NEUTROSOPHIC INTERVAL BIALGEBRAIC STRUCTURES

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PREFACE

In this book the authors for the first time introduce the notion of neutrosophic intervals and study the algebraic structures using them. Concepts like groups and fields using neutrosophic intervals are not possible. Pure neutrosophic intervals and mixed neutrosophic intervals are introduced and by the very structure of the interval one can understand the category to which it belongs.

We in this book introduce the notion of pure (mixed) neutrosophic interval bisemigroups or neutrosophic biinterval semigroups. We derive results pertaining to them. The new notion of quasi bisubsemigroups and ideals are introduced. Smarandache interval neutrosophic bisemigroups are also introduced and analysed. Also notions like neutrosophic interval bigroups and their substructures are studied in section two of this chapter. Neutrosophic interval bigroupoids and the identities satisfied by them are studied in section three of this chapter.

The final section of chapter one introduces the notion of neutrosophic interval biloops and studies them. Chapter two of this book introduces the notion of neutrosophic interval birings and bisemirings. Several results in this direction are derived and described. Even new bistructures like neutrosophic interval ring-semiring or neutrosophic interval semiring-ring are introduced and analyzed. Further in this chapter the concept of neutrosophic biinterval vector spaces or neutrosophic interval bivector spaces are introduced and their properties are described.

In the third chapter we introduce the notion of neutrosophic interval n-structures or neutrosophic interval n-structures. Over 60 examples are given and various types of n-structures are studied. Possible applications of these new structures are given in chapter four. The final chapter suggests over hundred problems some of which are at research level.

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W.B.VASANTHA KANDASAMY
FLORENTIN SMARANDACHE

Chapter One

BASIC CONCEPTS

In this chapter we first introduce the notion of neutrosophic intervals and special neutrosophic intervals. We built in this chapter interval neutrosophic bistructures with single binary operation; we call them also as biinterval neutrosophic algebraic structures.

$N((R^+ \cup \{0\})I) = R^+ I \cup \{0\} = \{[0, aI] \mid a \in R^+ \cup \{0\}\}$ denotes the pure neutrosophic intervals of reals.

$N(Q^+I \cup \{0\}) = N((Q^+ \cup \{0\})I) = Q^+ I \cup \{0\} = \{[0, aI] \mid a \in Q^+ \cup \{0\}\} = \{[0, aI] \mid a \in Q^+ \cup \{0\}\}$ denotes the pure neutrosophic intervals of rationals.

$N(Z^+I \cup \{0\}) = \{[0, aI] \mid a \in Z^+ \cup \{0\}\}$ denotes the intervals of pure neutrosophic integers. $N(Z_nI) = \{[0, aI] \mid a \in Z_n\}$ denotes the pure neutrosophic interval of modulo integers, we can define now neutrosophic interval modulo integers as $N(\langle Z_n \cup I\rangle) = \{[0, a+bI] \mid a, b \in Z_n\}$. $N(\langle Q^+ \cup I\rangle) \cup \{0\}) = \{[0, a+bI] \mid a, b \in Q^+ \cup \{0\}\}$ is the rational neutrosophic interval.

$N(\langle R^+ \cup I\rangle \cup \{0\}) = \{[0, a + bI] \mid a, b \in R^+ \cup \{0\}\}$ is the neutrosophic interval of reals.

Finally $N(\langle Z^+ \cup I\rangle \cup \{0\}) = \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}\}$ is the neutrosophic interval of integers.

Now we will be using these intervals and work with our results. However by the context the reader can understand whether we are working with pure neutrosophic intervals or neutrosophic of rationals or integers or reals or modulo integers. This chapter has four sections. Section one introduces neutrosophic interval bisemigroups, neutrosophic interval bigroups are introduced in section two. Section three defines biinterval neutrosophic bigroupoids. The final section gives the notion of neutrosophic interval biloops.

1.1 Neutrosophic Interval Bisemigroups

In this section we define the notion of pure neutrosophic interval bisemigroup, neutrosophic interval bisemigroup, neutrosophic - real interval bisemigroup, quasi neutrosophic interval bisemigroup and quasi neutrosophic - real interval bisemigroup using Z_n or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$. We give some examples and describe their properties.

DEFINITION 1.1.1: *Let $S = S_1 \cup S_2$ where S_1 and S_2 are interval pure neutrosophic semigroups such that S_1 and S_2 are distinct, then we define S to be a pure neutrosophic interval bisemigroup or pure neutrosophic biinterval semigroup.*

We will illustrate this situation by some examples.

Example 1.1.1: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_3, +\} \cup \{[0, bI] \mid b \in Z^+ \cup \{0\}, +\}$ be the pure neutrosophic interval bisemigroup.

Example 1.1.2: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_{12}, \times\}$ be a pure neutrosophic interval bisemigroup.

We see both the bisemigroups given in examples 1.1.1 and 1.1.2 are of infinite order.

Example 1.1.3: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z_{40}, +\} \cup \{[0, aI] \mid a \in Z_{25}, \times\}$ be a pure neutrosophic interval bisemigroup of finite order.

Example 1.1.4: Let $W = W_1 \cup W_2 = \{[0, aI] \mid a \in Q^+I \cup \{0\}, \times\} \cup \{[0, bI] \mid b \in Q^+ \cup \{0\}, +\}$ be a pure neutrosophic interval bisemigroup of infinite order.

Example 1.1.5: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in R^+I \cup \{0\}, +\} \cup \{[0, bI] \mid b \in Q^+I \cup \{0\}, \times\}$ be a pure neutrosophic interval bisemigroup of infinite order.

Example 1.1.6: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z_5, +\} \cup \{[0, bI] \mid b \in Z_7, \times\}$ be a pure neutrosophic interval bisemigroup of finite order.

Clearly order of T; o (T) = 5 . 7 = 35.

We can define the notion of pure neutrosophic interval subbisemigroup or pure neutrosophic biinterval subsemigroup or pure neutrosophic interval bisubsemigroup in the usual way. This task is left as an exercise to the reader.

We give only examples of them.

Example 1.1.7: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z^+I \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Q^+I \cup \{0\}, \times\}$ be a pure neutrosophic interval bisemigroup. Consider $H = H_1 \cup H_2 = \{[0, aI] \mid a \in 3Z^+I \cup \{0\}, +\} \cup \{[0, bI] \mid b \in Z^+I \cup \{0\}, \times\} \subseteq T_1 \cup T_2$; H is a pure neutrosophic interval bisubsemigroup of T. Infact T has infinitely many such pure neutrosophic interval bisubsemigroups.

Example 1.1.8: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{12}\} \cup \{[0, aI] \mid a \in Z^+ \cup \{0\}\}$ be a pure neutrosophic interval bisemigroup. Consider $S = S_1 \cup S_2 = \{[0, aI] \mid a \in \{0, 2, 4, 6, 8, 10\} \subseteq Z_{12}\} \cup \{[0, aI] \mid a \in 5Z^+ \cup \{0\}\} \subseteq V_1 \cup V_2$. S is a pure neutrosophic interval bisubsemigroup of V. We see they are pure interval bisemigroup both under addition and multiplication. Of course only one operation will be used at a time.

Example 1.1.9: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{40}\} \cup \{[0, bI] \mid b \in Z_{28}\}$ be a pure neutrosophic interval bisemigroup. Consider $H = H_1 \cup H_2 = \{[0, aI] \mid a \in \{0, 10, 20, 30) \subseteq Z_{40}\} \cup \{[0, bI] \mid$

$b \in \{0, 2, 4, \ldots, 26\} \subseteq Z_{28}\} \subseteq P_1 \cup P_2$ is a pure neutrosophic interval bisubsemigroup of P.

Example 1.1.10: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in R^+ \cup \{0\}, +\} \cup \{[0, bI] \mid b \in Q^+ \cup \{0\}, \times\}$ be a pure neutrosophic interval bisemigroup. $T = T_1 \cup T_2 = \{[0, aI] \mid a \in \left\{\frac{1}{(2)^n} \middle| n = 0, 1, 2, \ldots, \infty, +\right\}\} \cup \{[0, bI] \mid b \in 13Z^+ \cup \{0\}\} \subseteq P_1 \cup P_2$ is pure neutrosophic interval bisubsemigroup of P.

We can define ideals in case of these structures also. This is direct and hence left for the reader as an exercise. However we give examples of them.

Example 1.1.11: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times\} \cup \{[0, bI] \mid b \in Z_{12}, \times\}$ be a pure neutrosophic interval bisemigroup.

Take $P_1 \cup P_2 = P = \{[0, aI] \mid a \in 3Z^+ \cup \{0\}\} \cup \{[0, bI] \mid b \in \{0, 3, 6, 9\} \subseteq Z_{12}\} \subseteq M_1 \cup M_2$; it is easily verified P is a biideal of M.

It is important to mention here that every pure neutrosophic interval bisubsemigroup of a pure neutrosophic interval bisemigroup need not in general be a pure neutrosophic interval biideal, however every pure neutrosophic interval biideal of a pure neutrosophic interval bisemigroup is a bisubsemigroup. We see the bisubsemigroup given in example 1.1.10 is not a biideal.

Example 1.1.12: Consider $H = H_1 \cup H_2 = \{[0, aI] \mid a \in Z_{13}, +\} \cup \{[0, bI] \mid b \in Z_{29}, +\}$ a pure neutrosophic interval bisemigroup. Clearly H has no pure neutrosophic interval bisubsemigroups hence H has no pure neutrosophic biideals.

We call a pure neutrosophic interval bisemigroup S to be bisimple if it has no proper bisubsemigroups. We call S to be biideally simple if it has no biideals.

We will give examples and prove a few results in this direction.

Example 1.1.13: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{17}, \times\} \cup \{[0, bI] \mid b \in Z_{23}, \times\}$ be a pure neutrosophic interval bisemigroup. Clearly P has no ideals.

However consider $H = H_1 \cup H_2 = \{[0, I], [0, 16I], 0\} \cup \{[0, I], 0, [0, 22I]\} \subseteq P_1 \cup P_2$ is a pure neutrosophic interval bisubsemigroup of P and is not a biideal of P.

One can just think of for any pure neutrosophic interval bisemigroup $S = S_1 \cup S_2$; $H = S_1 \cup \{0\} \subseteq S_1 \cup S_2$ is a biideal of S. Also $T = \{0\} \cup S_2 \subseteq S_1 \cup S_2$ is again a biideal we choose to call these biideals as trivial biideals of S.

THEOREM 1.1.1: *Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_p, +\} \cup \{[0, bI] \mid b \in Z_p, +\}$ where p and q are two distinct primes; be a pure neutrosophic interval bisemigroup. S is simple, hence S is biidealy simple.*

The proof is direct and hence left as a simple exercise.

THEOREM 1.1.2: *Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_p, \times\} \cup \{[0, bI] \mid a \in Z_q, \times\}$ be a pure neutrosophic interval bisemigroup p and q primes. S is only a biideally simple bisemigroup but is not a simple bisemigroup.*

Proof: Follows from the fact that $H = H_1 \cup H_2 = \{[0, aI] \mid a \in \{0, p-1\}\} \cup \{[0, bI] \mid b \in \{0, q-1\}\} \subseteq S_1 \cup S_2$ is a pure neutrosophic interval subbisemigroup of S but is not a biideal of S.

Hence the claim.

However if $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_5, +\} \cup \{[0, bI] \mid b \in Z_{19}, \times\}$ be a pure neutrosophic interval bisemigroup still S is both simple and ideally simple. In view of this we have the following corollary, the proof of which is direct.

COROLLARY 1.1.1: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z_p, +\} \cup \{[0, bI] \mid b \in Z_q, \times\}$, p and q primes be a pure neutrosophic interval bisemigroup.

T is simple and ideally simple it has only trivial or interval quasi bisubsemigroups and ideals. For $P = \{[0, aI] \mid a \in Z_p, +\}$

$\cup$ {0} = $P_1 \cup P_2$, V = $V_1 \cup V_2$ = {0} $\cup$ {[0, bI] | b $\in Z_q$, $\times$} are trivial ideals.

M = $M_1 \cup M_2$ = {0} $\cup$ {[0, aI] | a $\in$ {0, q-1}, $\times$} be a pure quasi interval bisubsemigroup of T which is not an ideal.

Now we bring out the fact that Lagrange's theorem in general is not true in case of pure neutrosophic interval bisemigroups.

This is proved by the following examples.

Example 1.1.14: Let S = $S_1 \cup S_2$ = {[0, aI] | a $\in Z_{17}$, $\times$} $\cup$ {[0, bI] | b $\in Z_{29}$, $\times$} be a pure neutrosophic interval semigroup. Take T = $T_1 \cup T_2$ = {[0, I], [0, 16I], [0, 0]} $\cup$ {[0, I], [0, 28I]} $\subseteq S_1 \cup S_2$ be a pure neutrosophic interval bisemigroup of S. o (S) = 17 $\times$ 29 and o (T) = 3 $\times$ 2. Clearly o (T) $\nmid$ o (S). Consider P = $P_1 \cup P_2$ = {[0, I], 0, [0, 16I]} $\cup$ {[0, I], 0, [0, 28I]} = $S_1 \cup S_2$ is again a pure neutrosophic interval bisubsemigroup of S which is not a biideal of S. Further o (P) = 3 $\times$ 3 and 3 $\times$ 3 $\nmid$ 17 $\times$ 29.

Thus in general the Lagrange theorem is not true in case of pure neutrosophic interval bisemigroup of finite order.

Example 1.1.15: Let S = $S_1 \cup S_2$ = {[0, aI] | a $\in Z_{12}$} $\cup$ {[0, bI] | b $\in Z_{25}$} be a pure neutrosophic interval bisemigroup of order 12 $\times$ 25. Consider M = $M_1 \cup M_2$ = {[0, aI] | a $\in$ {0, 2, 4, 6, 8, 10} $\subseteq Z_{12}$} $\cup$ {[0, bI] | b $\in$ {0, 5, 10, 15, 20} $\subseteq Z_{25}$} $\subseteq$ S = $S_1 \cup S_2$; M is a pure neutrosophic interval bisubsemigroup of S. Now o (M) = 6 $\times$ 5 and we see o (M) / o (S). Thus Lagrange's theorem for finite group is true for this bisubsemigroup M of S.

Now we can define the notion of Smarandache pure neutrosophic bisemigroup S = $S_1 \cup S_2$ if S_1 and S_2 are Smarandache bisemigroups if only one of S_1 or S_2 is a Smarandache bisemigroup then we call S to be a quasi Smarandache pure neutrosophic interval bisemigroup. We give examples of Smarandache pure neutrosophic interval bisemigroup.

Example 1.1.16: Let V = $V_1 \cup V_2$ = {[0, aI] | a $\in Z_{23}$, $\times$} $\cup$ {[0, bI] | b $\in Q^+ \cup$ {0}, $\times$} be a pure neutrosophic interval bisemigroup. $H = \{[0, I], [0, 22I]\} \cup \{[0, 2^n I], [0, \frac{1}{2^m} I] \mid m, n \in Z^+\} \subseteq V_1 \cup V_2 = V$; is a interval group which is pure neutrosophic hence V is a S-pure neutrosophic interval bisemigroup.

Example 1.1.17: Let $V = \{[0, aI] \mid a \in Z^+ \cup \{0\}\} \cup \{[0, aI] \mid a \in Z_7, +\}$ be a pure neutrosophic interval bisemigroup. Clearly V is not a Smarandache pure neutrosophic interval bisemigroup.

We have the following interesting theorems the proof of which is left as an exercise.

THEOREM 1.1.3: *Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_p, +\} \cup \{[0, bI] \mid b \in Z_q, +\}$ (p and q are two distinct primes) be a pure neutrosophic interval bisemigroup. V is not a S-pure neutrosophic interval bisemigroup.*

THEOREM 1.1.4: *Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_p, \times\} \cup \{[0, bI] \mid b \in Z_q, \times\}$ (p and q two distinct primes) be a pure neutrosophic interval bisemigroup. V is a S-pure neutrosophic interval bisemigroup.*

Proof: Take $H = H_1 \cup H_2 = \{[0, aI] \mid a \in \{1, p-1\}, \times\} \cup \{[0, bI] \mid b \in \{1, q-1\}, \times\} \subseteq V$ is a pure neutrosophic interval bigroup of V. Also $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_p \setminus \{0\}, \times\} \cup \{[0, bI] \mid b \in Z_p \setminus \{0\}, \times\} \subseteq V$ is a pure neutrosophic interval bigroup. Thus V is a S-pure neutrosophic biinterval semigroup.

We have a class of S-pure neutrosophic biinterval semigroup as well as a pure neutrosophic interval bisemigroup which is not Smarandache.

THEOREM 1.1.5: *Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in R^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_p, +\}$ (p a prime) be a pure neutrosophic interval bisemigroup. S is not a S-pure neutrosophic interval bisemigroup.*

THEOREM 1.1.6: *Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, +\} \cup \{[0, bI] \mid b \in Z_q, +\}$ (q a prime) be a pure neutrosophic* *interval bisemigroup. M is not a S-pure neutrosophic interval bisemigroup.*

Now we proceed onto define the concept of homomorphisms of pure neutrosophic interval bisemigroups using the pure neutrosophic intervals.

Let $V = V_1 \cup V_2$ and $S = S_1 \cup S_2$ be any two pure neutrosophic interval bisemigroups. A bimap $\eta = \eta_1 \cup \eta_2 = V \to S$ such that $\eta_1 : V_1 \to S_1$ and $\eta_2 : V_2 \to S_2$ are pure neutrosophic homomorphisms will be known as the pure neutrosophic interval bisemigroup homomorphisms.

Consider $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_{12}, \times\}$ be a pure neutrosophic interval bisemigroup. $W = W_1 \cup W_2 = \{[0, aI] \mid a \in Q^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_{24}, \times\}$ be a pure neutrosophic interval bisemigroup. $\eta : V \to W$ defined by $\eta = \eta_1 \cup \eta_2 : V_1 \cup V_2 \to W_1 \cup W_2$ given by $\eta_1 : V_1 \to W_1$ and $\eta_2 : V_2 \to W_2$ such that $\eta_1 ([0, aI]) \mapsto [0, aI]$ and $\eta_2 ([0, aI]) \mapsto [0, 2aI]$).

It is easily verified $\eta = \eta_1 \cup \eta_2$ is a pure neutrosophic interval bisemigroup homomorphism. Interested reader can define bikernel of a pure neutrosophic interval bisemigroup homomorphism. Now we call $S = \{([0, a_1I], \ldots, [0, a_nI])$ where $a_i \in Z_n$ or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}\}$ as a pure neutrosophic interval row matrix, $1 \leq i \leq n$. Clearly $(S, +)$ is a semigroup. $(S, \times)$ is also a semigroup.

But if we consider $H = \left\{\begin{bmatrix} [0, a_1I] \\ [0, a_2I] \\ \vdots \\ [0, a_nI] \end{bmatrix}\right|$ where $a_i \in Z_n$ or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$; $1 \leq i \leq n\}$ then H is only a pure neutrosophic semigroup under addition as multiplication is not compatible.

Suppose $M = \left\{ \left[\begin{array}{ccc} [0, a_1 I] & \ldots & [0, a_n I] \\ [0, b_1 I] & \ldots & [0, b_n I] \\ \vdots & & \vdots \\ [0, m_1 I] & \ldots & [0, m_n I] \end{array} \right] \right|$ $a_i, b_i, \ldots, m_i \in Z_n$ or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}\}$ be a collection of $m \times n$ pure neutrosophic interval matrices $m \neq n$ then M is only a interval pure neutrosophic semigroup under addition and under multiplication M is not compatible. If however $m = n$, M will be a semigroup under addition as well as multiplication.

Let $P = \left\{ \sum_{i=0}^{\infty} [0, aI] x^i \right|$ $a_i \in Z_n$ or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}\}$, P is a pure neutrosophic interval polynomial semigroup under addition as well as multiplication. Take $K = \left\{ \sum_{i=0}^{n} [0, aI] x^i \right|$ $n < \infty$, $a_i \in Z_n$ or $Q^+ \cup \{0\}$ or $Z^+ \cup \{0\}$ or $R^+ \cup \{0\}$; $0 \leq i \leq n\}$; K is only a pure neutrosophic interval semigroup under addition. However if we impose the condition $x^n = 1$ then K is closed under multiplication.

We will be using these types of pure neutrosophic interval semigroups to construct bisemigroups. The definition needs no modification. We give only examples of them.

Example 1.1.18: Let $P = P_1 \cup P_2 = \{([0, a_1I], \ldots, [0, a_8I]) \mid a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 8, \times\} \cup \left\{ \left[\begin{array}{c} [0, a_1 I] \\ [0, a_2 I] \\ \vdots \\ [0, a_{10} I] \end{array} \right] \right|$ $a_i \in Z_{25}$, $1 \leq i \leq 10, +\}$ be the pure neutrosophic interval bisemigroup.

Example 1.1.19: Let $K = K_1 \cup K_2 =$

$$\left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] & [0,a_4I] \\ [0,a_8I] & [0,a_5I] & [0,a_6I] & [0,a_7I] \\ [0,a_{10}I] & [0,a_9I] & [0,a_{11}I] & [0,a_{12}I] \end{bmatrix} \right| a_i \in Z_{12}; 1 \leq i \leq 12, +\right\}$$

$$\cup \left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \\ \vdots & \vdots \\ [0,a_{17}I] & [0,a_{18}I] \end{bmatrix} \right| a_i \in Z_{15}; 1 \leq i \leq 18, +\right\}$$

be a pure neutrosophic interval bisemigroup. Clearly K is commutative and is of finite order.

Example 1.1.20: Let $K = K_1 \cup K_2 =$

$$\left\{\left.\sum_{i=0}^{20}[0,a_iI]x^i \right| a_i \in Q^+ \cup \{0\}; 0 \leq i \leq 20, +\right\} \cup$$

$$\left\{\left.\begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ \vdots \\ [0,a_{25}I] \end{bmatrix} \right| a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 25, +\right\}$$

be a pure neutrosophic interval bisemigroup of infinite order.

Example 1.1.21: Let $G = G_1 \cup G_2 =$

$$\left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_8I] & [0,a_7I] & [0,a_9I] \end{bmatrix} \right| a_i \in Z_5; 1 \leq i \leq 9, +\right\}$$

$$\cup \left\{\left.\sum_{i=0}^{7}[0,a_iI]x^i \right| a_i \in Z_8, 0 \leq i \leq 7, +\right\}$$

be a pure neutrosophic interval bisemigroup. P is of finite order and is commutative.

Example 1.1.22: Let W = $W_1 \cup W_2 = \left\{\sum_{i=0}^{20}[0, a_iI]x^i \,\middle|\, a_i \in Z^+ \cup \right.$ $\{0\}$, +, $0 \le i \le 20\} \cup \left\{\sum_{i=0}^{35}[0, a_iI]x^i \,\middle|\, a_i \in Z_9; 0 \le i \le 35, +\right\}$ be a pure neutrosophic interval bisemigroup of infinite order. We can define substructures. This is a matter of routine and hence is left as an exercise to the reader. However we give examples.

Example 1.1.23: Let V = $\left\{\begin{bmatrix}[0, a_1I]\\ [0, a_2I]\\ \vdots \\ [0, a_{15}I]\end{bmatrix} \,\middle|\, a_i \in Q^+ \cup \{0\}, 1 \le i \le 15, +\right\} \cup \{([0, a_1I], [0, a_2I], \ldots, [0, a_{10}I]) \mid a_i \in Q^+ \cup \{0\}, 1 \le i \le 10, \times\}$ be a pure neutrosophic interval bisemigroup.

Take $M = M_1 \cup M_2 = \left\{\begin{bmatrix}[0, a_1I]\\ 0\\ 0\\ 0\\ 0\\ [0, a_2I]\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ [0, a_3I]\end{bmatrix} \,\middle|\, a_i \in Q^+ \cup \{0\}, 1 \le i \le 3, +\right\} \cup \{([0, a_1I], [0, a_2I], [0, a_3I], 0 \ldots 0) \mid a_i \in Q^+ \cup \{0\},$

$1 \leq i \leq 3, \times\} \subseteq V$ is a pure neutrosophic interval bisubsemigroup of V. Infact M is not a pure neutrosophic interval biideal of V.

$$P = P_1 \cup P_2 = \left\{ \begin{bmatrix} [0, a_1 I] \\ [0, a_2 I] \\ \vdots \\ [0, a_{15} I] \end{bmatrix} \middle| \; a_i \in Z^+ \cup \{0\},\ 1 \leq i \leq 15, +\right\} \cup$$

$\{([0, a_1I], [0, a_2I], \ldots, [0, a_{10}I]) \mid a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 10, \times\} \subseteq$ V is a pure neutrosophic interval bisubsemigroup of V and is not a biideal of V. Thus we have bisubsemigroups which are not biideals of V.

Example 1.1.24: Let $M = M_1 \cup M_2 =$

$$\left\{ \begin{bmatrix} [0, a_1 I] & [0, a_2 I] \\ [0, a_3 I] & [0, a_4 I] \end{bmatrix} \middle| \; a_i \in Z_{12}, 1 \leq i \leq 4, \times \right\} \cup$$

$$\left\{ \begin{bmatrix} [0, a_1 I] & [0, a_2 I] & [0, a_3 I] \\ 0 & [0, a_4 I] & [0, a_5 I] \\ 0 & 0 & [0, a_6 I] \end{bmatrix} \middle| \; a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 6, \times \right\}$$ be a pure neutrosophic interval bisemigroup.

Consider $P = P_1 \cup P_2 =$

$$\left\{ \begin{bmatrix} [0, a_1 I] & 0 \\ 0 & [0, a_2 I] \end{bmatrix} \middle| \; a_i \in Z_{12}, 1 \leq i \leq 2, \times \right\} \cup$$

$$\left\{ \begin{bmatrix} [0, a_1 I] & [0, a_2 I] & [0, a_3 I] \\ 0 & [0, a_4 I] & [0, a_5 I] \\ 0 & 0 & [0, a_6 I] \end{bmatrix} \middle| \; a_i \in 5Z^+ \cup \{0\}, 1 \leq i \leq 6, \times \right\} \subseteq$$

$M_1 \cup M_2 = M$, P is not only a pure neutrosophic interval subsemigroup of M but is also a pure neutrosophic interval biideal of M.

Example 1.1.25: Let B = $B_1 \cup B_2$ =

$$\left\{\sum_{i=0}^{\infty}[0,a_iI]x^i \,\middle|\, a_i \in Z^+ \cup \{0\}, \times\right\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_7I] & [0,a_8I] & [0,a_9I] \\ [0,a_{10}I] & [0,a_{11}I] & [0,a_{12}I] \\ [0,a_{13}I] & [0,a_{14}I] & [0,a_{15}I] \end{bmatrix} \,\middle|\, a_i \in Z_{90}, 1 \le i \le 15, +\right\}$$ be a pure

neutrosophic interval bisemigroup. Take T = $T_1 \cup T_2$ =

$$\left\{\sum_{i=0}^{\infty}[0,a_iI]x^i \,\middle|\, a_i \in 5Z^+ \cup \{0\}, \times\right\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1I] & 0 & [0,a_2I] \\ 0 & [0,a_3I] & 0 \\ [0,a_4I] & 0 & [0,a_5I] \\ 0 & [0,a_6I] & 0 \\ [0,a_7I] & 0 & [0,a_8I] \end{bmatrix} \,\middle|\, a_i \in Z_{90}, 1 \le i \le 8, +\right\} \subseteq B =$$

$B_1 \cup B_2$ is only a pure neutrosophic interval subbisemigroup of B. Now we can build quasi interval pure neutrosophic bisemigroup S = $S_1 \cup S_2$ by taking only one of S_1 or S_2 to be pure neutrosophic interval semigroup and the other to be just a pure neutrosophic semigroup.

We will give some examples.

Example 1.1.26: Let V = $V_1 \cup V_2$ =

$$\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \\ [0,a_6I] & [0,a_5I] \end{bmatrix} \,\middle|\, a_i \in Z^+ \cup \{0\}, 1 \le i \le 6, +\right\}$$ $\cup$ {(a_1I, a_2I, a_3I, a_4I) | $a_i \in Z^+ \cup \{0\}$, $1 \le i \le 4$, $\times$} be a quasi interval pure neutrosophic bisemigroup.

Example 1.1.27: Let $V = V_1 \cup V_2 = \{([0, a_1I], [0, a_2I], \ldots, [0, a_9I] \mid a_i \in Z_{40}, 1 \leq i \leq 90, \times\} \cup$

$$\left\{\left.\begin{bmatrix} a_1I & a_2I & a_3I & a_4I \\ a_5I & a_6I & a_7I & a_8I \\ a_9I & a_{10}I & a_{11}I & a_{12}I \\ a_{13}I & a_{14}I & a_{15}I & a_{16}I \\ a_{17}I & a_{18}I & a_{19}I & a_{20}I \\ a_{21}I & a_{22}I & a_{23}I & a_{24}I \end{bmatrix}\right| a_i \in Z_{240}, 1 \leq i \leq 24, +\right\}$$

be a quasi interval pure neutrosophic bisemigroup.

Example 1.1.28: Let $M = M_1 \cup M_2 =$

$$\left\{\left.\begin{bmatrix} a_1I \\ a_2I \\ \vdots \\ a_{12}I \end{bmatrix}\right| a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 12, +\right\} \cup \{([0, a_1I], \ldots, [0, a_{25}I]) \mid a_i$$

$\in Z^+ \cup \{0\}, 1 \leq i \leq 25, +\}$ be a quasi interval pure neutrosophic bisemigroup.

Example 1.1.29: Let $P = P_1 \cup P_2 = \{$all $n \times n$ pure neutrosophic matrices with entries from $Z_{15}I$ under $\times\} \cup \{$all $n \times n$ pure neutrosophic interval matrices with entries from $Z^+I \cup \{0\}$ under $\times\}$ be a quasi interval pure neutrosophic bisemigroup.

Example 1.1.30: Let $M = M_1 \cup M_2 = \{Z_{219}I, \times\} \cup \{[0, aI] \mid a \in Z_{40}, \times\}$ be a quasi interval pure neutrosophic bisemigroup of finite order.

Example 1.1.31: Let $M = M_1 \cup M_2 =$

$$\left\{\left.\sum_{i=0}^{8}[0, a_iI]x^i \right| a_i \in Z^+ \cup \{0\}, +\right\} \cup$$

$$\left\{\sum_{i=0}^{\infty}[0,a_iI]x^i \;\middle|\; a_i \in Z^+ \cup \{0\}, \times\right\}$$

be a quasi interval pure neutrosophic bisemigroup of infinite order.

Now having seen examples of quasi interval pure neutrosophic bisemigroups we can now proceed onto define substructures in them. This is infact a matter of routine and hence left for the reader. We give a few examples of them.

Example 1.1.32: Let $V = V_1 \cup V_2 =$

$$\left\{\begin{bmatrix}[0,a_1I] & [0,a_2I]\\ [0,a_3I] & [0,a_4I]\end{bmatrix} \;\middle|\; a_i \in Z^+ \cup \{0\}, 1 \le i \le 4, +\right\} \cup$$

$$\left\{\sum_{i=0}^{25} a_iIx^i \;\middle|\; a_i \in Z^+ \cup \{0\}, 0 \le i \le 25, +\right\}$$

be a quasi interval pure neutrosophic bisemigroup of infinite order. Consider

$$H = H_1 \cup H_2 = \left\{\begin{bmatrix}[0,a_1I] & [0,a_2I]\\ 0 & [0,a_3I]\end{bmatrix} \;\middle|\; a_i \in 3Z^+ \cup \{0\}, 1 \le i \le 3, +\right\} \cup$$

$$\left\{\sum_{i=0}^{10} a_iIx^i \;\middle|\; a_i \in 5Z^+ \cup \{0\}, 0 \le i \le 10, +\right\} \subseteq V_1 \cup V_2 = V;$$

H is a quasi interval pure neutrosophic bisubsemigroup of V and is not a biideal of V.

Example 1.1.33: Let $P = P_1 \cup P_2 =$

$$\left\{\begin{bmatrix}[0,a_1I] & [0,a_2I] & [0,a_3I]\\ [0,a_4I] & [0,a_5I] & [0,a_6I]\\ [0,a_7I] & [0,a_8I] & [0,a_9I]\end{bmatrix} \;\middle|\; a_i \in Z^+ \cup \{0\}, 1 \le i \le 9, \times\right\} \cup$$

$$\left\{\begin{bmatrix}a_1I & a_2I & a_3I & a_4I\\ a_5I & a_6I & a_7I & a_8I\\ a_9I & a_{10}I & a_{11}I & a_{12}I\\ a_{13}I & a_{14}I & a_{15}I & a_{16}I\end{bmatrix} \;\middle|\; a_i \in Z_{240}, 1 \le i \le 16, \times\right\}$$

be a quasi interval pure neutrosophic bisemigroup. Consider H = $H_1 \cup H_2$ =

$$\left\{\begin{bmatrix} [0,a_1I] & 0 & 0 \\ 0 & [0,a_2I] & 0 \\ 0 & 0 & [0,a_3I] \end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}, 1 \le i \le 3, \times \right\} \cup$$

$$\left\{\begin{bmatrix} a_1I & a_2I & a_3I & a_4I \\ a_5I & a_6I & a_7I & a_8I \\ a_9I & a_{10}I & a_{11}I & a_{12}I \\ a_{13}I & a_{14}I & a_{15}I & a_{16}I \end{bmatrix} \middle| a_i \in \{0, 2, 4, 6, 8, \ldots, 236, 238\} \subseteq\right.$$

Z_{240}, $1 \le i \le 16, \times\} \subseteq P_1 \cup P_2 = P$ be a quasi interval pure neutrosophic bisubsemigroup of P. Infact H is also a quasi interval pure neutrosophic biideal of P.

Now we can define quasi pure neutrosophic interval bisemigroup, $S = S_1 \cup S_2$ to be a bisemigroup in which one of S_1 or S_2 is a pure neutrosophic interval semigroup where as the other is just an interval semigroup.

We will illustrate this situation by an example or two.

Example 1.1.34: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times\} \cup \{[0, a] \mid a \in Q^+ \cup \{0\}, +\}$ be a quasi pure neutrosophic interval bisemigroup of infinite order.

Example 1.1.35: Let $N = N_1 \cup N_2 = \{([0, a_1I], [0, a_2I], [0, a_3I])$

$$\mid a_i \in Z_{20}, 1 \le i \le 30, \times\} \cup \left\{\begin{bmatrix} [0,a_1] \\ [0,a_2] \\ [0,a_3] \\ [0,a_4] \\ [0,a_5] \\ [0,a_6] \end{bmatrix} \middle| a_i \in Z_{12}, 1 \le i \le 6, + \right\} \text{ be a}$$

quasi pure neutrosophic interval bisemigroup of finite order.

Example 1.1.36: Let $V = V_1 \cup V_2 =$

$$\left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] & [0,a_4I] & [0,a_5I] \\ [0,a_6I] & [0,a_7I] & [0,a_8I] & [0,a_9I] & [0,a_{10}I] \end{bmatrix}\right| a_i \in Z_{40}, 1 \leq i \leq 10, + \right\}$$

$\cup$ {5 × 5 interval matrices with intervals of the form [0, a] where a $\in Z_5$ under multiplication} be a finite quasi pure neutrosophic interval bisemigroup which is non commutative.

Substructures can be defined and illustrated by any interested reader as it is direct and simple. Now if in a bisemigroup $S = S_1 \cup S_2$ one of S_1 or S_2 is a pure neutrosophic interval semigroup and the other is just a semigroup then we call $S = S_1 \cup S_2$ to be a quasi pure neutrosophic quasi interval bisemigroup.

We will illustrate this situation by some examples.

Example 1.1.37: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Q^+ \cup \{0\}, \times\} \cup \{Z^+ \cup \{0\}, +\}$ be a quasi pure neutrosophic quasi interval bisemigroup.

Example 1.1.38: Let $M = M_1 \cup M_2 = \{([0, aI], [0, bI], [0, cI], [0, dI]) \mid a, b, c, d \in Z_{40}, \times\} \cup \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix}\right.$ where $a_i \in Z_{15}$, $1 \leq i \leq 6$, $+\}$ be a quasi pure neutrosophic quasi interval bisemigroup.

Example 1.1.39: Let $T = T_1 \cup T_2 =$

$$\left\{\left.\begin{bmatrix} [0,a_1I] & \dots & [0,a_5I] \\ [0,a_6I] & \dots & [0,a_{10}I] \\ \vdots & & \vdots \\ [0,a_{21}I] & \dots & [0,a_{25}I] \end{bmatrix}\right| a_i \in Z_{40}, 1 \leq i \leq 25, \times \right\} \cup$$

{All 8 × 8 matrices with entries from Z_{25} under multiplication} be a non commutative finite quasi pure neutrosophic quasi interval bisemigroup.

Example 1.1.40: Let $M = M_1 \cup M_2 =$

$$\left\{\sum_{i=0}^{\infty}[0, aI]x^i \,\middle|\, a \in Q^+ \cup \{0\}, \times\right\} \cup$$

{all 8 × 8 matrices with entries from $Q^+ \cup \{0\}$ under product} be a quasi pure neutrosophic quasi interval bisemigroup of finite order which is non commutative.

Let $W = W_1 \cup W_2 =$

$$\left\{\sum_{i=0}^{\infty}[0, aI]x^i \,\middle|\, a \in Z^+ \cup \{0\}, \times\right\}$$

$\cup$ {all 8 × 8 matrices with entries from $Z^+ \cup \{0\}$, ×} $\subseteq M_1 \cup M_2 = M$ be a quasi interval quasi pure neutrosophic bisubsemigroup of M, this is also non commutative.

Example 1.1.41: Let $V = V_1 \cup V_2 =$

$$\left\{\sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in Q^+ \cup \{0\}, 0 \leq i \leq 9, +\right\} \cup$$

$$\left\{\sum_{i=0}^{20}[0, aI]x^i \,\middle|\, a_i \in Z^+ \cup \{0\}, 0 \leq i \leq 6, +\right\}$$

be a quasi pure neutrosophic quasi interval bisemigroup.

$M = M_1 \cup M_2 =$

$$\left\{\sum_{i=0}^{6} a_i x^i \,\middle|\, a_i \in Z^+ \cup \{0\}, 0 \leq i \leq 6, \times\right\} \cup$$

$$\left\{\sum_{i=0}^{10}[0, a_i I]x^i \,\middle|\, a \in 3Z^+ \cup \{0\}, 0 \leq i \leq 20\right\}$$

be a quasi pure neutrosophic quasi interval bisubsemigroup of V. Clearly M is not a quasi pure neutrosophic quasi interval biideal of V.

Example 1.1.42: Let $M = \{[0, aI] \mid a \in Q^+ \cup \{0\}, +\} \cup \{[0, a] \mid a \in R^+ \cup \{0\}, +\}$ be a quasi pure neutrosophic interval bisemigroup. Clearly M has no biideals but infinitely many subbisemigroups.

We now proceed onto define the notion of mixed neutrosophic interval bisemigroup.

Let $S = S_1 \cup S_2$ if S_1 and S_2 are mixed neutrosophic distinct interval semigroups then we call S to be a mixed neutrosophic interval bisemigroup. Even if we do not use the term mixed but say neutrosophic interval bisemigroup still the term will mean only mixed neutrosophic intervals and not pure neutrosophic intervals.

We will illustrate this situation by some examples.

Example 1.1.43: Let $V = V_1 \cup V_2 = \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{12}, \times\}$ be the neutrosophic interval bisemigroup. Clearly V is of infinite order and is commutative.

Example 1.1.44: Let $M = M_1 \cup M_2 = \{([0, a_1+b_1I], [0, a_2+b_2I], [0, a_3+b_3I]) \mid a_i, b_i \in Z^+ \cup \{0\}, 1 \leq i \leq 3, +\} \cup$

$$\left\{\begin{bmatrix}[0,a_1+b_1I]\\ [0,a_2+b_2I]\\ \vdots\\ [0,a_9+b_9I]\end{bmatrix} \middle| a_i, b_i \in R^+ \cup \{0\}, 1 \leq i \leq 9, +\right\}$$

be an infinite commutative neutrosophic interval bisemigroup.

Example 1.1.45: Let $M = M_1 \cup M_2 =$

$$\left\{\sum_{i=0}^{\infty}[0,a+bI]x^i \middle| a,b \in Z^+ \cup \{0\}, \times\right\} \cup$$

$$\left\{\begin{bmatrix}[0,a_1+b_1I] & [0,a_2+b_2I]\\ [0,a_3+b_3I] & [0,a_4+b_4I]\\ [0,a_5+b_5I] & [0,a_6+b_6I]\\ [0,a_7+b_7I] & [0,a_8+b_8I]\end{bmatrix} \middle| a_i, b_i \in Z_{90}, 1 \leq i \leq 8, +\right\}$$

be a neutrosophic interval bisemigroup.

We can in case of neutrosophic interval bisemigroups have the concept of 6 types of bisubsemigroups.

Suppose $S = S_1 \cup S_2$ is a neutrosophic interval bisemigroup and if $P = P_1 \cup P_2 \subseteq S_1 \cup S_2 = S$ where P_1 and P_2 are mixed neutrosophic interval semigroups then P is a neutrosophic interval bisubsemigroup of S. If $M = M_1 \cup M_2$ is such that $M_i \subseteq S_i$ is a pure neutrosophic interval subsemigroup for i=1, 2 then we call M to be a pseudo pure neutrosophic interval subbisemigroup of S.

Let $T = T_1 \cup T_2 \subseteq S_1 \cup S_2$ be such that both T_1 and T_2 are just interval semigroups then we define T to be a pseudo interval subsemigroup of S. Take $V = V_1 \cup V_2 \subseteq S_1 \cup S_2$ where one of V_1 or V_2 is a mixed neutrosophic interval semigroup and other a pure neutrosophic interval semigroup then we call V to be a mixed pure neutrosophic interval subbisemigroup.

Consider $B = B_1 \cup B_2 \subseteq S_1 \cup S_2$; where B_1 or B_2 is a mixed neutrosophic interval semigroup and the other is just a interval semigroup then we call B to be a mixed interval neutrosophic and interval bisubsemigroup of S.

Likewise if $C = C_1 \cup C_2$ where C_1 is a pure neutrosophic interval semigroup and C_2 is just a interval semigroup then we call $C = C_1 \cup C_2$ to be a pure neutrosophic interval and interval subbisemigroup.

We will illustrate these six types of subbisemigroups by some examples.

Example 1.1.46: Let $V = V_1 \cup V_2 = \{([0, a_1+b_1I], [0, a_2+b_2I], [0, a_3+b_3I], [0, a_4+b_4I], [0, a_5+b_5I]) \mid a_i, b_i \in Z_{10}, 1 \leq i \leq 5, +\} \cup$

$$\left\{\left.\begin{bmatrix} [0,a_1+b_1I] & [0,a_2+b_2I] & [0,a_3+b_3I] \\ [0,a_4+b_4I] & [0,a_5+b_5I] & [0,a_6+b_6I] \\ [0,a_7+b_7I] & [0,a_8+b_8I] & [0,a_9+b_9I] \\ [0,a_{10}+b_{10}I] & [0,a_{11}+b_{11}I] & [0,a_{12}+b_{12}I] \end{bmatrix}\right| a_i, b_i \in Z_{15}, 1 \leq i \leq 12, +\right\}$$

be a neutrosophic interval bisemigroup.

Consider $H = H_1 \cup H_2 = \{([0, a_1+b_1I], [0, a_2+b_2I], [0, a_3+b_3I], [0, a_4+b_4I], [0, a_5+b_5I]) \mid a_i, b_i \in \{0, 2, 4, 6, 8\} \subseteq Z_{10},$

$1 \leq i \leq 5, +\} \cup \left\{\left.\begin{bmatrix} [0, a_1 + b_1 I] & [0, a_2 + b_2 I] & [0, a_3 + b_3 I] \\ \vdots & \vdots & \vdots \\ [0, a_{10} + b_{10} I] & [0, a_{11} + b_{11} I] & [0, a_{12} + b_{12} I] \end{bmatrix}\right|\right.$

$a_i, b_i \in \{0, 3, 6, 9, 12\} \subseteq Z_{15}, 1 \leq i \leq 12, +\} \subseteq V_1 \cup V_2 = V$ is a neutrosophic interval bisubsemigroup of V.

