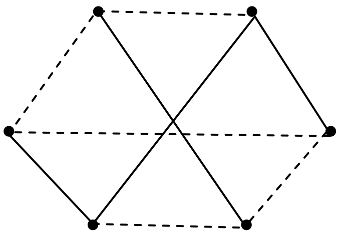


Figure 5.30

55. Let G be a neutrosophic graph.

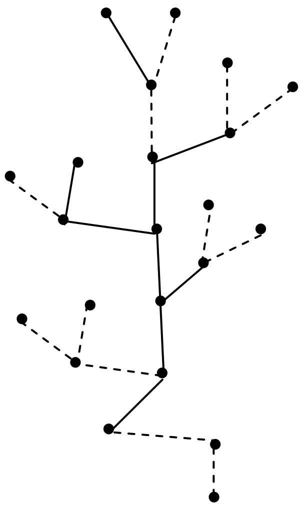


Figure 5.31

Find the number of colors needed to edge colors G.

56. Let G be the planar connected neutrosophic graph with n vertices. Find the number of colors needed to edge color G.

57. Let G be a connected non planar neutrosophic graph with n vertices. How many colors are needed to edge color G.

58. Let $G_1, \ldots, G_9$ be the neutrosophic graphs given in the following. Find the number of colors needed to edge column $G_1, \ldots, G_9$.

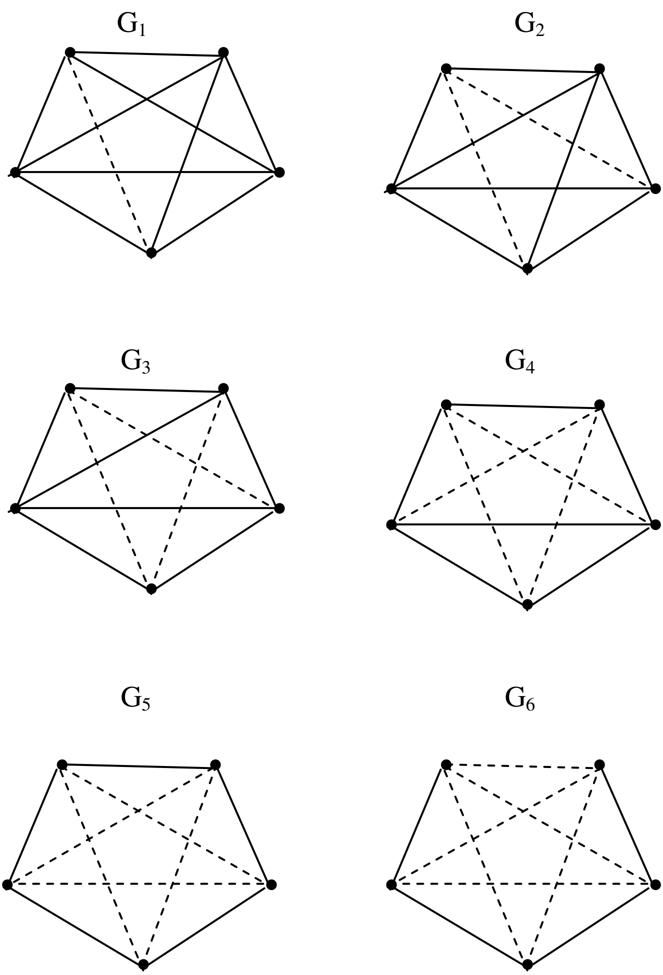


Figure 5.32

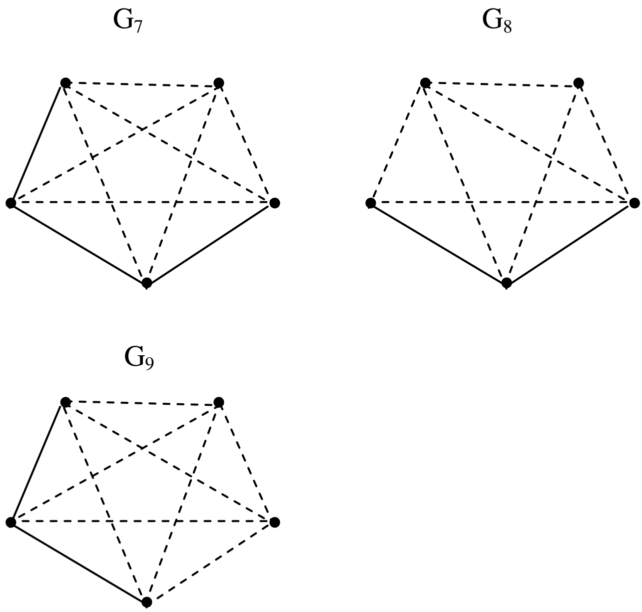


Figure 5.32

i. Will any of the G_i's need same number of colors to edge color G_i's.

59. Study problem for neutrosophic graphs with n-vertices.

60. Let G be a neutrosophic graph which is as follows:

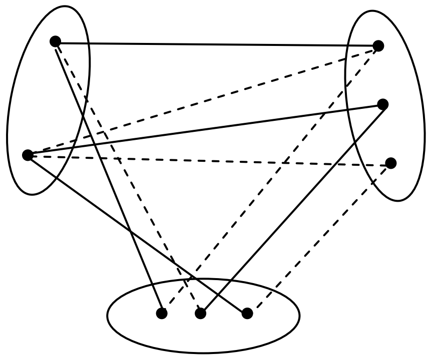


Figure 5.33

Find the number of colors needed to edge color G.

FURTHER READING

1. Harary, F., Graph Theory, Addison Wesley, 1969.

2. Smarandache, F. (editor), Proceedings of the First International Conference on Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics, Univ. of New Mexico – Gallup, 2001.