Take $T = T_1 \cup T_2 = H_1 \cup \left\{\left.\begin{bmatrix} [0, a_1 I] & [0, a_2 I] & [0, a_3 I] \\ [0, a_4 I] & [0, a_5 I] & [0, a_6 I] \\ [0, a_7 I] & [0, a_8 I] & [0, a_9 I] \\ [0, a_{10} I] & [0, a_{11} I] & [0, a_{12} I] \end{bmatrix}\right| a_i\right.$

$\in Z_{15}, 1 \leq i \leq 12, +\} \subseteq V_1 \cup V_2 = V$, T is a mixed pure neutrosophic interval subbisemigroup of V.

Let $W = W_1 \cup W_2 = \{([0, a_1 I], [0, a_2 I], [0, a_3 I], [0, a_4 I], [0, a_5 I]) \mid a_i \in Z_{10}, 1 \leq i \leq 5, +\} \cup T_2 \subseteq V_1 \cup V_2 = V$ be a pseudo pure neutrosophic interval subbisemigroup of V.

Consider $L = L_1 \cup L_2 = \{([0, a_1], [0, a_2], [0, a_3], [0, a_4],$

$[0, a_5]) \mid a_i \in Z_{10}, 1 \leq i \leq 5, +\} \cup \left\{\left.\begin{bmatrix} [0, a_1] & [0, a_2] & [0, a_3] \\ [0, a_4] & [0, a_5] & [0, a_6] \\ [0, a_7] & [0, a_8] & [0, a_9] \\ [0, a_{10}] & [0, a_{11}] & [0, a_{12}] \end{bmatrix}\right|\right.$

$a_i \in Z_{15}, 1 \leq i \leq 12, +\} \subseteq V_1 \cup V_2 = V$ is a pseudo neutrosophic interval bisubsemigroup.

Consider $N = N_1 \cup N_2 = L_1 \cup T_2 \subseteq V_1 \cup V_2$ is a interval pure neutrosophic interval subbisemigroup of V.

Take $R = R_1 \cup R_2 = H_1 \cup L_2 \subseteq V_1 \cup V_2$, R is a mixed interval neutrosophic interval subbisemigroup of V.

Example 1.1.47: Let $V = V_1 \cup V_2 = \{([0, a+bI] \mid a, b \in Z^+ \cup \{0\}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{24}, +\}$ be a mixed neutrosophic interval bisemigroup.

i) Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in 3Z^+ \cup \{0\}, +\} \cup \{[0, a+bI] \mid a, b \in \{0, 2, 4, 6, 8, 10, \ldots, 22\} \subseteq Z_{24}, +\} \subseteq V_1 \cup V_2$ be a mixed neutrosophic interval bisubsemigroup of V.

ii) Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_{24}; +\} \subseteq V_1 \cup V_2$ be a pseudo pure neutrosophic interval bisubsemigroup of V.

iii) Consider $S = S_1 \cup S_2 = \{[0, a] \mid Z^+ \cup \{0\}, +\} \cup \{[0, b] \mid b \in Z_{24}; +\} \subseteq V_1 \cup V_2 = V$, S is the pseudo neutrosophic interval bisubsemigroup of S.

iv) Take $W = W_1 \cup W_2 = \{[0, a+bI] \mid a, b \in 5Z^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in \{2Z_{24}\} \subseteq Z_{24}, +\} \subseteq V_1 \cup V_2$, W is a mixed- pure neutrosophic interval bisubsemigroup.

v) Let $N = N_1 \cup N_2 = \{[0, a+bI] \mid a, b \in 5Z^+ \cup \{0\}, +\} \cup \{[0, a] \mid a \in Z_{24}, +\} \subseteq V_1 \cup V_2$, N is a mixed neutrosophic interval interval subbisemigroup.

vi) $D = D_1 \cup D_2 = \{[0, a] \mid a \in 7Z^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_{24}, +\} \subseteq V_1 \cup V_2$ is a interval neutrosophic pure-interval bisubsemigroup.

It is pertinent to mention here that all types of neutrosophic interval bisemigroups cannot be biideals.

For instance consider the following types of bisubsemigroup of a mixed neutrosophic interval bisemigroup.

(1) $S = S_1 \cup S_2$ be a mixed neutrosophic interval bisemigroup.

$P = P_1 \cup P_2 = \{[0, a] \mid a$ is real$\} \cup \{$any real interval subsemigroup $\}$ of $S_1 \cup S_2$. P is only a bisubsemigroup and never a biideal of S.

(2) If $T = T_1 \cup T_2 = \{$real interval semigroup$\} \cup \{$pure neutrosophic interval semigroup$\}$ is a bisubsemigroup, then T is not a biideal.

(3) $W = W_1 \cup W_2 = \{$real interval semigroup$\} \cup \{$mixed neutrosophic interval semigroup$\} \subseteq S_1 \cup S_2$ is only a bisubsemigroup and never an ideal.

However the mixed neutrosophic interval bisubsemigroup, pseudo pure neutrosophic interval bisubsemigroup and mixed pure interval neutrosophic interval bisubsemigroup can be sometimes biideals.

We will give some examples of them before we proceed onto work with other types of interval bisemigroups.

Example 1.1.48: Let $V = V_1 \cup V_2 = \{[0, aI+b] \mid a, b \in Z^+ \cup \{0\}, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{40}, \times\}$ be a mixed neutrosophic interval bisemigroup. $H = H_1 \cup H_2 = \{[0, a+bI] \mid a, b \in 5Z^+ \cup \{0\}, \times\} \cup \{[0, aI+b] \mid a, b \in \{0, 4, 8, \ldots, 36\} \subseteq Z_{40}, \times\} \subseteq V_1 \cup V_2$ be a mixed neutrosophic interval bisubsemigroup of V. Clearly H is a biideal of V.

$T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times\} \cup \{[0, bI] \mid b \in Z_{40}, \times\} \subseteq V_1 \cup V_2$ be a pseudo pure neutrosophic interval bisubsemigroup of V. T is also a biideal of V. Consider $L = L_1 \cup L_2 = \{[0, a+bI] \mid a, b \in 3Z^+ \cup \{0\}, \times\} \cup \{[0, aI] \mid a \in Z_{40}, \times\} \subseteq V_1 \cup V_2$ be a mixed pure neutrosophic interval bisubsemigroup. L is a biideal of V.

Now $R = R_1 \cup R_2 = \{[0, a] \mid a \in Z^+ \cup \{0\}, \times\} \cup \{[0, a] \mid a \in Z_{40}, \times\} \subseteq V_1 \cup V_2$ is a bisubsemigroup of V but R is not a biideal. Likewise $D = D_1 \cup D_2 = \{[0, a] \mid a \in Z^+ \cup \{0\}, \times\} \cup \{[0, a+bI] \mid a, b \in 2Z_{40}, \times\}$ is a subbisemigroup which is not a biideal of V.

Also $P = P_1 \cup P_2 = \{[0, a] \mid a \in Z^+ \cup \{0\}, \times\} \cup \{[0, a] \mid a \in Z_{40}, \times\} \subseteq V_1 \cup V_2$ is only a bisubsemigroup of V and is not a biideal of V.

Now we can develop all other properties in case of mixed neutrosophic interval bisemigroup as in case of pure neutrosophic interval bisemigroup.

However we can define mixed - neutrosophic interval - pure neutrosophic interval bisemigroup or mixed - pure neutrosophic interval bisemigroup.

Example 1.1.49: Let $V = V_1 \cup V_2 =$

$$\left\{\begin{pmatrix} [0,aI] & [0,bI] \\ [0,cI] & [0,dI] \end{pmatrix} \middle| a,b,c,d \in Z_{20}, \times \right\} \cup \{([0, a_1+b_1I], [0, a_2+b_2I],$$

$\ldots, [0, a_8+b_8I])$ where $a_i, b_i \in Z^+ \cup \{0\}, \times, 1 \leq i \leq 8\}$ be the pure - mixed neutrosophic interval bisemigroup.

Real interval mixed neutrosophic interval bisemigroup is illustrated by the following example.

Example 1.1.50: Let M = $M_1 \cup M_2$ =

$$\left\{\begin{bmatrix} [0,a] & [0,b] & [0,c] & [0,d] \\ [0,e] & [0,f] & [0,g] & [0,h] \end{bmatrix} \middle| a,b,c,d,e,f,g,h \in Z^+ \cup \{0\}, + \right\}$$

$$\cup \left\{\begin{bmatrix} [0,a_1+b_1I] \\ [0,a_2+b_2I] \\ \vdots \\ [0,a_9+b_9I] \end{bmatrix} \middle| a_i, b_i \in Q^+ \cup \{0\}, +; 1 \le i \le 9 \right\}$$

be the real interval mixed neutrosophic bisemigroup.

We can still have mixed neutrosophic interval pure neutrosophic bisemigroup or quasi interval mixed pure neutrosophic bisemigroup. We have illustrated them by the following examples.

Example 1.1.51: Let V = $V_1 \cup V_2$ =

$$\left\{\begin{bmatrix} a_1I & a_2I & a_3I & a_4I \\ a_5I & a_6I & a_7I & a_8I \\ a_9I & a_{10}I & a_{11}I & a_{12}I \\ a_{13}I & a_{14}I & a_{15}I & a_{16}I \end{bmatrix} \middle| a_i \in Z_{25}, 1 \le i \le 16, + \right\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1+b_1I] \\ [0,a_2+b_2I] \\ \vdots \\ [0,a_{12}+b_{12}I] \end{bmatrix} \middle| a_i + b_i \in Z_{45}, 1 \le i \le 12, + \right\}$$

be a quasi interval mixed pure neutrosophic bisemigroup.

Example 1.1.52: Let $M = M_1 \cup M_2 =$

$$\left\{\left.\begin{bmatrix} a_1+b_1I & a_2+b_2I & a_3+b_3I \\ a_4+b_4I & a_5+b_5I & a_6+b_6I \\ a_7+b_7I & a_8+b_8I & a_9+b_9I \\ a_{10}+b_{10}I & a_{11}+b_{11}I & a_{12}+b_{12}I \\ a_{13}+b_{13}I & a_{14}+b_{14}I & a_{15}+b_{15}I \\ a_{16}+b_{16}I & a_{17}+b_{17}I & a_{18}+b_{18}I \end{bmatrix}\right| a_i, b_i \in Q^+ \cup \{0\}, +, 1 \le i \le 18\right\}$$

$$\cup \left\{\left.\left[\sum_{i=0}^{\infty}[0, aI]x^i\right]\right| a \in Q^+ \cup \{0\}, \times\right\}$$

be a quasi interval mixed - pure neutrosophic interval bisemigroup.

Example 1.1.53: Let $L = L_1 \cup L_2 =$

$$\left\{\left.\left[\sum_{i=0}^{\infty}[0, a_i]x^i\right]\right| a_i \in Z^+ \cup \{0\}, \times\right\} \cup$$

$$\left\{\left.\begin{bmatrix} a_1+b_1I & a_2+b_2I \\ a_3+b_3I & a_4+b_4I \\ a_5+b_5I & a_6+b_6I \\ a_7+b_7I & a_8+b_8I \\ a_9+b_9I & a_{10}+b_{10}I \\ a_{11}+b_{11}I & a_{12}+b_{12}I \end{bmatrix}\right| a_i, b_i \in Q^+ \cup \{0\}, +, 1 \le i \le 12, +\right\}$$

be a quasi interval real - mixed neutrosophic bisemigroup.

Example 1.1.54: Let $M = M_1 \cup M_2 = \{8 \times 8$ matrix with entries from $R^+ \cup \{0\}, \times\} \cup$

$$\left\{\left.\begin{bmatrix} a_1+b_1I & a_2+b_2I \\ a_3+b_3I & a_4+b_4I \\ a_5+b_5I & a_6+b_6I \\ a_7+b_7I & a_8+b_8I \\ a_9+b_9I & a_{10}+b_{10}I \\ a_{11}+b_{11}I & a_{12}+b_{12}I \end{bmatrix}\right| a_i, b_i \in Q^+ \cup \{0\}, +, 1 \le i \le 12, +\right\}$$ be a

quasi interval real - mixed neutrosophic bisemigroup.

Having seen examples of mixed neutrosophic interval bisemigroup and their generalization we proceed onto discuss and define neutrosophic interval bigroups.

1.2 Neutrosophic Interval Bigroups

In this section we proceed onto define the notion of pure neutrosophic interval bigroups and mixed neutrosophic interval bigroups and describe and define their related properties. It is important to mention neutrosophic groups in general need not have a group structure.

DEFINITION 1.2.1: *Let $G = G_1 \cup G_2$ where both G_1 and G_2 are pure neutrosophic interval groups and $G_1 \neq G_2$ or $G_1 \not\subseteq G_2$ or $G_2 \not\subseteq G_1$. Then we define G to be pure neutrosophic interval bigroup.*

We will illustrate this situation by some examples.

Example 1.2.1: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{20}, +\} \cup \{[0, bI] \mid b \in Q^+, \times\}$ be a pure neutrosophic interval bigroup of infinite order and is abelian.

Example 1.2.2: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_7 \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in R^+, \times\}$ be the pure neutrosophic interval bigroup of infinite order.

Example 1.2.3: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_{20}, +\} \cup \{[0, aI] \mid a \in Z_{11} \setminus \{0\}, \times\}$ be the pure neutrosophic interval bigroup of finite order.

Example 1.2.4: Let $M = M_1 \cup M_2 = \{([0, a_1I], [0, a_2I], \ldots, [0, a_{12}I]) \mid a_i \in Z_{40}, 1 \leq i \leq 12, +\} \cup \left\{\left.\begin{bmatrix} [0, a_1I] \\ [0, a_2I] \\ \vdots \\ [0, a_{15}I] \end{bmatrix} \right| a_i \in Z_{40}, 1 \leq i \leq 15, +\right\}$ be a pure neutrosophic interval bisemigroup of finite order.

Example 1.2.5: Let V = $V_1 \cup V_2$ =
$$\left\{\begin{bmatrix}[0,a_1I] & [0,a_2I]\\ [0,a_3I] & [0,a_4I]\end{bmatrix} \middle| \, |A| \neq 0, a_i \in Z_{19} \setminus \{0\}, 1 \leq i \leq 4, \times\right\} \cup \{[0, aI] \mid$$
$a \in R^+$, $\times$} be a pure neutrosophic interval bigroup which is non commutative.

Example 1.2.6: Let T = $T_1 \cup T_2$ = {([0, a_1I], [0, a_2I], …, [0, a_{25}I]) | $a_i \in Z_{31} \setminus \{0\}$, ×, 1 ≤ i ≤ 25, +} ∪
$$\left\{\begin{bmatrix}[0,a_1I] & [0,a_2I] & [0,a_3I] & [0,a_4I] & [0,a_5I]\\ [0,a_6I] & [0,a_7I] & [0,a_8I] & [0,a_9I] & [0,a_{10}I]\\ [0,a_{11}I] & [0,a_{12}I] & [0,a_{13}I] & [0,a_{14}I] & [0,a_{15}I]\\ [0,a_{16}I] & [0,a_{17}I] & [0,a_{18}I] & [0,a_{19}I] & [0,a_{20}I]\\ [0,a_{21}I] & [0,a_{22}I] & [0,a_{23}I] & [0,a_{24}I] & [0,a_{25}I]\end{bmatrix} \middle| \, a_i \in Z_{200},\right.$$
1 ≤ i ≤ 25, +} be a pure neutrosophic interval bigroup of finite order and is commutative.

We can define subbigroups, this is direct and hence left as an exercise to the reader.

We will give examples of them.

Example 1.2.7: Let G = $G_1 \cup G_2$ = {[0, aI] | a ∈ Z_{40}, +} ∪ {[0, bI] | b ∈ $Z_{43} \setminus \{0\}$, ×} be a pure neutrosophic interval bigroup. Take P = $P_1 \cup P_2$ = {[0, aI] | a ∈ $2Z_{40}$, +} ∪ {[0, bI] | b ∈ {1, 42}, ×} ⊆ $G_1 \cup G_2$ = G, P is a pure neutrosophic interval subbigroup of G.

Example 1.2.8: Let M = $M_1 \cup M_2$ = {[0, aI] | a ∈ Q^+, ×}
$$\cup \left\{\begin{bmatrix}[0,a_1I]\\ [0,a_2I]\\ [0,a_3I]\\ [0,a_4I]\\ [0,a_5I]\end{bmatrix} \middle| \, a_i \in Z_{25}, 1 \leq i \leq 5, + \right\}$$
be a pure neutrosophic interval bigroup. Take P = $P_1 \cup P_2$ = {[0, aI] | a ∈ {2^n, $\frac{1}{2^n}$ | n =

$0, 1, 2, \ldots, \infty\} \subseteq Q^+, \times\} \cup \left\{ \begin{bmatrix} [0, a_1I] \\ 0 \\ [0, a_2I] \\ 0 \\ [0, a_3I] \end{bmatrix} \middle| a_i \in Z_{25}, 1 \le i \le 3, + \right\} \subseteq$

$M_1 \cup M_2 = M$, P is a pure neutrosophic interval subbigroup of M.

Example 1.2.9: Let $M = M_1 \cup M_2 = \{([0, a_1I], [0, a_2I], \ldots,$

$[0, a_{24}I]) \mid a_i \in R^+, \times\} \cup \left\{ \begin{bmatrix} [0, a_1I] & [0, a_2I] \\ [0, a_3I] & [0, a_4I] \\ \vdots & \vdots \\ [0, a_{29}I] & [0, a_{30}I] \end{bmatrix} \middle|\right.$ where $a_i \in Z_{45}$,

$1 \le i \le 30, +\}$ be a pure neutrosophic interval bigroup. We take $S = S_1 \cup S_2 = \{([0, a_1I], [0, a_2I], \ldots, [0, a_{24}I]) \mid a_i \in Q^+, \times\}$

$\cup \left\{ \begin{bmatrix} [0, a_1I] & 0 \\ 0 & [0, a_2I] \\ [0, a_3I] & 0 \\ 0 & [0, a_4I] \\ [0, a_5I] & 0 \\ \vdots & \vdots \\ 0 & [0, a_{14}I] \\ [0, a_{15}I] & 0 \end{bmatrix} \middle|\right.$ $a_i \in Z_{45}$; $1 \le i \le 15, +\} \subseteq M_1 \cup M_2 =$

M; S is a pure neutrosophic interval bisubgroup of S.

Example 1.2.10: Let $V = V_1 \cup V_2 = \{([0, a_1I], [0, a_2I], [0, a_3I]) \mid a_i \in Z_{12}; 1 \le i \le 3, +\} \cup \{([0, a_1I], [0, a_2I], [0, a_3I]) \mid a_i \in Z_{13} \setminus \{0\}; 1 \le i \le 3, \times\}$ be a pure neutrosophic interval bigroup. Consider $M = M_1 \cup M_2 = \{([0, a_1I], [0, a_2I], [0, a_3I]) \mid a_i \in 2Z_{12}; 1 \le i \le 3, +\} \cup \{([0, a_1I], [0, a_2I], [0, a_3I]) \mid a_i \in \{1, 12\} \subseteq Z_{13}; 1 \le i \le 3, \times\} \subseteq V_1 \cup V_2$; M is a pure neutrosophic interval bisubgroup of V.

Now we can define normal bisubgroups as in case of usual bigroup. Let us define the order of a pure neutrosophic interval bigroup $G = G_1 \cup G_2$ to be of finite order if both G_1 and G_2 are finite; even if one of G_1 or G_2 are of infinite order then we define G to be of infinite order.

If both G_1 and G_2 are of finite order we denote the biorder of G by $|G_1| . |G_2|$ or $o(G_1) \times o(G_2)$.

It can be easily proved that if $H = H_1 \cup H_2 \subseteq G_1 \cup G_2 = G$ where G is of finite biorder then o(H) / o(G). We can define homomorphism and isomorphism of pure neutrosophic interval bigroups as in case of pure neutrosophic interval bisemigroups.

However in case of pure neutrosophic interval bigroups G we see bikernel of a homomorphism is a pure neutrosophic interval normal subbigroup of G. All these are direct and hence is left as an exercise to the reader. However we see we can define the notion of quasi interval pure neutrosophic bigroup. Suppose $G = G_1 \cup G_2$ where only one of G_1 or G_2 is pure neutrosophic interval group and the other is just a pure neutrosophic group then we define G to be a quasi interval pure neutrosophic bigroup. We will give some examples of them.

Example 1.2.11: Let $G = G_1 \cup G_2 = \{Z_{25}I, +\} \cup \{[0, aI] \mid a \in Q^+, \times\}$ be a quasi interval pure neutrosophic bigroup of infinite order.

Example 1.2.12: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{20}, +\} \cup \{Q^+I, \times\}$ be a quasi interval pure neutrosophic bigroup of infinite order.

We can replace in the definition the pure neutrosophic intervals by mixed neutrosophic intervals and study these structures. We will illustrate this situation by some examples.

Example 1.2.13: Let $V = \{[0, a+bI] \mid a, b \in Z_{90}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{45}, +\}$ be a mixed neutrosophic interval bigroup.

Example 1.2.14: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{24}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{12}, +\}$ be a mixed neutrosophic interval bigroup of finite order.

We can find substructures.

Example 1.2.15: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{120}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{45}, +\}$ be a mixed neutrosophic interval bigroup of finite order. Consider $H = H_1 \cup H_2 = \{[0, a+bI] \mid a, b \in \{0, 10, 20, \ldots, 110\} \subseteq Z_{120}, +\} \cup \{[0, a+bI] \mid a, b \in \{0, 5, 10, 15, 20, \ldots, 40\} \subseteq Z_{45}, +\} \subseteq M_1 \cup M_2$, H is a mixed neutrosophic interval subbigroup of finite order.

We cannot construct mixed neutrosophic interval bigroups using R^+ or Q^+ for finding inverse of a + bI is not a easy task.

Now we can define different types of interval bigroups. The following examples express their properties.

Example 1.2.16: Let $V = V_1 \cup V_2 = \{[0, a] \mid a \in R^+, \times\} \cup \{[0, aI] \mid a \in Z_{20}, +\}$ be a real-pure neutrosophic interval bigroup of infinite order.

Example 1.2.17: Let $M = M_1 \cup M_2 = \{([0, a_1I], [0, a_2I], [0, a_3I]) \mid a_i \in Z_{40}, 1 \leq i \leq 3, +\} \cup \{[0, a] \mid a \in Q^+, \times\}$ be a pure neutrosophic - real interval bigroup of infinite order.

Example 1.2.18: Let $P = P_1 \cup P_2 = S_5 \cup \{[0, a] \mid a \in Z_{40}, +\}$ be a quasi interval quasi pure neutrosophic bigroup of finite order which is non commutative.

Example 1.2.19: Let $M = M_1 \cup M_2 = \left\{ A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_6 & a_7 & a_8 \end{bmatrix} \right| |A|$ $\neq 0$ with $a_i \in Z_{25}, 1 \leq i \leq 9, \times\} \cup \{[0, aI] \mid a \in Z_{25}, +\}$ be a quasi interval pure neutrosophic bigroup of finite order.

Example 1.2.20: Let $M = M_1 \cup M_2 = \{S_{12}\} \cup \{[0, a+bI] \mid a, b \in Z_{10}, +\}$ be a quasi interval mixed neutrosophic bigroup of finite order.

Example 1.2.21: Let $S = S_1 \cup S_2 = \{ZI, +\} \cup \{[0, a+bI] \mid a, b \in Z_{14}, +\}$ be a quasi interval neutrosophic bigroup of infinite order.

Example 1.2.22: Let $M = M_1 \cup M_2 = \{Q^+I, \times\} \cup \{0, a+bI] \mid a, b \in Z_{28}, +\}$ be the quasi interval neutrosophic bigroup of infinite order.

Example 1.2.23: Let $P = P_1 \cup P_2 = \{[0, a] \mid a \in Q^+, \times\} \cup \{[0, aI] \mid a \in Q^+, \times\}$ be the quasi neutrosophic interval bigroup of infinite order.

Now we can define pure neutrosophic interval group - semigroup, mixed neutrosophic interval group - semigroup, quasi interval neutrosophic group - semigroup and so on.

From the very structure one can easily understand the algebraic structure, so we would give only examples of them.

Example 1.2.24: Let $V = V_1 \cup V_2 = \{[0, a] \mid a \in Z^+ \cup\{0\}, \times\} \cup \{[0, aI] \mid a \in Z_{20}, +\}$ be a pure neutrosophic interval semigroup - group of infinite order.

Example 1.2.25: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{40}, \times\} \cup \{([0, a_1I], [0, a_2I], [0, a_3I], [0, a_4I], [0, a_5I]) \mid a_i \in Z_5, 1 \leq i \leq 5\}$ be a pure neutrosophic interval semigroup - group of finite order which is commutative.

Example 1.2.26: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times\} \cup \{[0, aI] \mid a \in Q^+, \times\}$ be a pure neutrosophic interval semigroup - group of infinite order.

Example 1.2.27: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{45}, \times\} \cup \{[0, aI] \mid a \in R^+, \times\}$ be a pure neutrosophic interval semigroup - group of infinite order.

Example 1.2.28: Let $M = M_1 \cup M_2 = \{Z^+I \cup \{0\}, \times\} \cup \{[0, aI] \mid a \in Q^+ \cup \{0\}, \times\}$ be a quasi interval neutrosophic semigroup group of infinite order.

Example 1.2.29: Let $V = V_1 \cup V_2 = \{Q^+I \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_{25}, +\}$ be a quasi interval pure neutrosophic semigroup - group of infinite order.

Example 1.2.30: Let $V = V_1 \cup V_2 = \{Z_{25}I, +\} \cup \{[0, aI] \mid a \in Z_{12}, \times\}$ be a quasi interval pure neutrosophic group - semigroup of finite order.

Example 1.2.31: Let $M = M_1 \cup M_2 = \{Z_{12}I, \times\} \cup \{[0, aI] \mid a \in Q^+, \times\}$ be a quasi interval pure neutrosophic semigroup - group of infinite order.

We can define substructure, if it has no subsemigroup - subgroup we call it simple if it has one of subsemigroup or subgroup (or in the mutually exclusive sense) we call it a quasi simple. We will give examples of these concepts.

Example 1.2.32: Let $V = V_1 \cup V_2 = \{Z_{12}I, \times\} \cup \{[0, aI] \mid a \in Z_{30}, +\}$ be a quasi interval pure neutrosophic semigroup - group of finite order.

Consider $M = M_1 \cup M_2 = \{2Z_{12}, \times\} \cup \{[0, aI] \mid a \in \{0, 3, 6, \ldots, 27\} \subseteq Z_{30}, +\} \subseteq V_1 \cup V_2$, M is a quasi interval pure neutrosophic subsemigroup - subgroup of V.

Example 1.2.33: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times\} \cup \{Q^+I, \times\}$ be a quasi interval pure neutrosophic semigroup - group. Consider $S = S_1 \cup S_2 = \{[0, aI] \mid a \in 5Z^+ \cup \{0\}, \times\} \cup \left\{\frac{1}{2^n}, 2^n \,\middle|\, n = 0, 1, 2, \ldots, \infty, \times\right\} \subseteq M = M_1 \cup M_2$, M is a quasi interval pure neutrosophic subsemigroup - subgroup of M.

Example 1.2.34: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{11}, +\} \cup \{3Z_6I = \{0, 3I\}, \times\}$ be a quasi interval pure neutrosophic group - semigroup. Clearly V is simple.

Example 1.2.35: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{23}, +\} \cup \{Z_{20}I, \times\}$ be a quasi interval pure neutrosophic group - semigroup.

Clearly V is a quasi interval pure neutrosophic quasi subgroup - subsemigroup as V has $W = \{V_1\} \cup \{2\, Z_{20}I, \times\} \subseteq V_1 \cup V_2$ is a quasi group - semigroup.

Example 1.2.36: Let M= $M_1 \cup M_2$ = $\{Z_{40}I, +\} \cup \{[0, aI] \mid a \in \{0, 5\} \subseteq Z_{10}, \times\}$ be a quasi interval pure neutrosophic group - semigroup. M is only a quasi group-semigroup, for M_1 has subgroups but M_2 has no subsemigroups.

Inview of these results we have the following theorems the proof of them are direct.

THEOREM 1.2.1: *Let V = $\{Z_pI, +\} \cup \{[0, aI] \mid a \in Z_n, \times\}$ be a quasi interval pure neutrosophic group - semigroup. V is a quasi interval pure neutrosophic quasi group - semigroup.*

Hint: $(Z_pI, +)$ has no subgroups.

THEOREM 1.2.2: *Let V = $\{[0, aI] \mid a \in Z_pI, +\} \cup \{Z_nI, \times\}$ be a interval pure neutrosophic group - semigroup. V is quasi interval pure neutrosophic quasi group - semigroup.*

THEOREM 1.2.3: *Let V = $\{Z^+I \{0\}, \times\} \cup \{[0, aI] \mid a \in Z_n, +\}$ be a quasi interval pure neutrosophic semigroup-group. V is not simple or quasi simple.*

Now we will give examples of quasi neutrosophic interval group.

Example 1.2.37: Let V = $V_1 \cup V_2$ = $\{[0, a] \mid a \in Z_{20}, +\} \cup \{[0, aI] \mid a \in Q^+, \times\}$ be a quasi neutrosophic interval bigroup.

Example 1.2.38: Let M = $M_1 \cup M_2$ = $\{[0, aI] \mid a \in Q^+, \times\} \cup \{[0, aI] \mid a \in Z_{40}, +\}$ be a quasi neutrosophic interval bigroup.

Example 1.2.39: Let L = $L_1 \cup L_2$ = $\{[0, aI] \mid a \in Z_{23} \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in Z_{47} \setminus \{0\}, \times\}$ be a quasi neutrosophic interval bigroup.

Example 1.2.40: Let V = $V_1 \cup V_2$ = $\{S_5\} \cup \{[0, aI] \mid a \in Q^+, \times\}$ be a quasi neutrosophic quasi interval bigroup.

Example 1.2.41: Let S = $S_1 \cup S_2$ = $\{Z, +\} \cup \{[0, aI] \mid a \in Z_7 \setminus \{0\}, \times\}$ be a quasi neutrosophic quasi interval group-semigroup.

Example 1.2.42: Let $S = S_1 \cup S_2 = \{ZI, +\} \cup \{[0, a] \mid a \in Q^+, \times\}$ be a quasi interval quasi neutrosophic bigroup.

Example 1.2.43: Let $P = P_1 \cup P_2 = \{Q^+I, \times\} \cup \{[0, a] \mid a \in Z_{45}, +\}$ be a quasi interval quasi neutrosophic bigroup.

Example 1.2.44: Let $W = W_1 \cup W_2 = \{S(20)\} \cup \{[0, aI] \mid aI \in Q^+, \times\}$ be a quasi interval quasi neutrosophic semigroup - group.

Example 1.2.45: Let $V = V_1 \cup V_2 = \{Z, +\} \cup \{[0, aI] \mid a \in 3Z_{120}, \times\} \subseteq V_1 \cup V_2 = V$ be a quasi interval quasi neutrosophic group - semigroup.

Example 1.2.46: Let $T = T_1 \cup T_2 = \{Z_{47} \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in Z_{50}, \times\}$ be a quasi interval quasi pure neutrosophic group - semigroup of finite order.

$o(T) = 46 \times 50$.

Consider $W = W_1 \cup W_2 = \{\{1, 46\} \subseteq Z_{47} \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in \{0, 5, 10, 15, 20, \ldots, 45\}, \times\} \subseteq T_1 \cup T_2 = T$; W is a quasi interval quasi neutrosophic subgroup - subsemigroup of finite order.

$o(W) = 2 \times 10$ and $o(W) / o(T)$.

Now having seen examples of a quasi interval quasi pure neutrosophic group - semigroups we proceed onto give examples of quasi interval mixed neutrosophic group - semigroups and so on.

Example 1.2.47: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{25}, +\} \cup \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, \times\}$ be a neutrosophic interval group - semigroup.

Example 1.2.48: Let $T = T_1 \cup T_2 = \{[0, a+bI] \mid a, b \in Z_{50}, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{29}, +\}$ be a neutrosophic interval semigroup - group.

Example 1.2.49: Let $F = F_1 \cup F_2 = \{[0, a+bI] \mid a, b \in Q^+ \cup \{0\}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{15}, +\}$ be a neutrosophic interval semigroup - group. Consider $H = H_1 \cup H_2 = \{[0, a+bI] \mid a, b \in$

$Z^+ \cup \{0\}, +\} \cup \{[0, a+bI] \mid a, b \in \{0, 3, 6, 9, 12\}, +\} \subseteq F_1 \cup F_2 = F$, H is a neutrosophic interval subsemigroup - subgroup of F.

Example 1.2.50: Let $M = M_1 \cup M_2 = \{[0, a] \mid a \in Z_{25}, +\} \cup \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, \times\}$ be a quasi neutrosophic interval group - semigroup.

Example 1.2.51: Let $T = T_1 \cup T_2 = \{[0, a+bI] \mid a, b \in Z_{10}, +\} \cup \{Z, \times\}$ be a quasi neutrosophic quasi interval group - semigroup.

Example 1.2.52: Let $B = B_1 \cup B_2 = \{[0, aI+b] \mid a, b \in Q^+ \cup \{0\}, +\} \cup \{Z_{25}I, +\}$ be a quasi interval neutrosophic semigroup - group.

Example 1.2.53: Let $V = V_1 \cup V_2 = \{S_{20}\} \cup [0, a+bI] \mid a, b \in R^+ \cup \{0\}, +\}$ be a quasi neutrosophic quasi interval group - semigroup.

Example 1.2.54: Let $M = M_1 \cup M_2 = \{Z_{45}I, \times\} \cup [0, a] \mid a \in Z_{40}, +\}$ be a quasi interval quasi neutrosophic semigroup group.

Now having seen examples of neutrosophic interval bistructures using groups, semigroups and group-semigroup, we proceed onto define the notion of biinterval neutrosophic groupoids or neutrosophic interval bigroupoids.

1.3 Neutrosophic Biinterval Groupoids

In this section we introduce the notions of neutrosophic biinterval groupoids or neutrosophic interval bigroupoids and discuss some of their important properties.

DEFINITION 1.3.1: *Let $G = G_1 \cup G_2$ where both G_1 and G_2 are pure neutrosophic interval bigroupoids such that $G_1 \neq G_2$; $G_1 \not\subseteq G_2$ and $G_2 \not\subseteq G_1$. G will also be known as pure neutrosophic biinterval groupoids.*

We will illustrate this situation by some examples.

Example 1.3.1: Let $G = G_1 \cup G_2 = \{[0, aI] \mid aI \in Z_8I; *, (t, u) = (3, 2), t, u \in Z_8\} \cup \{[0, bI] \mid b \in Z_{40}, *, (8, 19)\}$ be a pure neutrosophic interval bigroupoid of finite order.

Example 1.3.2: Let $M = M_1 \cup M_2 = \{[0, aI] \mid aI \in Z^+ \cup \{0\}, *, (8, 17)\} \cup \{[0, aI] \mid a \in Z_{14}, *, (0, 3)\}$ be a pure neutrosophic biinterval groupoid of infinite order.

Example 1.3.3: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{15}, (3, 7), *\} \cup \{[0, aI] \mid a \in Z_7, (2, 3), *\}$ be a pure neutrosophic interval bigroupoid of finite order.

Example 1.3.4: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in R^+ \cup \{0\}, (3/4, \sqrt{2}), *\} \cup \{[0, aI] \mid a \in Q^+ \cup \{0\}, (27,4/11), *\}$ be a pure neutrosophic interval bigroupoid of infinite order. Clearly G is non commutative.

Example 1.3.5: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Q^+ \cup \{0\}, *, (5, 8)\} \cup \{[0, aI] \mid a \in Z_{28}, (5, 8), *\}$ be a pure neutrosophic interval bigroupoid of infinite order.

Example 1.3.6: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{52}, *, (3, 11)\} \cup \{[0, aI] \mid a \in Z_{25}, *, (11, 3)\}$ be a pure neutrosophic interval bigroupoid of finite order.

$o(M) = 52 \times 25.$

Example 1.3.7: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z_{45}, (8, 9), *\} \cup \{[0, aI] \mid a \in Z_{45}, (9, 8)*\}$ be a pure neutrosophic interval bigroupoid of order 45×45.

Example 1.3.8: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_7, (3, 3), *\} \cup \{[0, aI] \mid a \in Z_{11}, (5, 5), *\}$ be a pure neutrosophic interval bigroupoid of biorder $7 \times 11 = 77$.

We can define substructures on them.

DEFINITION 1.3.2: *Let $G = G_1 \cup G_2$ be a pure neutrosophic interval bigroupoid and $H = H_1 \cup H_2 \subseteq G_1 \cup G_2$ be a proper bisubset of G. If H itself is a pure neutrosophic interval bigroupoid under the operations of G then we define H to be a pure neutrosophic interval subbigroupoid of G.*

We will illustrate this situation by an example.

Example 1.3.9: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_9, *, (5, 3)\} \cup \{[0, aI] \mid a \in Z_6, (2, 4), *\}$ be a pure neutrosophic interval bigroupoid. Take $M = M_1 \cup M_2 = \{[0, aI] \mid a \in \{1, 2, 4, 5, 7, 8\} \subseteq Z_9, *, (5, 3)\} \cup \{[0, aI] \mid a \in \{0, 3\} \subseteq Z_6, *, (2, 4)\} \subseteq V_1 \cup V_2$ is a pure neutrosophic interval subbigroupoid of V.

Interested reader can give more examples of them.

We can define Smarandache pure neutrosophic interval bigroupoid as a pure neutrosophic interval bigroupoid as a pure neutrosophic interval bigroupoid $G = G_1 \cup G_2$, were both G_1 and G_2 are Smarandache pure neutrosophic interval groupoids.

We will illustrate this situation by some examples.

Example 1.3.10: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{10}, (5, 6), *\} \cup \{[0, aI] \mid a \in Z_{12}, (3, 9), *\}$ be a pure neutrosophic interval bigroupoid, clearly G is a S-pure neutrosophic interval bigroupoid.

Example 1.3.11: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_6, *, (3, 5)\} \cup \{[0, aI] \mid a \in Z_4, (2, 3), *\}$; M is a Smarandache pure neutrosophic interval bigroupoid.

We can define special identities on pure neutrosophic interval bigroupoids.

Example 1.3.12: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{10}, (1, 2), *\} \cup \{[0, aI] \mid a \in Z_8, (1, 6), *\}$ be a pure neutrosophic interval bigroupoid. V is a S-pure neutrosophic interval bigroupoid.

We can define pure neutrosophic interval biideal of a bigroupoid. We give an example of it. The biideal can be a left - right ideal of a right - left ideal or just a biideal.

Example 1.3.13: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_4, *, (2, 3)\} \cup \{[0, aI] \mid a \in Z_6, *, (4, 5)\}$ be a pure neutrosophic interval bigroupoid. Consider $I = I_1 \cup I_2 = \{[0, aI] \mid a \in \{1, 3\} \subseteq Z_4, *, (2, 3)\} \cup \{[0, aI] \mid a \in \{1, 3, 5\} \subseteq Z_6, *, (4, 5)\} \subseteq V_1 \cup V_2$, I is a pure neutrosophic interval left biideal of V.

We will give examples and results of pure neutrosophic interval bigroupoids which satisfy different identities.

Example 1.3.14: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{12}, (5, 8), *\} \cup \{[0, aI] \mid a \in Z_{45}, (20, 26), *\}$ be a pure neutrosophic interval bigroupoid. G is an idempotent bigroupoid.

Example 1.3.15: Let $W = W_1 \cup W_2 = \{[0, aI] \mid a \in Z_{15}, (6, 10), *\} \cup \{[0, aI] \mid a \in Z_{21}, (15, 7), *\}$ be a pure neutrosophic interval bigroupoid. W is a idempotent bigroupoid as both W_1 and W_2 are idempotent groupoids.

In view of this we have the following theorem.

THEOREM 1.3.1: *Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_n, (t, u); *\} \cup \{[0, bI] \mid b \in Z_m, (r, s); *\}$ be a pure neutrosophic interval bigroupoid. V is a pure neutrosophic interval idempotent bigroupoid if and only if $t + u \equiv 1 \pmod n$ and $r + s \equiv 1 \pmod m$.*

Proof is simple and is left as an exercise to the reader [11].

Example 1.3.16: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_6, (4, 3), *\} \cup \{[0, aI] \mid a \in Z_{10}, (5, 6), *\}$ be a pure neutrosophic interval bigroupoid. Infact V is a pure neutrosophic interval bisemigroup.

Example 1.3.17: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{14}, (8, 7), *\} \cup \{[0, aI] \mid a \in Z_{22}, (11,12), *\}$ be a pure neutrosophic interval bigroupoid which is a pure neutrosophic interval bisemigroup.

Inview of this we have the following theorem the proof of which is direct [ ].

THEOREM 1.3.2: *Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_n, (t, u), *\} \cup \{[0, aI] \mid a \in Z_m, (r, s), *\}$ be a pure neutrosophic interval* *bigroupoid. V is a pure neutrosophic interval bisemigroup if and only if $t^2 \equiv t \pmod n$, $u^2 \equiv u \pmod n$ for $t, u \in Z_n \setminus \{0\}$ with $(t, u) = 1$ and $r^2 \equiv r \pmod m$, $s^2 \equiv s \pmod m$ $(r, s) = 1$ and $r, s \in Z_m \setminus \{0\}$.*

Now we will give examples of pure neutrosophic interval bigroupoids which are bisimple.

Example 1.3.18: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{21}, (19, 2), *\} \cup \{[0, aI] \mid a \in Z_{25}, (23, 2), *\}$ be a pure neutrosophic interval bigroupoid, V is bisimple.

Example 1.3.19: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{19}, (17, 2), *\} \cup \{[0, aI] \mid a \in Z_{29}, *, (22, 7)\}$ be a pure neutrosophic interval bigroupoid which is bisimple.

THEOREM 1.3.3: *Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_n, (t, u), *\} \cup \{[0, aI] \mid a \in Z_m, (r, s), *\}$ be a pure neutrosophic interval bigroupoid. If $n = t+u$ and $m=r+s$ with t, u, r and s primes then V is bisimple.*

The proof is direct and uses simple number theoretic techniques only.

THEOREM 1.3.4: *Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_p, (t, u), *\} \cup \{[0, aI] \mid a \in Z_q, (r,s), *\}$ where p and q are primes be a pure neutrosophic interval bigroupoid. If $t + u = p$, $(t, u) = 1$ and $r + s = q$ and $(s, r) = 1$ then V is bisimple.*

For proof is simple and direct and hence left as an exercise to the reader.

Example 1.3.20: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{20}, (13, 7), *\} \cup \{[0, bI] \mid a \in Z_{43}, (23, 20), *\}$ be a pure neutrosophic interval bigroupoid $\{0\} \cup \{0\}$ is not a biideal of V.

Thus we can say if $V = V_1 \cup V_2$ is a pure neutrosophic interval bigroupoid with $V_1 = \{[0, aI] \mid a \in Z_n, (t, u) = 1, *\}$ and $V_2 = \{[0, bI] \mid b \in Z_m, (r, s) = 1, *\}$ then $\{0\} \cup \{0\}$ is not a biideal of V.

We can define notion of normal bigroupoid as follows. If $V = V_1 \cup V_2$ is a pure neutrosophic interval bigroupoid, we say $G = G_1 \cup G_2 \subseteq V_1 \cup V_2$ to be a pure neutrosophic interval normal subbigroupoid if

(a) $(a_1 \cup a_2)(G_1 \cup G_2) = a_1G_1 \cup a_2 G_2$

(b) $((G_1 \cup G_2)(x_1 \cup x_2))(y_1 \cup y_2)$
$= (G_1 \cup G_2)[(x_1 \cup x_2)(y_1 \cup y_2)]$
That is $G_1 (x_1 y_1) \cup G_2 (x_2 y_2)$
$= (G_1 x_1) y_1 \cup (G_2 x_2)y_2$

(c) $((x_1 \cup x_2)(y_1 \cup y_2))(G_1 \cup G_2)$
$= (x_1 \cup x_2)((y_1 \cup y_2) G)$.
That is $(x_1 y_1) G_1 \cup (x_2 y_2) G_2 = x_1 (y_1 G_1) \cup x_2 (y_2 G_2)$ for all $x_1, y_1, a_1 \in G_1$ and $x_2, y_2, a_2 \in G_2$.

If $V = V_1 \cup V_2$ the pure neutrosophic interval bigroupoid satisfies (a), (b) and (c) that is if G is replaced by V) then we say the bigroupoid to be binormal.

We give an example of normal bigroupoids.

Example 1.3.21: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_7, (3, 3), *\} \cup \{[0, bI] \mid a \in Z_{19}, (11, 11), *\}$ be a pure neutrosophic interval bigroupoid. V is a normal bigroupoid.

In view of this example we have the following theorem.

THEOREM 1.3.5: *Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_p, (t, t), *, t < p\} \cup \{[0, bI] \mid b \in Z_q, (s, s), *, s < q\}$, p and q primes be a pure neutrosophic interval bigroupoid. V is a normal interval bigroupoid.*

Proof is direct and simple [11]. We can define P-bigroupoids as in case of pure neutrosophic interval bigroupoid if both the interval groupoids are P-groupoids. We will provide some examples.

Example 1.3.22: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{42}, (7, 7), *\} \cup \{[0, aI] \mid a \in Z_{12}, (5, 5), *\}$ be a pure neutrosophic interval bigroupoid. G is a P-bigroupoid.

Example 1.3.23: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_{19}, *, (10, 10)\} \cup \{[0, aI] \mid a \in Z_{43}, *, (8, 8)\}$ be a pure neutrosophic interval bigroupoid. S is a P-bigroupoid of finite order.

In view of this we have the following theorem the proof of which is direct and hence left as an exercise to the reader.

THEOREM 1.3.6: *Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_n, (t, t); 0 < t < n, *\} \cup \{[0, aI] \mid a \in Z_m, (s, s), 0 < s < m, *\}$ be a pure neutrosophic interval bigroupoid. P is a pure neutrosophic interval P-bigroupoid of order mn.*

Note n can be prime or composite still the conclusions of the theorem is true.

We will now provide examples of alternative bigroupoids and bigroupoids which are not alternative.

Example 1.3.24: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_{26}, *, (14, 14)\} \cup \{[0, aI] \mid a \in Z_{46}, *, (24, 24)\}$ be a pure neutrosophic interval bigroupoid. S is a pure neutrosophic interval alternative bigroupoid.

Example 1.3.25: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{22}, *, (12, 12)\} \cup \{[0, aI] \mid a \in Z_{30}, *, (10, 10)\}$ be a pure neutrosophic interval bigroupoid. It is easily verified P is a pure neutrosophic interval alternative bigroupoid.

In view of this we have the following theorem.

THEOREM 1.3.7: *Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_n, (t, t), *\} \cup \{[0, aI] \mid a \in Z_m, (s, s), *\}$ be a pure neutrosophic interval bigroupoid where m and n are primes. V is a pure neutrosophic interval alternative bigroupoid if and only if $t^2 \equiv t \pmod{n}$ and $s^2 \equiv s \pmod{m}$.*

Proof is direct however the interested reader can refer [11]. We give now examples of pure neutrosophic interval bigroupoids which are not alternative.

Example 1.3.26: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{43}, *, (8, 8)\} \cup \{[0, aI] \mid a \in Z_{53}, *, (12, 12)\}$ be a pure neutrosophic interval bigroupoid. M is not an alternative bigroupoid.

Example 1.3.27: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{13}, *, (5, 5)\} \cup \{[0, aI] \mid a \in Z_{23}, *, (8, 8)\}$ be a pure neutrosophic interval bigroupoid. P is not an alternative bigroupoid.

Inview of this we have the following theorem which guarantees a non empty class of bigroupoids which are not alternative.

THEOREM 1.3.8: *Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_p, *, (t, t)\} \cup \{[0, aI] \mid a \in Z_q, *, (s, s)\}$ ($1 < t < p$, $1 < s < q$ where p and q are primes) be a pure neutrosophic interval bigroupoid. P is not a pure neutrosophic interval alternative bigroupoid.*

We just give an hint to the proof of the theorem.

Given $1 < t < p$, $1 < s < q$. Consider the alternative identity $(x*y)*y = x* (y*y)$, for $x, y \in P$. We see equality can never be achieved.

Let $x = [0, a_1I] \cup [0, a_2I]$ and
$y = [0, b_1I] \cup [0, b_2I]$ be in P.

It is simple to show $x* (y*y) \neq (x*y)*y$
(For refer [11]).

Example 1.3.28: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{46}, *, (0, 24)\} \cup \{[0, bI] \mid a \in Z_6, *, (0, 3)\}$ be a pure neutrosophic interval bigroupoid. G is both an alternative bigroupoid and P-bigroupoid.

Example 1.3.29: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_{10}, *, (0, 6)\} \cup \{[0, aI] \mid a \in Z_{30}, *, (0, 10)\}$ be a pure neutrosophic interval bigroupoid. Clearly S is both an alternative bigroupoid and P-bigroupoid.

In view of these examples we have the following theorem the proof of which is left as an exercise to the reader.

THEOREM 1.3.9: *Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_n, (0, t), *\} \cup \{[0, aI] \mid a \in Z_m, (0, s), *\}$ be a pure neutrosophic interval bigroupoid. P is an alternative bigroupoid and a P-bigroupoid if and only if $t^2 \equiv t \pmod n$ and $s^2 \equiv s \pmod m$.*

We still have an interesting theorem which states as follows:

THEOREM 1.3.10: *Every interval subbigroupod of a Smarandache pure neutrosophic interval bigroupoid S need not in general be a Smarandache pure neutrosophic interval subbigroupoid of S.*

The proof is by counter example.

Example 1.3.30: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_6, *, (4, 5)\} \cup \{[0, aI] \mid a \in Z_6, *, (2, 4)\}$ be a pure neutrosophic interval bigroupoid. Consider $H = H_1 \cup H_2 = \{[0, aI] \mid a \in \{0, 2, 4\} \subseteq Z_6, *, (4, 5)\} \cup \{[0, aI] \mid a \in \{0, 2, 4\} \subseteq Z_6, *\} \subseteq G_1 \cup G_2 = G$; H is a pure neutrosophic interval subbigroupoid of G but is not a Smarandache pure neutrosophic interval subbigroupoid of G.

The notion of Smarandache conjugate subbigroupoid in case of pure neutrosophic interval bigroupoids can be defined as a matter of routine [11].

We define a pure neutrosophic interval bigroupoid $G_1 \cup G_2 = G$ to be a Smarandache Moufang bigroupoid if there exists Smarandache subbigroupiod $H = H_1 \cup H_2$ in $G_1 \cup G_2$ which satisfies the Moufang identity.

If every Smarandache pure neutrosophic interval subbigroupoid satisfies the Moufang identity then we define G to be a Smarandache strong Moufang bigroupoid.

We will illustrate this by some examples.

Example 1.3.31: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{10}, *, (5, 6)\} \cup \{[0, aI] \mid a \in Z_{12}, *, (3, 9)\}$ be a pure neutrosophic interval bigroupoid. G is a Smarandache pure neutrosophic interval Moufang bigroupoid.

On similar lines we can define the notion of Smarandache pure neutrosophic interval Bol bigroupoid, Smarandache strong pure neutrosophic interval Bol bigroupoid, Smarandache pure neutrosophic interval P-bigroupoid and Smarandache strong pure neutrosophic interval P-bigroupoid.

Example 1.3.32: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{12}, *, (3, 4)\} \cup \{[0, aI] \mid a \in Z_4, *, (2, 3)\}$ be a pure neutrosophic interval bigroupoid. P is a Smarandache pure neutrosophic interval Bol bigroupoid.

Example 1.3.33: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_6, (4, 3), *\} \cup \{[0, aI] \mid a \in Z_4, (2, 3), *\}$ be a pure neutrosophic interval bigroupoid. G is a Smarandache pure neutrosophic interval strong P-bigroupoid of order 6×4.

Example 1.3.34: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{14}, (7, 8), *\} \cup \{[0, aI] \mid a \in Z_{12}, (1, 6), *\}$ be a pure neutrosophic interval bigroupoid. G is a Smarandache pure neutrosophic interval strong alternative bigroupoid.

We define a map η from two Smarandache pure neutrosophic interval bigroupoids $V = V_1 \cup V_2$ and $G = G_1 \cup G_2$ if η is a bisemigroup homomorphism or semigroup bihomomorphism from $A = A_1 \cup A_2$ to $B = B_1 \cup B_2$ where $A \subseteq V$ and $B \subseteq G$ are pure neutrosophic interval bisemigroup of V and G respectively.

That is $\eta : V \to G$ such that $\eta ((a_1 \cup a_2) * (x_1 \cup x_2))$

$= \eta (a_1 \cup a_2) * \eta (x_1 \cup x_2)$ where $a_i, x_i \in A_i$; i=1,2.

Clearly a_i, x_i are neutrosophic intervals of the form $[0, t_iI]$ and $[0, p_iI]$, i=1,2.

Interested reader can construct such interval; bigroupoid bihomomorphism.

Example 1.3.35: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_5, (1, 3), *\} \cup \{[0, aI] \mid a \in Z_5, (2, 1), *\}$ be a pure neutrosophic interval bigroupoid. Clearly G is not a Smarandache pure neutrosophic interval bigroupoid.

Example 1.3.36: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_7, (5, 3), *\} \cup \{[0, aI] \mid a \in Z_4, (3, 2), *\}$ be a pure neutrosophic interval bigroupoid. Clearly P is a Smarandache pure neutrosophic interval bigroupoid.

In view of the above examples we have the following theorem.

THEOREM 1.3.11: *Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_n, (t, u), *\} \cup \{[0, aI] \mid a \in Z_m, (r, s), *\}$ be a pure neutrosophic interval bigroupoid. G is a Smarandache pure neutrosophic interval bigroupoid if $(t, u) = 1$, $t \neq u$, $t+u \equiv 1 \pmod n$ and $(r, s) = 1$, $s \neq r$, $r+s \equiv 1 \pmod m$.*

The proof is direct uses only simple number theoretic techniques.

THEOREM 1.3.12: *Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{2p}, (1, 2), *\} \cup \{[0, bI] \mid b \in Z_{2q}, (1, 2), *\}$ where p and q two distinct odd primes. G is a pure neutrosophic interval bigroupoid which is a Smarandache bigroupoid.*

We just give an hint for the proof. Consider $S = \{[0, pI]\} \cup \{[0, qI]\} \subseteq G_1 \cup G_2$; G is a pure neutrosophic interval bisemigroup in G. Hence the claim.

COROLLARY 1.3.1: *Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{3p}, *, (1, 3)\} \cup \{[0, bI] \mid b \in Z_{3q}, (1,3), *\}$ (where p and q are odd primes) be a pure neutrosophic interval bigroupoid. Clearly G is a Smarandache bigroupoid.*

COROLLARY 1.3.2: *Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_n\ *, (1, p); p$ a prime and $p/n\} \cup \{[0, bI] \mid b \in Z_m, *, (1,q), q$ a prime and $q/m\}$ be a pure neutrosophic interval bigroupoid. Then G is a Smarandache pure neutrosophic interval bigroupoid.*

THEOREM 1.3.13: *Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_n, (t, u), *\} \cup \{[0, bI] \mid b \in Z_m, (s, r), *\}$ be a pure neutrosophic interval bigroupoid. If $t+u \equiv 1 \pmod n$ and $r+s \equiv 1 \pmod m$ then G is a Smarandache pure neutrosophic interval idempotent bigroupoid.*

THEOREM 1.3.14: *Let $G = G_1 \cup G_2 = [0, aI] \mid a \in Z_n, (t, u), *\} \cup \{[0, bI] \mid b \in Z_m, (r, s), *\}$ with $t + u \equiv 1 \pmod n$ and $r+s \equiv 1 \pmod m$ be a Smarandache neutrosophic interval P - bigroupoid if and only if $t^2 \equiv t \pmod n$, $r^2 \equiv r \pmod m$ $u^2 \equiv u \pmod n$, $s^2 \equiv s \pmod m$.*

Simple number theoretic methods yield the proof of the theorem.

THEOREM 1.3.15: *Let* $G = G_1 \cup G_2 = [0, aI] \mid a \in Z_n , (t, u), *\} \cup \{[0, bI] \mid b \in Z_m, (r, s), *\}$ *with* $t+u \equiv 1 \ (mod\ n)$ *and* $r+s \equiv 1 \ (mod\ m)$*. G is a Smarandache strong pure neutrosophic Bol bigroupoid if and only if* $t^3 \equiv t \ (mod\ n)$, $u^2 \equiv u \ (mod\ n)$, $r^3 \equiv r \ (mod\ m)$ *and* $s^2 \equiv s \ (mod\ m)$.

This proof also is direct and the reader is expected to prove the theorem.

Similarly it is easily verified that pure neutrosophic interval bigroupoid satisfying condition of theorem 1.3.14 is Smarandache strong Moufang interval bigroupoid.

THEOREM 1.3.16: *Let* $G = G_1 \cup G_2 = [0, aI] \mid a \in Z_n , (m, m), *\} \cup \{[0, aI] \mid b \in Z_t, (u, u), *\}$ *with* $m+m \equiv 1 \ (mod\ n)$ *and* $u+u \equiv 1 \ (mod\ t)$ *with* $m^2 \equiv m \ (mod\ n)$ *and* $u^2 \equiv u \ (mod\ t)$ *be a Smarandache pure neutrosophic interval bigroupoid. Then*

1. *G is a Smarandache idempotent bigroupoid.*
2. *G is a Smarandache strong P-bigroupoid.*
3. *G is a Smarandache strong Bol bigroupoid.*
4. *G is a Smarandache strong Moufang bigroupoid.*
5. *G is a Smarandache strong alternative bigroupoid.*

The proof is left as an exercise to the reader.

Interested reader can derive more properties in this direction. We can define pure neutrosophic interval groupoid - semigroup which is illustrated by some examples.

Example 1.3.37: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{40}, \times\} \cup \{[0, bI] \mid b \in Z_{19}, (3, 8), *\}$ be a pure neutrosophic interval semigroup - groupoid.

Example 1.3.38: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times\} \cup [0, bI] \mid b \in Z^+ \cup \{0\}, (3, 8), *\}$ be a pure neutrosophic interval semigroup - groupoid.

Example 1.3.39: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{125}, \times\} \cup \{[0, aI] \mid a \in Z_{45}, (3, 7), *\}$ be a pure neutrosophic interval semigroup - groupoid.

We can define substructures in them which is a matter of routine.

We can also define Smarandache pure neutrosophic interval groupoid - semigroup $G = G_1 \cup G_2$ as one in which both the groupoid and the semigroup are Smarandache. If only one of them (semigroup or groupoid) is Smarandache then we define that G to be a quasi Smarandache pure neutrosophic interval groupoid - semigroup.

We will now illustrate both the situations by some examples.

Example 1.3.40: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{40}, \times\} \cup \{[0, aI] \mid a \in Z_{90}, (45, 45), *\}$ be a Smarandache pure neutrosophic interval semigroup - groupoid.

Example 1.3.41: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_6, (3, 4), *\} \cup \{[0, aI] \mid a \in Z_{48}, \times\}$ be a Smarandache pure neutrosophic interval groupoid - semigroup.

Example 1.3.42: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_7, (5, 3), *\} \cup \{[0, aI] \mid a \in Z_{120}, \times\}$ be a Smarandache pure neutrosophic interval groupoid - semigroup.

Example 1.3.43: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{43}, \times\} \cup \{[0, aI] \mid a \in Z_9, (5,3), *\}$. G is only a quasi Smarandache pure neutrosophic interval groupoid - semigroup. The groupoid is not a Smarandache pure neutrosophic interval groupoid.

Example 1.3.44: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_8, (1, 6), *\}$ be a pure neutrosophic interval semigroup - groupoid. G is only a quasi Smarandache pure neutrosophic interval semigroup - groupoid as the semigroup is not a Smarandache semigroup.

In general all pure neutrosophic interval groupoid - semigroup need not be Smarandache pure neutrosophic interval groupoid - semigroup.

This is established by the following examples.

Example 1.3.45: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_9, (5, 3), *\}$ be a pure neutrosophic interval semigroup - groupoid. G is not a Smarandache pure neutrosophic interval semigroup - groupoid.

Example 1.3.46: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in R^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Z_5, (2, 1), *\}$ be a pure neutrosophic interval semigroup - groupoid. Clearly M is not Smarandache.

Now we can also define pure neutrosophic quasi interval bigroupoid $G = G_1 \cup G_2$ where only one of G_1 or G_2 is a pure neutrosophic interval groupoid.

We will illustrate this situation by some examples.

Example 1.3.47: Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_9, (5, 3), *\} \cup \{Z_{10}I, (1, 2), *\}$ be a pure neutrosophic quasi interval bigroupoid.

Example 1.3.48: Let $F = F_1 \cup F_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, (9, 4), *\} \cup \{Z_{15}I, *, (7, 11)\}$ be a pure neutrosophic quasi interval bigroupoid of infinite order.

Example 1.3.49: Let $V = V_1 \cup V_2 = \{Z_{125}I, *, (43, 17)\} \cup \{[0, aI] \mid a \in Z_{125}, \times\}$ be a pure neutrosophic quasi interval groupoid - semigroup.

We now also define the notion of quasi pure neutrosophic interval bigroupoids etc.

Example 1.3.50: Let $G = G_1 \cup G_2 = \{[0, a] \mid a \in Z_{120}, (1, 5), *\} \cup \{[0, aI] \mid a \in Z_{45}, (3, 17), *\}$ be a quasi pure neutrosophic interval bigroupoid.

Example 1.3.51: Let $M = M_1 \cup M_2 = \{[0, a] \mid a \in Z_{19}, *, (0, 7)\} \cup \{[0, aI] \mid a \in Z_{127}, (3, 0), *\}$ be a quasi pure neutrosophic interval bigroupoid.

Example 1.3.52: Let $T = T_1 \cup T_2 = \{[0, a] \mid a \in Z^+ \cup \{0\}, *, (6, 13)\} \cup \{[0, aI] \mid a \in Z_{40}, (3, 19), *\}$ be a quasi pure neutrosophic interval bigroupoid of infinite order.

Example 1.3.53: Let $R = R_1 \cup R_2 = \{[0, aI] \mid a \in Z_{19}, *, (3, 3)\} \cup \{[0, a] \mid a \in Z_{19}, (2, 13), *\}$ be a quasi pure neutrosophic interval bigroupoid.

We can define quasi pure neutrosophic quasi interval bigroupoid also. We will give examples of them.

Example 1.3.54: Let $V = V_1 \cup V_2 = \{Z_{19}, (3, 11), *\} \cup \{[0, aI] \mid a \in Z_{45}, *, (8, 23)\}$ be a quasi pure neutrosophic quasi interval bigroupoid of finite order.

Example 1.3.55: Let $M = M_1 \cup M_2 = \{Z_{23}I, *, (11,13)\} \cup \{[0, a] \mid a \in Z_{43}, *, (8, 11)\}$ be a quasi pure neutrosophic quasi interval bigroupoid.

These combined quasi structures can be Smarandache strong Bol bigroupoid, Smarandache strong Moufang bigroupoid, Smarandache strong P-bigroupoid and so on. All properties can be derived with appropriate modifications.

Now we proceed onto give examples of quasi pure neutrosophic quasi interval groupoid - semigroup.

Example 1.3.56: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_{40}, \times\} \cup \{Z_{19}I, (8, 11), *\}$ be a quasi pure neutrosophic quasi interval semigroup - groupoid.

Example 1.3.57: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{20}, \times\} \cup \{Z_{43}, (3, 17), *\}$ be a quasi pure neutrosophic quasi interval semigroup - groupoid.

Example 1.3.58: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{43}, (8, 12), *\} \cup \{Z, \times\}$ be a quasi pure neutrosophic quasi interval groupoid - semigroup.

We also state here that we can replace pure neutrosophic intervals [0, aI] in all these algebraic structures by the mixed neutrosophic algebraic structures [0, a+bI] and get all related results with simple, appropriate modifications.

Now we can also define the notion of pure neutrosophic interval group - groupoid and their quasi analogue. We only give illustrative examples and leave the work of defining this structure to the reader.

Example 1.3.59: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{40}, +\} \cup \{[0, aI] \mid a \in Z_{40}, *, (3, 7)\}$ be a pure neutrosophic interval group - groupoid.

Thus by defining this mixed neutrosophic bistructure we get an associative and non associative neutrosophic algebraic structure.

Example 1.3.60: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in R^+, \times\} \cup \{[0, aI] \mid a \in Z_{43}, *, (7, 11)\}$ be a pure neutrosophic interval group - groupoid of infinite order.

Example 1.3.61: Let $V = V_1 \cup V_2 = \{[0, a+bI] \mid a, b \in Z_{25}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{25}, (3, 7), *\}$ be a neutrosophic interval group-groupoid.

This contains $H = H_1 \cup H_2 = \{[0, aI] \mid a \in Z_{25}, +\} \cup \{[0, aI] \mid a \in Z_{25}, (3, 7), +\} \subseteq V_1 \cup V_2 = V$ the pure neutrosophic interval group - groupoid as well as $S = S_1 \cup S_2 = \{[0, a] \mid a \in Z_{25}, +\} \cup \{[0, aI] \mid a \in Z_{25}, *, (3, 7)\} \subseteq V_1 \cup V_2 = V$ which is a interval group - groupoid as its substructure. Thus one of the advantages of studying these mixed neutrosophic interval bistructures is that they contain both pure neutrosophic interval bistructure as well as interval bistructure as its subbistructure.

Example 1.3.62: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z_{40}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{40}, (8, 5), *\}$ be a neutrosophic interval group-groupoid. T has pure neutrosophic interval subgroup - subgroupoid given by $S = S_1 \cup S_2 = \{[0, aI] \mid a \in \{2Z_{40}\}, +\} \cup \{[0, bI] \mid a \in Z_{40}, (8, 5), *\} \subseteq T_1 \cup T_2$ however T has no interval subgroup - subgroupoid.

Interested reader can derive related results in this direction. We also can have mixed quasi neutrosophic quasi interval groupoid - groups which is illustrated by the following example.

Example 1.3.63: Let V = $V_1 \cup V_2$ = {[0, aI+b] | a, b $\in$ Z_{43}, (7, 11), *} $\cup$ {S_3} be a quasi mixed neutrosophic quasi interval groupoid - group.

Example 1.3.64: Let S = $S_1 \cup S_2$ = {[0, a+bI] | a, b $\in$ Z_{120}, +} $\cup$ {Z_{27} (3, 8), *} be a quasi mixed neutrosophic quasi interval group - groupoid.

Example 1.3.65: Let M = $M_1 \cup M_2$ = {[0, aI+b] | a, b $\in$ Z_{27}, (8, 14), *} $\cup$ {Z_{144}, +} be a quasi mixed neutrosophic quasi interval groupoid - group.

1.4 Neutrosophic Interval Biloops

In this section we proceed onto define the notion of neutrosophic interval biloops and their generalization. For loops please refer [9, 13].

DEFINITION 1.4.1: *Let L = {[0, a] | a $\in$ Z_n or $R^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $Z^+ \cup \{0\}$} be the collection of intervals. If a binary operation * on L be defined so that L is a loop; that is * satisfies the following condition.*

(a) For every [0, a], [0, b] in L
*[0, a] * [0, b] $\in$ L.*

*(b) [0, a] * ([0, b] * [0, c]) $\neq$ ([0, a] * [0, b]) * [0, c] for atleast some [0, a], [0, b] and [0, c] in L.*

(c) There exists an element [0, e] in L such that
*[0, a] * [0, e] = [0, e] * [0, a] = [0, a] for all [0, a] in L called the identity element of L.*

(d) For every pair ([0, a], [0, b]) in L $\times$ L there exists a unique pair ([0, x], [0, y]) in L $\times$ L such that
*[0, a] * [0, x] = [0, b] and*
*[0, y] * [0, a] = [0, b].*

We will illustrate this situation by some examples.

Example 1.4.1: Let L = {[0, a] | a $\in$ L_5 (2), *} be an interval loop of order five given by the following table.

*	[0,e]	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
[0,e]	[0,e]	[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
[0,1]	[0,1]	[0,e]	[0,3]	[0,5]	[0,2]	[0,4]
[0,2]	[0,2]	[0,5]	[0,e]	[0,4]	[0,1]	[0,3]
[0,3]	[0,3]	[0,4]	[0,1]	[0,e]	[0,5]	[0,2]
[0,4]	[0,4]	[0,3]	[0,5]	[0,2]	[0,e]	[0,1]
[0,5]	[0,5]	[0,2]	[0,4]	[0,1]	[0,3]	[0,e]

For more about loop $L_n(m)$ refer [9, 13].

Example 1.4.2: Let L = {[0, a] | a ∈ {e, 1, 2, …, 15}, 7, *} be an interval loop of order 16.

DEFINITION 1.4.2: *Let L = {[0, aI] | a ∈ Z_n or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$, *} be a loop. We define L to be a pure neutrosophic interval loop. If we replace [0, aI] by [0, x+yI] x and y in Z_n or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$ then we call L to be a mixed neutrosophic interval loop.*

We will give examples of them.

DEFINITION 1.4.3: *Let $L = L_1 \cup L_2$ where L_1 and L_2 are neutrosophic interval loops such that $L_1 \neq L_2$, $L_1 \not\subseteq L_2$ and $L_2 \not\subseteq L_1$. Then we define L to be a neutrosophic interval biloop.*

We will for sake of completeness recall the definition of interval loop and neutrosophic interval loop.

Example 1.4.3: Let V = {[0, a+bI] | a, b ∈ {e, 1, 2, …, 17}, 5, *} be a mixed neutrosophic interval loop of order 18^2.

Example 1.4.4: Let M = {[0, a+bI] | a, b ∈ {e, 1, 2, …, 13}, 8, *} be a mixed neutrosophic interval loop of order 14^2.

Now we can define the notion of neutrosophic interval biloop, which is a matter of routine [9, 13]. Just like a neutrosophic group does not satisfy all group axioms so also a neutrosophic loop may not in general satisfy all axioms of a loop.

We will illustrate this situation by some examples.

Example 1.4.5: Let L = $L_1 \cup L_2$ = {[0, aI] | a ∈ {e, 1, 2, …, 23}, *, 8} ∪ {[0, bI] | b ∈ {e, 1, 2, …, 19}, 8, *} be a pure neutrosophic interval biloop of order 24 × 20.

Example 1.4.6: Let L = $L_1 \cup L_2$ = {[0, aI] | a ∈ {e, 1, 2, …, 45}, *, 16} ∪ {[0, aI] | a ∈ {e, 1, 2, …, 27}, *, 10} be a pure neutrosophic interval biloop of order 46 × 28.

Example 1.4.7: Let L = $L_1 \cup L_2$ = {[0, aI] | a ∈ {e, 1, 2, …, 11}, 8, *} ∪ {[0, aI] | a ∈ {e, 1, 2, …, 13}, 8, *} be a pure neutrosophic interval biloop of order 12 × 14.

Inview of this we have the following theorem.

THEOREM 1.4.1: *Let L = $L_1 \cup L_2$ = {[0, aI] | a ∈ {e, 1, 2, …, n}, n odd n > 3 and *, m, (m, n) = 1 (m–1, n) = 1 m < n} ∪ {[0, aI] | a ∈ {e, 1, 2, …, s}; *, s > 3, s odd, t, (t,s)=1, (t–1,s) = 1, t < s}. L is a pure neutrosophic interval biloop of order (n+1) (s+1).*

Proof is direct, if need be refer [9, 13]. We call a pure neutrosophic interval biloop to be a Smarandache pure neutrosophic interval biloop if each of the pure neutrosophic interval loop is Smarandache.

We will give examples of them.

Example 1.4.8: Let L = $L_1 \cup L_2$ = {[0, aI] | a ∈ {e, 1, 2, …, 13}, 8, *} ∪ {[0, aI] | a ∈ {e, 1, 2, …, 17}, 12, *} be a pure neutrosophic interval biloop of order 14 × 18.

THEOREM 1.4.2: *Let L = $L_1 \cup L_2$ = {[0, aI] | a ∈ {e, 1, 2, …, n} n odd n > 3, m, m<n with (m, n) = 1 = (m–1, n), *} ∪ {[0, aI] | a ∈ {e, 1, 2, …, s}, s > 3, s odd; t < s, (t, s) = (t–1, s) = 1, *} be a pure neutrosophic interval biloop. Then the following are true.*

1. *L is of even biorder.*

2. *$|L| = 2^2N$ (N a positive number ≥ 9).*

3. *L is a Smarandache pure neutrosophic interval biloop.*

This proof involves only simple number theoretic techniques [9, 13]. We say a pure neutrosophic interval biloop is commutative if both the pure neutrosophic interval loops are commutative. If only one of them is commutative we say the interval biloop is quasi commutative. We will give examples of them.

Example 1.4.9: Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 17\}, *, 9\} \cup \{[0, aI] \mid b \in \{e, 1, 2, \ldots, 25\}, *, 13\}$ be a pure neutrosophic interval biloop. L is a commutative biloop.

Example 1.4.10: Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 17\}, *, 7\} \cup \{[0, aI] \mid b \in \{e, 1, 2, \ldots, 25\}, 8, *\}$ be a pure neutrosophic interval biloop. L is a non commutative biloop.

Example 1.4.11: Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 23\}, *, 8\} \cup \{[0, aI] \mid b \in \{e, 1, 2, \ldots, 15\}, *, 14\}$ be a pure neutrosophic interval biloop. L is a quasi commutative pure neutrosophic interval biloop.

THEOREM 1.4.3: *Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, n\}, \frac{n+1}{2}, *\} \cup \{[0, bI] \mid b \in \{e, 1, 2, \ldots, m\}, \frac{m+1}{2}, *\}$ be a pure neutrosophic interval biloop. L is a commutative.*

This proof also requires only simple number theoretic techniques.

We can define several identities in case of pure neutrosophic interval biloops as in case of usual biloops.

$L_{ns}(I) = \{[0, a] \mid a \in \{e, 1, 2, \ldots, n\}$ $n > 3$, n odd; $m < n$ $(m-1, n) = (m, n) = 1 \in \{[0, bI] \mid b \in \{e, 1, 2, \ldots, s\}$, $s > 3$, s odd, $t < s$, $(t, s) = (t-1, s) = 1\}$ denote the class of pure neutrosophic interval biloops each of order $(n+1) \times (s+1)$.

Further if $n = p_1^{\alpha_1} \ldots p_k^{\alpha_k}$ and $s = q_1^{\beta_1} \ldots q_b^{\beta_b}$ then the number of pure neutrosophic interval biloops in L_{ns} (I) is

$$\left(\prod_{i=1}^{k}(p_i - 2)p_i^{\alpha_i - 1}\right) \times \left(\prod_{j=1}^{b}(q_j - 2)q_j^{\beta_j - 1}\right).$$

Thus we will denote the class of pure neutrosophic biloop by L_{ns} (I); n and s odd greater than three.

Example 1.4.12: Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 19\}, 2, *\} \cup \{[0, bI] \mid b \in \{e, 1, 2, \ldots, 23\}, 2, *\}$ be a pure neutrosophic interval biloop. L is a pure neutrosophic interval right alternative biloop.

Example 1.4.13: Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 47\}, *, 46\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 53\}, *, 52\}$ be a pure neutrosophic interval biloop. L is a left alternative interval biloop which is not right alternative.

Inview of this we have the following theorem.

THEOREM 1.4.4: *Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, n\}, 2, *\} \cup \{[0, bI] \mid b \in \{e, 1, 2, \ldots, s\}, *, 2\}$ where $n > 3$, $s > 3$, n and s odd be a pure neutrosophic interval biloop. L is a right alternative pure neutrosophic interval biloop.*

The proof is got by using simple number theoretic techniques.

THEOREM 1.4.5: *Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, n\}, n > 3\ n - odd, *, n-1\} \cup \{[0, bI] \mid b \in \{e, 1, 2, \ldots, m\}, m > 3, m\ odd; m-1\}$ be a pure neutrosophic interval biloop. L is a left alternative pure neutrosophic interval biloop.*

This proof is left as an exercise to the reader .

COROLLARY 1.4.1: *Let $L_{ns}(I)$ be the class of pure neutrosophic interval biloops $n>3$ and $s>3$, n and s odd. $L_{ns}(I)$ has exactly one left alternative interval biloop and only one right alternative interval biloop and no alternative interval biloop.*

Infact the class of pure neutrosophic interval biloops $L_{ns}(I)$ does not contain (i) any moufang interval biloop (ii) any Bol interval biloop or Burck biloop.

We can define pure neutrosophic interval bisubloop (subbiloop) $H_1 \cup H_2$ Further if this pure neutrosophic interval subbiloop $H_1 \cup H_2$ contains a proper subset $T = T_1 \cup T_2$ such that T is a pure neutrosophic interval bigroup then we define $H_1 \cup H_2$ to be a Samarandache pure neutrosophic interval subbiloop.

We have following theorem the proof of which is direct.

THEOREM 1.4.6: *Let $L = L_1 \cup L_2$ be a pure neutrosophic interval biloop. If L has $H = H_1 \cup H_2$ to be a pure neutrosophic interval subbiloop which is Smarandache then L itself is a Smarandache pure neutrosophic interval biloop.*

If a pure neutrosophic interval biloop $L = L_1 \cup L_2$ has only pure neutrosophic interval bisubgroups then we define L to be a Smarandache pure neutrosophic interval subbigroup biloop.

Example 1.4.14: Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 7\}, 4, *\} \cup \{[0, bI] \mid b \in \{e, 1, 2, \ldots, 17\}, 9, *\}$ be a pure neutrosophic interval biloop. L is a Smarandache pure neutrosophic interval subbigroup biloop.

In view of this we have the following theorem.

THEOREM 1.4.7: *Let $L_{pq}(I)$ be the class of pure neutrosophic interval biloops p and q primes greater than three. The class of biloops $L_{pq}(I)$ is a Smarandache pure neutrosophic interval subbigroup biloop.*

Proof follows by using simple number theoretic techniques. We can define the notion of Smarandache pure neutrosophic interval normal subbiloop if the subbiloop is normal. If the pure neutrosophic interval biloop has no normal subbiloop we define them to be Smarandache simple pure neutrosophic interval biloop.

Example 1.4.15: Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e,1,2,\ldots, 19\},9, *\} \cup \{[0,bI] \mid b \in \{e,1,2, \ldots, 19\},12,*\}$ be a pure neutrosophic interval biloop. L is S-simple.

Inview of this we have the following theorem which shows the existence of a class of S-simple pure neutrosophic interval biloops.

THEOREM 1.4.8: *Let $L_{mn}(I)$ be the class of pure neutrosophic interval biloops; $L_{mn}(I)$ is Smarandache simple.*

The proof is direct hence left as an exercise to the reader.

Let $L = L_1 \cup L_2$ be a S-pure neutrosophic interval biloop.

Let $A = A_1 \cup A_2 \subseteq L_1 \cup L_2$ be a pure neutrosophic interval subbigroup of L . $a = a_1 \cup a_2 \in A = A_1 \cup A_2$ is said to be a Smarandache Cauchy bielement of L. $([0, a_1I] \cup [0, a_2I])^r = ([0, a_1I]^r \cup [0, a_2I]^r) = [0, eI] \cup [0, eI]$; otherwise a in A is not a S-Cauchy bielement of L.

Example 1.4.16: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in \{e,1,2,..., 11\}, * 8\} \cup \{[0, bI] \mid b \in \{e,1,2,..., 15\}, 8, * \}$ be a S- pure neutrosophic interval biloop. Consider $x = [0, 3I] \cup [0, 10I] \in V$; we see $x^2 = ([0, 3I] \cup [0, 10I])^2 = [0, eI] \cup [0, eI]$.

Thus x is a S- Cauchy bielement of V.

Note: If in a S- pure neutrosophic interval biloop every bielement is a S-Cauchy bielement then we define the biloop to be a S-pure neutrosophic Cauchy interval biloop or S-Cauchy pure neutrosophic interval biloop or pure neutrosophic interval S- Cauchy biloop.

We will give an example of a S-pure neutrosophic interval Cauchy biloop.

Example 1.4.17: Let $L = L_1 \cup L_2 = \{[0,aI] \mid a \in \{e,1,2,..., 27\}, 11, *\} \cup \{[0,aI] \mid a \in \{e,1,2,..., 47\}, 12, *\}$ be a pure neutrosophic interval biloop. L is a S-Cauchy pure neutrosophic interval biloop.

Example 1.4.18: Let $L = L_1 \cup L_2 = \{[0, aI] \mid a \in \{e,1,2,..., 29\}, 12, *\} \cup \{[0, aI] \mid a \in \{e,1,2,..., 33\}, *, 17\}$ be a pure neutrosophic interval biloop which is S-Cauchy biloop.

We in the following theorem show there exists a class of pure neutrosophic interval biloops of even order which are S-Cauchy.

THEOREM.1.4.9: *Let $L_{mn}(I) = \{[0,aI] \mid a \in \{e,1,2,..., m\}, t < m$, m odd and $m < 3$; $[t,m] = [t-1, m] = 1, *\} \cup \{[0, aI] \mid a \in \{e,1, 2,..., n\}; s < n$, n odd and $n < 3$; $[s, n] = [s-1,n] = 1, *\}$ be the class of pure neutrosophic interval biloops . $L_{mn}(I)$ is a class of S-Cauchy pure neutrosophic interval biloops.*

This proof is also simple directly follows from the definition of loops [9, 13].

We can as in case of biloops define the notion of Smarandache Lagrange interval biloop and Smarandache weakly Lagrange interval biloop which is direct and hence is left as an exercise to the reader [9, 13].

We provide examples of them.

Example 1.4.19: Let $L = L_1 \cup L_2 = \{[0,aI] \mid a \in \{e,1,2,..., 19\}, 8, *\} \cup \{[0,aI] \mid a \in \{e,1,2,..., 53\}, 9, *\}$ be a pure neutrosophic interval biloop. L is a Smarandache pure neutrosophic interval Lagrange biloop or pure neutrosophic interval Smarandache Lagrange biloop.

Example 1.4.20: Let $L = L_1 \cup L_2 = \{[0,aI] \mid a \in \{e,1,2,..., 45\}, 14, *\} \cup \{[0, aI] \mid a \in \{e,1,2,..., 25\}, *,7\}$ be a pure neutrosophic interval biloop. L is a Smarandache weakly Lagrange pure neutrosophic interval biloop.

We have a class of pure neutrosophic interval biloops which are Smarandache Lagrange and a class of pure neutrosophic interval biloop which is Smarandache weakly Lagrange.

These situations are given by the following theorems.

THEOREM 1.4.10: *Let $L_{pq}(I)= \{[0,aI] \mid a \in\{e,1,2,..., p\}, s <p, s;$ $* \; p$ an odd prime greater than 3$\} \cup \{[0,aI] \mid a \in\{e,1,2,..., q\},$ $r<p, r,*;$ q an odd prime greater than three$\}$ be a class of pure neutrosophic interval biloops. Every biloop in this class of interval biloops $L_{pq}(I)$ is a Smarandache Lagrange pure neutrosophic interval biloop.*

The proof follows from the fact the order of every biloop is (p+1) (q+1)=2 m (m<q). Hence these biloops have only subbigroups of order 2.2.

THEOREM 1.4.11: *Let $L_{mn}(I)= \{[0,aI] \mid a \in \{e,1,2,..., n\}, *, n$ odd $n > 3,t; t<n, (t-1,n)= (t,n)=1, *\} \cup \{[0,aI] \mid a \in\{e,1,2,...,$ $m\},*, m$ odd, $m > 3, s$ $s<m; (s-1, m)= (s,m)= 1,*\}$ be the class of pure neutrosophic interval biloops. Every interval biloop in this class is a Smarandache weakly Lagrange interval biloop.*

Proof follows from the fact every $\{[0,aI] \cup [0, bI], [0, eI] \cup [0, eI]\} \subseteq L = L_1 \cup L_2 \in L_{mn}(I)$ is a subbigroup of order four and o (L) = 2^2.m ($m \geq 9$).

Now we can define the notion of Smarandache (p_1, p_2) Sylow interval subbiloops of a pure neutrosophic interval biloop.

Suppose $L = L_1 \cup L_2$ is a pure neutrosophic interval biloop of finite order. Let p_1, p_2 be two primes (distinct or otherwise) such that $p_1\ p_2$ / o (L). Suppose $A = A_1 \cup A_2 \subseteq L_1 \cup L_2$ be a Smarandache pure neutrosophic subbiloop of L of order mm_1 and if $B = B_1 \cup B_2 \subseteq A_1 \cup A_2 = A$ is a pure neutrosophic interval subbigroup of A of biorder p_1p_2 and if p_1p_2 / m_1m_2, then we say L is a Smarandache (p_1, p_2)- Sylow pure neutrosophic interval bisubgroup.

Example 1.4.21: Let $L= L_1 \cup L_2 = \{[0,aI] \mid a \in \{e,1,2,..., 23\},8, *\} \cup \{[0,aI] \mid a \in \{e,1,2,..., 43\}, 8, *\}$ be a pure neutrosophic interval biloop, L is a Smarandache (2,2)- Sylow biloop.

We say a finite biorder Smarandache strong (p_1, p_2)- Sylow biloop if every subbigroup is of a prime power biorder and divides biorder of L. We will give example of it.

Example 1.4.22: Let $L = L_1 \cup L_2 = \{[0,aI] \mid a \in \{e,1,2,..., 11\},*, 9\} \cup \{[0,aI] \mid a \in \{e,1,2,..., 11\}, *,10\}$ be a pure neutrosophic interval biloop. L is a Smarandache strong (2, 2) Sylow interval biloop.

In view of this we have the following theorem.

THEOREM 1.4.12: *Let $L_{pq}(I) = \{[0,aI] \mid a \in \{e,1,2,...,p\}$, p an odd prime greater than 3,t, $t < p$; $(t,p) = (t-1, p) = 1, *\} \cup \{[0,aI] \mid a \in \{e,1,2,..., q\}$, q an odd prime greater than 3,s, $s<q$, $(s,q)= (s-1,q)= 1,*\}$ be the class of pure neutrosophic interval biloops. Every biloop in $L_{pq}(I)$ is a Smarandache strong (2,2) Sylow interval biloop.*

We can define as in case of biloops in case of pure neutrosophic interval biloops also the notion biassociator, Smarandache bicyclic and Smarandache strong bicyclic biloops. We call a pure neutrosophic interval biloop to be a

Smarandache cyclic biloop if $L = L_1 \cup L_2$ contains a bisubset A $= A_1 \cup A_2$ such that A is a cyclic bigroup. If every proper bisubset A of L which is a subbigroup is bicyclic then we say the interval biloop is a Smarandache strong pure neutrosophic interval bicyclic loop. We will first provide examples of this concept.

Example 1.4.23: Let $L = L_1 \cup L_2 = \{[0,aI] \mid a \in \{e,1,2,..., 27\}, * \ 8\} \cup \{[0,aI] \mid a \in \{e,1,2,..., 30\}, *, 14\}$ be a pure neutrosophic interval biloop. L is a Smarandache cyclic pure neutrosophic interval biloop.