3. Smarandache, F., Neutrosophic Logic, A Generalization of Fuzzy logic, http://gallup.unm.edu/~smarandache/NeutLog.txt

4. Smarandache, F., Special Algebraic Structures, in Collected Papers II, Abaddaba, Oradea, 78-91, 2000.

5. Vasantha Kandasamy, W. B. and Florentin Smarandache, Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps, Xiquan, Phoenix, 2003.

6. Vasantha Kandasamy, W. B. and Florentin Smarandache, Neutrosophic algebraic structures and Neutrosophic N-algebraic structures, Xiquan, Phoenix, 2003.

7. Vasantha Kandasamy, W. B. and Florentin Smarandache, Fuzzy Relational Equations and Neutrosophic Relational Equations Hexis, Church Rock, USA, 2004.

8. Vasantha Kandasamy, W. B. and Sultana Yasmin, Knowledge Processing Using Fuzzy Relational Maps, Ultra Sciences, 12, 242-245, 2000.

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Dr.W.B.Vasantha Kandasamy is a Professor in the Department of Mathematics, Indian Institute of Technology Madras, Chennai. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 694 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 103th book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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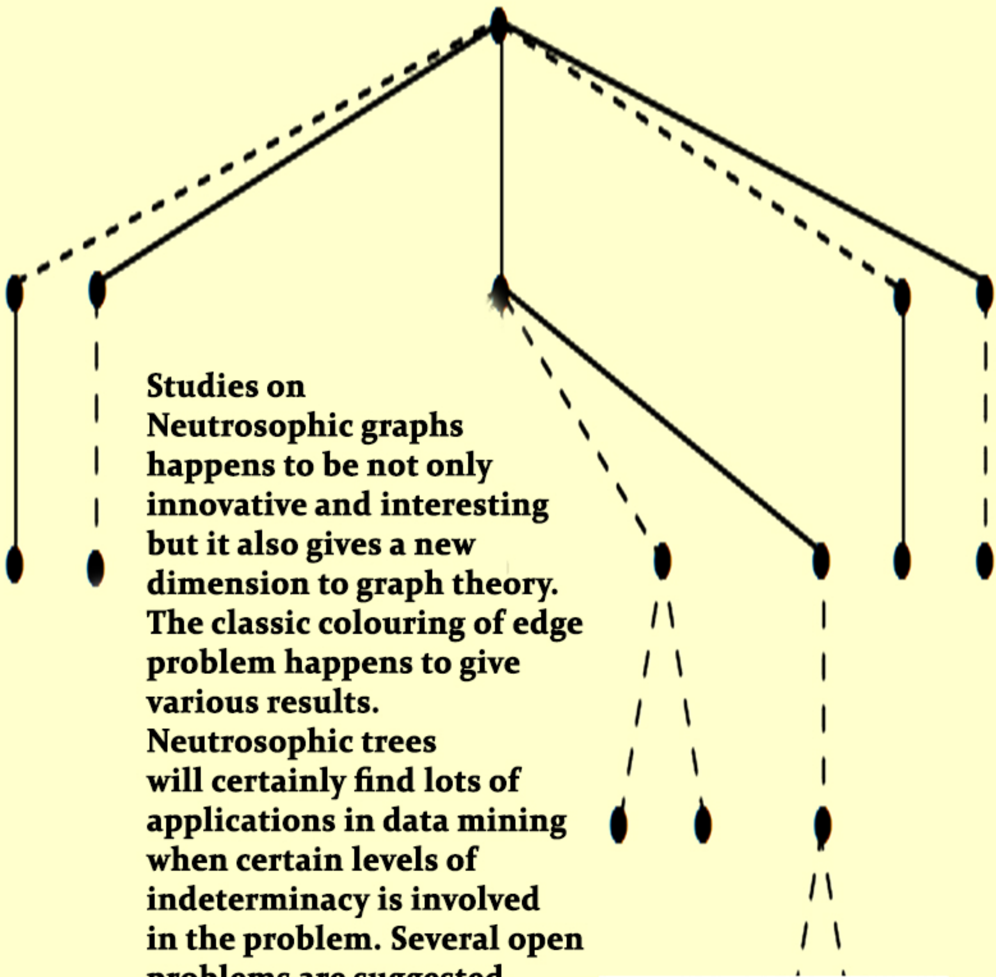
Dr. Florentin Smarandache is a Professor of Mathematics at the University of New Mexico in USA. He published over 75 books and 200 articles and notes in mathematics, physics, philosophy, psychology, rebus, literature. In mathematics his research is in number theory, non-Euclidean geometry, synthetic geometry, algebraic structures, statistics, neutrosophic logic and set (generalizations of fuzzy logic and set respectively), neutrosophic probability (generalization of classical and imprecise probability). Also, small contributions to nuclear and particle physics, information fusion, neutrosophy (a generalization of dialectics), law of sensations and stimuli, etc. He got the 2010 Telesio-Galilei Academy of Science Gold Medal, Adjunct Professor (equivalent to Doctor Honoris Causa) of Beijing Jiaotong University in 2011, and 2011 Romanian Academy Award for Technical Science (the highest in the country). Dr. W. B. Vasantha Kandasamy and Dr. Florentin Smarandache got the 2012 New Mexico-Arizona and 2011 New Mexico Book Award for Algebraic Structures. He can be contacted at smarand@unm.edu



Studies on Neutrosophic graphs happens to be not only innovative and interesting but it also gives a new dimension to graph theory. The classic colouring of edge problem happens to give various results. Neutrosophic trees will certainly find lots of applications in data mining when certain levels of indeterminacy is involved in the problem. Several open problems are suggested.

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