Example 1.4.24: Let $P=P_1 \cup P_2 = \{[0,aI] \mid a \in \{e,1,2,..., 47\}, *,9\} \cup \{[0,aI] \mid a \in \{e,1,2,..., 53\}, *, 9\}$ be a pure neutrosophic interval biloop. P is a Smarandache strong cyclic interval biloop.

In view of this we have the following theorem.

THEOREM 1.4.13: *Let $L_{pq}(I)= \{[0,aI] \mid a \in\{e,1,2,..., p\}$; p a prime greater than 3 and $n < p$; n, $*\} \cup \{[0,aI] \mid a \in\{e,1,2,..., q\}$, q a prime, $q>3$; m; $m<q$, $*\}$ be a class of pure neutrosophic interval biloops. Every pure neutrosophic interval biloop in $L_{pq}(I)$ is a Smarandache strongly cyclic biloop.*

The proof is direct follows from the fact that every distinct bipair generates a bisubloop or the whole biloop. We can define for a pure neutrosophic interval biloop $L= L_1 \cup L_2$ the notion of biassociator of L denoted by $A(L) = A(L_1) \cup A(L_2)$.

Example 1.4.25: Let $L = L_1 \cup L_2 =\{[0,aI] \mid a \in \{e,1,2,..., 27\}, 20, *\} \cup \{[0,aI] \mid a \in\{e, 1, 2, ..., 23\}, *, 20\}$ be a pure neutrosophic interval biloop. The associator of L is $A(L) = A(L_1) \cup A(L_2) = L_1 \cup L_2 =L$.

In view of this we have the following theorem.

THEOREM 1.4.14: *Let $L_{mn}(I)= \{[0,aI] \mid a \in \{e,1,2, ..., m\}$, s, $s <m$, $*\} \cup \{[0,aI] \mid a \in\{e,1,2,..., n\}$, t, $t < n$, $*\}$ m and n odd greater than three be the class of pure neutrosophic interval biloops. Every pure neutrosophic interval biloop $L= L_1 \cup L_2$ in*

$L_{mn}(I)$ is such that $A(L)=A(L_1 \cup L_2) = A(L_1) \cup A(L_2) = L_1 \cup L_2 = L$.

The proof is direct based on the definition of associator [9, 13]. We can define for these biloops the notion of first and second binormalizers.

In particular the first binormalizer in general is not equal to the second binormalizer. Interested reader can supply with examples. Several interesting properties derived for biloops can also be derived for pure neutrosophic interval biloops with appropriate modifications. Further these properties can be easily extended in case of mixed neutrosophic interval biloops. We will only give examples of them.

Example 1.4.26: Let $L = L_1 \cup L_2 = \{[0,a+bI] \mid a, b \in Z_7, 3, *\} \cup \{[0, a + bI] \mid a, b \in Z_{19}, *, 8\}$ be a mixed neutrosophic interval biloop of finite order.

Clearly this biloop contains as a subbiloop both pure neutrosopbic interval biloop as well as just interval biloop. We now proceed on to define quasi neutrosophic interval biloop. We call $L= L_1 \cup L_2$ to be a quasi neutrosophic interval biloop if one of L_1 or L_2 is a pure neutrosophic or a mixed neutrosophic interval loop and the other is just an interval loop. We will illustrate this situation by some examples.

Example 1.4.27: Let $V= V_1 \cup V_2 = \{[0,a] \mid a \in Z_{15}, *, 8\} \cup \{[0,aI] \mid a \in Z_{43},*,8\}$ be a quasi neutrosophic interval biloop of finite order.

Example 1.4.28: Let $L = L_1 \cup L_2 = \{[0,aI+b] \mid a,b \in Z_{47},*, 9\} \cup \{[0,a] \mid a \in Z_{19}, *,9\}$ be a quasi neutrosophic interval biloop of finite order.

Example 1.4.29: Let $L= L_1 \cup L_2 = \{[0,a] \mid a \in Z_{17},*, 8\} \cup \{[0,aI] \mid a \in Z_{17}, 12\}$ be a quasi neutrosophic interval biloop of finite order.

Example 1.4.30: Let $L = L_1 \cup L_2 = \{[0,a] \mid a \in Z_{29}, * 7\} \cup \{[0,a+bI] \mid a, b \in Z_{29}, *,21\}$ be a quasi neutrosophic interval biloop of finite order.

For these class of biloops also. We define pure neutrosophic or mixed neutrosophic quasi interval biloops as follows. Let L= $L_1 \cup L_2$ if only one of L_1 or L_2 is a neutrosophic interval loop and the other is just a neutrosophic loop.

We will give examples of this structure .

Example 1.4.31: Let L = $L_1 \cup L_2$= $\{LI_{25}(9)\} \cup \{[0,aI] \mid a \in \{e,1,2,..., 29,*,9\}$ be a neutrosophic quasi interval biloop of finite order where $L_1 = \{aI \mid a \in \{e,1,2,..., 25\}, 9, *\}$.

Example 1.4.32: Let $L = L_1 \cup L_2 = \{[0,aI] \mid a \in \{e,1,2,..., 23\}, *,19\} \cup \{aI \mid a \in \{e,1,2,..., 23\}, *, 22\}$ be a neutrosophic quasi interval biloop of order 24×24.

Example 1.4.33: Let $L = L_1 \cup L_2 = \{[0,a+bI] \mid a, b \in \{e,1,2, ..., 26, 27\},11,*\} \cup \{aI \mid a\in \{e,1,2,..., 47\}, 11,*\}$ be a neutrosophic quasi interval biloop of finite order 28×48.

Example 1.4.34: Let $L = L_1 \cup L_2 = \{aI \mid a \in \{e,1,2, ..., 13\}, 9,*\} \cup \{[0,a+bI] \mid a,b \in \{e,1,2,..., 13\}, 9, * \}$ be a neutrosophic quasi interval biloop of order $14^2 =196$.

We can derive almost all the results discussed in this section about pure neutrosophic interval biloops to the class of quasi neutrosophic interval biloops to the class of quasi neutrosophic interval biloops with simple modifications. We can still define another type of biloop which we choose to call as quasi neutrosophic quasi interval biloop if only one of L_1 or L_2 is a interval loop other just a loop and only one of them is neutrosophic other just not neutrosophic then we call L= $L_1 \cup L_2$ to be a quasi neutrosophic quasi interval biloop. We will now proceed on to give examples of them.

Example 1.4.35: Let L= $L_1 \cup L_2 = \{L_9(8)\} \cup \{[0, aI] \mid a \in \{e, 1, 2, ..., 47\}, 8, *\}$ be a quasi neutrosophic quasi interval biloop.

Example 1.4.36: Let L= $L_1 \cup L_2 = \{[0,a] \mid a \in \{e, 1, 2, ..., 43\}, 10, *\} \cup \{aI \mid a \in \{e, 1, 2, ..., 23\}, *, 10\}$ be a quasi neutrosophic quasi interval biloop of finite order.

Example 1.4.37: Let $V = V_1 \cup V_2 = \{[0,a+bI] \mid a,b \in \{e,1,2,..., 11\},8,*\} \cup \{a \mid a \in \{e,1,2,..., 13\}, 8,*\}$ be a quasi neutrosophic quasi interval biloop.

Consider $M = M_1 \cup M_2 = \{[0,a+bI] \mid a, b \in \{e,1,2,..., 5\},*,3\} \cup \{a \mid a \in \{1,2,..., 5\}, *, 3\}$ is not a quasi neutrosophic quasi interval biloop as $M_2 \subseteq M_1$.

However if $P = P_1 \cup P_2 = \{[0, a+bI] \mid a,b \in \{e,1,2, ..., 5\},*, 3\} \cup \{a \mid a \in \{e,1,2,..., 5\}, *, 4\}$ is a quasi neutrosophic quasi interval biloop. Now one can define neutrosophic interval loop-group, quasi neutrosophic interval loop-group and quasi neutrosophic quasi interval loop-group.

We only give examples of them.

Example 1.4.38: Let $L = L_1 \cup L_2 = \{[0,aI] \mid a \in \{e,1,2,..., 25\},*, 12\} \cup \{[0,a+bI] \mid a,b \in Z_{25}, +\}$ be a neutrosophic interval loop-group of finite order.

Example 1.4.39: Let $P = P_1 \cup P_2 = \{[0,aI+b] \mid a,b \in \{e,1,2, . . . 29\}, *, 12\} \cup \{[0,aI] \mid a \in Z_{45}, + \}$ be a neutrosophic interval loop-group of finite order.

Example 1.4.40: Let $M = M_1 \cup M_2 = \{[0,a+bI] \mid a,b \in Z_{280}, +\} \cup \{[0,a+bI] \mid a, b \in \{e,1,2,..., 13\}, 12, *\}$ be a neutrosophic interval group-loop.

Example 1.4.41: Let $M = M_1 \cup M_2 = \{[0,a] \mid a \in Z_{45}, +\} \cup \{[0,a+bI] \mid a,b \in \{e,1,2,..., 43\},8,*\}$ be a quasi neutrosophic interval group-loop of finite order.

Example 1.4.42: Let $R = R_1 \cup R_2 = \{[0,a+bI] \mid a,b \in Z_{25},+\} \cup \{[0,a] \mid a \in \{e,1,2,..., 47\}, 19,*\}$ be a quasi neutrosophic interval group-loop.

Example 1.4.43: Let $T = T_1 \cup T_2 = \{[0,aI] \mid a \in Z_5I \setminus \{0\}, \times\} \cup \{[0,a] \mid a \in \{e,1,2,...,43\}, 19,*\}$ be a quasi neutrosophic interval group-loop.

Example 1.4.44: Let $T = T_1 \cup T_2 = \{[0,a] \mid a \in Z_{19} \setminus \{0\}, \times\} \cup \{[0,a+bI] \mid a,b \in \{e,1,2,..., 33\}, 14, *\}$ be a quasi neutrosophic interval group-loop of finite order.

Example 1.4.45: Let $V=V_1 \cup V_2 = \{[0,a+bI] \mid a, b \in Z_{40},+\} \cup \{aI \mid a \in \{e,1,2,..., 43\},*, 8\}$ be a neutrosophic quasi interval group- loop.

Example 1.4.46: Let $M=M_1 \cup M_2=\{[0,a+bI] \mid a, b \in \{e,1,2,..., 43\},29,*\} \cup \{aI \mid a \in Z_{13} \setminus \{0\}, \times\}$ be a neutrosophic quasi interval loop-group of finite order.

Example 1.4.47: Let $B = B_1 \cup B_2 = \{[0,a+bI] \mid a, b \in Z_{425},+\}\cup\{[aI \mid a \in \{e,1,2,..., 47\}, 9,*\}$ be a neutrosophic quasi interval group-loop of finite order.

Example 1.4.48: Let $V=V_1 \cup V_2 = \{[0,aI] \mid a \in\{e,1,2,..., 15\}, 8, *\} \cup \{Z_{148},+\}$ be a quasi neutrosophic quasi interval loop-group.

Example 1.4.49: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{145}, +\} \cup \{L_{29}(8), *\}$ be a quasi neutrosophic quasi interval group-loop.

Example 1.4.50: Let $P = P_1 \cup P_2 = \{[0, a+bI] \mid a, b \in \{e, 1, 2, ..., 29\}, 19, *\} \cup \{Z_{19} \setminus \{0\}, \times\}$ be a quasi neutrosophic quasi interval loop-group.

Now the notion of substructures can be easily derived and described by any interested reader.

We can define the new notions of neutrosophic interval loop - semigroup, quasi neutrosophic interval loop - semigroup, neutrosophic quasi interval loop - semigroup and quasi neutrosophic quasi interval loop - semigroup, it is left to the reader, however we give examples of them.

Example 1.4.51: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in \{e, 1, 2, ..., 23\}, 10, *\} \cup \{[0, a+bI] \mid a, b \in Z_{40}, \times\}$ be a neutrosophic interval loop - semigroup.

Example 1.4.52: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, \times\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, …, 41\}, 8, *\}$ be a neutrosophic interval loop-semigroup.

Example 1.4.53: Let $M = M_1 \cup M_2 = \{([0, a_1I], [0, a_2I], [0, a_3I], [0, a_4I]) \mid a_i \in Z^+ \cup \{0\}, \times; 1 \leq i \leq 4\} \cup \{[0, a+bI] \mid a, b \in$

$\{e, 1, 2, ..., 13\}, 8, *\}$ be a neutrosophic interval semigroup - loop.

Example 1.4.54: Let $P = P_1 \cup P_2 = \{[0, a] \mid a \in \{e, 1, 2, \ldots, 19\}, 8, *\} \cup \{[0, a+bI] \mid a, b \in Z_{24}, \times\}$ be a quasi neutrosophic interval loop - semigroup.

Example 1.4.55: Let $S = S_1 \cup S_2 = \{[0, a] \mid a \in Z_{45}, \times\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 23\}, 18, *\}$ be a quasi neutrosophic interval semigroup - loop.

Example 1.4.56: Let $P = P_1 \cup P_2 = \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 43\}, 10, *\} \cup \{[0, a] \mid a \in Z_{42}, \times\}$ be a quasi neutrosophic interval loop - semigroup.

Example 1.4.57: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{40}, \times\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 29\}, 20, *\}$ be a neutrosophic interval semigroup - loop.

Example 1.4.58: Let $M = M_1 \cup M_2 = \{Z_{45}I, \times\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 23\}, *, 8\}$ be a neutrosophic quasi interval semigroup - loop.

Example 1.4.59: Let $G = G_1 \cup G_2 = \{[aI] \mid a \in \{e, 1, 2, \ldots, 23\}, 12, *\} \cup \{[0, a+bI] \mid a, b \in Z_{40}, \times\}$ be a neutrosophic quasi interval loop - semigroup.

Example 1.4.60: Let $P = P_1 \cup P_2 = \{Z_{40}I, \times\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 41\}, 9, *\}$ be a neutrosophic quasi interval semigroup - loop.

Example 1.4.61: Let $B = B_1 \cup B_2 = \{Z_{44}, \times\} \cup \{[0, aI+b] \mid a, b \in \{e, 1, 2, \ldots, 47\}, *, 12\}$ be a quasi neutrosophic quasi interval semigroup - loop.

Example 1.4.62: Let $C = C_1 \cup C_2 = \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, \times\} \cup \{L_{27}(8)\}$ be a quasi interval semigroup - loop.

Now having seen quasi non associative bistructure we now proceed onto give examples of the non associative bistructures viz. loop - groupoids of various types.

Example 1.4.63: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{25}, (3, 8), *\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 45\}, 8, *\}$ be a neutrosophic interval groupoid - loop of finite order.
Example 1.4.64: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{140}, *, (8, 17)\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 15\}, 8, *\}$ be a neutrosophic interval groupoid - loop.

Example 1.4.65: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 19\}, 18, *\} \cup \{[0, aI] \mid a \in Z_{480}, (17, 11), *\}$ be a neutrosophic interval loop - groupoid.

Example 1.4.66: Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_{42}, *, (11, 19)\} \cup \{[0, a] \mid a \in \{e, 1, 2, \ldots, 23\}, *, 18\}$ be a quasi neutrosophic interval groupoid - loop.

Example 1.4.67: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z_{43}, *, (12, 11)\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 43\}, *, 15\}$ be a quasi neutrosophic interval groupoid - loop.

Example 1.4.68: Let $C = C_1 \cup C_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 23\}, *, 8\} \cup \{[0, aI] \mid a \in Z_{41}, *, (3, 8)\}$ be a neutrosophic quasi interval loop - groupoid.

Example 1.4.69: Let $R = R_1 \cup R_2 = \{[0, aI] \mid a \in Z_{48}, *, (8, 1)\} \cup \{[aI] \mid a \in \{e, 1, 2, \ldots, 49\}, 9, *\}$ be a neutrosophic quasi interval groupoid - loop.

Example 1.4.70: Let $N = N_1 \cup N_2 = \{[0, a] \mid a \in Z_{490}, *, (23, 140\} \cup \{[aI] \mid a \in \{e, 1, 2, \ldots, 29\}, 9, *\}$ be a quasi neutrosophic quasi interval groupoid - loop.

Example 1.4.71: Let $V = V_1 \cup V_2 = \{[aI] \mid a \in Z_{489}, *, (19, 29\} \cup \{[0, a] \mid a \in \{e, 1, 2, \ldots, 53\}, 8, *\}$ be a quasi neutrosophic quasi interval groupoid - loop.

Since groupoid - loop is a non associative structure all identities and all properties associated with these structures can be studied with appropriate modifications. We now proceed of to define interval bistructures using matrices and polynomials.

Recall $\sum_{i=0}^{\infty}[0, aI]x^i$ is a neutrosophic interval polynomial in the variable x where $a_i \in Z_n$ or $Z^+ \cup \{0\}$ or $R^+ \cup \{0\}$ or $Q^+ \cup \{0\}$. We can give on the collection of neutrosophic intervals polynomials semigroup structures or group structure or groupoid structure or loop structure when Z_n is used only for semigroup - groupoid structure $Z_I^+ \cup \{0\}$ or $R_I^+ \cup \{0\}$ or $Q_I^+ \cup \{0\}$ coefficients are used as the interval coefficients. We will only give examples [9, 13].

Example 1.4.72: Let $V = V_1 \cup V_2 =$

$$\left\{ \begin{bmatrix} [0, a_1I] \\ [0, a_2I] \\ \vdots \\ [0, a_8I] \end{bmatrix} \middle| a_i \in Z_{40}, + \right\} \cup$$

$$\left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| a_i = [0, x_iI] \text{ where } x_i \in \{e, 1, 2, \ldots, 43\}, 8, * \right\}$$

be a neutrosophic interval semigroup - groupoid.

Example 1.4.73: Let $M = M_1 \cup M_2 =$

$$\left\{ \sum_{i=0}^{\infty}[0, aI]x^i \middle| a \in \{e, 1, 2, \ldots, 19\}*, 8 \right\}$$

$$\cup \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| a_i = [0, x_iI]; x_i \in Z_{42}, + \right\}$$

be a neutrosophic interval groupoid - semigroup.

Example 1.4.74: Let S = $S_1 \cup S_2$ =

$$\left\{\sum_{i=0}^{\infty}[0, a_iI]x^i \,\middle|\, a_i \in Z^+ \cup \{0\}, \times\right\} \cup$$

$$\left\{\left.\begin{bmatrix} a_1 & a_5 \\ a_2 & a_6 \\ a_3 & a_7 \\ a_4 & a_8 \end{bmatrix}\right| a_i = [0, x_iI] \text{ where } x_i \in Z_{20}, (8, 2), *\right\}$$

be a neutrosophic interval semigroup - groupoid.

Example 1.4.75: Let X = $X_1 \cup X_2$ = {[0, aI] | a ∈ {e, 1, 2, …, 31}, 9, *} ∪ $\left\{\sum_{i=0}^{\infty}[0, a_iI]x^i \,\middle|\, a_i \in Z_{20}, *, (3,10)\right\}$ be a neutrosophic interval loop - groupoid.

We can build several types of them like quasi neutrosophic or quasi neutrosophic quasi interval bistructures.

Example 1.4.76: Let L = $L_1 \cup L_2$ = {[0, a] | a ∈ {e, 1, 2, …, 37}, 9, *} ∪ $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}\right| a_i = [0, x_iI] \text{ where } x_i \in Z_{40}, (3, 17), *\right\}$ be a quasi neutrosophic interval loop - groupoid.

Example 1.4.77: Let S = $S_1 \cup S_2$ = {[0, aI] | a ∈ {e, 1, 2, …, 37}, 28, *} ∪ $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \end{bmatrix}\right| a_i = [0, x_iI] \text{ where } x_i \in Z_{37}, (2, 18), *\right\}$ is a neutrosophic interval loop-groupoid.

We can build bistructures in them and work as in case of other interval bistructures.

Chapter Two

NEUTROSOPHIC INTERVAL BIRINGS AND NEUTROSOPHIC INTERVAL BISEMIRINGS

In this chapter we for the first time introduce the notion of neutrosophic interval birings, neutrosophic interval bisemirings, neutrosophic interval bivector spaces and neutrosophic interval bisemivector spaces study and describe their properties. This chapter has three sections. In section one the notion of neutrosophic interval birings are introduced.

Neutrosophic interval bisemirings are introduced in section two. In section three neutrosophic interval bivector spaces and neutrosophic interval bisemivector spaces are introduced and studied.

2.1 Neutrosophic Interval Birings

In this section we introduce the notion of neutrosophic interval birings and study their properties.

DEFINITION 2.1.1: *Let $R = R_1 \cup R_2$ where R_1 and R_2 are distinct with R_i a collection of neutrosophic intervals of the* *special form [0, aI] with a ∈ Z_{n_i} $n_i < \infty$ which is a ring for i=1, 2. We define R = $R_1 \cup R_2$ to be a pure neutrosophic interval biring.*

If instead of [0, aI] we use [0, a+bI]; a, b ∈ Z_n we call R to be a mixed neutrosophic interval biring or just neutrosophic interval biring. Clearly using $Z^+ \cup \{0\}$ or $R^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ in the place of Z_{n_i} will not give R a ring structure.

We will illustrate this situation by some examples.

Example 2.1.1: Let R = $R_1 \cup R_2$ = {[0, aI] | a ∈ Z_{20}, +, ×} ∪ {[0, bI] | b ∈ Z_{42}, +, ×} be a pure neutrosophic interval biring of order 20 × 42.

Clearly R is commutative with [0, I] ∪ [0, I] as its multiplicative identity.

Example 2.1.2: Let R = $R_1 \cup R_2$ = {[0, aI] | a ∈ Z_{19}, +, ×} ∪ {[0, bI] | b ∈ Z_{13}, +, ×} be a pure neutrosophic interval biring of finite order.

Clearly R has no zero divisors. Infact R has no idempotents. R is a neutrosophic interval bifield of order 19 × 13.

Example 2.1.3: Let B = $B_1 \cup B_2$ = {[0, aI] | a ∈ Z_{40}, +, ×} ∪ {[0, bI] | b ∈ Z_{12}, +, ×} be a neutrosophic interval biring. B has bizero divisors. For take

x = [0, 10I] ∪ [0, 3I]

and y = [0, 4I] ∪ [0, 4I] in B.

Clearly x.y = [0] ∪ [0]. B has biunits. For take x = [0, 39I] ∪ [0, 11I] in B; we get x^2 = [0, I] ∪ [0, I]. B has biidempotents. Consider x = [0, 16I] ∪ [0, 4I] in B, we see x^2 = x hence the claim.

Thus these rings have bizero divisors, biunits and biidempotents.

Example 2.1.4: Let M = $M_1 \cup M_2$ = {[0, aI] | a ∈ Z_{40}, +, ×} ∪ {[0, aI] | a ∈ Z_{41}, +, ×} be a neutrosophic interval biring. M has no biidempotents but has biidempotents of the form x = [0, 16I] $\cup$ [0, I] in M, as x^2 = x. We call this type of biidempotents as quasi biidempotents.

M has also quasi binilpotents, for take y = [0, 20I] $\cup$ [0, 0] we see y^2 = [0, 0] $\cup$ [0, 0].

We call a neutrosophic interval biring S = $S_1 \cup S_2$ in which one of S_1 or S_2 is a neutrosophic interval field as a quasi neutrosophic interval bifield.

Example 2.1.5: Let V = $V_1 \cup V_2$ = {[0, aI] | a $\in Z_{24}$, +, $\times$} $\cup$ {[0, aI] | a $\in Z_7$, +, $\times$} be a quasi pure neutrosophic interval bifield.

Inview of this we have the following theorem.

THEOREM 2.1.1: *Let R = $R_1 \cup R_2$ = {[0, aI] | a $\in Z_n$; n a non prime, +, $\times$} $\cup$ {[0, aI] | a $\in Z_p$, p a prime, +, $\times$} be a pure neutrosophic interval biring. R is a pure neutrosophic quasi interval bifield.*

The proof is direct from the very definition.

We can define pure neutrosophic interval subbiring and biideal which is simple hence left as an exercise to the reader.

We give examples of them.

Example 2.1.6: Let R = $R_1 \cup R_2$ = {[0, aI] | a $\in Z_{12}$, +, $\times$} $\cup$ {[0, aI] | a $\in Z_{10}$, +, $\times$} be a pure neutrosophic interval biring. Consider P = $P_1 \cup P_2$ = {[0, aI] | a $\in$ {0, 2, 4, 6, 8, 10} $\subseteq Z_{12}$, $\times$, +} $\cup$ {[0, aI] | a $\in$ {0, 5} $\subseteq Z_{10}$, $\times$, +} $\subseteq R_1 \cup R_2$. Clearly P is a pure neutrosophic interval biring and is nothing but P is a pure neutrosophic interval bisubring of R. We can easily verify that P is infact a biideal of R.

Example 2.1.7: Let R = $R_1 \cup R_2$ = {[0, aI] | a $\in Z_{23}$, +, $\times$} $\cup$ {[0, aI] | a $\in Z_{23}$, +, $\times$} be a pure neutrosophic interval biring. It is a bifield and hence has no ideals.

We can study the notion of quotient birings in case of neutrosophic interval birings.

Let $R = R_1 \cup R_2$ be a pure neutrosophic interval biring. $J = I_1 \cup I_2$ be a pure neutrosophic interval biideal of R. $R/J = R_1/ I_1 \cup R_2/I_2$ is the pure neutrosophic interval quotient biring.

We will illustrate this situation by some examples.

Example 2.1.8: Let $R = R_1 \cup R_2 = \{[0, aI] \mid a \in Z_{12}, +, \times\} \cup \{[0, aI] \mid a \in Z_{15}, +, \times\}$ be a pure neutrosophic interval biring. Let $J = I_1 \cup I_2 = \{[0, aI] \mid a \in \{0, 2, \ldots, 10\} \subseteq Z_{12}, +, \times\} \cup \{[0, aI] \mid a \in \{0, 3, 6, 9, 12\} \subseteq Z_{15}, +, \times\}$ be a pure neutrosophic interval biideal of R.

Consider $R/J = R_1/ I_1 \cup R_2/I_2$.

$= \{I_1, [0,I] + I_1\} \cup \{I_2, [0,I] + I_2, [0, 2I], I_2\}$. We see R/I is a pure neutrosophic interval bifield isomorphic with $\{[0, aI] \mid a \in Z_2, +, \times\} \cup \{[0, aI] \mid a \in Z_3, +, \times\}$. We can also have the quotient rings as pure neutrosophic interval birings.

Consider $J = I_1 \cup I_2 = \{[0, aI] \mid a \in \{0, 4, 8, 12, 16, 20\}, \times, +\} \cup \{[0, aI] \mid a \in \{0, 10, 20\}, +, \times\} \subseteq R_1 \cup R_2$, be a pure neutrosophic biideal of R.

Take $R/J = R_1/ I_1 \cup R_2/I_2 = \{I_1, [0, I] + I_1, [0, 2I] + I_1 [0, 3I] + I_1\} \cup \{I_2, [0, I] + I_2, [0, 2I] + I_2, \ldots., [0, aI] + I_2\}$ to be the quotient interval biring.

Clearly $R/J \cong Z_4 \cup Z_{10}$ and is not a pure neutrosophic interval bifield only a pure neutrosophic interval biring. We cannot define biring structures using $R^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $Z^+ \cup \{0\}$.

We define $S = S_1 \cup S_2$ to be a quasi neutrosophic interval biring if S_1 is a pure neutrosophic interval ring and S_2 is just an interval ring. We will give examples of them.

Example 2.1.9: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{43}, +, \times\} \cup \{[0, a] \mid a \in Z_{42}, +, \times\}$ be a quasi neutrosophic interval biring.

Example 2.1.10: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{11}, +, \times\} \cup \{[0, a] \mid a \in Z_7, +, \times\}$ be a quasi neutrosophic interval bifield.

Example 2.1.11: Let $T = T_1 \cup T_2 = \{[0, a] \mid a \in Z_{43}, +, \times\} \cup \{[0, aI] \mid a \in Z_{240}, +, \times\}$ be a quasi neutrosophic interval quasi bifield.

Example 2.1.12: Let $V = V_1 \cup V_2 = \{[0, a] \mid a \in Z_{24}, +, \times\} \cup \{[0, aI] \mid a \in Z_{23}, +, \times\}$ be a quasi neutrosophic interval quasi bifield.

Quasi neutrosophic interval birings also contain bizero divisors, biunits biidempotents and quotient birings can be constructed using biideals. This is a matter of routine and hence is left as an exercise to the reader.

We see however we can construct neutrosophic quasi interval birings. We call $R = R_1 \cup R_2$ to be a neutrosophic quasi interval biring if R_1 is just a neutrosophic ring and R_2 is a neutrosophic interval ring.

We will give examples of such birings.

Example 2.1.13: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{42}, +, \times\} \cup \{[Z_{15}I, +, \times\}$ be a neutrosophic quasi interval biring. V has bizero divisors and biunits. We see biunit element of V is $[0, I] \cup I$. Consider $x = [0, 41I] \cup 14I$ in V, $x^2 = [0, I] \cup I$. Take $x = [0, 21I] \cup 5I$ and $y = [0, 2I] \cup 3I$ in V $xy = 0 \cup 0$.

If $P = P_1 \cup P_2 = \{[0, aI] \mid a \in 2Z_{42}, +, \times\} \cup \{3Z_{15}I, +, \times\} \subseteq V_1 \cup V_2 = V$; P is a neutrosophic quasi interval bisubring of V as well as neutrosophic quasi interval biideal of V.

Example 2.1.14: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_7, +, \times\} \cup \{Z_{13}I, +, \times\}$ be a pure neutrosophic quasi interval bifield.

M has no biideals or subbirings or bizero divisors.

Example 2.1.15: Let $V = V_1 \cup V_2 = \{Z_{19}I, +, \times\} \cup \{[0, aI] \mid a \in Z_{40}, +, \times\}$ be a neutrosophic quasi interval quasi bifield. This V has only neutrosophic quasi interval quasi biideals, quasi biunits and quasi bizero divisors.

Example 2.1.16: Let $M = M_1 \cup M_2 = \{Z_{25}I, +, \times\} \cup \{[0, aI] \mid a \in Z_{23}, +, \times\}$ be a neutrosophic quasi interval quasi bifield. M has only quasi biideals given by $I = I_1 \cup I_2 = \{5Z_{25}I, +, \times\} \cup \{0\}$. Now we say a biring $S = S_1 \cup S_2$ is a quasi neutrosophic quasi interval biring if one of S_1 or S_2 is just a interval ring.

We will give examples of them.

Example 2.1.17: Let $V = V_1 \cup V_2 = \{Z_{40}, +, \times\} \cup \{[0, aI] \mid a \in Z_{27}, +, \times\}$ be the quasi neutrosophic quasi interval biring.

This biring has biideals, bizero divisors, bisubrings etc.

Example 2.1.18: Let $V = V_1 \cup V_2 = \{Z_{140}I, \times, +\} \cup \{[0, a] \mid a \in Z_{20}, +, \times\}$ be a quasi neutrosophic quasi interval biring.

Example 2.1.19: Let $M = M_1 \cup M_2 = \{Z_{17}I, +, \times\} \cup \{[0, a] \mid a \in Z_{43}, +, \times\}$ be a quasi neutrosophic quasi interval bifield.

Example 2.1.20: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{19}, +, \times\} \cup \{Z_{23}, +, \times\}$ be a quasi neutrosophic quasi interval bifield.

Example 2.1.21: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{13}, +, \times\} \cup \{Z_{45}, +, \times\}$ be a quasi neutrosophic quasi interval quasi bifield. This M has only quasi biideals, quasi biidempotents and quasi biunits.

Example 2.1.22: Let $T = T_1 \cup T_2 = \{Z_{17}, +, \times\} \cup \{[0, aI] \mid a \in Z_{12}, +, \times\}$ be a quasi neutrosophic quasi interval quasi bifield.

Example 2.1.23: Let $P = P_1 \cup P_2 = \{Z_{23}I, +, \times\} \cup \{[0, a] \mid a \in Z_{420}, +, \times\}$ be a quasi neutrosophic quasi interval bifield.

Example 2.1.24: Let $R = R_1 \cup R_2 = \{[0, aI] \mid a \in Z_{53}, +, \times\} \cup \{Z_{425}I, +, \times\}$ be a quasi neutrosophic quasi interval quasi bifield.

Now we can study the notions by replacing the pure neutrosophic intervals [0, aI] by [0, a+bI] and derive interesting results.

We give examples and indicate how it differ from pure neutrosophic bistructures.

Example 2.1.25: Let $R = R_1 \cup R_2 = \{[0, a+bI] \mid a, b \in Z_{40}, \times, +\} \cup \{[0, a+bI] \mid a, b \in Z_{12}, \times, +\}$ be a neutrosophic interval biring. Clearly R is not a pure neutrosophic interval biring but R has pure neutrosophic interval subbiring and just interval biring, given by $P = P_1 \cup P_2 = \{[0, bI] \mid b \in Z_{40}, +, \times\} \cup \{[0, bI]$

| b ∈ Z_{12}, +, ×} ⊆ R is a pure neutrosophic interval subbiring which is also a pure neutrosophic interval biideal.

Take S = $S_1 \cup S_2$ = {[0, a] | a ∈ Z_{40}, +, ×} ∪ {[0, a] | a ∈ Z_{12}, +, ×} ⊆ R, S is a interval subbiring which is not an interval biideal.

Apart from this if we take W = $W_1 \cup W_2$ = {[0, a+bI] | a, b ∈ $2Z_{40}$, +, ×} ∪ {[0, a+bI] | a,b ∈ $2Z_{12}$, +, ×} ⊆ $R_1 \cup R_2$ = R, W is a neutrosophic interval subbiring which is a biideal of R.

Example 2.1.26: Let S = $S_1 \cup S_2$ = {[0, a+bI] | a, b ∈ Z_{246}, ×, +} ∪ {[0, a+bI] | a, b ∈ Z_9, ×, +} be a neutrosophic interval biring. This has bisubrings which are not biideals and bisubrings which are biideals.

Inview of this we have the following theorem.

THEOREM 2.1.2: *Let M = $M_1 \cup M_2$ = {[0, a+bI] | a, b ∈ Z_n, ×, +} ∪ {[0, a+bI] | a, b ∈ Z_m, ×, +} m and n are distinct non prime numbers. M has P = $P_1 \cup P_2$ = {[0, aI] | a ∈ Z_n, ×, +} ∪ {[0, bI] | b ∈ Z_m, ×, +} ⊆ M, P is a subbiring as well as subbiideal.*

But T = $T_1 \cup T_2$ = {[0, a] | a ∈ Z_n, ×, +} ∪ {[0, b] | b ∈ Z_m, ×, +} ⊆ M is only a subbiring and is not a biideal of M.

Proof is straight forward hence left as an exercise to the reader.

Example 2.1.27: Let P = $P_1 \cup P_2$ = {[0, a+bI] | a, b ∈ Z_9, +, ×} 8∪ {[0, a+bI] | a, b ∈ Z_{12}, +, ×} be a neutrosophic interval biring.

This has biideals and bisubrings. Take S = $S_1 \cup S_2$ = {[0, a+bI] | a, b ∈ {0, 3, 6} ⊆ Z_9, ×, +} ∪ {[0, a+bI] | a, b ∈ {0, 4, 8}, +, ×} ⊆ $P_1 \cup P_2$. S is a biideal of P.

Take W = $W_1 \cup W_2$ = {[0, a] | a, b ∈ {0, 3, 6} ⊆ Z_9, ×, +} ∪ {[0, a] | a ∈ {0, 4, 8}, +, ×} ⊆ $P_1 \cup P_2$ = P; W is only a bisubring of P and is not a biideal of P. Thus P has subbirings which are not biideals.

Consider P/S = $P_1/S_1 \cup P_2/S_2$ = {S_1, [0, a+bI] + S_1 | a, b ∈ {0, 1, 2}, +, ×} ∪ {S_2, [0, a+bI] + S_2 | a, b ∈ {0, 1, 2, 3}, +, ×} is a biring. The order of P/S denoted by |P/S| = 9 ∪ 16.

Now $B = B_1 \cup B_2 = \{[0, aI] \mid a \in Z_9, +, \times\} \cup \{[0, bI] \mid b \in Z_{12}, +, \times\} \subseteq P_1 \cup P_2$ is a biideal of P.

Consider $P/B = P_1/B_1 \cup P_2/B_2 = \{B_1, [0,1] + B_1, \ldots, [0, 8] + B_1\} \cup \{B_2, [0,1] + B_2, [0,2] + B_2, \ldots, [0, 11] + B_2\}$ is the quotient neutrosophic interval biring or neutrosophic interval quotient biring.

These neutrosophic interval birings also contain zerobidivisors, biunits etc.

Example 2.1.28: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{11}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{13}, +, \times\}$ be a neutrosophic interval biring. This has biideals. For take $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{11}, +, \times\} \cup \{[0, aI] \mid a \in Z_{13}, +, \times\} \subseteq M_1 \cup M_2 = M$ is neutrosophic interval biideal. The quotient biring $M/P = M_1/P_1 \cup M_2/P_2 = \{P_1, [0,1] + P_1, \ldots, [0, 10] + P_1, +, \times\} \cup \{P_2, [0,1] + P_2, \ldots, [0, 12] + P_2, +, \times\} \cong Z_{11} \cup Z_{13}$.

Studies in this direction can be carried out by the interested reader.

Example 2.1.29: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{12}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{20}, +, \times\}$ be a neutrosophic interval biring. M has biideals, bisubrings, which are not biideals, biunits, bizero divisors and biidempotents.

We can also define $S = S_1 \cup S_2$ where S_1 is pure neutrosophic and S_2 mixed neutrosophic still we call interval biring.

We will give some examples of them.

Example 2.1.30: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{12}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{12}, +, \times\}$ be the neutrosophic interval biring.

Consider $S = S_1 \cup S_2 = \{[0, aI] \mid a \in Z_{14}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_4, +, \times\}$, S is not a neutrosophic interval biring of $S_1 \subseteq S_2$.

Example 2.1.31: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{24}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{49}, +, \times\}$ be the neutrosophic interval biring.

This biring has biideals, bisubrings, bizero divisors and so on.

Example 2.1.32: Let P = $P_1 \cup P_2 = \{Z_5I, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_7, +, \times\}$ be a neutrosophic quasi interval biring.

Example 2.1.33: Let P = $P_1 \cup P_2 = \{[0, a+bI] \mid a, b \in Z_{14}, +, \times\} \cup \{Z_{15}, +, \times\}$ be a quasi neutrosophic quasi interval biring.

We can define special type of neutrosophic interval birings.

Example 2.1.34: Let

$$T = T_1 \cup T_2 = \left\{ \sum_{i=0}^{\infty} [0, aI]x^i \;\middle|\; a \in Z_7, +, \times \right\} \cup \left\{ \sum_{i=0}^{\infty} [0, a+bI]x^i \;\middle|\; a, b \in Z_{10}, +, \times \right\}$$

be a neutrosophic interval polynomial biring.

Example 2.1.35: Let

$$S = S_1 \cup S_2 = \left\{ \sum_{i=0}^{\infty} [0, a]x^i \;\middle|\; a \in Z_{40}, +, \times \right\} \cup \left\{ \sum_{i=0}^{\infty} [0, a+bI]x^i \;\middle|\; a, b \in Z_{20}, +, \times \right\}$$

be a quasi neutrosophic interval polynomial biring.

Example 2.1.36: Let

$$S = S_1 \cup S_2 = \left\{ \sum_{i=0}^{\infty} aIx^i \;\middle|\; a \in Z_{26}, +, \times \right\} \cup \left\{ \sum_{i=0}^{\infty} [0, a+bI]x^i \;\middle|\; a, b \in Z_{40}, +, \times \right\}$$

be a neutrosophic quasi interval polynomial biring.

Example 2.1.37: Let

$$A = A_1 \cup A_2 = \left\{ \sum_{i=0}^{\infty} [0, a+bI]x^i \;\middle|\; a, b \in Z_{28}, +, \times \right\} \cup$$

$$\left\{ \sum_{i=0}^{\infty} [0, a+bI]x^i \;\middle|\; a, b \in Z_{48}, +, \times \right\}$$

be a quasi neutrosophic quasi interval polynomial biring.

Now likewise we can define neutrosophic interval matrix birings.

Example 2.1.38: Let

$$M = M_1 \cup M_2 = \left\{ \begin{bmatrix} [0, a_1 I] & [0, a_2 I] \\ [0, a_3 I] & [0, a_4 I] \end{bmatrix} \;\middle|\; a \in Z_6, +, \times \right\} \cup$$

$\{P = (p_{ij})_{5\times5}$ where $p_{ij} = [0, a_{ij}I]$ with $a_{ij} \in Z_{12}$, $1 \leq i, j \leq 5$, $\times$, $+\}$ be a neutrosophic interval matrix biring.

Example 2.1.39: Let $P = P_1 \cup P_2$ = {all 3×3 neutrosophic interval matrices with intervals of the form [0, a+bI] where a, b $\in Z_{120}$, +, ×} ∪ {all 8×8 neutrosophic interval matrices with intervals of the form [0, aI] with a $\in Z_{48}$, +, ×} be a neutrosophic interval matrix biring.

Example 2.1.40: Let $M = M_1 \cup M_2$ = {all 10×10 neutrosophic interval matrices with intervals of the form [0, a+bI] where a, b $\in Z_{27}$, +, ×} ∪ {all 6×6 neutrosophic interval matrices with intervals of the form [0, a] with a $\in Z_{48}$, +, ×} be a quasi neutrosophic interval matrix biring.

Example 2.1.41: Let $P = P_1 \cup P_2$ = {all 3×3 interval matrices with intervals of the form [0, a] where a $\in Z_{40}$, +, ×} ∪ {all 10×10 neutrosophic interval matrices with intervals of the form [0, aI] with a $\in Z_{12}$, +, ×} be a quasi neutrosophic interval matrix biring.

Example 2.1.42: Let $T = T_1 \cup T_2$ = {All 5×5 matrices with pure neutrosophic entries from $Z_{29}I$} ∪ {all 20×20 neutrosophic interval matrices with intervals of the form [0, a+bI], a, b $\in Z_{12}$, +, ×} be the neutrosophic quasi interval matrix biring.

Example 2.1.43: Let P = $P_1 \cup P_2$ = {All 3×3 matrices with entries form Z_{42}, +, ×} ∪ {all 2×2 neutrosophic interval matrices with intervals of the form [0, a+bI] where a, b $\in Z_{20}$, +, ×} be a quasi neutrosophic quasi interval matrix biring.

For these special type of birings also bisubstructures, bizero divisors, biunits etc can be defined and studied as a matter of routine.

2.2 Neutrosophic Interval Bisemirings

In this section we define the notion of neutrosophic interval bisemirings. It is important to note that in case of neutrosophic interval birings we could not use $Z^+ \cup \{0\}$ or $R^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ as they are not rings but in case of neutrosophic interval bisemirings we can make use of these positive reals, positive rationals and positive integers apart from the modulo integers Z_n.

DEFINITION 2.2.1: *Let P = $P_1 \cup P_2$ = {[0, aI] | a $\in Z_n$ or $Z^+ \cup \{0\}$ or $R^+ \cup \{0\}$ or $Q^+ \cup \{0\}$} ∪ {[0, aI] | a $\in Z_n$ or $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$} (or used in the mutually exclusive sense).*

P_1 and P_2 are closed with respect + and ×. So P_1 and P_2 are interval semirings. If $P_1 \neq P_2$ or $P_1 \not\subseteq P_2$ or $P_2 \not\subseteq P_1$ then we define P to be a neutrosophic interval bisemiring.

In other words P = $P_1 \cup P_2$ where P_1 and P_2 are two distinct neutrosophic interval semirings, then P is defined as the neutrosophic interval bisemiring.

We give examples of them.

Example 2.2.1: Let V = $V_1 \cup V_2$ = {[0, aI] | a $\in Z^+ \cup \{0\}$, +, ×} ∪ {[0, aI] | a $\in Z_9$, +, ×} be a pure neutrosophic interval bisemiring.

Example 2.2.2: Let T = $T_1 \cup T_2$ = {[0, aI] | a $\in R^+ \cup \{0\}$, +, ×} ∪ {[0, aI] | a $\in Z_{42}$, +, ×} be a pure neutrosophic interval bisemiring. Elements in T will be of the form $[0, aI] \cup [0, bI]$ where $a \in R^+ \cup \{0\}$ and $b \in Z_{42}$.

Note: When we use $\{[0, aI] \mid a \in Z_9, +, \times\}$ as a semiring '+' denotes max and '×' denotes min operation.

Example 2.2.3: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{42}, +, \times\} \cup \{[0, bI] \mid b \in Z_{27}, +, \times\}$ be the pure neutrosophic interval bisemiring.

Example 2.2.4: Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in 3Z^+ \cup \{0\}, +, \times\} \cup \{[0, aI] \mid a \in 5Z^+ \cup \{0\}, +, \times\}$ be the pure neutrosophic interval bisemiring.

Example 2.2.5: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in 9Z^+ \cup \{0\}, +, \times\} \cup \{[0, aI] \mid a \in 8Z^+ \cup \{0\}, +, \times\}$ be the neutrosophic interval bisemiring of infinite order.

Now having seen neutrosophic interval bisemiring we can define bisubstructures in them [8].

We see in general the interval bisemirings are interval bisemifields. This is due to the fact that $Z^+ \subseteq Q^+ \subseteq R^+$ so we cannot define bisemifields which are distinct. Further if the entries are from Z_n we see they are not strict interval bisemirings hence cannot be bisemifields.

We can of course define mixed neutrosophic interval bisemirings.

We will give only examples of this structure.

Example 2.2.6: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{20}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{49}, +, \times\}$ be a neutrosophic interval bisemiring.

Example 2.2.7: Let $P = P_1 \cup P_2 = \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{25}, +, \times\}$ be the neutrosophic interval bisemiring.

Example 2.2.8: Let $V = V_1 \cup V_2 = \{[0, a+bI] \mid a, b \in 3Z^+ \cup \{0\}, +, \times\} \cup \{[0, aI] \mid a \in R^+ \cup \{0\}, +, \times\}$ be a neutrosophic interval bisemiring. V is not a bisemifield.

$M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in 9Z^+ \cup \{0\}, +, \times\} \cup \{[0, aI] \mid a \in Q^+ \cup \{0\}, +, \times\} \subseteq V_1 \cup V_2$ is a neutrosophic interval bisubsemiring.

Take $T = T_1 \cup T_2 = \{[0, a] \mid a \in 3Z^+ \cup \{0\}, +, \times\} \cup \{[0, aI] \mid a \in Q^+ \cup \{0\}, +, \times\} \subseteq V_1 \cup V_2$; T is a quasi neutrosophic interval bisubsemiring of V. V is not a Smarandache neutrosophic interval bisemiring.

Example 2.2.9: Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times, +\} \cup \{[0, aI] \mid a \in Z_{45}, +, \times\}$ be a neutrosophic interval bisemiring.

Example 2.2.10: Let $B = B_1 \cup B_2 = \{[0, aI] \mid a \in Z_7, +, \times\} \cup \{[0, aI] \mid a \in Z_5, +, \times\}$ be a neutrosophic interval bisemiring of finite order which is not a bisemifield.

We can as in case of brings define the notion of quasi neutrosophic interval bisemirings and neutrosophic quasi interval bisemirings.

We give only examples of them.

Example 2.2.11: Let $C = C_1 \cup C_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}\} \cup \{[0, a] \mid a \in R^+ \cup \{0\}\}$ be a quasi neutrosophic interval bisemiring. Clearly C is a quasi neutrosophic interval bisemifield. $[0, I] \cup [0, 1]$ is the biidentity element of C with respect of multiplication.

Example 2.2.12: Let $C = C_1 \cup C_2 = \{[0, a + bI] \mid a, b \in Z^+ \cup \{0\}, +, \times\} \cup \{[0, a] \mid a \in Q^+ \cup \{0\}, \times, +\}$ be a quasi neutrosophic interval bisemiring. C is a quasi neutrosophic interval bisemifield.

Example 2.2.13: Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{25}, +, \times\} \cup \{[0, a] \mid a \in Z^+ \cup \{0\}, +, \times\}$ be a quasi neutrosophic interval bisemiring but is not a bisemifield. Infact M is a quasi neutrosophic interval quasi bifield.

Example 2.2.14: Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{20}, +, \times\} \cup \{[0, aI] \mid a \in Z^+ \cup \{0\}, +, \times\}$ be a neutrosophic interval bisemiring of finite order. M is a neutrosophic interval quasi bifield.

Example 2.2.15: Let $M = M_1 \cup M_2 = \{Z+ \cup \{0\}, +, \times\} \cup \{[0, aI] \mid a \in Z_{45}, +, \times\}$ be a quasi neutrosophic quasi interval quasi bifield.

Example 2.2.16: Let $M = M_1 \cup M_2 = \{Z^+I \cup \{0\}, +, \times\} \cup \{[0, aI] \mid a \in Z_{20}, +, \times\}$ be a neutrosophic quasi interval quasi bifield.

Example 2.2.17: Let $V = V_1 \cup V_2 = \{Z^+I \cup \{0\}, +, \times\} \cup \{[0, a] \mid a \in Z_{40}, +, \times\}$ be a quasi neutrosophic quasi interval bisemiring. Clearly V is a quasi neutrosophic quasi interval quasi field.

Example 2.2.18: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{20}, +, \times\} \cup \{Z^+ \cup \{0\}, \times, +\}$, V is also quasi neutrosophic quasi interval quasi bifield.

Now having seen examples of interval bisemirings and their generalization, we leave it for the reader to prove related results and properties associated with bisemirings [8].

Example 2.2.19: Let $L = L_1 \cup L_2 = \{Q^+ \cup \{0\}, \times, +\} \cup \{[0, a] \mid a \in R^+ \cup \{0\}, +, \times\}$ be a quasi neutrosophic quasi interval biring. Infact L is a quasi neutrosophic quasi interval bifield which has subbifields.

Example 2.2.20: Let $M = M_1 \cup M_2 = \{Z^+ \cup \{0\}, +, \times\} \cup \{[0, aI] \mid a \in Z_{25}, +, \times\}$ be a quasi interval quasi neutrosophic quasi bifield.

Now we can construct bistructures using neutrosophic interval rings and semirings.

Let $L = L_1 \cup L_2$ where L_1 is a neutrosophic interval ring and L_2 is a neutrosophic interval semiring. We define $L = L_1 \cup L_2$ to be a neutrosophic interval ring-semiring.

We will give some examples of them.

Example 2.2.21: Let $W = W_1 \cup W_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times, +\} \cup \{[0, aI] \mid a \in Z_{20}, +, \times\}$ be a neutrosophic interval semiring - ring of infinite order. (Here $Z_{20}, +, \times$ is modulo addition and multiplication)

Example 2.2.22: Let $M = M_1 \cup M_2 = \{[0, aI+b] \mid a, b \in Z_{40}, +, \times\} \cup \{[0, a+bI] \mid a, b \in R^+ \cup \{0\}, +, \times\}$ be a neutrosophic interval ring - semiring.

Example 2.2.23: Let $W = W_1 \cup W_2 = \{[0, aI] \mid a \in Z_{51}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, +, \times\}$ be the neutrosophic interval ring - semiring.

We can use distributive lattices for semirings as all distributive lattices are semirings.

Example 2.2.24: Let $L = L_1 \cup L_2 = \{[0, aI]$, aI in chain lattice $C_n = \{0 < a_1I < a_2I \ldots < a_nI\}\} \cup \{[0, aI] \mid a \in Z_{40}, +, \times\}$ be a neutrosophic interval semiring - ring.

Example 2.2.25: Let

$$L = L_1 \cup L_2 = \left\{\begin{bmatrix}[0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I\end{bmatrix} \middle| a_i \in Z_{42},+,\times;1 \leq i \leq 4\right\} \cup$$

$\{([0, a_1I], \ldots, [0, a_{10}I]) \mid a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 10, +, \times\}$ be a neutrosophic interval semiring - ring.

Example 2.2.26: Let

$$L = L_1 \cup L_2 = \left\{\sum_{i=0}^{\infty}[0,a_i]x^i \middle| a_i \in R^+ \cup \{0\}\right\} \cup$$

$$\left\{\sum_{i=0}^{\infty}[0,a_i]x^i \middle| a_i \in Z_{17}\right\}$$

be a neutrosophic interval semiring - ring.

Example 2.2.27: Let M = $M_1 \cup M_2$ = {all 10 × 10 interval matrices with intervals of the form {[0, aI] where a ∈ $Z^+ \cup$ {0}} $\cup$ $\left\{\sum_{i=0}^{\infty}[0, aI]x^i \,\middle|\, a_i \in Z_{25}\right\}$ be the neutrosophic interval semiring - ring.

Substructures can be defined in these cases which is a matter of routine. We will give one or two examples before we proceed onto give generalized forms of neutrosophic interval ring - semiring.

Example 2.2.28: Let

$$V = V_1 \cup V_2 = \left\{\sum_{i=0}^{\infty}[0, aI]x^i \,\middle|\, a_i \in R^+ \cup \{0\}, +, \times\right\} \cup$$

{([0, aI], [0, bI], [0, cI], [0, dI]) | a, b, c, d ∈ Z_{240}, +, ×} be a neutrosophic interval semiring - ring. Consider

$$W = W_1 \cup W_2 = \left\{\sum_{i=0}^{\infty}[0, aI]x^i \,\middle|\, a_i \in Q^+ \cup \{0\}, +, \times\right\} \cup$$

{([0, aI], 0, 0, [0, bI]) | a, b ∈ Z_{240}, +, ×} ⊆ $V_1 \cup V_2$. W is a neutrosophic interval subsemiring - subring of V.

Example 2.2.29: Let M = $M_1 \cup M_2$ = {([0, a_1I], …, [0, a_{12}I]) | a_i ∈ $R^+ \cup$ {0}, 1 ≤ i ≤ 12, +, × } ∪ {([0, aI], [0, bI]) | a, b ∈ Z_{25}, +, ×} be a neutrosophic interval semiring - ring. Consider S = $S_1 \cup S_2$ = {([0, a_1I], 0, 0, 0, 0, 0, 0, [0, a_8I], 0, 0, 0, [0, a_{12}I], where a_i ∈ $R^+ \cup$ {0}, i = 1, 8, 12} ∪ {([0, aI), [0, bI] | a, b ∈ {0, 5, 10, 15, 20} ⊆ Z_{25}} ⊆ $M_1 \cup M_2$ is a neutrosophic interval subsemiring - subring of M, which is also a biideal of M. In general all subsemiring-subring of M need not be biideals of M.

For take T = $T_1 \cup T_2$ = {([0, a_1I], …, [0, a_{12}I]) | a_1, …, a_{12} ∈ $Z^+ \cup$ {0}} ∪ {([0, aI], 0) | a ∈ Z_{25}} ⊆ $M_1 \cup M_2$ = M is only a neutrosophic interval subsemiring - subring and not a biideal of M.

We can define bizero divisor, biidempotents etc in case of neutrosophic interval semiring - ring. We can also define the notion of Smarandache concepts of these bistructures. All of them are direct and hence left as an exercise to the reader. We now proceed give examples of quasi bistructure which can be easily understood by the reader.

Example 2.2.30: Let $V = V_1 \cup V_2 = \{[0, a] \mid a \in Z^+ \cup \{0\}\} \cup \{[0, aI] \mid a \in Z_{40}\}$ be the quasi neutrosophic interval semiring - ring.

Example 2.2.31: Let $R = R_1 \cup R_2 = \{([0, a_1], [0, a_2], \ldots, [0, a_9]) \mid a_i \in Z_{420}, 1 \leq i \leq 9\} \cup \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Q^+ \cup \{0\}\right\}$ be a quasi neutrosophic quasi interval ring - semiring.

Example 2.2.32: Let

$$M = M_1 \cup M_2 = \left\{\sum_{i=0}^{\infty} [0, aI]x^i \,\middle|\, a \in Z^+ \cup \{0\}\right\} \cup$$

$\{[0, a] \mid a \in Z_{200}\}$ be a quasi neutrosophic interval semiring - ring.

Example 2.2.33: Let

$$T = T_1 \cup T_2 = \left\{\sum_{i=0}^{\infty} a_i x^i \,\middle|\, a_i \in Z_{30}\right\} \cup$$

$$\left\{\sum_{i=0}^{\infty} [0, aI]x^i \,\middle|\, a \in Z^+ \cup \{0\}\right\}$$

be a quasi neutrosophic quasi interval ring - semiring.

Example 2.2.34: Let

$$W = W_1 \cup W_2 = \left\{\sum_{i=0}^{\infty} [0, a_i I]x^i \,\middle|\, a \in Z^+ \cup \{0\}\right\} \cup$$

{All 10×10 neutrosophic interval matrices with intervals of the form $[0, aI]$ with $a \in Z_{100}$} be a neutrosophic quasi interval semiring - ring.

Example 2.2.35: Let

$$T = T_1 \cup T_2 = \left\{ \sum_{i=0}^{\infty} [0, aI]x^i \,\middle|\, a \in Q^+ \cup \{0\} \right\} \cup \left\{ \sum_{i=0}^{\infty} aIx^i \,\middle|\, a \in Z_{28} \right\}$$

be a neutrosophic quasi interval semiring - ring.

Now having seen examples of these we can define subbistructure, Smarandache notions on them and study them; which can be thought as a matter of routine. We can also define all properties related with rings and semirings on these bistructures with appropriate modifications. We leave all these task to the reader. We will be using these bistructures to build bivector spaces, bisemivector spaces and vector space - semivector space.

Example 2.2.36: Let

$$V = V_1 \cup V_2 = \left\{ \sum_{i=0}^{\infty} [0, a_i]x^i \,\middle|\, a_i \in Q^+ \cup \{0\} \right\} \cup$$

$\{[0, aI] \mid a \in Z_{24}\}$ be a quasi neutrosophic interval semiring – ring. This has no bizero divisors but has quasi bizero divisors given by $0 \cup [0, 12I] = x$ and $y = 0 \cup [0, 2I] \in V$, we see $xy = 0 \cup 0$. Likewise quasi biunits given by $x = [0, 1] \cup [0, 23I] \in V$ is such that $x^2 = [0, 1] \cup [0, I]$. It is clearly $[0, 1] \cup [0, I]$ is the identity bielement of V.

We see V has only quasi biidempotents for $x = [0, 1] \cup [0, 16I] \in V$ such that $x^2 = x$. V has also binilpotents. Consider $x = [0, 0] \cup [0, 12I]$ in V. We see $x^2 = [0, 0] \cup [0, 0]$ as $[0, 12I]$ $[0, 12I] = [0, 144\, I] = [0, 0]$ (mod 24).

Thus we can have notion of quasi Smarandache bizero divisors, Smarandache biidempotents, Smarandache biunits and so on.

2.3 Neutrosophic Interval Bivector Spaces and their Generalization

In this section we for the first time we define the notion of neutrosophic interval bivector spaces, neutrosophic interval bisemivector spaces give their generalization. We also describe some of their properties associated with them.

DEFINITION 2.3.1: *Let $V = V_1 \cup V_2$ be a neutrosophic interval commutative bigroup under addition. Let F be a field if V_i is a vector space over F for i = 1, 2 then, we define V to be neutrosophic interval bivector space over the field F.*

It is important to mention here that $F = Z_p$, p a prime for we see none of our bigroups can take intervals from $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$ as they are not groups under addition. Thus when we speak of neutrosophic interval bivector spaces we only take over the field Z_p, p a prime.

We give examples of them.

Example 2.3.1: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_5, +\} \cup$

$$\left\{\begin{bmatrix}[0,aI]\\ [0,bI]\\ [0,cI]\end{bmatrix} \middle| \; a, b, c \in Z_5, +\right\}$$

be an additive abelian bigroup. V is a pure neutrosophic interval bigroup over the field $Z_5 = F$.

Example 2.3.2: Let $M = M_1 \cup M_2 = \{[0, aI + b] \mid a, b \in Z_7, +\} \cup \{([0, aI], [0, bI], [0, cI], [0, dI]) \mid a, b, c, d \in Z_7, +\}$ be a neutrosophic interval bigroup; V is a neutrosophic interval bivector space over the field $F = Z_7$.

Example 2.3.3: Let

$$V = V_1 \cup V_2 = \left\{\sum_{i=0}^{21}[0,aI]x^i \middle| \; a \in Z_{13}, +\right\} \cup$$

$$\left\{\sum_{i=0}^{40}[0, aI]x^{2i} \middle| a \in Z_{13}, +\right\}$$

be a neutrosophic interval bigroup. V is a neutrosophic interval bivector space over the field Z_{13}.

Example 2.3.4: Let M = $M_1 \cup M_2$ =

$$\left\{\begin{bmatrix} [0,aI] & [0,bI] & [0,cI] & [0,dI] \\ [0,eI] & [0,fI] & [0,gI] & [0,hI] \\ [0,jI] & [0,iI] & [0,kI] & [0,lI] \end{bmatrix} \middle| a,b,c,\ldots,k,l \in Z_{23}, +\right\} \cup$$

{([0, aI], [0, bI], …, [0, tI]) | a, b, …, t ∈ Z_{23}, +} be a pure neutrosophic interval bivector space over Z_{23}.

Example 2.3.5: Let M = $M_1 \cup M_2$ = {all neutrosophic interval 3×7 matrices with intervals of the form [0, aI] where a ∈ Z_{43}} $\cup$ $\left\{\sum_{i=0}^{70}[0, a+bI]x^{i} \middle| a, b \in Z_{43}\right\}$ be a neutrosophic interval bivector space over the field Z_{43}.

Example 2.3.6: Let

$$M = M_1 \cup M_2 = \left\{\sum_{i=0}^{20}[0, aI]x^{i} \middle| a \in Z_{23}\right\} \cup$$

{([0, aI], [0, bI], [0, cI]) | a, b, c ∈ Z_{23}}; M is a pure neutrosophic interval bivector space over the field F = Z_{23}.

Infact bidimension of M over F is (21 ∪ 3). The bibasis of M over F is given by {[0, I], [0, I]x, [0, I] x^2, …, [0, I]x^{20}} ∪ {([0, I], 0, 0), (0, [0, I], 0), (0, 0, [0, I])}. Take

$$T = \left\{\sum_{i=0}^{10}[0, aI]x^{i} \middle| a \in Z_{23}\right\} \cup$$

{([0, aI], [0, bI], 0) | a, b ∈ Z_{23}} ⊆ M; T is of bidimension 11 ∪ 2 and a bibasis for T is given by B_1 = {[0, I], [0, I] x, …, [0, I] x^{10}} ∪ {[0, I], 0, 0), (0, [0, I], 0)}. Thus T is a neutrosophic interval subbivector space of bidimension 11 ∪ 2 and B_1 is a bibasis of T.

It is important to note that the basis of interval bivector spaces are intervals and not elements of $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$ or Z_p. Infact the base elements do not belong to Z_p, which is the case of usual bivector spaces.

Example 2.3.7: Let $V = V_1 \cup V_2 =$

$$\left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_7I] & [0,a_8I] & [0,a_9I] \\ [0,a_{10}I] & [0,a_{11}I] & [0,a_{12}I] \end{bmatrix} \right| a_i \in Z_{47}, 1 \leq i \leq 12 \right\} \cup$$

$$\left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \end{bmatrix} \right| a_i \in Z_{47}, 1 \leq i \leq 4 \right\}$$

be a pure neutrosophic interval vector space over the field Z_{47}. V is a bidimension 12 ∪ 4 and a interval bibasis for V is given by B =

$$\left\{ \begin{bmatrix} [0,I] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & [0,I] & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & [0,I] \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \right.$$

$$\begin{bmatrix} 0 & 0 & 0 \\ [0,I] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & [0,I] & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & [0,I] \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ [0,I] & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & [0,I] & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & [0,I] \\ 0 & 0 & 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ [0,I] & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & [0,I] & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & [0,I] \end{bmatrix} \Bigg\} \cup$$

$$\left\{ \begin{bmatrix} [0,I] & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & [0,I] \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ [0,I] & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & [0,I] \end{bmatrix} \right\}$$ is a bibasis of V over Z_{47}.

$$\text{Let } J = \left\{ \begin{bmatrix} [0,a_1I] & 0 & 0 \\ 0 & [0,a_2I] & 0 \\ 0 & 0 & [0,a_3I] \\ 0 & [0,a_4I] & 0 \end{bmatrix} \middle| a_i \in Z_{47}, 1 \le i \le 4 \right\} \cup$$

$$\left\{ \begin{bmatrix} [0,a_1I] & [0,a_2I] \\ 0 & [0,a_3I] \end{bmatrix} \middle| a_i \in Z_{47}, 1 \le i \le 3 \right\} \subseteq V_1 \cup V_2,$$

J is pure neutrosophic interval bivector subspace of V over the field Z_{43}.

The bidimension of J is $\{4\} \cup \{3\}$. The interval bibasis of J over Z_{43} is given by

$C = C_1 \cup C_2 =$

$$\left\{ \begin{bmatrix} [0,I] & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & [0,I] & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & [0,I] \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & [0,I] & 0 \end{bmatrix} \right\}$$

$$\cup \left\{ \begin{bmatrix} [0,I] & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & [0,I] \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & [0,I] \end{bmatrix} \right\} \subseteq V_1 \cup V_2,$$ is a bibasis of C over Z_{43}. The bidimension of C is $\{4\} \cup \{3\}$. Here also we see the bibase elements of C is only intervals.

Example 2.3.8: Let V = $V_1 \cup V_2$ =

$$\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] & [0,a_4I] \\ [0,a_5I] & [0,a_6I] & [0,a_7I] & [0,a_8I] \end{bmatrix} \middle| a_i \in Z_5, 1 \le i \le 8\right\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ [0,a_3I] \\ [0,a_4I] \\ [0,a_5I] \\ [0,a_6I] \\ [0,a_7I] \end{bmatrix} \middle| a_i \in Z_5, 1 \le i \le 7\right\}$$

be a pure neutrosophic interval bivector space over Z_5.

$$P = \left\{\begin{bmatrix} 0 & [0,a_1I] & 0 & [0,a_2I] \\ [0,a_3I] & 0 & [0,a_4I] & 0 \end{bmatrix} \middle| a_i \in Z_5, 1 \le i \le 4\right\}$$

$$\cup \left\{\begin{bmatrix} [0,a_1I] \\ 0 \\ [0,a_2I] \\ 0 \\ [0,a_3I] \\ 0 \\ [0,a_4I] \end{bmatrix} \middle| a_i \in Z_5, 1 \le i \le 4\right\} \subseteq V_1 \cup V_2 = V$$

is a pure neutrosophic interval bivector subspace of V over Z_5.

The bidimension of V is {8} ∪ {7} and a interval bibasis of V is

B =

$$\left\{\begin{bmatrix} [0,I] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & [0,I] & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & [0,I] & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\right.$$

$$\begin{bmatrix} 0 & 0 & 0 & [0,I] \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ [0,I] & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & [0,I] & 0 & 0 \end{bmatrix},$$

$$\left.\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & [0,I] & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & [0,I] \end{bmatrix}\right\} \cup$$

$$\left\{\begin{bmatrix} [0,I] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ [0,I] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ [0,I] \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ [0,I] \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ [0,I] \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ [0,I] \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ [0,I] \end{bmatrix}\right\}.$$

Now an interval bibasis of P is as follows:

$$C = \left\{\begin{bmatrix} 0 & [0,I] & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & [0,I] \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ [0,I] & 0 & 0 & 0 \end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & [0,I] & 0 \end{bmatrix}\right\} \cup \left\{\begin{bmatrix} [0,I] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ [0,I] \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ [0,I] \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ [0,I] \end{bmatrix}\right\} \subseteq B \text{ is}$$

an interval bibasis of P over Z_5.

Now we can define the notion of interval bilinear transformation of neutrosophic interval bivector spaces defined over the same field F.

Let V and W be any two neutrosophic interval bivector spaces defined over the field F. Let $T = T_1 \cup T_2 : V \to W$ that is $T = T_1 \cup T_2 : V_1 \cup V_2 \to W_1 \cup W_2$ where $T_i : V_i \to W_i$ is a linear interval vector transformation. The only condition being on T_i in case of neutrosophic interval linear transformation is that T ([0, aI]) $\to$ [0, bI], $b \neq 0$ that $aI \mapsto bI$ a can be equal to b but $b \neq 0$ for every $a \neq 0$; i=1, 2.

We will give some examples of them.

Example 2.3.9: Let $V = V_1 \cup V_2 = \{([0, aI], [0, bI], [0, cI], [0, dI]) \mid a, b, c, d \in Z_{23}\} \cup \left\{\begin{bmatrix}[0,a_1I]\\ [0,a_2I]\\ [0,a_3I]\end{bmatrix} \middle| a_1, a_2, a_3 \in Z_{23}\right\}$ and $W = W_1 \cup W_2 = \left\{\begin{bmatrix}[0,a_1I] & [0,a_2I]\\ [0,a_3I] & [0,a_4I]\end{bmatrix} \middle| a_i \in Z_{23}, 1 \le i \le 4\right\} \cup \{([0, a_1I], [0, a_2I], \ldots, [0, a_6I]) \mid a_i \in Z_{23}, 1 \le i \le 6\}$ be two pure neutrosophic interval bivector spaces over the field $F = Z_{23}$. Define $T = T_1 \cup T_2 : V = V_1 \cup V_2 \to W = W_1 \cup W_2$ as follows.

$T_1 : V_1 \to W_1$ and

$T_2 : V_2 \to W_2$ is defined by $T_1 ([0, aI], [0, bI], [0, cI], [0, dI]) = \left(\begin{bmatrix}[0,aI] & [0,bI]\\ [0,cI] & [0,dI]\end{bmatrix}\right)$ and $T_2\left(\begin{bmatrix}[0,a_1I]\\ [0,a_2I]\\ [0,a_3I]\end{bmatrix}\right) = ([0, a_1I], 0, [0, a_2I], 0, [0, a_3I], 0)$. $T = T_1 \cup T_2$ is a bilinear transformation of V to W.

We can define bikernel etc as in case of usual vector spaces.

The following theorem is simple and direct and hence left as an exercise to the reader.

THEOREM 2.3.1 : *Let $V = V_1 \cup V_2$ and $W = W_1 \cup W_2$ be neutrosophic interval bivector spaces defined over the same field F. Let T be a linear bitransformation from V into W. Suppose V is finite dimensional then*

birank T + binullity T = dim $V_1 \cup$ dim V_2 = bidim V.

that is (rank $T_1 \cup$ rank T_2) + (nullity $T_1 \cup$ nullity T_2) = dim $V_1 \cup$ dim V_2 = (rank T_1 + nullity T_1) $\cup$ (rank T_2 + nullity T_2) = dim $V_1 \cup$ dim V_2

Further for any two neutrosophic interval bivector spaces $V = V_1 \cup V_2$ and $W = W_1 \cup W_2$ defined over the field F if T_1 and T_2 are linear bitransformations of V into W then

$(T_1 + T_2)$ is again a linear bitransformation of V into W defined by

$(T_1 + T_2)(\alpha \cup \beta) = T_1(\alpha \cup \beta) + T_2(\alpha \cup \beta)$
where $T_1 = T_1^1(\alpha) + T_1^2(\beta) \cup T_1^2(\alpha) + T_2^2(\beta)$.

Let V and W be neutrosophic interval bivector spaces defined over the same field F. We say $T : V \to W$, a linear bitransformation from V to W to be invertible if there exists a linear bitransformation U such that UT is the identity linear bitransformation (bifunction) on V and TU is identity linear bifunction (bitransformation) on W.

In other words if $T = T_1 \cup T_2 : V_1 \cup T_2 \to W_1 \cup W_2$ and $U = U_1 \cup U_2 : V_1 \cup V_2 \to W_1 \cup W_2$ where $T_1 : V_1 \to W_1$, $T_2 : V_2 \to W_2$

$U_1 : V_1 \to W_1$ and $U_2 : V_2 \to W_2$ with $T_1 U_1$ is the identity function on V_1 and T_2U_2 is the identity function on V_2. That is $T_1 U_1 \cup T_2U_2$ is the identity bifunction on $V = V_1 \cup V_2$ and similarly $U_1 T_1 \cup U_2 T_2$ is the identity bifunction on $W = W_1 \cup W_2$. We denote U by T^{-1} that is $U = U_1 \cup U_2 = T_1^{-1} \cup T_2^{-1}$.

Thus we say a linear bitransformation $T = T_1 \cup T_2$ is invertible if and only if

(i) T is a one to one that is $T(\alpha \cup \beta) = (T_1 \cup T_2)(\alpha \cup \beta) = T_1(\alpha) \cup T_2(\beta) = T(a \cup b) = (T_1 \cup T_2)(a \cup b) = T_1(a) \cup T_2(b)$ implies $\alpha \cup \beta = a \cup b$.

(ii) T is onto, that is the range of T $(=T_1 \cup T_2)$ is also $W = W_1 \cup W_2$. We say $T : V \to W$ that is $T = T_1 \cup T_2 : V = V_1 \cup V_2 \to W = W_1 \cup W_2$ is the linear bitransformation for which $T^{-1} = T_1^{-1} \cup T_2^{-1}$ exists that is T is non singular if $T\alpha = T_1(\alpha_1) \cup T_2(\alpha_2)$ $(\alpha = \alpha_1 \cup \alpha_2) = 0 \cup 0$ implies $\alpha = \alpha_1 \cup \alpha_2 = 0 \cup 0$.

Further if both $V = V_1 \cup V_2$ and $W = W_1 \cup W_2$ are finite dimensional pure neutrosophic interval bivector spaces over the field F such that bidim V = bidim W = dim $V_1 \cup$ dim V_2 = dim $W_1 \cup$ dim W_2. If $T = T_1 \cup T_2$ is a linear bitransformation from V into W, the following are equivalent.

(i) $T = T_1 \cup T_2$ is biinvertible.
(ii) T is non-bisingular (i.e., T_1 and T_2 are non singular).
(iii) $T = T_1 \cup T_2$ is onto that is birange of T is $W = W_1 \cup W_2$.

We see all our neutrosophic interval bivector spaces can only be over the field Z_p, of prime characteristic p i.e., over finite fields.

Further all results and properties true in case of vector spaces over finite characteristic fields is true and can be proved with simple and appropriate modifications.

Let $T = T_1 \cup T_2 : V_1 \cup V_2 \to W_1 \cup W_2$ be a linear bitransformation of the neutrosophic interval bivector spaces $V = V_1 \cup V_2$ into the neutrosophic interval bivector space $W = W_1 \cup W_2$ defined over the field F. If we take $W = W_1 \cup W_2$ to be the same as $V = V_1 \cup V_2$ that is V = W then we define the linear bitransformation T to be a linear bioperator on V.

We will give some examples of linear bioperators.

Example 2.3.10: Let

$$V = V_1 \cup V_2 = \left\{ \left. \begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \\ [0,a_5I] & [0,a_6I] \end{bmatrix} \right| a_i \in Z_{29}, 1 \le i \le 6 \right\} \cup$$
$$\left\{ \left. \sum_{i=0}^{5} [0,a_i]x^i \right| a_i \in Z_{29}; 0 \le i \le 5 \right\}$$

be a neutrosophic interval bivector space over the field Z_{29}.

Let $T = T_1 \cup T_2 : V = V_1 \cup V_2 \to V = V_1 \cup V_2$ defined by

$T(v) = T(v_1 \cup v_2) = T_1(v_1) \cup T_2(v_2)$

where $T_1 \left(\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \\ [0,a_5I] & [0,a_6I] \end{bmatrix} \right) = \begin{bmatrix} [0,a_1I] & 0 \\ 0 & [0,a_2I] \\ [0,a_3I] & 0 \end{bmatrix}$ and

$$T_2 \left(\sum_{i=0}^{5} [0,a_iI]x^i \right) = [0, a_0I] + [0, a_2I]x^2 + [0, a_4I]x^4$$

is a linear bioperator on $V = V_1 \cup V_2$. We can derive almost all properties regarding bioperators with appropriate modifications. We can define the notion of characteristic bivalue and characteristic bivector. Here it is pertinent to mention that the characteristic value c will be of the form [0, aI] where $a \in Z_p$ the field over which $V = V_1 \cup V_2$ is defined.

For consider the neutrosophic interval bimatrix;

$S = S_1 \cup S_2$

$$= \begin{bmatrix} [0,2I] & 0 & [0,I] \\ [0,7I] & [0,I] & 0 \\ 0 & 0 & [0,5I] \end{bmatrix} \cup \begin{bmatrix} [0,I] & 0 \\ [0,2I] & [0,4I] \end{bmatrix}$$

if we want to find the characteristic bivalues of S. Consider

$$| S - \lambda I_{n\times n} | = | S_1 - \lambda_1 I_{3\times 3} | \cup | S_2 - \lambda_2 I_{2\times 2} |$$

$$\left| \begin{bmatrix} [0,2I] & 0 & [0,I] \\ [0,7I] & [0,I] & 0 \\ 0 & 0 & [0,5I] \end{bmatrix} - \begin{bmatrix} \lambda_1[0,I] & 0 & 0 \\ 0 & \lambda_1[0,I] & 0 \\ 0 & 0 & \lambda_1[0,I] \end{bmatrix} \right| \cup$$

$$\left| \begin{bmatrix} [0,I] & 0 \\ [0,2I] & [0,4I] \end{bmatrix} - \begin{bmatrix} \lambda_2[0,I] & 0 \\ 0 & \lambda_2[0,I] \end{bmatrix} \right|$$

$$= \begin{vmatrix} [0,(2-\lambda_1)I] & 0 & [0,I] \\ [0,7I] & [0,(1-\lambda_1)I] & 0 \\ 0 & 0 & [0,(5-\lambda_1)I] \end{vmatrix} \cup$$

$$\begin{vmatrix} [0,(1-\lambda_2)I] & 0 \\ [0,2I] & [0,(4-\lambda_2)I] \end{vmatrix}$$

$$= \{[0, (2-\lambda_1)I] \begin{vmatrix} [0,(1-\lambda_1)I] & 0 \\ 0 & [0,(5-\lambda_1)I] \end{vmatrix} +$$

$$[0, I] \left. \begin{vmatrix} [0,7I] & [0,(1-\lambda_1)I] \\ 0 & 0 \end{vmatrix} \right\} \cup |[0, (1-\lambda_2)I] \times [0, (4-\lambda_2)I]|$$

$= [0, (2–\lambda_1)I]\ [0, (1–\lambda_1)I]\ [0, (5–\lambda_1)I] \cup [0, (1–\lambda_2)\ (4–\lambda_2)I]$
$= [0, (2–\lambda_1)\ (1–\lambda_1)\ (5–\lambda_1)I] \cup [0, (1–\lambda_2)\ (4–\lambda_2)I]$
$= \{0\} \cup \{0\}$ gives the biroots as

$\{[0, 2I], [0, I], [0, 5I]\} \cup \{[0, I], [0, 4I]\}$.

So the biroots are

[0, 2I] ∪ [0, I], [0, 2I] ∪ [0, 4I], [0, I] ∪ [0, I], [0, I] ∪ [0, 4I], [0, 5I] ∪ [0, I] and [0, 5I] ∪ [0, 4I].

Thus whenever the biequations are solvable over the finite characteristic field Z_p (p a prime) we have the bicharacteristic values associated with the neutrosophic interval bimatrix and the bisolution or the biroot is not in Z_p but in the set $\{[0, aI] \mid a \in Z_p\}$; if we assume the bispace is pure neutrosophic otherwise in the set $\{[0, a+bI] \mid a, b \in Z_p\}$; if we assume the bispace is just neutrosophic. If the biroot exists then alone we get the characteristic bivalues. Interested reader can study in this direction. We can derive atmost all results in this direction with simple and appropriate modifications.

If $T : V = V_1 \cup V_2 \to V_1 \cup V_2 = V$ is a linear bioperator on V; $V = V_1 \cup V_2$ a neutrosophic interval bivector space over the field $F = Z_p$. Let $W = W_1 \cup W_2 \subseteq V_1 \cup V_2$ be a neutrosophic interval bivector subspace of V. We say W is biinvariant under the bioperator $T = T_1 \cup T_2$ on V if $T(W) \subseteq V$ that is $T_1 (W_1) \subseteq V_1$ and $T_2 (W_2) \subseteq V_2$.

Let $V = V_1 \cup V_2$ be a neutrosophic interval bivector space over the field $F = Z_p$. Let $W^1, W^2, \ldots, W^k$ be neutrosophic interval bivector subspaces of V over the field $F = Z_p$. We say $W^1, W^2, \ldots, W^k$ are biindependent if

$\alpha^1 + \alpha^2 + \ldots + \alpha^k = 0$; $\alpha^i \in W^i$ that is $\alpha^i = \alpha_1^i \cup \alpha_2^i \in W_1^i \cup W_2^i$; $1 \leq i \leq k$ implies $\alpha^i = 0 \cup 0$ that is $\alpha_1^i = 0$ and $\alpha_2^i = 0$. Here $W^i = W_1^i \cup W_2^i$; i=1,2,…,k.

Let $V = V_1 \cup V_2$ be a finite dimensional neutrosophic interval bivector space over the field $F = Z_p$. Let $W^1, W^2, \ldots, W^k$ ($W^i = W_1^i \cup W_2^i$; i=1,2, …, k) be bisubspaces of V and let $W = W^1 + \ldots + W^k$ that is $W = W_1 \cup W_2 = W_1^1 \cup W_2^1 + \ldots +$

$W_1^k \cup W_2^k = \left(W_1^1 + ... + W_1^k\right) \cup \left(W_2^1 + ... + W_2^k\right)$. Then we have the following three conditions to be equivalent.

(i) W^1, W^2, …, W^k are biindependent; that is $W_1^1, W_1^2, ..., W_1^k$ are independent and $W_2^1, W_2^2, ..., W_2^k$ are independent.

(ii) For each j, $2 \leq j \leq k$ we have $W^j \cap (W^1 + \ldots + W^{j-1}) = \{0\}$ that is $W_1^j \cap \left(W_1^1 + ... + W_1^{j-1}\right) = \{0\}$ and $W_2^j \cap \left(W_2^1 + ... + W_2^{j-1}\right) = \{0\}$

(iii) If β^i is a bibasis of W^i, $1 \leq i \leq k$ then the bisequence B $= (\beta^1, \ldots, \beta^k)$ is a basis for $W = W_1 \cup W_2$.

Here $\beta^i = \beta_1^i \cup \beta_2^i$ where β_t^i is a basis of W_t^i; t = 1, 2.

Thus we say $W = W^1 + \ldots + W^k$ is the bidirect sum or W is a bidirect sum of W^1, W^2, …, W^k that is W_1 is the direct sum of $W_1^1, ..., W_1^k$, thus

$W_1 = W_1^1 \oplus ... \oplus W_1^k$ and W_2 is the direct sum of

$W_2^1, ..., W_2^k$ and $W_2 = W_2^1 \oplus ... \oplus W_2^k$.

If W = V then we say V is the bidirect sum of W_1, …, W_k and each $V_1 = W_1^1 \oplus ... \oplus W_1^k$ and $V_2 = W_2^1 \oplus ... \oplus W_2^k$.

Now we will proceed onto define the notion of biprojection. Let $V = V_1 \cup V_2$ be a neutrosophic interval bivector space, a biprojection of V is a linear bioperator $E = E_1 \cup E_2$ on V such that $E^2 = E_1^2 \cup E_2^2 = E_1 \cup E_2$. We have $E : V \to V$ that is $E = E_1 \cup E_2 : V = V_1 \cup V_2 \to V = V_1 \cup V_2$ is a biprojection, with $R = R_1 \cup R_2$ is the birange of E and $B = N_1 \cup N_2$ is the binull space of E.

Thus $\beta = \beta_1 \cup \beta_2$ is in $R_1 \cup R_2$ if and only if $E\beta = E_1\beta_1 \cup E_2\beta_2 = \beta_1 \cup \beta_2 = \beta$.

Conversely if $\beta = \beta_1 \cup \beta_2 = E\beta = E_1\beta_1 \cup E_2\beta_2$ then β is in the birange of E.

Further $V = R \oplus N$, that is $V = V_1 \cup V_2 = R_1 \oplus N_1 \cup R_2 \oplus N_2$.

We can write every bivector as

$\alpha = \alpha_1 \cup \alpha_2 = E_1 \alpha_1 + (\alpha_1 - E_1 \alpha_1) \cup E_2 \alpha_2 + (\alpha_2 - E_2 \alpha_2)$

We can derive all the results related with projections.

Let $V = V_1 \cup V_2$ be a neutrosophic interval bivector space.

$V = V_1 \cup V_2 = W^1 \oplus \ldots \oplus W^k$

$= W_1^1 \oplus \ldots \oplus W_1^k \cup W_2^1 \oplus \ldots \oplus W_2^k$.

For each j we define $E^j = E_1^j \cup E_2^j$ on $V = V_1 \cup V_2$ a bioperator on V.

For every $\alpha^j = \alpha_1^j \cup \alpha_2^j$ in $W^j = W_1^j \cup W_2^j$ we have $E^j \alpha^j = E_1^j \alpha_1^j \cup E_2^j \alpha_2^j = \alpha_1^j \cup \alpha_2^j$. Then E^j is a well defined rule. Further $E_j \alpha^j = 0 \cup 0$ simply means $\alpha^j = \alpha_1^j \cup \alpha_2^j = 0 \cup 0$.

For each $\alpha \in V = V_1 \cup V_2$ we have $\alpha = E^1\alpha + \ldots + E^k\alpha$

$= E_1^1\alpha_1^1 \cup E_2^1\alpha_2^1 + \ldots + E_1^k\alpha_1^k \cup E_2^k\alpha_2^k$.

We have $I = I_1 \cup I_2$.

$= \left(E_1^1 + \ldots + E_1^k\right) \cup \left(E_2^1 + \ldots + E_2^k\right)$.

If $i \neq j$ we have $E^i \,.\, E^j = 0 \cup 0$ that is $E_1^i E_1^j \cup E_2^i E_2^j = 0 \cup 0$.

Thus the birange of E^j is the bisubspace $W^j = W_1^j \cup W_2^j$, which is in the null space of E^i.

THEOREM 2.3.2: *Let $V = V_1 \cup V_2$ be a neutrosophic interval bivector space. Suppose $V = W^1 \oplus \ldots \oplus W^k$.*

$= V_1 \cup V_2 = W_1^1 \cup W_2^1 \oplus \ldots \oplus W_1^k \cup W_2^k$

$= (W_1^1 \oplus \ldots \oplus W_1^k) \cup (W_2^1 \oplus \ldots \oplus W_2^k)$ *then there exists k linear bioperators $E^1, E^2, \ldots, E^k$ that is $E_1^1 \cup E_2^1, E_1^2 \cup E_2^2, \ldots, E_1^k \cup E_2^k$ on $V = V_1 \cup V_2$ such that*

(i) Each $E^i = E_1^i \cup E_2^i$ is a biprojection $((E^i)^2 = E^i)$; $1 \leq i \leq k$.

(ii) $E^i E^j = (0)$ if $i \neq j$, $1 \leq j$, $1 \leq i, j \leq k$.

(iii) $I = I_1 \cup I_2 = E^1 + \ldots + E^k = E_1^1 \cup E_2^1 + \ldots + E_1^k \cup E_2^k$

$= \left(E_1^1 + \ldots + E_1^k\right) \cup \left(E_2^1 + \ldots + E_2^k\right)$.

(iv) The birange of $E^i = E_1^i \cup E_2^i$ is $W^i = W_1^i \cup W_2^i$; that is birange of E_t^i is W_t^i; t=1, 2.

Conversely if E^1, E^2 , …, E^k are linear bioperators on $V = V_1 \cup V_2$ which satisfy conditions (i), (ii) and (iii) and if W_i is the birange of E^i then $V = W^1 \oplus \ldots \oplus W^k$ that is $V = V_1 \cup V_2 = (W_1^1 \oplus \ldots \oplus W_1^k) \cup (W_2^1 \oplus \ldots \oplus W_2^k)$.

The proof can be obtained as a matter of routine with appropriate modifications.

Note under the conditions of the above theorem if $V = V_1 \cup V_2$ the neutrosophic interval bivector space where $V = W^1 \oplus \ldots \oplus W^k = (W_1^1 \oplus \ldots \oplus W_1^k) \cup (W_2^1 \oplus \ldots \oplus W_2^k)$ and for E^1, E^2, …, E^k given as in the above theorem, the necessary and sufficient condition that each bisubspace W^i be invariant under T is that $T = T_1 \cup T_2$ commute with each of the projections $E^j = E_1^j \cup E_2^j$ that is $TE^j = E^jT = T_1 E_1^j \cup T_2 E_2^j = E_1^j T_1 \cup E_2^j T_2$ for j = 1, 2, …, k. This can be easily verified.

If $T = T_1 \cup T_2$ is a linear bioperator on a finite dimensional bispace $V = V_1 \cup V_2$; V a neutrosophic interval bivector space then $\text{Hom}_F(V,V) = \{T : V \to V\}$.

Study the algebraic structure enjoyed by $\text{Hom}_F(V,V)$.

Now we can proceed onto define the notion of neutrosophic interval linear bialgebra.

DEFINITION 2.3.2: *Let $V = V_1 \cup V_2$ be a neutrosophic interval bivector space over the field F. If V is such that V_i closed with respect to product and the product is associative then we define V to be neutrosophic interval linear bialgebra over the field $F = Z_p$ (p a prime).*

We give some examples of them.

Example 2.3.11: Let $V = V_1 \cup V_2 = \{([0, aI]\ [0, bI]\ [0, cI]) \mid a, b, c \in Z_7\} \cup \left\{ \begin{bmatrix} [0,aI] & [0,bI] \\ [0,cI] & [0,dI] \end{bmatrix} \middle| a,b,c,d \in Z_7 \right\}$ be a pure neutrosophic interval linear bialgebra over the field Z_7.

Example 2.3.12: Let $V = V_1 \cup V_2 = \left\{\sum_{i=0}^{\infty}[0, aI]x^i \,\middle|\, a \in Z_{23}\right\} \cup$ $\{([0, aI], [0, bI], [0, cI]) \mid a, b, c \in Z_{23}\}$ be a pure neutrosophic interval linear bialgebra over the field Z_{23}.

We can as in case of linear bialgebra derive all the properties with appropriate changes or modifications. This work is left as exercise to the reader.

Now we can also define the notion of Special neutrosophic interval bivector space as follows:

DEFINITION 2.3.3: *Let $V = V_1 \cup V_2$ be an additive abelian neutrosophic interval bigroup. $F = F_1 \cup F_2$ be a bifield (neutrosophic or otherwise). If V_i is a neutrosophic interval vector space over F_i ; i=1, 2, then we define V to be a special neutrosophic interval bivector space over the bifield F.*

We will give examples of them.

Example 2.3.13: Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z_{23}\} \cup \{[0, aI] \mid a \in Z_{43}\}$ be a special neutrosophic interval bivector space over the bifield $F = Z_{23} \cup Z_{43}$.

Example 2.3.14: Let $M = M_1 \cup M_2 = \{([0, aI], [0, bI], [0, cI]) \mid a, b, c \in Z_7\} \cup \left\{\begin{bmatrix}[0,a_1I]\\ [0,a_2I]\\ [0,a_3I]\\ [0,a_4I]\\ [0,a_5I]\\ [0,a_6I]\end{bmatrix} \,\middle|\, a_i \in Z_5, 1 \leq i \leq 6\right\}$ be a special neutrosophic interval bivector space over the bifield $Z_7 \cup Z_5 = F$.

Example 2.3.15: Let $V = V_1 \cup V_2 = \{[0, aI+b] \mid a, b \in Z_{53}\} \cup \{[0, aI] \mid a, b \in Z_{47}\}$ be a special neutrosophic interval bivector space over the bifield $F = Z_{53} \cup Z_{47}$.

We can define bisubstructures, bibasis, bidimension as in case of neutrosophic interval bivector spaces.

We give one or two examples before we proceed to define other new structures.

Examples 2.3.16: Let $V = V_1 \cup V_2 = \{([0, aI], [0, bI], [0, cI]) \mid a, b, c \in Z_{53}\} \cup \left\{\left.\begin{bmatrix} [0,aI] & [0,eI] \\ [0,bI] & [0,fI] \\ [0,cI] & [0,gI] \\ [0,dI] & [0,hI] \end{bmatrix} \right| a,b,c,d,e,f,g,h \in Z_{23}\right\}$ be a special neutrosophic interval bivector space over the bifield $F = Z_{53} \cup Z_{23}$.

Consider $M = M_1 \cup M_2 = \{([0, aI], 0, 0) \mid a \in Z_{53}\} \cup$

$$\left\{\left.\begin{bmatrix} 0 & [0,aI] \\ [0,bI] & 0 \\ & [0,cI] \\ [0,dI] & 0 \end{bmatrix} \right| a,b,c,d \in Z_{23}\right\} \subseteq V_1 \cup V_2 = V.$$

M is a special neutrosophic interval bivector subspace of V over the bifield $F = Z_{53} \cup Z_{23}$.

Now $B = \{([0, I], 0, 0), (0, [0, I], 0). (0, 0, [0, I])\} \cup$

$$\left\{\begin{bmatrix} [0,I] & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & [0,I] \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ [0,I] & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & [0,I] \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\right.$$

$$\left.\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ [0,I] & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & [0,I] \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ [0,I] & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0] \\ 0 & 0 \\ 0 & [0,I] \end{bmatrix}\right\} =$$

$B_1 \cup B_2$ is a special bibasis of V over the bifield $F = Z_{53} \cup Z_{23}$. Clearly the special bidimension of V over $F = Z_{53} \cup Z_{23}$ is $\{3\} \cup \{8\}$.

We can define special linear bitransformation of two special bivector spaces only if they are defined over the same bifield, otherwise special linear bitransformation cannot be defined.

We will give an example of it.

Example 2.3.17: Let V = $V_1 \cup V_2$ = {([0, aI], [0, bI], [0, cI], [0, dI]) | a, b, c,d ∈ Z_7} ∪

$$\left\{\left.\begin{bmatrix}[0,aI] & [0,bI] & [0,cI] \\ [0,dI] & [0,eI] & [0,nI] \\ [0,mI] & [0,pI] & [0,qI] \\ [0,sI] & [0,tI] & [0,rI]\end{bmatrix}\right| a,b,c,...,t,r \in Z_{29}\right\}$$ be a special neutrosophic interval bivector space over the bifield F = $Z_7 \cup Z_{29}$. Take W = $W_1 \cup W_2$ = $\left\{\left.\begin{bmatrix}[0,aI] & [0,bI] \\ [0,cI] & [0,dI]\end{bmatrix}\right| a,b,c,d \in Z_7\right\} \cup$

$$\left\{\left.\begin{bmatrix}[0,aI] & [0,bI] & [0,dI] & [0,cI] & [0,rI] & [0,pI] \\ [0,eI] & [0,tI] & [0,qI] & [0,mI] & [0,nI] & [0,sI]\end{bmatrix}\right| a,b,c,...,t,r \in Z_{29}\right\}$$

be a special neutrosophic interval bivector space over the bifield F = $Z_7 \cup Z_{29}$. We can define a special linear bitransformation of V into W.

Let T = $T_1 \cup T_2$; $V_1 \cup V_2 \to W_1 \cup W_2$ where $T_1 : V_1 \to W_1$ and $T_2 : V_2 \to W_2$ given by

$$T_1 ([0, aI], [0, bI], [0, cI], [0, dI]) = \begin{bmatrix}[0,aI] & [0,bI] \\ [0,cI] & [0,dI]\end{bmatrix} \text{ and}$$

$$T_2 \left(\begin{bmatrix}[0,aI] & [0,bI] & [0,cI] \\ [0,dI] & [0,eI] & [0,nI] \\ [0,mI] & [0,pI] & [0,qI] \\ [0,sI] & [0,tI] & [0,rI]\end{bmatrix}\right) =$$

$$= \left(\begin{bmatrix}[0,aI] & [0,bI] & [0,dI] & [0,cI] & [0,rI] & [0,pI] \\ [0,eI] & [0,tI] & [0,qI] & [0,mI] & [0,nI] & [0,sI]\end{bmatrix}\right).$$

T is a special linear bioperator from V to W.

Let T : V → V, where V is a space of special neutrosophic bivector space defined over the bifield. If T is a function such that $T = T_1 \cup T_2$ and $T_1 : V_1 \to V_1$ and $T_2 : V_2 \to V_2$ where both T_1 and T_2 are linear transformations (operators) then we define T to be a special linear bioperator on V.

Define $T = T_1 \cup T_2 : V = V_1 \cup V_2 \to V = V_1 \cup V_2$ where $T_1 : V_1 \to V_1$ and $T_2 : V_2 \to V_2$ given by

T_1 (([0, aI], [0, bI], [0, cI], [0, dI])) = ([0, aI], 0, [0, cI], 0) and

$$T_2\left(\begin{bmatrix} [0,aI] & [0,bI] & [0,cI] \\ [0,eI] & [0,fI] & [0,sI] \\ [0,mI] & [0,nI] & [0,pI] \\ [0,tI] & [0,rI] & [0,sI] \end{bmatrix}\right) =$$

$$\left(\begin{bmatrix} [0,aI] & [0,bI] & [0,cI] \\ 0 & [0,dI] & [0,fI] \\ 0 & 0 & [0,sI] \\ 0 & 0 & [0,tI] \end{bmatrix}\right).$$

T is a special linear bioperator on $V = V_1 \cup V_2$.

All properties associated with usual bivector spaces / vector spaces can be derived for special neutrosophic bivector spaces with appropriate modifications.

We give examples of special neutrosophic interval bivector spaces.

Example 2.3.18: Let $V = V_1 \cup V_2$ = {([0, a+bI], [0, c+dI] [0, e+fI]) | a, b, c, d, e, f $\in Z_7$} $\cup$

$$\left\{\begin{bmatrix} [0,a_1+b_1I] \\ [0,a_2+b_2I] \\ [0,a_3+b_3I] \\ [0,a_4+b_4I] \\ \vdots \\ [0,a_{10}+b_{10}I] \end{bmatrix} \middle| a_i, b_i \in Z_{11}; 1 \le i \le 10\right\}$$

be a special neutrosophic interval bivector space defined over the bifield $F = Z_7 \cup Z_{11}$.

Thus V has special pure neutrosophic interval bisubspace as well as special interval vector bisubspace given by $P = P_1 \cup P_2 = \{([0, aI], [0, bI]\ [0, cI]) \mid a, b, c \in Z_7\} \cup \left\{\begin{bmatrix}[0,a_1I]\\ [0,a_2I]\\ \vdots \\ [0,a_{10}I]\end{bmatrix} \middle| a_i \in Z_{11}; 1 \leq i \leq 10\right\} \subseteq V_1 \cup V_2$ is a special pure neutrosophic interval bisubspace of V.

The bidimension of P is $\{3\} \cup \{10\}$.

Further $T = T_1 \cup T_2 = \{([0, aI], [0, bI], [0, cI]) \mid a, b, c \in Z_7\} \cup \left\{\begin{bmatrix}[0,a_1I]\\ 0\\ [0,a_2I]\\ 0\\ [0,a_3I]\\ 0\\ [0,a_4I]\\ 0\\ [0,a_5I]\\ 0\end{bmatrix} \middle| a_i \in Z_{11}; 1 \leq i \leq 5\right\} \subseteq V$ is also special pure neutrosophic interval vector bisubspace of V over the bifield $F = Z_7 \cup Z_{11}$ and the bidimension of T is $\{3\} \cup \{5\}$.

Consider $R = R_1 \cup R_2 = \{([0, a], [0, b], [0, c]) \mid a, b, c \in Z_7\} \cup \left\{\begin{bmatrix}[0,a_1]\\ [0,a_2]\\ \vdots \\ [0,a_{10}]\end{bmatrix} \middle| a_i \in Z_{11}; 1 \leq i \leq 10\right\} \subseteq V_1 \cup V_2 = V$. R is a special interval bivector subspace of V over the bifield $Z_7 \cup Z_{11} = F$.

Further $S = S_1 \cup S_2 = \{([0, a], 0, [0, b])$ where $a, b \in Z_7\} \cup$

$$\left\{ \begin{bmatrix} [0,a] \\ 0 \\ [0,b] \\ [0,c] \\ 0 \\ 0 \\ 0 \\ 0 \\ [0,d] \\ 0 \end{bmatrix} \middle| a,b,c,d \in Z_{11} \right\} \subseteq V_1 \cup V_2$$

is again a special interval bivector subspace of V over the bifield $F = Z_7 \cup Z_{11}$.

Clearly bidimension of S is $\{2\} \cup \{4\}$.

Now we define quasi neutrosophic interval bivector space $V = V_1 \cup V_2$ as an interval bivector space where V_1 is a interval vector space and V_2 is a neutrosophic interval vector space.

We give examples of them.

Example 2.3.19: Let $V = V_1 \cup V_2 = \{[0, a] \mid a \in Z_{23}\} \cup \{([0, aI], [0, bI], [0, cI], [0, dI], [0, eI]) \mid a, b,. c, d, e, \in Z_{23}\}$ be a quasi neutrosophic interval bivector space over the field Z_{23}.

Example 2.3.20: Let $M = M_1 \cup M_2 =$

$$\left\{ \begin{bmatrix} [0,a_1] \\ [0,a_2] \\ [0,a_3] \\ [0,a_4] \\ \vdots \\ [0,a_{15}] \end{bmatrix} \middle| a_i \in Z_{43}; 1 \leq i \leq 15 \right\} \cup$$

$\{[0, a+bI] \mid a, b \in Z_{43}\}$ be a quasi neutrosophic interval bivector space over Z_{23}. We see bidimension of M is $\{15\} \cup \{1\}$.

Example 2.3.21: Let

$$V = V_1 \cup V_2 = \left\{ \sum_{i=0}^{29} [0, aI]x^i \,\middle|\, a \in Z_{53} \right\} \cup \left\{ \sum_{i=0}^{59} aIx^i \,\middle|\, a \in Z_{53} \right\}$$

be a neutrosophic quasi interval bivector space over the field Z_{53}. Bidimension of V is {30} ∪ {60}.

Example 2.3.22: Let

$$V = V_1 \cup V_2 = \left\{ \begin{bmatrix} [0,aI] \\ [0,bI] \\ [0,cI] \\ [0,dI] \\ [0,eI] \end{bmatrix} \,\middle|\, a,b,c,d,e \in Z_{11} \right\} \cup \left\{ \begin{bmatrix} a_1I & a_2I & a_3I & a_4I & a_5I \\ a_6I & a_7I & a_8I & a_9I & a_{10}I \\ a_{11}I & a_{12}I & a_{13}I & a_{14}I & a_{15}I \\ a_{16}I & a_{17}I & a_{18}I & a_{19}I & a_{20}I \\ a_{21}I & a_{22}I & a_{23}I & a_{24}I & a_{25}I \end{bmatrix} \,\middle|\, a_i \in Z_{11}; 1 \leq i \leq 25 \right\}$$

be a neutrosophic quasi interval bivector space over the field Z_{11}.

Example 2.3.23: Let

$$M = M_1 \cup M_2 = \left\{ \sum_{i=0}^{20} a_iIx^i \,\middle|\, a_i \in Z_{59} \right\} \cup \left\{ \begin{bmatrix} [0,a_1I] & \ldots & [0,a_7I] \\ [0,a_8I] & \ldots & [0,a_{14}I] \\ [0,a_{15}I] & \ldots & [0,a_{21}I] \\ \vdots & & \vdots \\ [0,a_{57}I] & \ldots & [0,a_{63}I] \end{bmatrix} \,\middle|\, a_i \in Z_{59}; 1 \leq i \leq 63 \right\}$$

be a neutrosophic quasi interval bivector space over Z_{59}. Bidimension of V over Z_{59} is $\{21\} \cup \{63\}$.

Example 2.3.24: Let $V = V_1 \cup V_2$ = {all 10×12 matrices with entries from Z_{43}} $\cup$ $\left\{ \sum_{i=0}^{12} [0, a_i I]x^i \,\middle|\, a_i \in Z_{43} \right\}$ be a quasi neutrosophic quasi interval bivector subspace of V over Z_{43}. The bidimension of V is $\{120\} \cup \{13\}$.

Example 2.3.25: Let $P = P_1 \cup P_2 = \left\{ \sum_{i=0}^{9} a_i x^i \,\middle|\, a_i \in Z_{13} \right\}$ $\cup$ {all neutrosophic interval 6×3 matrices with intervals of the form [0, a+bI] where a, b $\in Z_{13}$} be a quasi neutrosophic quasi interval bivector space over the field Z_{13} of finite bidimension.

Example 2.3.26: Let

$$T = T_1 \cup T_2 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{15} \end{bmatrix} \,\middle|\, a_i \in Z_{47}; 1 \leq i \leq 15 \right\} \cup$$

$$\left\{ \sum_{i=0}^{9} [0, aI]x^i \,\middle|\, a \in Z_{47} \right\}$$

be a quasi neutrosophic quasi interval bivector space over the field Z_{47} of bidimension $\{15\} \cup \{30\}$.

All properties related with bivector spaces / interval bivector spaces/ vector spaces can be derived with simple appropriate modifications.

We can define quasi special neutrosophic interval bistructure also which can be easily understood from the examples.

Example 2.3.27: Let $V = V_1 \cup V_2 = \left\{ \sum_{i=0}^{8} [0, aI]x^i \,\middle|\, a \in Z_{13} \right\}$ $\cup$ $\{([0, a_1], [0, a_2], \ldots, [0, a_9]) \mid a_i \in Z_{43}; 1 \leq i \leq 9\}$ be a special quasi neutrosophic interval bivector space over the bifield $F = Z_{13} \cup Z_{43}$.

Example 2.3.28: Let $P = \{([0, a_1+b_1I], \ldots,[0, a_{15}+b_{15}I]) \mid a_i\ b_i \in Z_{47}.\ 1 \leq i \leq 15\} \cup$

$$\left\{\left.\begin{bmatrix} [0,a_1] & [0,a_2] & [0,a_3] \\ [0,a_4] & [0,a_5] & [0,a_6] \\ \vdots & \vdots & \vdots \\ [0,a_{31}] & [0,a_{32}] & [0,a_{33}] \end{bmatrix}\right| a_i \in Z_{23}; 1 \leq i \leq 33\right\}$$

be a special quasi neutrosophic quasi interval bivector space of finite bidimension over the bifield $F = Z_{47} \cup Z_{23}$.

Example 2.3.29: Let

$$P = P_1 \cup P_2 = \left\{\left.\sum_{i=0}^{25} a_i I x^i \right| a_i \in Z_7\right\} \cup$$

$$\left\{\left.\begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ \vdots \\ [0,a_{12}I] \end{bmatrix}\right| a_i \in Z_{13}; 1 \leq i \leq 12\right\}$$

be a special neutrosophic quasi interval bivector space of bidimension $\{26\} \cup \{12\}$ over the bifield $F = Z_7 \cup Z_{13}$.

Example 2.3.30: Let $M = M_1 \cup M_2$ = {all 5×5 neutrosophic interval matrices with intervals of the form $[0, a+bI] \mid a, b \in Z_{59}\}$ $\cup$ {all 8×2 matrices with entries from RI, R-reals} be a special neutrosophic quasi interval bivector space over the bifield $F = Z_{59} \cup R$.

Example 2.3.31: Let $T = T_1 \cup T_2 = \{[0, a+bI] \mid a, b \in Z_{13}\} \cup \left\{\left.\sum_{i=0}^{40} aIx^i \right| a \in Q\right\}$ be a special neutrosophic quasi interval bivector space over the bifield $F = Z_{13} \cup Q$.

Example 2.3.32: Let

$$P = P_1 \cup P_2 = \left\{ \sum_{i=0}^{\infty} a_i I x^i \,\middle|\, a_i \in Q \right\} \cup \left\{ \sum_{i=0}^{20} [0, a] x^i \,\middle|\, a \in Z_{43} \right\}$$

be a special quasi neutrosophic quasi interval bivector space of infinite dimension over the bifield $F = Q \cup Z_{43}$.

Example 2.3.33: Let $M = M_1 \cup M_2 = \left\{ \sum_{i=0}^{15} [0, aI] x^i \,\middle|\, a \in Z_{13} \right\} \cup$ {all 10×10 matrices with entries from Q} be the special quasi neutrosophic quasi interval bivector space of finite bidimension over the bifield $F = Z_{13} \cup Q$.

All properties associated with interval bivector spaces / vector spaces can be derived for the special quasi neutrosophic interval bivector spaces or special neutrosophic quasi interval bivector spaces with simple appropriate modifications which is left as an exercise to the reader.

Also it can be said that without any difficulty special neutrosophic interval linear bialgebras can be defined. We give some examples of special neutrosophic interval linear bialgebras.

Example 2.3.34: Let

$$T = T_1 \cup T_2 = \left\{ \sum_{i=0}^{\infty} [0, a_i I] x^i \,\middle|\, a_i \in Z_{11} \right\} \cup$$

$$\left\{ \begin{bmatrix} [0, a_1 I] & [0, a_6 I] & \ldots & [0, a_{21} I] \\ \vdots & \vdots & & \vdots \\ [0, a_5 I] & [0, a_{10} I] & \ldots & [0, a_{25} I] \end{bmatrix} \,\middle|\, a_i \in Z_7; 1 \leq i \leq 25, + \right\}$$

be a special neutrosophic interval linear bialgebra over the bifield $S = Z_{11} \cup Z_7$.

Example 2.3.35: Let $S = S_1 \cup S_2 = \{[0, a+bI] \mid a, b \in Z_{19}\} \cup$ {All 15×15 neutrosophic intervals with intervals of the form $[0, a+bI] \mid a, b \in Z_3$} be a special neutrosophic interval linear bialgebra over the bifield $F = Z_{19} \cup Z_3$.

Example 2.3.36: Let

$$M = M_1 \cup M_2 = \left\{ \sum_{i=0}^{\infty} a_i I x^i \,\middle|\, a_i \in Q \right\} \cup$$

$\{([0, a_1 + b_1I], \ldots, [0, a_{10}+b_{10}I]) \mid a_i, b_i \in Z_{17}, 1 \leq i \leq 10\}$ be a special neutrosophic quasi interval linear bialgebra of infinite dimension over the bifield $F = Q \cup Z_{17}$.

Example 2.3.37: Let

$$S = S_1 \cup S_2 = \left\{ \sum_{i=0}^{\infty} [0, a] x^i \,\middle|\, a \in R \right\} \cup$$

{All 3×3 neutrosophic interval matrices with intervals of the form $[0, a+bI]$ with a, b from Z_2} be a special quasi neutrosophic quasi interval linear bialgebra of infinite dimension over the bifield $S = R \cup Z_2$.

Now we proceed onto define neutrosophic interval bisemivector spaces.

DEFINITION 2.3.4: *Let $V = V_1 \cup V_2$ be an additive abelian neutrosophic interval bisemigroup with $0 \cup 0$ as its identity. Let F be a semifield if V_i is a neutrosophic interval semivector space over F; i = 1, 2, then we define V to be a neutrosophic interval bisemivector space over the semifield F.*

We will illustrate this situation by some examples.

Example 2.3.38: Let

$$V = V_1 \cup V_2 = \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}\} \cup$$

$$\left\{ \begin{bmatrix} [0, aI] \\ [0, bI] \\ [0, cI] \\ [0, dI] \end{bmatrix} \,\middle|\, a, b, c, d \in Z^+ \cup \{0\} \right\}$$

be a neutrosophic interval semibivector space (bisemivector space) over the semifield $S = Z^+ \cup \{0\}$.

Example 2.3.39: Let

$$S = S_1 \cup S_2 = \left\{ \sum_{i=0}^{25} [0, aI]x^i \,\middle|\, a \in Q^+ \cup \{0\} \right\} \cup$$

{All 10 × 2 neutrosophic interval matrices with intervals of the form {[0, a+bI] where a, b ∈ $Q^+ \cup \{0\}$} be a neutrosophic interval bisemivector space over F = $Z^+ \cup \{0\}$, the semifield.

Example 2.3.40: Let W = $W_1 \cup W_2$ = {All 8 × 3 neutrosophic interval matrices with intervals of the form [0, aI] with a ∈ $Z^+ \cup \{0\}$} ∪ {All 3 × 3 neutrosophic interval matrices with intervals from $Z^+ \cup \{0\}$} be a neutrosophic interval bisemivector space over the semifield S = $Z^+ \cup \{0\}$.

It is interesting to note W is not a neutrosophic interval bisemivector space over the semifield $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$. Further the dimension of a neutrosophic interval bisemivector space also depends on the semifield over which it is defined.

Example 2.3.41: Let M = $M_1 \cup M_2$ =

$$\left\{ \begin{bmatrix} [0,a_1I] & [0,a_{10}I] \\ [0,a_2I] & [0,a_{11}I] \\ \vdots & \vdots \\ [0,a_9I] & [0,a_{18}I] \end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}, 1 \le i \le 18 \right\} \cup$$

$$\left\{ \begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \end{bmatrix} \middle| a_i \in Q^+ \cup \{0\}, 1 \le i \le 4 \right\}$$

be a neutrosophic interval bisemivector space over the semifield S = $Z^+ \cup \{0\}$.

Clearly M is of bidimension {18} ∪ {∞} over S = $Z^+ \cup \{0\}$.

Further M is not defined over the semifield $Q^+ \cup \{0\}$.

Example 2.3.42: Let $V = V_1 \cup V_2 =$

$$\left\{\left.\begin{bmatrix} [0, a_1 + b_1I] \\ [0, a_2 + b_2I] \\ [0, a_3 + b_3I] \\ \vdots \\ [0, a_{10} + b_{10}I] \end{bmatrix}\right| a_i, b_i \in Z^+ \cup \{0\}, 1 \leq i \leq 10\right\} \cup$$

$\{([0, aI], [0, a_1 + b_1I], [0, c_1 + d_1I], [0, bI], [0, cI]) \mid a, b_1, c_1, d_1, c \in Z^+ \cup \{0\}\}$ be a neutrosophic interval bisemivector space over the semifield $S = Z^+ \cup \{0\}$.

$$\text{Take } M = M_1 \cup M_2 = \left\{\left.\begin{bmatrix} 0 \\ 0 \\ [0, aI] \\ 0 \\ 0 \\ [0, bI] \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right| a, b \in Z^+ \cup \{0\}\right\} \cup$$

$\{(0, [0, aI], 0, [0, bI], [0, dI]) \mid a, b, d \in Z^+ \cup \{0\}\} \subseteq V_1 \cup V_2 = V$; M is a neutrosophic interval bisemivector subspace of V over the semifield $S = Z^+ \cup \{0\}$.

Example 2.3.43: Let $T = T_1 \cup T_2 = \left\{\left.\sum_{i=0}^{27} [0, aI]x^i \right| a \in Q^+ \cup \{0\}\right\}$

$\cup \left\{\begin{bmatrix} [0, aI] & [0, eI] \\ [0, bI] & [0, fI] \\ [0, cI] & [0, mI] \\ [0, dI] & [0, nI] \end{bmatrix}\right.$ where a, b, c, d, e, f, m, n are in $Q^+ \cup$

{0}} be a neutrosophic interval bisemivector space over the semifield F = $Q^+ \cup \{0\}$.

We see bidimension of T on F is {28} ∪ {8}. If $Q^+ \cup \{0\}$ is replaced by $Z^+ \cup \{0\}$ the bidimension is infinite. Infact T is not defined over the semifield $R^+ \cup \{0\}$.

Example 2.3.44: Now consider P = $P_1 \cup P_2$ =

$$\left\{\left[\begin{array}{cccc} [0,a_1+b_1I] & [0,a_2+b_2I] & [0,a_3+b_3I] & [0,a_4+b_4I] \\ [0,a_5+b_5I] & [0,a_6+b_6I] & [0,a_7+b_7I] & [0,a_8+b_8I] \\ [0,a_9+b_9I] & [0,a_{10}+b_{10}I] & [0,a_{11}+b_{11}I] & [0,a_{12}+b_{12}I] \end{array}\right]\right|$$

$a_i, b_i \in Z^+ \cup \{0\}, 1 \le i \le 12\} \cup$

$$\left\{\left[\begin{array}{cc} [0,a_1I] & [0,a_7I] \\ [0,a_2I] & [0,a_8I] \\ \vdots & \vdots \\ [0,a_6I] & [0,a_{12}I] \end{array}\right] \middle| a_i \in Q^+ \cup \{0\}; 1 \le i \le 12 \right\}$$

be a neutrosophic interval bisemivector space over the semifield S = $Z^+ \cup \{0\}$ of infinite bidimension over S.

We can define subbistructures bibasis, linear bitransformation and linear bioperator, which is a matter of routine and left as exercise to the reader.

Example 2.3.45: Let M = $M_1 \cup M_2$ = {all 4 × 4 neutrosophic interval matrices with intervals of the form [0, a+bI] where a, b $\in Z^+ \cup \{0\}\} \cup \left\{\sum_{i=0}^{29} [0, a+bI]x^i \middle| a, b \in Z^+ \cup \{0\}\right\}$ be a neutrosophic interval bisemivector space defined over the semifield S = $Z^+ \cup \{0\}$. Take W = $W_1 \cup W_2$ = {collection of all upper triangular 4 × 4 pure neutrosophic interval matrices with intervals of the form [0, aI] with a $\in Z^+ \cup \{0\}\} \cup$

$$\left\{ \sum_{i=0}^{12} [0, aI]x^i \,\middle|\, a \in Z^+ \cup \{0\} \right\} \subseteq M_1 \cup M_2;$$

W is a pure neutrosophic interval bisubsemivector space of M over the semifield S = $Z^+ \cup \{0\}$. Clearly bidimension of W is $\{10\} \cup \{13\}$.

We can define pure neutrosophic interval subbisemivector spaces of M of bidimension less than or equal to $\{16\} \cup \{30\}$.

We can also have pure neutrosophic interval bisemivector subspaces of bidimension $\{1\} \cup \{1\}$. We have several such bisemivector subspaces.

We also can define quasi neutrosophic interval semibivectors spaces over a semifield, this task is left as an exercise to the reader. We however give examples of them.

Example 2.3.46: Let $V = V_1 \cup V_2 = \left\{ \begin{bmatrix} [0, a_1] \\ [0, a_2] \\ \vdots \\ [0, a_9] \end{bmatrix} \right.$ where $a_i \in Z^+ \cup \{0\}; 1 \le i \le 9\} \cup$ {All 3 × 3 neutrosophic interval matrices with intervals of the form [0, a+bI], a, b $\in Z^+ \cup \{0\}$} be a quasi neutrosophic interval bisemivector space over the semifield S = $Z^+ \cup \{0\}$.

Example 2.3.47: Let T = {([0, a_1], [0, a_2], …, [0, a_{12}]) | $a_i \in Z^+ \cup \{0\}; 1 \le i \le 12\} \cup \left\{ \sum_{i=0}^{20} [0, a_i I]x^i \,\middle|\, a_i \in Z^+ \cup \{0\} \right\}$ be a quasi neutrosophic interval bisemivector space over the field S = $Z^+ \cup \{0\}$.

Example 2.3.48: Let

$$W = W_1 \cup W_2 = \left\{ \sum_{i=0}^{21} [0, aI]x^i \,\middle|\, a \in Z^+ \cup \{0\} \right\} \cup$$

$$\left\{ \begin{bmatrix} a_1I & a_2I \\ a_3I & a_4I \\ a_5I & a_6I \\ a_7I & a_8I \end{bmatrix} \text{ where } a_i \in Z^+ \cup \{0\};\ 1 \leq i \leq 8 \right\}$$

be neutrosophic quasi interval bisemivector space over the semifield $Z^+ \cup \{0\}$.

Example 2.3.49: Let

$$T = T_1 \cup T_2 = \left\{ \sum_{i=0}^{12} [0, aI]x^i \,\middle|\, a \in Z^+ \cup \{0\} \right\} \cup$$

$$\left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_{17} \\ a_5 & a_6 & a_7 & a_8 & a_{18} \\ a_9 & a_{10} & a_{11} & a_{12} & a_{19} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{20} \end{bmatrix} \,\middle|\, a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 20 \right\}$$

be a quasi neutrosophic quasi interval bisemivector space over the semifield $Z^+ \cup \{0\}$.

Now having seen the quasi types of bisemivector spaces, the authors leave the task of studying these bistructures to the reader as it is simple and straight forward. Now we define neutrosophic interval semivector space set vector space $V = V_1 \cup V_2$ over the semifield $S = Z^+ \cup \{0\}$ as follows: V_1 is a interval semivector space over the semifield $Z^+ \cup \{0\}$ and V_2 is just a set vector space over the same semifield $Z^+ \cup \{0\}$ realized as a set we give examples of them.

Example 2.3.50: Let $V = V_1 \cup V_2 =$

$$\left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_{11}I] & \ldots & [0,a_{61}I] \\ [0,a_2I] & [0,a_{12}I] & \ldots & [0,a_{62}I] \\ \vdots & \vdots & & \vdots \\ [0,a_{10}I] & [0,a_{20}I] & \ldots & [0,a_{70}I] \end{bmatrix}\right| a_i \in Z^+ \cup \{0\}; 1 \le i \le 70\right\} \cup$$

$$\left\{\begin{bmatrix} [0,aI] & [0,cI] \\ [0,bI] & [0,dI] \end{bmatrix}, \begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ \vdots \\ [0,a_{20}I] \end{bmatrix}, ([0, a_1I], [0, a_2I, \ldots, [0, a_{12}I]) \mid\right.$$

a, b, c, d, a_i, $\in Z^+ \cup \{0\}$; $1 \le i \le 20\}$ be a neutrosophic interval semivector space - set vector space over the semifield $S = Z^+ \cup \{0\}$.

Example 2.3.51: Let

$$V = V_1 \cup V_2 = \left\{\left.\sum_{i=0}^{45} [0,aI]x^i \right| a_i \in Q^+ \cup \{0\}\right\} \cup$$

$$\left\{\sum_{i=0}^{49} [0,aI]x^i, \begin{bmatrix} [0,aI] & [0,bI] \\ [0,cI] & [0,dI] \\ [0,eI] & [0,fI] \\ [0,gI] & [0,hI] \\ [0,mI] & [0,nI] \end{bmatrix}, \left.\begin{pmatrix} [0,a_1I] & \ldots & [0,a_6I] \\ [0,a_7I] & \ldots & [0,a_{12}I] \end{pmatrix}\right| a_i \in Z^+\right.$$

$\cup \{0\}$;a, a_i, b, c, d, e, f, g, h, m, n, $\in Z^+ \cup \{0\}$; $1 \le i \le 12\}$ is a neutrosophic interval semivector space - set vector space over the semifield $Z^+ \cup \{0\}$.

We can define all properties associated with this bistructure also with appropriate modifications.

We can define all notions related with neutrosophic interval structures in case of neutrosophic interval bistructures which is very simple with straight forward modifications. This task is also left as an exercise to the reader.

Let $V = V_1 \cup V_2$ if V_1 is a semivector space of neutrosophic intervals over the semifield S and V_2 is a neutrosophic interval of vector space over the field F_1 then we define $V = V_1 \cup V_2$ to be a special neutrosophic interval semivector - vector space over the semifield - field.

We will illustrate this situation by some simple examples.

Example 2.3.52: Let

$$V = V_1 \cup V_2 = \left\{ \sum_{i=0}^{120} [0, aI]x^i \,\middle|\, a \in Z^+ \cup \{0\} \right\} \cup$$

$$\left\{ \begin{bmatrix} [0,aI] & [0,bI] \\ [0,cI] & [0,dI] \\ [0,eI] & [0,fI] \end{bmatrix} \,\middle|\, a,b,c,d,e,f \in Z_{43}, + \right\}$$

be a special neutrosophic interval semivector - vector space defined over the semifield - field; $Z^+ \cup \{0\} \cup Z_{43}$.

Example 2.3.53: Let

$$M = M_1 \cup M_2 = \left\{ \sum_{i=0}^{5} [0, a+bI] \,\middle|\, a, b \in Z_{23} \right\} \cup$$

$$\left\{ \sum_{i=0}^{\infty} [0, a+bI]x^{2i} \,\middle|\, a, b \in Q^+ \cup \{0\} \right\}$$

be a special neutrosophic interval vector space - semivector space over the field - semifield $F = F_1 \cup F_2 = Z_{23} \cup Q^+ \cup \{0\}$.

Example 2.3.54: Let $V = V_1 \cup V_2 =$

$$\left\{ \left. \begin{bmatrix} [0,a_1I] & [0,a_2I] & ... & [0,a_9I] \\ [0,a_{10}I] & [0,a_{11}I] & ... & [0,a_{18}I] \\ [0,a_{19}I] & [0,a_{20}I] & & [0,a_{27}I] \\ [0,a_{28}I] & [0,a_{29}I] & ... & [0,a_{36}I] \end{bmatrix} \right| a_i \in Q^+ \cup \{0\}; 1 \leq i \leq 36 \right\} \cup$$

$$\left\{ \sum_{i=0}^{12} [0,aI]x^i \,\middle|\, a \in Z_7 \right\}$$

be a special neutrosophic interval semivector space - vector space over the semifield – field; $S = S_1 \cup S_2 = (Q^+ \cup \{0\}) \cup Z_7$.

Take $W = W_1 \cup W_2 =$

$$\left\{ \left. \begin{bmatrix} [0,a_1I] & 0 & 0 & 0 & 0 & [0,a_5I] & 0 & 0 & [0,a_9I] \\ [0,a_2I] & 0 & 0 & 0 & 0 & [0,a_6I] & 0 & 0 & 0 \\ [0,a_3I] & 0 & 0 & 0 & 0 & [0,a_7I] & 0 & 0 & [0,a_{10}I] \\ [0,a_4I] & 0 & 0 & 0 & 0 & [0,a_8I] & 0 & 0 & 0 \end{bmatrix} \right| a_i \in Q^+ \cup \right.$$

$\{0\};\ 1 \leq i \leq 10\} \cup \left\{ \sum_{i=0}^{6} [0,aI]x^i \,\middle|\, a \in Z_7 \right\} \subseteq V_1 \cup V_2 = V$ be a special neutrosophic interval subsemivector space - subvector space over the semifield - field $(Q^+ \cup \{0\}) \cup Z_7$.

Example 2.3.55: Let $V = V_1 \cup V_2 =$ {All 5×5 interval matrices with intervals of the form $[0, a]$ where $a \in Z_3$} $\cup$ $\left\{ \sum_{i=0}^{40} [0,aI]x^i \,\middle|\, a \in Z^+ \cup \{0\} \right\}$ be a special quasi neutrosophic interval vector space - semivector space over the field - semifield $S = Z_3 \cup Z^+ \cup \{0\}$.

Example 2.3.56: Let $T = T_1 \cup T_2 =$ {All 5×10 matrices with entries from $Z^+ \cup \{0\}$} $\cup$ {All 10×5 neutrosophic interval matrices with entries from $Z_{11}I$} be a special quasi neutrosophic quasi interval semivector space - vector space over the semifield - field $(Z^+ \cup \{0\}) \cup Z_{11}$.

Example 2.3.57: Let $P = P_1 \cup P_2 =$

$$\left\{\begin{bmatrix} a_1I & a_2I & a_3I & a_4I \\ a_5I & a_6I & a_7I & a_8I \\ a_9I & a_{10}I & a_{11}I & a_{12}I \\ a_{13}I & a_{14}I & a_{15}I & a_{16}I \end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}; 1 \leq i \leq 16 \right\} \cup$$

$$\left\{ \sum_{i=0}^{42} [0, a]x^i \middle| a \in Z_{13} \right\}$$

be a special quasi neutrosophic quasi interval semivector space - vector space over the semifield - field; $S = S_1 \cup S_2 = (Z^+ \cup \{0\}) \cup Z_{13}$.

All other properties can be derived for these bistructures with simple appropriate modifications without any difficulty.

Chapter Three

NEUTROSOPHIC n-INTERVAL STRUCTURES (NEUTROSOPHIC INTERVAL n-STRUCTURES)

In this chapter we introduce for the first time the new notion of neutrosophic n-structures and mixed neutrosophic n-structures and discuss various properties enjoyed by them.

DEFINITION 3.1: *Let $S = S_1 \cup S_2 \cup \ldots \cup S_n$ $(n \geq 3)$ where each S_i is a neutrosophic interval semigroup such that $S_i \neq S_j$; if $i \neq j$, $S_i \underline{\not\subset} S_j$ or $S_j \underline{\not\subset} S_i$; $1 \leq i, j \leq n$. Then we define S to be a n - neutrosophic interval semigroup or neutrosophic n-interval semigroup or neutrosophic interval n-semigroup.*

We will give examples of them.

Just we mention if n = 3 we can call them as neutrosophic interval trisemigroup or neutrosophic triinterval semigroup.

Example 3.1: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 = \{[0, aI] \mid a \in Z_5, \times\} \cup \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, +\} \cup \{[0, a+bI] \mid a, b \in Z_6, \times\} \cup$

$$\left\{\begin{bmatrix}[0,aI]\\ [0,bI]\\ [0,cI]\\ [0,dI]\end{bmatrix} \middle| a,b,c,d \in Z_{24},+\right\}$$

be a 4-neutrosophic interval semigroup.

Clearly S is commutative but of infinite order.

Example 3.2: Let M = $M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 \cup M_6$ =

$$\left\{\sum_{i=0}^{6}[0,aI]x^i \middle| a \in Z_3\right\} \cup \left\{\begin{bmatrix}[0,aI] & [0,cI]\\ [0,bI] & [0,dI]\end{bmatrix} \middle| a,b,c,d \in Z_4,\times\right\} \cup$$

$$\left\{\begin{bmatrix}[0,aI]\\ [0,bI]\\ [0,cI]\\ [0,dI]\\ [0,eI]\\ [0,fI]\end{bmatrix} \middle| a,b,c,d,e,f \in Z_6,+\right\} \cup$$

{([0, aI], [0, bI + c], [0, dI]) | a, b, c $\in$ Z_8, $\times$} $\cup$ {3 $\times$ 8 neutrosophic interval matrices with intervals of the form [0, aI] | a $\in$ Z_{14}, +} $\cup$ $\left\{\begin{bmatrix}[0,a+bI] & [0,aI] & [0,dI]\\ [0,d+eI] & [0,bI] & 0\end{bmatrix} \middle| a,b,c,d,e \in Z_2,+\right\}$

be a neutrosophic 6-interval semigroup of finite order.

We can define n-substructures like n-ideals and n-subsemigroup. Also these n-semigroups can contain n-zero divisors, n-units, n-idempotents and so on. We will give some examples of them as the definition is a matter of routine.

Example 3.3: Let T = $T_1 \cup T_2 \cup T_3$ =

$$\left\{\begin{bmatrix}[0,aI] & [0,bI]\\ [0,dI] & [0,cI]\end{bmatrix} \middle| a,b,c,d \in Z_{24},\times\right\} \cup$$

{All 5 $\times$ 5 neutrosophic interval matrices with entries from Z_{10}, $\times$} $\cup$ {3 $\times$ 3 neutrosophic interval matrices from Z_6, $\times$} be a pure neutrosophic triinterval semigroup or pure neutrosophic interval 3 - semigroup. Consider S = $S_1 \cup S_2 \cup S_3$ =

$$\left\{\begin{bmatrix}[0,aI] & [0,bI]\\ [0,cI] & [0,dI]\end{bmatrix} \middle| a,b,c,d \in 2Z_{24}, \times\right\} \cup$$

{all 5 × 5 neutrosophic interval matrices with entries from {0, 5} ⊆ Z_{10}, ×} ∪ {all 3 × 3 neutrosophic interval matrices with entries from {0, 2, 4} ⊆ Z_6, ×} ⊆ $T_1 \cup T_2 \cup T_3$ = T; S is a neutrosophic interval 3-subsemigroup of T.

Further it is easily verified T has 3-zero divisors and 3 - idempotents.

Example 3.4: Let P = $P_1 \cup P_2 = \left\{\sum_{i=0}^{20}[0,aI]x^i \middle| a \in Z^+ \cup \{0\}, +\right\} \cup$ {([0, aI], [0, aI], [0, aI], [0, aI], [0, aI]) | a ∈ $Z^+ \cup \{0\}$, ×} ∪ $\left\{\begin{bmatrix}[0,aI]\\ [0,aI]\\ [0,aI]\\ [0,aI]\\ [0,aI]\end{bmatrix} \middle| a \in Z^+ \cup \{0\}, +\right\}$ be a neutrosophic interval trisemigroup. P has no zero divisors, no idemponents but has triideals and trisubsemigroup.

Example 3.5: Let W = $W_1 \cup W_2 \cup W_3 \cup W_4$ = {[0, aI+b] | a, b ∈ Z_{12}, +} ∪ {[0, a+bI] | a, b ∈ Z_{15}, ×} ∪ {([0, aI], [0, a+bI] | a, b ∈ Z_8, ×} ∪ $\left\{\begin{bmatrix}[0,aI]\\ [0,bI]\\ [0,cI]\end{bmatrix} \middle| a,b,c \in Z_6, +\right\}$ be a neutrosophic interval 4-semigroup of finite order. W has 4-ideals, 4-subsemigroups, 4-zero divisors and 4 units.

We define a neutrosophic n-interval semigroup to be a Smarandache neutrosophic n-interval semigroup if each semigroup S_i in S = $S_1 \cup S_2 \cup \ldots \cup S_n$ is a Smarandache neutrosophic interval n-semigroup.

We will give examples of them.

Example 3.6: Let V = $V_1 \cup V_2 \cup V_3 \cup V_4$ = {[0, aI] | a ∈ Z_{12}, ×} ∪ $\left\{\begin{bmatrix}[0,a_1I] & [0,a_3I] \\ [0,a_2I] & [0,a_4I]\end{bmatrix} \middle| a \in Z_7, \times; 1 \le i \le 4\right\}$ ∪ {[0, aI] | a ∈ Z_{15}, ×} ∪ {([0, aI], [0, bI]) | a, b ∈ Z_9, ×} be the neutrosophic interval 4-semigroup.

Consider P = {[0, I], [0, 11I] | 1, 11 ∈ Z_{12}, ×} ∪ $\left\{A = \begin{bmatrix}[0,a_1I] & [0,a_3I] \\ [0,a_2I] & [0,a_4I]\end{bmatrix} \middle| a_i \in \{1,14\} \subseteq Z_{15}, \times; |A| \neq (0)\right\}$ ∪ {[0, aI] | a ∈ {0, 1} in Z_{15}} ∪ {([0, I], [0, 8I]), ([0, I] [0,I]), ([0, 8I], [0, 8I]), ([0, 8I], [0, I]) | 1, 8 ∈ Z_9, ×} is a neutrosophic 4-interval group, hence V is a Smarandache neutrosophic interval 4-semigroup.

Example 3.7: Let V = $V_1 \cup V_2 \cup V_3$ = {[0, aI] | a ∈ $Z^+ \cup \{0\}$} ∪ $\left\{\begin{bmatrix}[0,aI] \\ [0,bI] \\ [0,cI]\end{bmatrix} \middle| a \in Z^+ \cup \{0\}\right\}$ ∪ $\left\{\begin{bmatrix}[0,aI] & [0,aI] & \ldots & [0,aI] \\ [0,aI] & [0,aI] & \ldots & [0,aI]\end{bmatrix}\right.$ be a 2 × 8 neutrosophic interval matrices with a ∈ $Z^+ \cup \{0\}$,+} be a neutrosophic interval trisemigroup. V is not a Smarandache neutrosophic interval trisemigroup.

In view of this we have the following theorem.

THEOREM 3.1: *Let V = $V_1 \cup V_2 \cup \ldots \cup V_n$ be a neutrosophic n-interval semigroup. In general every V need not be a Smarandache neutrosophic n-interval semigroup.*

Proof follows from the example 3.7 as that neutrosophic interval 3-semigroup is not Smarandache. We now proceed on to define quasi n-interval semigroups.

Let V = $V_1 \cup V_2 \cup \ldots \cup V_n$, if only some of V_i's are neutrosophic interval semigroups i ≤ n and the rest just neutrosophic semigroups, we call V to be a neutrosophic quasi interval n-semigroup.

If in $V = V_1 \cup \ldots \cup V_n$ be such that some V_j's are neutrosophic interval semigroupsp $j \leq n$ and the rest just interval semigroups then we define V to be a quasi neutrosophic n-interval semigroup. Let $V = V_1 \cup V_2 \cup \ldots \cup V_n$ be such that some V_i's are neutrosophic interval semigroups, some V_j's neutrosophic semigroups, some V_k's interval semigroups and the rest semigroups then we call V to be a quasi neutrosophic quasi interval n-semigroup.

We give examples of these situations.

Example 3.8: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 = \{[0, aI] \mid a \in Z_8, \times\} \cup \{[0, a+bI] \mid a, b \in Z^+ \cup \{0\}, +\} \cup \{[0, aI] \mid a \in Q^+ \cup \{0\}, +\} \cup \{(a_1I, a_2I, .., a_nI) \mid a_i \in R^+ \cup \{0\}, \times\} \cup$ $\left\{\begin{bmatrix} a_1I & a_2I & a_3I \\ a_4I & a_6I & a_7I \\ a_8I & a_9I & a_5I \end{bmatrix} \middle| a_i \in Z_{45}, \times, 1 \leq i \leq 9\right\}$ be a neutrosophic quasi interval 5-semigroup.

Example 3.9: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 =$

$$\{[0, a] \mid a \in R^+ \cup \{0\}; \times\} \cup \{[0, aI] \mid a \in Q^+ \cup \{0\}\} \cup$$

$$\{([0, aI], [0, bI]) \text{ where } a, b \in R^+ \cup \{0\}, \times\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1] & [0,a_2] & \ldots & [0,a_{10}] \\ [0,a_{11}] & [0,a_{12}] & \ldots & [0,a_{20}] \end{bmatrix} \middle| a_i \in Z_{450}, \times, 1 \leq i \leq 20, +\right\}$$

be a quasi neutrosophic interval 4-semigroup of infinite order.

Example 3.10: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 =$

$$\{([0, a+bI], [0, c+dI]) \mid a, b \in R^+ \cup \{0\}, +\} \cup$$

$$\left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \text{ where } a_i \in R^+ \cup \{0\}, +; 1 \leq i \leq 12\right\} \cup$$

$$\left\{\left.\begin{bmatrix} a_1I & a_2I & ... & a_{12}I \\ a_{13}I & a_{14}I & ... & a_{24}I \\ a_{25}I & a_{26}I & ... & a_{36}I \\ a_{37}I & a_{38}I & ... & a_{48}I \end{bmatrix}\right| a \in Q^+ \cup \{0\}, 1 \leq i \leq 48, + \right\}$$

$\cup$ {[0, a] | a $\in$ Z_{28}, $\times$} $\cup$ $\left\{\sum_{i=0}^{25}[0, aI]x^i \middle| a \in Z_{47}, +\right\}$ be a quasi neutrosophic quasi interval 5-semigroup.

All properties related with neutrosophic interval n-semigroups, quasi neutrosophic interval n-semigroups and for other related n-structures can be easily extended, studied and defined with simple modifications but with appropriate working.

Now we proceed onto define Neutrosophic interval n-groupoids.

DEFINITION 3.2: *Let $G = G_1 \cup G_2 \cup \ldots \cup G_n$ be such that each G_i is a neutrosophic interval groupoid ($1 \leq i \leq n$) and each G_i is distinct that is $G_i \not\subseteq G_j$, $G_j \not\subseteq G_i$ if $i \neq j$. We define G to be a neutrosophic interval n-groupoid with the component wise operation is the operation on G.*

We will give examples of them.

Example 3.11: Let $L = L_1 \cup L_2 \cup L_3 \cup L_4$ = {[0, aI] | a $\in$ Z_9, *, (2, 7)} $\cup$ {[0, a+bI] | a, b $\in$ Z_{12}, *, (6, 2)} $\cup$ {[0, aI] | a $\in$ Z_{43}, (11, 12), *} $\cup$ {[0, aI] | a $\in$ Z_{24}, (8,15), *} be a neutrosophic interval 4-groupoid of finite order.

Example 3.12: Let $M = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5$ = $\left\{\sum_{i=0}^{20}[0, aI]x^i \middle| a \in Z_4, *, (7,13)\right\}$ $\cup$ {([0, aI], [0, d+cI], [0, bI]) | a, b, c, d $\in$ Z_7, *, (3,2)} $\cup$ $\left\{\left.\begin{bmatrix} [0, a_1I] \\ [0, a_2I] \\ \vdots \\ [0, a_8I] \end{bmatrix}\right| a_i \in Z_{11}, *, (7,3)\right\}$

$$\cup \left\{ \begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_6I] & [0,a_7I] \\ [0,a_8I] & [0,a_9I] & [0,a_5I] \end{bmatrix} \middle| a_i \in Z_{15}, (3,7), * \right\} \cup \{[0, a+bI] \mid$$

a, b ∈ Z_5, (2,3), *} be a neutrosophic interval 5-groupoid. We will show how the operation on M is done. Let x, y ∈ M where $x = x_1 \cup x_2 \cup x_3 \cup x_4 \cup x_5 = \{[0, 5I]x + [0, 2I]x^2 + [0, 4I]\} \cup$

$$([0, 2I], [0, 2+3I], [0,I]) \cup \left\{ \begin{bmatrix} [0,I] \\ 0 \\ [0,2I] \\ 0 \\ [0,5I] \\ 0 \\ 0 \\ [0,3I] \end{bmatrix} \right\} \cup$$

$$\begin{bmatrix} 0 & [0,I] & 0 \\ [0,2I] & 0 & [0,3I] \\ 0 & [0,4I] & 0 \\ [0,5I] & 0 & 0 \end{bmatrix} \cup \{[0, I+4]\} \text{ and}$$

$y = y_1 \cup y_2 \cup y_3 \cup y_4 \cup y_5 = ([0, 7I] + [0, 3I]\, x^8) \cup (0, 0, [0,$

$$6I]) \cup \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ [0,I] \\ 0 \\ 0 \\ [0,2I] \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & [0,I] \\ [0,2I] & 0 & 0 \\ 0 & [0,3I] & 0 \end{bmatrix} \cup \{[0, 3+2I]\}.$$

Now x * y = ([0, 5I] x = [0, 2I]x^2 + [0, 4I]) . ([0, 7I] + [0, 3I] x^8)

$$\cup ([0, 2I], [0, 2+3I], [0, I]) \times (0, 0, [0, 6I]) \cup \begin{bmatrix} [0,I] \\ 0 \\ [0,2I] \\ 0 \\ [0,5I] \\ 0 \\ 0 \\ [0,3I] \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ [0,I] \\ 0 \\ 0 \\ [0,I] \end{bmatrix}$$

$$\cup \begin{bmatrix} 0 & [0,I] & 0 \\ [0,2I] & 0 & [0,3I] \\ 0 & [0,4I] & 0 \\ [0,5I] & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & [0,I] \\ [0,2I] & 0 & 0 \\ 0 & [0,3I] & 0 \end{bmatrix} \cup$$

[0,4+I] × [0, 3+2I]

= ([0, 5I] * [0, 7I] x + [0, 2I] * [0, 7I]x^2 + [0, 4I] * [0, 7I] + [0, 5I] * [0, 3I] x^9 + [0, 2I] [0, 3I] x^{10} + [0, 4I] [0, 3I] x^8) ∪ ([0, 2I]

$$* \ 0, \ [0, 2+3I] * 0, \ [0, I] * [0, 6I]) \cup \begin{bmatrix} [0,I]*0 \\ 0*0 \\ [0,2I]*0 \\ 0*0 \\ [0,5I]*[0,I] \\ 0*0 \\ 0*0 \\ [0,3I]*[0,I] \end{bmatrix} \cup$$

$$\begin{bmatrix} 0*0 & [0,I]*0 & 0*0 \\ [0,2I]*0 & 0*0 & [0,3I]*[0,I] \\ 0*[0,2I] & [0,4I]*0 & 0*0 \\ [0,5I]*0 & 0*[0,3I] & 0*0 \end{bmatrix} \cup [0*0, [4+I]*(3+2I)]$$

= [0,6I]x + [0,25I]x^2 + [0, 39I] + [0, 34I]x^9 + [0, 23I] x^{10} + ([0, 6I], [0, 6+2I], [0, I])

$$\cup \begin{bmatrix} [0,7I] \\ 0 \\ [0,3I] \\ 0 \\ [0,5I] \\ 0 \\ 0 \\ [0,2I] \end{bmatrix} \cup \begin{bmatrix} 0 & [0,3I] & 0 \\ [0,6I] & 0 & [0,I] \\ [0,14I] & [0,12I] & 0 \\ [0,0] & [0,6I] & 0 \end{bmatrix} \cup [0, 2+3I] \in M_1 \cup M_2$$

$\cup\, M_3 \cup M_4 \cup M_5$.

Example 3.13: Let V = $V_1 \cup V_2 \cup V_3$ =

$$\left\{ \sum_{i=0}^{20} [0, a+bI]x^i \,\middle|\, a,b \in Z^+ \cup \{0\}, (3,8), * \right\} \cup \left\{ \begin{bmatrix} [0,a_1+b_1I] \\ [0,a_2+b_2I] \\ \vdots \\ [0,a_{12}+b_{12}I] \end{bmatrix} \right.$$

where a_i, b_i $\in$ $Z^+ \cup \{0\}$; $1 \leq i \leq 40$, +, (9, 41), *} $\cup$

$$\left\{ \begin{bmatrix} [0,a_1+b_1I] & [0,a_2+b_2I] & \ldots & [0,a_5+b_5I] \\ [0,a_6+b_6I] & [0,a_7+b_7I] & \ldots & [0,a_{10}+b_{10}I] \\ \vdots & \vdots & \vdots & \vdots \\ [0,a_{41}+b_{41}I] & [0,a_{42}+b_{42}I] & \ldots & [0,a_{45}+b_{45}I] \end{bmatrix} \right. \text{ where } a_i, b_i$$

$\in Z^+ \cup \{0\}$; $1 \leq i \leq 45$, (9, 41)} be a neutrosophic interval 3 - groupoid} of infinite order. V contain 3 - subgroups which are not 3 - ideals. V also contains 3-ideals. V has no 3-zero divisors and no 3-units.

Now having defined neutrosophic interval n-groupoid we can proceed onto define quasi structures as in case of n-semigroups.

Let V = $V_1 \cup V_2 \cup V_3 \cup \ldots \cup V_n$ where some of the V_i's are neutrosophic interval groupoids and the rest are just interval groupoids, then we define V to be a quasi neutrosophic interval n-groupoid.

Let $V = V_1 \cup V_2 \cup \ldots \cup V_n$ where some V_i's are neutrosophic interval groupoids $i < n$ and the rest of the groupoids are just neutrosophic groupoids. Then we define V to be a neutrosophic quasi interval groupoid. Take $V = V_1 \cup V_2 \cup \ldots \cup V_n$, where some V_i's are $(i<n)$ neutrosophic interval groupoids, some V_j's are neutrosophic groupoids $(j<n)$ some V_k's are interval groupoids $k<n$ and the rest are groupoids then we define V to be a quasi neutrosophic quasi interval groupoid.

We will illustrate all these situations by some examples.

Example 3.14: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 =$

$$\left\{ \sum_{i=0}^{8} [0, aI]x^i \,\middle|\, a \in Z_7, *, (2,5) \right\} \cup$$

$$\{([0, a_1], [0, a_2], \ldots, [0, a_{12}]) \mid a \in Z_9, *, (6, 3)\} \cup$$

$$\left\{ \begin{bmatrix} [0,a_1+b_1I] & [0,a_2+b_2I] & [0,a_3+b_3I] \\ [0,a_4+b_4I] & [0,a_5+b_5I] & [0,a_6+b_6I] \\ \vdots & \vdots & \vdots \\ [0,a_{28}+b_{28}I] & [0,a_{20}+b_{29}I] & [0,a_{30}+b_{30}I] \end{bmatrix} \text{ where } a_i, b_i \in Z_{47}\ *, (40, 7) \right\} \cup$$

$$\left\{ \begin{bmatrix} [0,a_1] & [0,a_2] & [0,a_3] & [0,a_4] \\ [0,a_5] & [0,a_6] & [0,a_7] & [0,a_8] \\ [0,a_9] & [0,a_{10}] & [0,a_{11}] & [0,a_{12}] \\ [0,a_{13}] & [0,a_{14}] & [0,a_{15}] & [0,a_{16}] \end{bmatrix} \middle| a_i \in Z_{25}, *, (7,11) \right\} \cup$$

$$\left\{ \begin{bmatrix} [0,a_1+b_1I] \\ [0,a_2+b_2I] \\ \vdots \\ [0,a_9+b_9I] \end{bmatrix} \middle| a_i, b_i \in Z_{50}, *, (2,7) \right\}$$

be a quasi neutrosophic interval 5-groupoid of finite order.

Example 3.15: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 =$
$\{[0, a+bI] \mid a, b \in Z_{27}, *, (8,11)\} \cup$

$$\left\{\sum_{i=0}^{7} aIx^i \middle| a \in Z_{17}, *, (4,11)\right\} \cup$$

$$\left\{\begin{bmatrix} a_1I \\ a_2I \\ a_3I \\ \vdots \\ a_{20}I \end{bmatrix} \middle| a_i \in Z_{19}, 1 \le i \le 20, *, (5,14)\right\} \cup$$

$\{([0, a_1+b_1I] \ldots [0, a_{10} + b_{10}I]) \mid a_i, b_i \in Z_{19}, *, (7, 11), 1 \le i \le 10\}$ be a neutrosophic quasi interval 4-groupoid of finite order.

Example 3.16: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 \cup V_6 = \{[0, a+bI] \mid a, b \in Z_{20}, *, (3, 13)\} \cup \{[0, a] \mid a \in Z_{49}, *, (2,17)\} \cup$

$$\left\{\sum_{i=0}^{20} aIx^i \middle| a \in Z_{42}, *, (13,11)\right\} \cup$$

$$\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \middle| a_i \in Z_{21}, *, (3,8), 1 \le i \le 25\right\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] & [0,a_4I] \\ [0,a_5I] & [0,a_6I] & [0,a_7I] & [0,a_8I] \\ [0,a_9I] & [0,a_{10}I] & [0,a_{11}I] & [0,a_{12}I] \end{bmatrix} \middle| a_i \in Z_{29}, 1 \le i \le 12, (7,23), *\right\}$$

$$\cup \left\{\sum_{i=0}^{9} a_ix^i \middle| a_i \in Z_{43}, *, (2,11)\right\}$$

be a quasi neutrosophic quasi interval 6-groupoid of finite order.

All properties discussed and described with neutrosophic interval bigroupoids can be derived in case of neutrosophic interval n-groupoids ($n \ge 3$) and obtain a class of neutrosophic interval n-groupoids which are Smarandache strong Bol

(Moufang or P-groupoid or idempotents, alternative and so on). Further all these results can also be extended as in case of quasi n-structures. These are left as exercise to the reader.

We define mixed neutrosophic interval (t, r) groupoid - semigroup. Let $V = V_1 \cup V_2 \cup \ldots \cup V_n$ if t of the V_i's are neutrosophic interval t-groupoid and the rest of the n-t = r of the V_j's are neutrosophic interval r-semigroups then we define V to be a (t, r) mixed neutrosophic interval groupoid - semigroup or mixed neutrosophic interval t - groupoid - r - semigroup or mixed neutrosophic (t, r) groupoid - semigroup. We will give examples of them. We can also define quasi mixed structures.

Example 3.17: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 \cup V_6$ =

$$\left\{\sum_{i=0}^{5}[0,aI]x^i \,\middle|\, a \in Z_{40},(3,2),*\right\} \cup \left\{\sum_{i=0}^{\infty}[0,aI]x^i \,\middle|\, a \in Z_8,\times\right\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & \ldots & [0,a_{10}I] \\ [0,a_{11}I] & [0,a_{12}I] & \ldots & [0,a_{20}I] \\ \vdots & \vdots & & \vdots \\ [0,a_{91}I] & [0,a_{92}I] & \ldots & [0,a_{100}I] \end{bmatrix} \text{ where } a_i \in Z_{25};\ 1 \leq i \leq 100,\right.$$

$\times\} \cup \{([0, a_1I], \ldots, [0, a_{40}I]) \mid a_i \in Z_7, *, (3,2)\} \cup \{$All 7×8 neutrosophic interval matrices with intervals of the form [0, aI] with $a \in Z_{42}$, *, (19, 13)$\}$ $\cup$ $\left\{\begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ \vdots \\ [0,a_{20}I] \end{bmatrix} \,\middle|\, a_i \in Z_{25},*,(12,13)\right\}$ be a mixed neutrosophic interval (4,2) groupoid - semigroup or mixed neutrosophic interval 4-groupoid-2-semigroup.

Example 3.18: Let $G = G_1 \cup G_2 \cup G_3 \cup G_4 \cup G_5$ =

$\left\{\sum_{i=0}^{21}[0,aI]x^i \,\middle|\, a \in Z_7,*,(3,2)\right\} \cup \{([0, aI], [0, bI], [0,cI]) \mid a, b, c$

$\in Z_{19}, *, (3,11)\} \cup \left\{\begin{bmatrix}[0,a_1+b_1I]\\ [0,a_2+b_2I]\\ \vdots \\ [0,a_9+b_9I]\end{bmatrix} \middle| a_i, b_i \in Z_{17}, *, 1 \leq i \leq 9; (9,6)\right\}$

$$\cup \left\{\begin{bmatrix}[0,a_1I] & [0,a_2I] & [0,a_3I]\\ [0,a_4I] & [0,a_5I] & [0,a_6I]\\ [0,a_7I] & [0,a_8I] & [0,a_9I]\end{bmatrix} \middle| a_i \in Z_{12}, *, (5,7)\right\} \cup$$

$\left\{\sum_{i=0}^{\infty}[0,a_i]x^i \middle| a \in Z^+ \cup \{0\}, \times\right\}$ be a mixed neutrosophic interval (4,1) - groupoid - semigroup of infinite order.

Now having seen examples of mixed neutrosophic interval (r,t) groupoid - semigroups we can introduce the quasi n-structure in them as in case of other n-structures. We will give examples of them which show how they are built and how they function.

Example 3.19: Let $M = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 \cup M_6 = \{[0, a] \mid a \in Z_7, *, (3, 6)\} \cup \{[0, aI] \mid a \in Z_{47}, \times\} \cup \left\{\begin{bmatrix}[0,a_1] & [0,a_2]\\ [0,a_3] & [0,a_4]\\ [0,a_5] & [0,a_6]\\ [0,a_7] & [0,a_8]\end{bmatrix} \middle| a_i \in Z_{40}, +, 1 \leq i \leq 8\right\} \cup \{([0, a_1I], [0, a_2I], [0, a_3I], [0, a_4I]) \mid a_i \in Z_{19}, *, (3, 11), 1 \leq i \leq 4\} \cup$

$$\left\{\begin{bmatrix}[0,a_1+b_1I] & [0,a_2+b_2I]\\ [0,a_3+b_3I] & [0,a_4+b_4I]\end{bmatrix} \middle| a_i, b_i \in Z_{12}, \times\right\} \cup$$

$$\left\{\begin{bmatrix}[0,a_1+b_1I]\\ [0,a_2+b_2I]\\ \vdots \\ [0,a_{12}+b_{12}I]\end{bmatrix} \middle| a_i, b_i \in Z_{18}, (3,11), *, 1 \leq i \leq 12\right\}$$

be a mixed quasi neutrosophic interval (3,3) groupoid - semigroup of finite order.

Example 3.20: Let T = $T_1 \cup T_2 \cup \ldots \cup T_8$ = $\{[0, a] \mid a \in Z_{12}, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{12}, (3, 5), *\} \cup \left\{\begin{bmatrix}[0,a_1I]\\ [0,a_2I]\\ \vdots \\ [0,a_7I]\end{bmatrix} \middle| a_i \in Z_{12}, +\right\}$

$\cup \left\{\begin{bmatrix}[0,a_1+b_1I] & \ldots & [0,a_8+b_8I]\\ [0,a_9+b_9I] & \ldots & [0,a_{16}+b_{16}I]\end{bmatrix} \middle| a,b\in Z_{12}, *, (1,11)\right\} \cup$

$\left\{\begin{bmatrix}[0,a_1] & [0,a_2] & [0,a_3] & [0,a_4]\\ [0,a_5] & [0,a_6] & [0,a_7] & [0,a_8]\\ [0,a_9] & [0,a_{10}] & [0,a_{11}] & [0,a_{12}]\\ [0,a_{13}] & [0,a_{14}] & [0,a_{15}] & [0,a_{16}]\end{bmatrix} \middle| a_i \in Z_{12}, \times, 1 \le i \le 16\right\} \cup$

$\left\{\sum_{i=0}^{20}[0,aI]x^i \middle| a\in Z_{12}, +\right\} \cup \{([0, a_1+b_1I]\ [0, a_2+b_2I], [0, a_3+b_3I]) \mid$ $a_i, b_i \in Z_{12}, (0, 7), *\} \cup \left\{\sum_{i=0}^{7}[0,a]x^i \middle| a\in Z_{12}, +\right\}$ be a mixed quasi neutrosophic interval (5,3) semigroup - groupoid of finite order.

Example 3.21: Let V = $V_1 \cup V_2 \cup V_3 \cup \ldots \cup V_7$ $= \left\{\sum_{i=0}^{12}a_iIx^i \middle| a_i \in Z_{24}, +\right\} \cup \left\{\sum_{i=0}^{7}[0,a_iI]x^i \middle| a_i \in Z_{24}, +\right\} \cup \{[0, aI+b]$ $\mid a, b \in Z_{24}, (11, 0), *\} \cup$

$\left\{\begin{bmatrix}[0,a_1+b_1I] & [0,a_2+b_2I]\\ [0,a_3+b_3I] & [0,a_4+b_4I]\\ \vdots & \vdots\\ [0,a_{11}+b_{11}I] & [0,a_{12}+b_{12}I]\end{bmatrix} \middle| a_i, b_i \in Z_{24}, (0,5), *\right\} \cup$

$\left\{\begin{bmatrix}a_1I & a_2I & a_3I\\ a_4I & a_5I & a_6I\\ a_7I & a_8I & a_9I\end{bmatrix} \middle| a\in Z_{20}, \times\right\} \cup \{([0, a_1I], \ldots, [0, a_{20}I]) \mid a_i \in$

Z_{12}, (11, 7), *} ∪ {Z_{20}I, *, (0, 11)} be a mixed neutrosophic quasi interval (3, 4) semigroup - groupoid of finite order which is non commutative.

Example 3.22: Let M = $M_1 \cup M_2 \cup \ldots \cup M_5$ = {Z_{40}I, *, (7, 8)} ∪ {Z_{40}I, ×} ∪ {[0, a+bI] | a, b ∈ Z_{40}, *, (3, 11)} ∪ {all 5 × 5 matrices with entries from Z_{40}I under matrix multiplication} ∪ {all 3 × 7 neutrosophic interval matrices with intervals of the form [0, aI] where a ∈ Z_{40} and (1, 29), *} be a mixed neutrosophic quasi interval (3, 2) groupoid - semigroup of finite order which is non commutative.

Example 3.23: Let P = $P_1 \cup P_2 \cup \ldots \cup P_9$ =
$\left\{\sum_{i=0}^{12}[0,a+bI]x^i \middle| a,b \in Z_7,+\right\} \cup \left\{\sum_{i=0}^{27}[0,a]x^i \middle| a \in Z_{15},+\right\}$ ∪ {Z_{15}, *, (3, 9)} ∪ {Z_{15}I, (4, 0), *} ∪ {7 × 7 neutrosophic interval matrices with intervals of the form [0, a+bI] with a, b ∈ Z_7 under matrix multiplication} ∪ {all 3 × 7 matrices with entries from Z_{12}, *, (0, 7)} ∪ {([0, a+bI] | a, b ∈ Z_{28}, (7, 8), *} ∪ {([0, a_1], …, [0, a_{10}]) | a_i ∈ Z_{11}, ×} ∪ {all 5 × 5 matrices with entries from Z_{12}, ×} be a mixed quasi neutrosophic quasi interval (5, 4) semigroup - groupoid of finite order.

Example 3.24: Let M = $M_1 \cup M_2 \cup \ldots \cup M_7$ = $\left\{\sum_{i=0}^{20}[0,aI]x^i \middle| a \in Z_{18},+\right\} \cup \left\{\sum_{i=0}^{8}a_i x^i \middle| a_i \in Z_{15},+\right\}$ ∪ {[0, a+bI] | a, b ∈ Z_{12}, (9, 0), *} ∪ {Z_{15}, (11, 4), *} ∪ {all 4 × 4 interval matrices with intervals of the form [0, a] with a ∈ Z_{11} under matrix multiplication} ∪ {3 × 6 interval matrices with intervals of the form [0, a] a ∈ Z_7, (2, 5), *} ∪ {[0, a+bI] | a, b ∈ Z_{19}, ×} be a mixed quasi neutrosophic quasi interval semigroup - groupoid of finite order.

Now having seen examples of these mixed structures one can define Smarandache mixed neutrosophic interval n-structures, sub n-structures, n-ideals, n-units and n-zero divisors and their Smarandache analogoue with simple appropriate modification all these tasks are left to the reader.

Now we proceed onto define neutrosophic interval n-groups, describe their quasi analogue and mixed n-structures.

DEFINITION 3.3: *Let $G = G_1 \cup G_2 \cup \ldots \cup G_n$ ($n \geq 3$) be such that each G_i is a neutrosophic interval group with $G_i \not\subseteq G_j$ or $G_j \not\subseteq G_i$ if $i \neq j$; $1 \leq i, j \leq n$. G with the inherited operations from each G_i component wise is defined as the neutrosophic interval n-group or neutrosophic n - interval group.*

We will illustrate this situation by some examples.

Example 3.25: Let $G = G_1 \cup G_2 \cup G_3 \cup G_4 = \{[0, aI] \mid a \in Z_{15}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{19}, +\} \cup \{[0, aI] \mid a \in Z_5 \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in Z_{43} \setminus \{0\}, \times\}$ be the neutrosophic interval 4-group of finite order. We can define 4 order as $15 \times 19 \times 4 \times 42$.

Example 3.26: Let $G = G_1 \cup G_2 \cup G_3 = \{[0, aI] \mid a \in Q^+, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{25}, +\} \cup \{[0, aI] \mid a \in Z_{53} \setminus \{0\}, \times\}$ be a neutrosophic interval 3-group or neutrosophic interval trigroup of infinite order.

It is both important interesting to note that all results regarding finite groups are true in case of neutrosophic interval n-groups of finite n-order. That is to be more specific, Lagrange's theorem, Cayley theorem, Cauchy theorem and Sylow theorems are true in case of finite neutrosophic interval n-groups.

The proofs of these theorems are also straight forward and simple and hence left as an exercise to the reader.

Example 3.27: Let $G = G_1 \cup G_2 \cup G_3 \cup G_4 = \{[0, aI] \mid a \in Z_6, +\} \cup \{[0, aI] \mid a \in Z_7 \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in Z_{13} \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in Z_{10}, +\}$ be a neutrosophic interval four group of order 5460.

Consider $H = H_1 \cup H_2 \cup H_3 \cup H_4 = \{[0, aI] \mid a \in \{0, 2, 4\}, +\} \cup \{[0, aI] \mid a \in \{1, 6\} \subseteq Z_7 \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in \{1, 12\} \subseteq Z_{13} \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in \{0, 2, 4, 6, 8\} \subseteq Z_{10}, +\} \subseteq G_1 \cup G_2 \cup G_3 \cup G_4$ be a neutrosophic interval 4-subgroup of G. Clearly o (H) = $3 \times 2 \times 2 \times 5 = 60$ and 60 / 5460.

Thus we can say Lagrange's theorem for finite group is true in case this finite interval n-group $G = G_1 \cup G_2 \cup G_3 \cup G_4$. Likewise all results can be verified and proved. We say if $G = G_1 \cup G_2 \cup \ldots \cup G_n$ is such that some G_i's are neutrosophic interval groups and the rest just neutrosophic groups then we define G to be a neutrosophic quasi interval n-group. Also if $G = G_1 \cup G_2 \cup \ldots \cup G_n$ be a n-group such that some of the G_i's are neutrosophic interval group and the rest just interval groups then we define G to be a quasi neutrosophic interval n-group. Suppose $G = G_1 \cup G_2 \cup \ldots \cup G_n$ be a n-group in which some G_i's are neutrosophic interval group some G_j's are neutrosophic groups the rest interval groups or groups then we define G to be a quasi neutrosophic quasi interval n-group. We give some examples of them.

Example 3.28: Let $G = G_1 \cup G_2 \cup G_3 \cup G_4 \cup G_5 = \{[0, aI] \mid a \in Z_{14}, +\} \cup \{[0, a] \mid a \in Z_{15}, +\} \cup \{Z_{19} I \setminus \{0\}, \times\} \cup \{QI, +\} \cup \{R \setminus \{0\}, \times\}$ be a quasi neutrosophic quasi interval 5-group of infinite order.

Example 3.29: Let $H = H_1 \cup H_2 \cup H_3 \cup H_4 = \{[0, aI] \mid a \in Z_{27}, +\} \cup \{[0, a] \mid a \in Z_{13} \setminus \{0\}, \times\} \cup \{Z_{45}I, +\} \cup \{[0, aI] \mid a \in Z_{28}, +\}$ be a quasi neutrosophic quasi interval 4-group of finite order.

Example 3.30: Let $G = G_1 \cup G_2 \cup \ldots \cup G_5 = \{[0, aI] \mid a \in Z_{15}, +\} \cup \{([0, a_1I], \ldots, [0, a_5I]) \mid a_i \in Z_{27}, +, 1 \leq i \leq 5\} \cup \{([0, a_1], [0, a_2], \ldots, [0, a_9] \mid a_i \in Z_{11} \setminus \{0\}, 1 \leq i \leq 9, \times\} \cup$

$$\left\{\left.\begin{bmatrix}[0, a_1 I] \\ \vdots \\ [0, a_5 I]\end{bmatrix}\right| a_i \in Z_{40}; 1 \leq i \leq 5, +\right\} \cup \left\{\sum [0, a_i]x^i \,\middle|\, a_i \in Z_{20}, +\right\}$$ be a

quasi neutrosophic interval 5-group of finite order.

Example 3.31: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 = \{[0, aI] \mid a \in Z_{40}, +\} \cup \{Z_{25}I, +\} \cup \{(a_1, \ldots, a_{12}) \mid a_i \in Z_{29}I \setminus \{0\}, \times\} \cup$

$\left\{\sum_{i=0}^{7}[0, aI]x^i \,\middle|\, a \in Z_{11}, +\right\}$ be a neutrosophic quasi interval 4-group of finite order.

We can define mixed structures like n-group - groupoid n-group - semigroup and n-group - groupoid - semigroup. We give only one example of each as it is simple and direct.

Example 3.32: Let G = $G_1 \cup G_2 \cup G_3 \cup G_4 \cup G_5 \cup G_6$ = {[0, aI] | a $\in$ Z_{15}, $\times$} $\cup$ $\left\{\sum_{i=0}^{10}[0, a_i]x^i \,\middle|\, a_i \in Z_{10}, +\right\} \cup$ $\left\{\begin{bmatrix}[0, a_1I]\\ \vdots \\ [0, a_{10}I]\end{bmatrix} \,\middle|\, a_i \in Z_{40}, 1 \le i \le 10, +\right\}$ $\cup$ {all 10 $\times$ 10 neutrosophic interval matrices with intervals of the form [0, a+bI] with a, b $\in$ Z_6 under matrix multiplication} $\cup$ {[0,a+bI] | a, b $\in$ Z_{18}, $\times$} $\cup$ $\left\{\begin{bmatrix}[0, aI] & [0, bI]\\ [0, cI] & [0, dI]\end{bmatrix} = A \,\middle|\, a, b, c, d \in Z_{15}, |A| \ne 0, \times\right\}$ be a mixed neutrosophic interval (3, 3) semigroup - group of finite order.

Example 3.33: Let G = $G_1 \cup G_2 \cup G_3 \cup G_4$ =
$\left\{\sum_{i=0}^{20}[0, a_iI]x^i \,\middle|\, a_i \in Z_{45}, +\right\}$ $\cup$ {[0, aI+b] | a, b $\in$ Z_{40}, *, (0, 11)} $\cup$

$$\left\{\begin{bmatrix}[0, a_1I]\\ [0, a_2I]\\ \vdots \\ [0, a_9I]\end{bmatrix} \,\middle|\, a_i \in Z_{20}, +, 1 \le i \le 9\right\} \cup$$

$$\left\{\begin{bmatrix}[0, aI] & [0, bI]\\ [0, cI] & [0, dI]\end{bmatrix} \,\middle|\, a, b, c, d \in Z_7, *, (0, 5)\right\}$$

be a mixed neutrosophic interval (2, 2) group - groupoid of finite order.

Example 3.34: Let $G = G_1 \cup G_2 \cup G_3 \cup \ldots \cup G_7 = \{([0, aI] \mid a \in Z_{22}, +\} \cup \{[0, a_1I], \ldots, [0, a_{12}I]) \mid a_i \in Z_{40}, *, (3, 7), 1 \leq i \leq 12\} \cup \{[0, a+bI] \mid a, b \in Z_{12}, \times\} \cup \{([0, a_1I], \ldots, [0, a_{15}I]) \mid a_i \in Z_{43} \setminus \{0\}; 1 \leq i \leq 15, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{29}, *, (8, 11)\} \cup$ {All 15 × 15 neutrosophic interval matrices with intervals of the form [0, aI] with $a \in Z_{42}$ under matrix multiplication} $\cup$

$$\left\{\left.\begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ \vdots \\ [0,a_{12}I] \end{bmatrix} \right| a_i \in Z_{25}, +, 1 \leq i \leq 12\right\}$$

be a mixed neutrosophic interval (3, 2, 2) group - groupoid - semigroup.

Quasi n-structures can also be defined and analysed by the interested reader. Also all results can be proved with direct and simple modifications some of these mixed structures are also non associative . We now define n-loops using neutrosophic intervals and give examples of mixed n-structures using loops.

DEFINITION 3.4: *Let $L = L_1 \cup L_2 \cup \ldots \cup L_n$, be such that each L_i is a neutrosophic interval loop where $L_i \not\subseteq L_j$, $L_j \not\subseteq L_i$, $i \neq j$, $1 \leq i, j \leq n$; on L with the inherited operation from each L_i; $1 \leq i \leq n$. We define L to be a neutrosophic interval n-loop.*

Example 3.35: Let $L = L_1 \cup L_2 \cup L_3 \cup L_4 \cup L_5 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 29\}, *, 9\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 31\}, 8, *\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 29\}, 19, *\} \cup \{[0, a+bI] \mid a \in \{e, 1, 2, \ldots, 49\}, 9, *\} \cup \{[0, a+bI] \mid a, b \in Z_5, *, 3\}$ be a neutrosophic interval 5-loop of finite order.

Example 3.36: Let $L = L_1 \cup L_2 \cup L_3 = \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 17\}, 9, *\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 45\}, 23, *\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 57\}, 29, *\}$ be a neutrosophic interval 3-loop.

Clearly L is a commutative 3-loop.

It is pertinent to mention here that all properties studied and described on neutrosophic interval biloops can be extended to n-loop without any difficulty. We can define quasi n-structures as in case of groups, semigroups etc. This task is also left for the reader as an exercise.

However we supply a few examples.

Example 3.37: Let $L = L_1 \cup L_2 \cup \ldots \cup L_6 = \{[0, aI] \mid a \in \{e, 1, 2, .., 27\}, 8, *\} \cup \{[0, aI+b] \mid a, b \in \{e, 1, 2, \ldots, 43\}, 25, *\} \cup \{[0, a] \mid a \in \{e, 1, 2, \ldots, 25\}, 8, *\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 53\}, 28, *\} \cup \{[0, aI+b] \mid a, b \in \{e, 1, 2, \ldots, 29\}, *, 12\} \cup \{[0, a] \mid a \in \{e, 1, 2, \ldots, 23\}, 8, *\}$ be a quasi neutrosophic interval 6-loop of finite order.

Example 3.38: Let $L = L_1 \cup L_2 \cup L_3 \cup L_4 \cup \ldots \cup L_7 = \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 23\}, 9, *\} \cup \{aI \mid a \in \{e, 1, 2, \ldots, 29\}, *, 23\} \cup \{aI \mid a \in \{e, 1, 2, \ldots, 49\}, 9, *\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 81\}, 41, *\} \cup \{aI \mid a \in \{e, 1, 2, \ldots, 25\}, 13, *\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 7\}, 3, *\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 17\}, 9, *\}$ be a neutrosophic quasi interval 7-loop of finite order.

Example 3.39: Let $L = L_1 \cup L_2 \cup \ldots \cup L_5 = \{[0, a] \mid a \in \{e, 1, 2, \ldots, 29\}, 23, *\} \cup \{ L_{27}^{(8)} \} \cup \{[0, aI+b] \mid a, b \in \{e, 1, 2, \ldots, 57\}, *, 5\} \cup \{aI \mid a \in \{e, 1, 2, \ldots, 19\}, *, 8\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 21\}, *, 11\}$ be a quasi neutrosophic quasi interval 5-loop of finite order.

We can also define mixed n-structures. We only give examples of them.

Example 3.40: Let $L = L_1 \cup L_2 \cup L_3 \cup L_4 \cup L_5 \cup L_6 = \{[0, aI] \mid a \in Z_{240}, +\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 23\}, *, 9\} \cup \{[0, a+bI] \mid a, b \in Z_{25}, +\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 27\}, *, 11\} \cup \{[0, aI] \mid a \in Z_{23} \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 65\}, *, 3\}$ be a mixed neutrosophic interval (3, 3) loop-group of finite order.

Example 3.41: Let $M = M_1 \cup M_2 \cup \ldots \cup M_5 = \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 23\}, 8, * \} \cup \{[0, a+bI] \mid a, b \in Z_{23}, \times\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 17\}, 9, *\} \cup \{[0, a_1+b_1I], \ldots, [0, a_8+b_8I] \mid a_i, b_i \in Z_{49}, *, 9\} \cup$ {all 6×6 neutrosophic interval matrices with intervals of the form [0, a+bI] where a, b $\in$ Z_8 under matrix multiplication be the mixed neutrosophic interval (3, 2) loop - semigroup.

Example 3.42: Let T = $T_1 \cup T_2 \cup \ldots \cup T_5$ = {[0, aI] | a $\in$ {e, 1, 2, …, 27}, 8, *} $\cup$ {[0, aI] | a $\in$ Z_{45}, *, (9, 8)} $\cup$ {[0, aI + b] | a, b $\in$ {e, 1, 2, …, 43}, *, 9} $\cup$ {[0, a_1I], …, [0, a_{12}I] | a_i $\in$ Z_{20}, (3, 7), *, $1 \leq i \leq 12$} $\cup$ {([0, a_1I], …, [0, a_6I]) | a_i $\in$ {e, 1, 2, …, 23}, *, 8, $1 \leq i \leq 6$} be a neutrosophic interval (3, 2) loop - groupoid of finite order.

Example 3.43: Let L = $L_1 \cup L_2 \cup L_3 \cup \ldots \cup L_9$ = {[0, a+bI] | a, b $\in$ {e, 1, 2, …, 29}, 9, *} $\cup$ $\left\{ \sum_{i=0}^{10} [0, aI]x^i \,\middle|\, a \in Z_{10}, + \right\}$ $\cup$ {([0, a_1I], …, [0, a_{10}I]) | a_i $\in$ Z_{15}, $1 \leq i \leq 10$, $\times$} $\cup$ {([0, a_1I], …, [0, a_{12}I]) where a_i $\in$ Z_{17}, (8, 9), *) $\cup$ {([0, a_1I] , …, [0, a_{20}I]) | a_i $\in$ $Z_{17} \setminus \{0\}$, $\times$} $\cup$ {([0, a_1I], …, [0, a_{12}I]$)^t$ | a_i $\in$ {e, 1, 2, …, 23}, 9, *} $\cup$ {all 3 $\times$ 3 neutrosophic interval matrices of the form [0, a+bI] where a, b $\in$ Z_{12} under matrix multiplication} $\cup$ $\left\{ \sum_{i=0}^{\infty} [0, a + bI]x^i \,\middle|\, a, b \in Z^+ \cup \{0\}, \times \right\}$ $\cup$ {[0, aI] | a $\in$ {e, 1, 2, …, 47}, 25, *} be a mixed neutrosophic interval (3, 2, 3, 1) loop - group - semigroup - groupoid of infinite order.

Now we can study the substructures of these mixed n-structures. Further quasi n-structures can be defined and described by the reader.

Now we can define n-rings.

DEFINITION 3.5: *Let R = $R_1 \cup R_2 \cup \ldots \cup R_n$, where each R_i is a neutrosophic interval ring such that $R_i \not\subseteq R_j$ or $R_j \not\subseteq R_i$, if $i \neq j$, $1 \leq i, j \leq n$. R inherits the operation from each R_i carried out componentwise. R is defined as the neutrosophic interval n-ring.*

We give examples of them.

Example 3.44: Let R = $R_1 \cup R_2 \cup R_3 \cup R_4 \cup R_5$ = {[0, aI] | a $\in$ Z_{25}, +, $\times$} $\cup$ {[0, a+bI] | a, b $\in$ Z_{47}, +, $\times$} $\cup${([0, a_1I], [0, a_2I],

…, [0, a_8I]) | $a_i \in Z_{50}$, +, ×, $1 \leq i \leq 8$} ∪ {([0, $a_1 + b_1$I], [0, a_2 + b_2I], [0, a_3+b_3I]) | $a_i \in Z_{220}$, +, ×; $1 \leq i \leq 3$} ∪ $\left\{\sum_{i=0}^{\infty}[0, aI] \middle| a \in Z_{40}, +, \times\right\}$ be a neutrosophic interval 5-ring.

Example 3.45: Let R = $R_1 \cup R_2 \cup R_3 \cup R_4$ = $\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_7I] & [0,a_8I] & [0,a_9I] \end{bmatrix} \middle| a_i \in Z_{12}, 1 \leq i \leq 9, +, \times\right\}$ ∪ {([0, a_1I+b_1], [0, a_2I + b_2], [0, a_3I + b_3], [0, a_4I, b_4], [0, a_5I + b_5]) | a_i, b_i $\in Z_{15}$, $1 \leq i \leq 5$, +, ×} ∪ $\left\{\sum_{i=0}^{\infty}[0, a_i] \middle| a_i \in Z_{20}, +, \times\right\}$ ∪ {[0, aI+b] | a, b $\in Z_{18}$, +, ×} be a neutrosophic interval 4-ring. R has zero divisors. R is a Smarandache 4-ring. R has idempotents and units.

Example 3.46: Let S = $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$ = {[0, aI] | a ∈ Z_{19}, ×, +} ∪ {[0, aI] | a ∈ Z_{17}, +, ×} ∪ {[0, aI] | a ∈ Z_{11}, +, ×} ∪ {[0, aI] | a ∈ Z_7, +, ×} ∪ {[0, aI] | a ∈ Z_{43}, +, ×} be a neutrosophic interval 5-ring. S has no 5-zero divisors or 5-idemponents. Infact we call S a neutrosophic 5-field.

THEOREM 3.2: *Let R = $R_1 \cup R_2 \cup R_3 \cup \ldots \cup R_n$ = {[0, a] | a ∈ Z_{p_1}, +, ×} ∪ {[0, aI] | a ∈ Z_{p_2}, +, ×} ∪ … ∪ {[0, aI] | a ∈ Z_{p_n}, +, ×} where $p_1, p_2, \ldots, p_n$ are n-distinct primes. R is a neutrosophic interval n-field.*

The proof is direct and hence left as an exercise to the reader.

THEOREM 3.3: *Let R = $R_1 \cup R_2 \cup R_3 \cup \ldots \cup R_n$ = {[0, a] | a ∈ Z_{n_1}, +, ×} ∪ {[0, aI] | a ∈ Z_{n_2}, +, ×} ∪ … ∪ {[0, aI] | a ∈ Z_{n_n}, +, ×} where $n_1, \ldots, n_n$ are n-distinct composite numbers of the form $2p_i = n_i$; i = 1, 2, …, n. R is a Smarandache neutrosophic interval n-ring.*

This proof is also direct and hence left as an exercise to the reader.

Note T = {[0, p_1], [0, 0]} ∪ … ∪ {[0, p_n], [0, 0]} ⊆ R is a neutrosophic interval n-field hence T is a Smarandache neutrosophic n-ring.

Example 3.47: Let S = $S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$ = {[0, aI] | a ∈ Z_6, ×, +} ∪ {[0, aI] | a ∈ Z_{10}, +, ×} ∪ {[0, aI] | a ∈ Z_{14}, +, ×} ∪ {[0, aI] | a ∈ Z_{22}, +, ×} ∪ {[0, aI] | a ∈ Z_{26}, +, ×} be a neutrosophic interval 5-ring. S is a Smarandache neutrosophic interval 5-ring. P = $P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5$ = {[0, 3I], 0, +, ×} ∪ {0, [0, 3I], +, ×} ∪ {[0, 7I], 0, +, ×} ∪ {[0, 11I], 0, +, ×} ∪ {[0, 13I], 0, +, ×} ⊆ S is a neutrosophic interval 5-field; hence S is a Smarandache neutrosophic interval 5-ring.

Example 3.48: Let R = $R_1 \cup R_2 \cup R_3 \cup R_4$ = {[0, aI] | a ∈ Z_{12}, +, ×} ∪ {[0, aI] | a ∈ Z_{30}, +, ×} ∪ {[0, aI] | a ∈ Z_{42}, +, ×} ∪ {[0, aI] | a ∈ Z_{66}, +, ×} be a neutrosophic interval 4-ring. R is a Smarandache neutrosophic interval 4-ring. For T = $T_1 \cup T_2 \cup T_3 \cup T_4$ = {[0, 4I], [0, 8I], 0, +, ×} ∪ {[0, 10I], [0, 20I], 0, +, ×} ∪ {[0, 14I], [0, 28I], 0, +, ×} ∪ {[0, 22I], [0,44I], 0, +, ×} ⊆ R = $R_1 \cup R_2 \cup R_3 \cup R_4$ is a neutrosophic interval 4-field. So R is a Smarandache neutrosophic interval 4-ring.

We can define in case of neutrosophic interval n-rings the notion of n-subrings and n-ideals. This task is simple and hence left as an exercise to the reader.

THEOREM 3.4: *Let S = $S_1 \cup S_2 \cup S_3 \cup \ldots \cup S_n$ = {[0, aI] | a ∈ Z_{p_1}, +, ×} ∪ {[0, aI] | a ∈ Z_{p_2}, +, ×} ∪ … ∪ {[0, aI] | a ∈ Z_{p_n}, +, ×} be a neutrosophic interval n-ring where p_j's are primes for j=1, 2, …, n. S has no n-ideal and no n-subrings.*

The proof is direct and hence is left as an exercise to the reader.

The major hint to be taken is that each $S_i \cong Z_{p_i}$ so is a field.

We cannot get using these types of neutrosophic intervals a ring of characteristic zero.

However we can find polynomial neutrosophic interval rings of the form $\sum_{i=0}^{\infty}[0, a_i I]x^i$ or $\sum_{i=0}^{\infty}[0, a_i + b_i I]x^i$ where $a_i, b_i \in Z_n$, $n < \infty$. We also can get the neutrosophic interval matrix ring using square neutrosophic interval matrices, the later ring is non commutative. Almost all properties can be derived without any difficulty in case of these neutrosophic interval n-rings.

Example 3.49: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 \cup S_6$

$= \left\{ \sum_{i=0}^{\infty}[0, a_i + b_i I]x^i \,\middle|\, a_i, b_i \in Z_{25}, +, \times \right\}$ $\cup$ {All 2×2 neutrosophic intervals of the form [0, a+bI] where a, b $\in Z_{10}$, +, ×} $\cup$ {([0, a_1I], [0, a_2I], …, [0, a_{12}I] | $a_i \in Z_{15}$, +, ×; $1 \leq i \leq 12$} $\cup$ $\left\{ \sum_{i=0}^{\infty}[0, a_i I]x^i \,\middle|\, a_i \in Z_{30}, +, \times \right\}$ $\cup$ {[0, a+bI] | a, b $\in Z_{35}$, +, ×} $\cup$ {5×5 upper triangular neutrosophic interval matrices with intervals of the form [0, a+bI] with a, b in Z_{40}, +, ×} is also a neutrosophic interval 6-ring of infinite order. This ring has 6-ideals, 6-subrings, 6-zero divisors and 6-units.

We can also define quasi n-rings as in case of other n-structures. We give one of two examples before we proceed to define n-semirings.

Example 3.50: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4$ =

$$\left\{ \sum_{i=0}^{\infty}[0, a_i + b_i I]x^i \,\middle|\, a_i, b_i \in Z_{12}, +, \times \right\} \cup$$

$$\left\{ \sum_{i=0}^{\infty}[0, a_i I]x^i \,\middle|\, a_i \in Z_{40}, +, \times \right\} \cup$$

{all 5×5 neutrosophic interval matrices with intervals of the form [0, a+bI]; a, b $\subseteq Z_{120}$ under matrix addition and multiplication} $\cup$ {all 7×7 lower triangular interval matrices with intervals of the form [0, a] with a $\in Z_{48}$, +, ×} be a quasi neutrosophic interval 4-ring.

Example 3.51: Let $M = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 =$

$$\left\{\sum_{i=0}^{\infty}[0, a_iI]x^i \middle| a_i \in Z_8\right\} \cup \left\{\sum_{i=0}^{\infty} a_iIx^i \middle| a_i \in Z_{12}\right\} \cup$$

{All 5 × 5 neutrosophic interval matrices with entries from Z_{15}, +, ×} $\cup \left\{\begin{bmatrix} a_1I & a_2I \\ a_3I & a_4I \end{bmatrix}\right.$ where $a_i \in Z_{18}$, $1 \leq i \leq 4$, +, ×} ∪ {([0, a_1+b_1I] [0, a_2+b_2I], …, [0,a_9+b_9I])| a_i, $b_i \in Z_{30}$, +, ×} be the neutrosophic quasi interval 5-ring of infinite order which is not commutative.

Example 3.52: Let $M = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 \cup M_6 \cup M_7$ = {([0, a_1], [0, a_2], …, [0, a_{12}]) | $a_i \in Z_{45}$, $1 \leq i \leq 12$, +, ×} ∪

$$\left\{\sum_{i=0}^{\infty} a_iIx^i \middle| a_i \in Z_{28}, +, \times\right\} \cup$$

$$\left\{\begin{bmatrix} [0, a_1I] & [0, a_2I] \\ [0, a_3I] & [0, a_4I] \end{bmatrix} \middle| a_i \in Z_7, 1 \leq i \leq 4, +, \times\right\} \cup$$

$$\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| a_i \in Z_{28}, 1 \leq i \leq 16, +, \times\right\} \cup$$

{([0, $a_1 + b_1I$], [0, $a_2 + b_2I$] , …, [0, $a_{11}+b_{11}I$]) | a_i, $b_i \in Z_{48}$, $1 \leq i \leq 11$; +, ×} ∪ {(a_1I, a_2I, …, a_8I) | $a_i \in Z_{248}$, +, ×, $1 \leq i \leq 8$} ∪

$$\left\{\sum_{i=0}^{\infty}[0, a_i]x^i \middle| a_i \in Z_{28}, +, \times\right\}$$

be the quasi neutrosophic quasi interval 7-ring.

It is important and interesting to note that all results on ring theory using Z_n's can be easily extended and studied with simple appropriate modifications. This task is left to the reader [8].

Now we proceed onto define neutrosophic interval n-semiring ($n \geq 3$).

DEFINITION 3.6: *Let $S = S_1 \cup S_2 \cup \ldots \cup S_n$, where each S_i is a neutrosophic interval semiring such that $S_i \not\subseteq S_j$ or $S_j \not\subseteq S_i$, if $i \neq j$, $1 \leq i, j \leq n$. S inherits the operation componentwise from each S_i, $i=1, 2, \ldots, n$. We define S to be a neutrosophic interval n-semiring.*

We give examples of them.

Example 3.53: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, +, \times\} \cup \{([0, a_1+b_1I], \ldots, [0, a_9 + b_9I]) \mid a_i, b_i \in Q^+ \cup \{0\}, 1 \leq i \leq 9, +, \times\} \cup \left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \end{bmatrix} \middle| a_i \in R^+ \cup \{0\}, 1 \leq i \leq 4, +, \times \right\}$ $\cup \{[0, a+bI] \mid a, b \in 5Z^+ \cup \{0\}\}$ be a neutrosophic interval 4-semiring.

Example 3.54: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 = \{[0, a+bI] \mid a, b \in Q^+ \cup \{0\}, \times, +\} \cup$

$$\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 4, +, \times \right\} \cup$$

$\{([0, a_1+b_1I], [0, a_2+b_2I], [0, a_3 + b_3I]) \mid a_i, b_i \in Z^+ \cup \{0\}, 1 \leq i \leq 3, +, \times\} \cup$ {All 10×10 upper triangular matrices with neutrosophic intervals $[0, a+bI]$ where $a, b \in R^+ \cup \{0\}, +, \times\} \cup$ {all 6×6 lower triangular neutrosophic intervals matrices with intervals of the form $[0,aI]$ where $a \in Q^+ \cup \{0\}, +, \times\}$ be a neutrosophic interval 5-semiring.

We can define as in case of usual semirings the notion of Smarandache n-semirings, n-subsemirings and so on with simple modifications. Further in case of neutrosophic interval n-semirings we can define the quasi structure.

Example 3.55: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 = \{[0, aI] \mid a \in R^+ \cup \{0\}\} \cup \{([0, a_1I], [0, a_2I], \ldots, [0, a_7I]), (0, 0, \ldots, 0) \mid a_i \in Z^+; 1 \leq i \leq 7, +, \times\} \cup \{(0,0, 0, 0), ([0, a_1 + b_1I], [0, a_2 + b_2I], [0, a_3+b_3I], [0, a_4 + b_4I]) \mid a_i, b_i \in Q^+, 1 \leq i \leq 4, +, \times\} \cup \{[0, a + bI] \mid a, b \in Z^+ \cup \{0\}\}$ be a neutrosophic interval 4-semiring which is also a neutrosophic interval 4-semifield.

We can also define the notion of quasi n-structures and mixed n-structures using these semirings. We only give examples of them as the definition is routine and direct.

Example 3.56: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 = \{[0, aI] \mid a \in Z^+ \cup \{0\}, +, \times\} \cup \{([0, a+bI] \mid a, b \in Z^+ \cup \{0\}, +, \times\} \cup \{ [0, a + bI] \mid a, b \in Q^+ \cup \{0\}, +, \times\} \cup \{[0, a] \mid a \in R^+ \cup \{0\}\} \cup \{([0, a_1], [0, a_2], \ldots, [0, a_{12}])) \mid a_i \in Q^+ \cup \{0\}, 1 \leq i \leq 12\}$ be a quasi neutrosophic interval 4-semiring.

Example 3.57: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 =$

$$\left\{\begin{bmatrix} [0,aI] & [0,bI] \\ [0,cI] & [0,dI] \end{bmatrix} \middle| a,b,c,d \in R^+ \cup \{0\},+,\times\right\} \cup$$

$$\{([0, aI], [0, bI], [0, cI]) \mid a, b, c \in Q^+ \cup \{0\}, +, \times\} \cup$$

$$\left\{\begin{pmatrix} a_1I & a_2I & a_3I \\ a_4I & a_5I & a_6I \\ a_7I & a_8I & a_9I \end{pmatrix} \middle| a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 9,+,\times\right\} \cup$$

$$\left\{\sum_{i=0}^{\infty}[0,a_iI]x^i \middle| a_i \in Q^+ \cup \{0\},+,\times\right\} \cup \left\{\sum_{i=0}^{\infty} a_iIx^i \middle| a_i \in Z^+ \cup \{0\}\right\}$$

be a neutrosophic quasi interval 5-semiring.

Example 3.58: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 \cup V_6 =$

$$\left\{\sum_{i=0}^{\infty}[0,a_iI]x^i \middle| a_i \in Q^+ \cup \{0\}\right\} \cup$$

$$\left\{\sum_{i=0}^{\infty} a_ix^i \middle| a_i \in R^+ \cup \{0\}\right\} \cup$$

$$\{([0, a_1+b_1I],\ldots,[0, a_9 + b_9I]) \mid a_i, b_i \in Z^+ \cup \{0\}; 1 \leq i \leq 9,+, \times\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1] & [0,a_2] & [0,a_3] & [0,a_4] \\ [0,a_5] & [0,a_6] & [0,a_7] & [0,a_8] \\ [0,a_9] & [0,a_{10}] & [0,a_{11}] & [0,a_{12}] \\ [0,a_{13}] & [0,a_{14}] & [0,a_{15}] & [0,a_{16}] \end{bmatrix} \middle| a_i \in Q^+ \cup \{0\}, 1 \leq i \leq 16,+,\times\right\}$$

$$\cup \left\{ \begin{pmatrix} a_1 I & a_2 I & a_3 I \\ a_4 I & a_6 I & a_5 I \\ a_7 I & a_8 I & a_9 I \end{pmatrix} \middle| a_i \in R^+ \cup \{0\}, 1 \le i \le 9, +, \times \right\} \cup$$

$$\left\{ \begin{pmatrix} a_1 & a_2 & a_3 \\ 0 & a_4 & a_5 \\ 0 & 0 & a_6 \end{pmatrix} \middle| a_i \in Z^+ \cup \{0\}, 1 \le i \le 6, +, \times \right\}$$

be a quasi neutrosophic quasi interval 6-semiring of infinite order.

All properties related with semirings can be derived for n-semirings without any difficulty.

Now we proceed onto define mixed n-structure with two binary operations.

DEFINITION 3.7: *Let $V = V_1 \cup V_2 \cup \ldots \cup V_n$ where some of the V_i's are neutrosophic interval rings and the rest are neutrosophic interval semirings. We call V the mixed neutrosophic interval ring - semiring.*

We illustrate this situation by some examples.

Example 3.59: Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 =$

$$\left\{ \sum_{i=0}^{\infty} [0, a_i + b_i I] x^i \middle| a_i, b_i \in Z_{20}, +, \times \right\} \cup$$

$$\{[0, a+bI] \mid a, b \in R^+ \cup \{0\}, +, \times\} \cup$$

$$\left\{ \sum_{i=0}^{\infty} [0, a_i I] x^i \middle| a \in Q^+ \cup \{0\}, +, \times \right\} \cup$$

$$\left\{ \begin{bmatrix} [0, aI] & [0, bI] \\ [0, cI] & [0, dI] \end{bmatrix} \middle| a, b, c, d \in Z_{45}, +, \times \right\} \cup$$

{all 9×9 upper triangular neutrosophic interval matrices with intervals of the form $[0, a+bI]$ where $a, b \in Z^+ \cup \{0\}, +, \times$} be a mixed neutrosophic interval (2, 3) ring - semiring.

Example 3.60: Let M = $M_1 \cup M_2 \cup M_3 \cup M_4$ = {[0, aI] | a ∈ Z_{30}, ×, +} $\cup \left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}, 1 \le i \le 4, +, \times \right\} \cup$

$$\left\{\begin{bmatrix} [0,a_1+b_1I] & [0,a_4+b_4I] & [0,a_7+b_7I] \\ [0,a_2+b_2I] & [0,a_5+b_5I] & [0,a_8+b_8I] \\ [0,a_3+b_3I] & [0,a_6+b_6I] & [0,a_9+b_9I] \end{bmatrix} \middle| a_i, b_i \in Q^+ \cup \{0\}, 1 \le i \le 9 \right\}$$

∪ {4 × 4 upper triangular interval matrices with intervals of the form [0, a+bI] with a, b ∈ Z_{320}} be a mixed neutrosophic interval ring - semiring.

Quasi mixed neutrosophic interval semiring – ring and other quasi types of semiring - ring are described by the following examples.

Example 3.61: Let V = $V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$ =

$$\left\{\sum_{i=0}^{\infty}[0,a_i]x^i \middle| a_i \in Z^+ \cup \{0\}, +, \times \right\} \cup$$

$$\left\{\sum_{i=0}^{\infty}[0,a+bI]x^i \middle| a, b \in Z_{45}, +, \times \right\} \cup$$

$\left\{\begin{bmatrix} [0,aI] & [0,bI] & [0,cI] \\ [0,dI] & [0,eI] & [0,fI] \\ [0,tI] & [0,sI] & [0,rI] \end{bmatrix}\right.$ where a, b, c, d, e, f, t, s and r are in Z_{20}, +, ×} ∪ {([0, a], [0, b], [0, c], [0, d]) | a, b, c, d ∈ Z_{42}, +, ×} $\cup \left\{\sum_{i=0}^{\infty}[0,a_i+b_iI]x^i \middle| a_i, b_i \in R^+ \cup \{0\}, +, \times \right\}$ be a mixed quasi neutrosophic interval (2, 3) semiring - ring.

Example 3.62: Let P = $P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5$ =

$$\left\{\sum_{i=0}^{\infty}[0,a_iI]x^i \middle| a_i \in Q^+ \cup \{0\}, +, \times \right\} \cup \left\{\sum_{i=0}^{\infty}a_iIx^i \middle| a_i \in Z_{42}, +, \times \right\} \cup$$

$$\left\{\left[\begin{array}{ccc} a_1I & a_2I & a_3I \\ a_4I & a_5I & a_6I \\ a_7I & a_8I & a_9I \end{array}\right] \middle| a_i \in Z^+ \cup \{0\}, +, \times \right\} \cup \{([0, a_1I + b_1], \ldots,$$

$[0, a_5I + b_5]) \mid a_i, b_i \in Z_{40}, +, \times\} \cup \{(a_1I, a_2I, \ldots, a_{12}I) \mid a_i \in Z_{25}, +, \times; 1 \leq i \leq 12\}$ be a neutrosophic quasi interval (2, 3) semiring - ring.

Example 3.63: Let $T = T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5 \cup T_6 =$

$$\left\{\left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{array}\right] \middle| a_i \in Z_{144}, 1 \leq i \leq 16, +, \times \right\} \cup$$

$$\left\{\left[\begin{array}{cc} [0, a_1 + b_1I] & [0, a_2 + b_2I] \\ [0, a_3 + b_3I] & [0, a_4 + b_4I] \end{array}\right] \middle| a_i, b_i \in Z^+ \cup \{0\}, 1 \leq i \leq 4, +, \times \right\} \cup$$

$$\left\{\sum_{i=0}^{\infty} [0, a_i]x^i \middle| a_i \in Z_{244}, +, \times \right\} \cup$$

$$\left\{\sum_{i=0}^{\infty} [0, a_i + b_iI]x^i \middle| a_i, b_i \in Z_{42}, +, \times \right\} \cup \{([0, a_1+b_1I], [0, a_2+b_2I],$$

$\ldots, [0, a_{10}+b_{10}I]$ where $a_i, b_i \in Z^+ \cup \{0\}, 1 \leq i \leq 10, +, \times\} \cup$

$$\left\{\left[\begin{array}{ccccc} a_1I & a_2I & a_3I & a_4I & a_5I \\ 0 & a_6I & a_7I & a_8I & a_9I \\ 0 & 0 & a_{10}I & a_{11}I & a_{12}I \\ 0 & 0 & 0 & a_{13}I & a_{14}I \\ 0 & 0 & 0 & 0 & a_{15}I \end{array}\right] \middle| a_i \in Q^+ \cup \{0\}, 1 \leq i \leq 15, +, \times \right\}$$

be a quasi neutrosophic quasi interval (3, 3) ring - semiring.

Now on similar lines we can define neutrosophic interval n-vector spaces, neutrosophic interval n-semivector spaces and their quasi structure and mixed structure. We give one or two examples of them. Further all results worked in case of bivector spaces and bisemivector spaces are true in case of n-vector spaces and n-semivector spaces.

Example 3.64: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 =$

$$\left\{ \sum_{i=0}^{20} [0, a_i + b_i I]x^i \,\middle|\, a_i, b_i \in Z_{19}, + \right\} \cup$$

$$\left\{ \begin{bmatrix} [0, a_1 I] \\ [0, a_2 I] \\ [0, a_3 I] \end{bmatrix} \,\middle|\, a_i \in Z_{19}, 1 \le i \le 3, + \right\} \cup$$

$\{([0, a_1+b_1I], [0,a_2+b_2I], [0, a_3+b_3I], [0, a_4+b_4I], [0, a_5+b_5I]) \mid a_i$, $b_i \in Z_{19}, +\} \cup \left\{ \begin{pmatrix} [0, a_1 I] & [0, a_2 I] & \ldots & [0, a_{10} I] \\ [0, a_{11} I] & [0, a_{12} I] & \ldots & [0, a_{20} I] \end{pmatrix} \right.$ where $a_i \in Z_{19}$, $1 \le i \le 20, +\}$ be a neutrosophic interval 4-vector space over the field Z_{19}.

Example 3.65: Let $V = V_1 \cup V_2 \cup V_3 =$

$$\left\{ \sum_{i=0}^{7} [0, a_i I]x^i \,\middle|\, a_i \in Z_7, + \right\} \cup$$

$$\left\{ \begin{pmatrix} [0, a_1 I] & [0, a_2 I] & \ldots & [0, a_8 I] \\ [0, a_9 I] & [0, a_{10} I] & \ldots & [0, a_{16} I] \\ [0, a_{17} I] & [0, a_{18} I] & \ldots & [0, a_{24} I] \end{pmatrix} \,\middle|\, a_i \in Z_{23}, 1 \le i \le 24, + \right\} \cup$$

$\{([0, a_1+b_1I], [0, a_2+b_2I], \ldots, [0, a_7+b_7I]) \mid a_i, b_i \in Z_{53}, 1 \le i \le 7, +\}$ be a special neutrosophic interval 3-vector space over the 3-field $F = F_1 \cup F_2 \cup F_3 = Z_7 \cup Z_{23} \cup Z_{53}$.

Example 3.66: Let $S = S_1 \cup S_2 \cup S_3 =$

$$\left\{ \sum_{i=0}^{20} [0, a_i I]x^i \,\middle|\, a_i \in Z^+ \cup \{0\}, + \right\} \cup$$

$$\left\{ \begin{bmatrix} [0, a_1 + b_1 I] & [0, a_2 + b_2 I] & [0, a_3 + b_3 I] \\ [0, a_4 + b_4 I] & [0, a_5 + b_5 I] & [0, a_6 + b_6 I] \\ [0, a_7 + b_7 I] & [0, a_8 + b_8 I] & [0, a_9 + b_9 I] \end{bmatrix} \,\middle|\, a_i, b_i \in Q^+ \cup \{0\}, 1 \le i \le 9, + \right\}$$

$$\cup \left\{ \begin{bmatrix} [0, a_1 + b_1 I] \\ [0, a_2 + b_2 I] \\ \vdots \\ [0, a_{12} + b_{12} I] \end{bmatrix} \middle| a_i, b_i \in Z^+ \cup \{0\}, 1 \le i \le 12, + \right\}$$

be a neutrosophic interval 3-semivector space over the semifield $F = Z^+ \cup \{0\}$. Clearly in case of n-semivector spaces we cannot define over finite semifields except over chain lattices.

Example 3.67: Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 \cup V_6 =$

$$\left\{ \sum_{i=0}^{24} [0, a_i I] x^i \middle| a_i \in Z_7, + \right\} \cup$$

{All 10×19 neutrosophic interval matrices with entries from Z_7 under addition} $\cup$

$$\left\{ \begin{bmatrix} [0, a_1 II] \\ [0, a_2 I] \\ \vdots \\ [0, a_{20} I] \end{bmatrix} \middle| a_i \in Z_7, 1 \le i \le 20, + \right\} \cup$$

$\{([0, a_1+b_1I], [0, a_2 + b_2I], \ldots, [0, a_7+b_7I]) \mid a_i, b_i \in Q^+ \cup \{0\}, 1 \le i \le 7\} \cup \left\{ \sum_{i=0}^{27} [0, a_i I] x^i \middle| a_i \in Z^+ \cup \{0\}, + \right\} \cup$ {all 8×3 neutrosophic interval matrices with entries from $Z^+ \cup \{0\}$, +} be a mixed neutrosophic interval (3, 3) vector space - semivector space over the field - semifield $F = Z_7 \cup (Z^+ \cup \{0\})$.

All properties related with bivector spaces, bisemivector spaces can be derived and proved in case of n-vector spaces and n-semivector spaces built using neutrosophic intervals.

We can replace N (Z_n) or N $(Z^+ \cup \{0\})$ or N $(R^+ \cup \{0\})$ or N $(Q^+ \cup \{0\})$ by $\langle [0, 1] \cup [0, I] \rangle$ and derive results which will be defined as algebraic structures using fuzzy neutrosophic intervals: In many cases min or max operations can be used. This is also a matter of routine and left for the reader to develop them.

Chapter Four

APPLICATIONS OF NEUTROSOPHIC INTERVAL ALGEBRAIC STRUCTURES

Neutrosophic interval algebraic structures can find applications in places / models where an element of indeterminacy is present. For instance in mathematical models we can use these structures. Also in finite element analysis if indeterminacy is present in those models we can use them so that caution can be applied in places where indeterminacy is present.

Neutrosophic interval matrices can be used in real world problems in the field of medicine or engineering or social issues when the data in hand is an unsupervised one.

When the study of eigen values or eigen vectors are expected to be an unsupervised one we can use these neutrosophic interval matrices or fuzzy - neutrosophic interval matrices. Further when n sets of simultaneous values are expected as the estimated / predicted values these n-neutrosophic interval matrices can be used. Also when the expert expects to study or obtain results in an interval one can use these interval models.

Since the research and the subject happens to be new and use new notions of neutrosophic intervals the authors are sure in due course of time these will find more and more applications.

Chapter Five

SUGGESTED PROBLEMS

In this chapter we suggest over hundred problems of which some are simple, some of them are difficult and some of them are research problems.

1. Give an example of a pure neutrosophic interval bisemigroup which is not commutative but of finite order.

2. Give some interesting properties related with pure neutrosophic interval bisemigroups of infinite order.

3. Give an example of a interval neutrosophic bigroup $G = G_1 \cup G_2$ of finite biorder and show the Lagrange theorem for G is true.

4. Let $G = S_3 \cup \{[0, aI] \mid a \in Z_{17}, +\}$ be a quasi interval quasi neutrosophic bigroup.
   a) Find the biorder of G.
   b) Can G have bisubgroups?
   c) Is G simple?

5. Let $V = V_1 \cup V_2 = S(5) \cup \{[0, aI] \mid a \in Z_{45}, \times\}$ be a quasi interval quasi neutrosophic bisemigroup
   a) Find biorder of V.
   b) Can V have bisubsemigroups?
   c) Does every biideal's biorder divide biorder of V?
   d) Is every bisubsemigroup a biideal in V?
   e) Is V a S-Lagrange bisemigroup?

6. Obtain some special properties enjoyed by pure neutrosophic interval bigroups of finite order.

7. Is all the classical theorems true in case of finite groups true in case of finite pure neutrosophic interval bigroup?

8. Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{43} \setminus \{0\}, \times\} \cup \{[0, aI] \mid a \in Z_{40}, +\}$ be a pure neutrosophic interval bigroup.
   a) Find biorder of G.
   b) Prove all classical theorems for finite groups are true in case of G.
   c) Find quotient interval bigroups of G.
   d) Find the order of $[0, 25I] \cup [0, 8I]$ in G.

9. Obtain some interesting properties about quasi interval quasi neutrosophic bigroupoids.

10. Prove every pure neutrosophic bigroupoid need not be a S-bigroupoid.

11. Let $G = G_1 \cup G_2 = \{[0, aI] \mid a \in Z_{41}, *, (2, 21)\} \cup \{[0, aI] \mid a \in Z_{41}, *, (3, 12)\}$ be a pure neutrosophic interval bigroupoid.
   i) Find the biorder of G.
   ii) Is G a S-bigroupoid?
   iii) Is G S-Moufang?
   iv) Does G satisfy Bol identity?

12. Obtain some interesting properties enjoyed by pure neutrosophic interval bisemirings of finite order.

13. Is every pure neutrosophic interval bisemiring built using $R^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $Z^+ \cup \{0\}$ a pure neutrosophic interval bisemifield?

14. Does there exist a pure neutrosophic interval bifield?

15. Define neutrosophic interval birings.

16. Give examples of S-neutrosophic interval bisemirings.

17. Is every pure neutrosophic interval bisemiring a Smarandache neutrosophic interval bisemiring?

18. Can neutrosophic interval birings be built using $Z^+ \cup \{0\}$ or $Q^+ \cup \{0\}$ or $R^+ \cup \{0\}$? Justify your answer.

19. Can we have a neutrosophic interval biring of prime biorder? Justify!

20. Prove we have neutrosophic interval birings of biorder 24.

21. How many neutrosophic interval birings of biorder 24 can be constructed.

22. Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_5, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_6, +, \times\}$ be a neutrosophic interval biring.
    i) Find biorder of M.
    ii) Find quasi biideals in M.
    iii) Can M have bisubrings?
    iv) Find quasi bizero divisors of M.
    v) Is M a S-biring?

23. Let $R = R_1 \cup R_2 = \{[0, aI] \mid a \in Z_{45}, +, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{24}, +, \times\}$ be a neutrosophic interval biring.
    a) Find the biorder of R.
    b) Find biideals in R.
    c) Does R have bisubrings which are not biideals?
    d) Find bizero divisors in R.
    e) Find biidempotents in R.

f) Find biunits in R.
g) Find a biisomorphism of R to R with nontrivial bikernel.

24. Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_9, *, (3, 4)\} \cup \{[0, aI] \mid a \in Z_9, *, (2, 1)\}$ be a neutrosophic interval bigroupoid?
i) Find biorder of P.
ii) Is P a S-bigroupoid?
iii) Does P have S-subbigroupoid?
iv) Does P satisfy any of the special identities?
v) Find biideals in P.
vi) Does P have subbigroupoids which are not Smarandache?

25. Let $T = T_1 \cup T_2 = \{[0, aI] \mid a \in Z_{12}, *, (7, 5)\} \cup \{[0, a+bI]$ where $a, b \in Z_{17}, *, (8, 9)\}$ be a neutrosophic interval bigroupiod.
i) Find biorder of T.
ii) Is T a S-bigroupoid?
iii) Does T have subbigroupodids?
iv) Can T have S-biideals?
v) Is every right biideal of T a left biideal of T?
vi) Is T a P-bigroupoid?
vii) Is T a S-Bol bigroupoid?
viii) Can T be a S-strong Moufang bigroupoid?
ix) Can T be a S-idempotent bigroupoid?

26. Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 29\}, *, 8\} \cup \{[0, a+bI] \mid a,b \in \{e, 1, 2, \ldots, 29\} *, 12\}$ be a neutrosophic interval biloop.
i) Find biorder of P.
ii) Find S-biloops of any?
iii) Is P a S-biloop?
iv) Does P satisfy any of the special identities?
v) Is P a S-strongly Lagrange biloop?
vi) Is P-S-strong Moufang?

27. Does there exist an interval biloop of biorder n; n a composite odd number?

28. Give an example of a Moufang neutrosophic interval biloop.

29. Construct a neutrosophic interval biloop which is a Bol biloop.

30. Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 33\}, 14, * \} \cup \{[0, a+bI] \mid a,b \in \{e, 1, 2, \ldots, 37\} \; *, 14\}$ be a neutrosophic interval biloop.
i) Find biorder of M.
ii) Is M S-strong Moufang?
iii) Is M a S-biloop?
iv) Is M simple?
v) Can M have bisubloops?
vi) Can M have S-bisubloops?
vii) Can M have bisubloops which are not Smarandache?
viii) Does M satisfy any of the special identities?
ix) Find the biisotope of M.
x) Is M a S-Lagrange biloop?

31. Let $V = V_1 \cup V_2 = \{[0, a+bI] \mid a, b \in Z_{15}, \times \} \cup \{[0, a+bI] \mid a,b \in Z_{15}, (7, 8), *\}$ be a neutrosophic interval semigroup - groupoid.
i) Find biorder of V.
ii) Find substructures of V.
iii) Does V have bizerodivisors?
iv) Is V a Smarandache semigroup-groupoid?

32. Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{14}, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{12}, \times\}$ be a neutrosophic interval bisemigroup.
i) Find biorder of V.
ii) Is M a S-bisemigroup?
iii) Does M contain S-biideals?
iv) Can M have S - bizerodivisors?
v) Can M have bisubsemigroups which are not biideals?

33. Let $S = S_1 \cup S_2 = \{[0, a+bI] \mid a, b \in Z_7, \times\} \cup \{[0, a+bI] \mid a, b \in Z_5, \times\}$ be a neutrosophic interval bisemigroup.

i) Find biorder of S.
ii) Is S a S-bisemigroup?
iii) Can S have bizero divisors?
iv) Can S have S-bizero divisors?
v) Can S have biideals?
vi) Can S have bisubsemigroups which are not biideals?
vii) Can S have biidempotents?
viii) Is S bisimple?

34. Let $V = V_1 \cup V_2 = \{[0, a+bI] \mid a, b \in Z_{40}, \times\} \cup \{[0, a+bI] \mid a, b \in Z_{40}, (3, 17), *\}$ be a neutrosophic interval semigroup - groupoid.
i) Find biorder of V.
ii) Is V a Smarandache semigroup - groupoid?
iii) Can V have biideals?
iv) Can V have bizero divisors?

35. Let $S = S_1 \cup S_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 23\}, 9, *\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 11\}, 9, *\}$ be a neutrosophic interval biloop.
i) Find biorder of S.
ii) Prove S is Smarandache.
iii) Prove S is S-simple.
iv) Is S simple?

36. Let $L = \{[0, aI] \mid a \in Z_{28}, *, (3, 2)\} \cup \{[0, a+bI] \mid a, b \in Z_{27}, *, 8\}$ be a neutrosophic interval groupoid - loop.
i) Does L satisfy any one of the standard identities?
ii) Find biorder of L.
iii) Is L Smarandache?
iv) Can L have Smarandache substructures?
v) Find any other interesting property associated with L.

37. Let $M = M_1 \cup M_2 = \{[0, a] \mid a \in Z_{48}, \times\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 47\}, *, 9\}$ be the neutrosophic interval semigroup - loop.
i) Find biorder of M.
ii) Find substructures of M.
iii) Is M Smarandache?
iv) Can M have Smarandache substructures?

38. Let $T = T_1 \cup T_2 = \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 43\}, *, 8\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 43\}, *, 9\}$ be a neutrosophic interval biloop.
   i) Is T a Smarandache biloop?
   ii) Does T satisfy any of the special identities?
   iii) Is T simple?
   iv) Can T have nontrivial bisubloops?
   v) Is T a S-strongly Lagrange biloop?
   vi) What is the biorder of T?
   vii) Find $f:T \to T$, a bihomomorphism such that bikernel of f is nontrivial.

39. Let $W = W_1 \cup W_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 81\}, *, 17\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 81\}, *, 23\}$ be a neutrosophic interval biloop.
   i) Find biorder of W.
   ii) Is W a S-biloop?
   iii) Does W contain S-subbiloops?
   iv) Is W simple?
   v) Does W satisfy any of the special identities?
   vi) Find the biisotope of W.

40. Let $M = M_1 \cup M_2 = \{[0, a+bI] \mid a, b \in Z_{16}, *, (3, 5)\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 15\}, 8, *\}$ be a neutrosophic interval groupoid - loop.
   i) Find the biorder of M
   ii) Is M a Smarandache structure?
   iii) Does M contain Smarandache substructure?
   iv) Does M satisfy any of the special identities?
   v) Does M contain a neutrosophic interval normal subgroupoid - normal subloop?

41. Let $V = V_1 \cup V_2 = \{[0, a+bI] \mid a, b \in Z_{90}, *, (3, 7)\} \cup \{[0, a+bI] \mid a, b \in Z_{90}, \times\}$ be a neutrosophic interval groupoid - semigroup.
   a) Find the biorder of V.
   b) Is V Smarandache?
   c) Can V have Smarandache substructures?

d) Prove V has pure neutrosophic interval subgroupoid - subsemigroup and interval subgroupoid S-ideals?
e) Does V contain S-biideals?

42. Let $M = M_1 \cup M_2 = \{[0, a] \mid a \in Z_{41}, \times\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 41\}, 9, *\}$ be a neutrosophic interval semigroup - loop.
a) Find the biorder of M.
b) Is M Smarandache?
c) Is $S = S_1 \cup S_2 = \{[0, I], [0, 40I], \times\} \cup \{[0, eI], [0, 9I]\} \subseteq M_1 \cup M_2$ a neutrosophic interval bigroup?
d) Is M a simple bistructure?

43. Obtain some interesting properties enjoyed by neutrosophic interval semiring - ring.

44. Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 29\}, 9, *\} \cup \{[0, aI] \mid a \in Z_{29}, (3, 20), *\}$ be a neutrosophic interval loop-groupoid.
i) Find biorder of M.
ii) Does M satisfy any of the special identities?
iii) Is M a Smarandache bistructure?
iv) Can M have S-bisubstructures?
v) Give a bisubstructure which is not a Smarandache subbistructure.

45. Let $W = W_1 \cup W_2 = \{Z_{40}, +\} \cup \{[0, aI] \mid a \in Z_{40}, \times\}$ be a quasi neutrosophic quasi interval group - loop.
i) Find biorder of W.
ii) Is W a Smarandache bistructure?
iii) Find subbistructures in S.

46. Let $V = V_1 \cup V_2 = \{Z_{25}, \times\} \cup \{[0, aI] \mid a \in Z_{25}, *, (3, 22)\}$ be a pure neutrosophic interval semigroup-groupoid.
i) Find biorder of V.
ii) Is V a Smarandache bistructure?
iii) Can V have S-zero divisors?
iv) Find S-units in V (if it exists).

v) Can V have idempotents which are not S-idempotents?

47. Let $P = P_1 \cup P_2 = \{[0, a + bI] \mid a, b \in Z_{47}, *, (3, 14)\} \cup \{[0, aI] \mid a \in Z_{47}, *, (0, 14)\}$ be a neutrosophic interval bigroupoid.
i) Find biorder of P.
ii) Can P have bizerodivisors?
iii) Can P have S-subbigroupoids?
iv) Can P have S-biideals?
v) Find S-quasi bisubstructures in P.
vi) Does P satisfy any of the special identities?
vii) Does P satisfy any of the Smarandache strong special identities like Bol, Moufang etc?

48. Let $P = P_1 \cup P_2 = \{[0, aI] \mid a \in Z_{40}, (3, 0), *\} \cup \{[0, aI] \mid a \in Z_{25}, (0, 3), *\}$ be a pure neutrosophic interval bigroupoid.
i) Find biorder of P.
ii) Is P Smarandache?
iii) Is P a Smarandache strong Bol bigroupoid?
iv) Is P a Smarandache strong P-bigroupoid?
v) Can P have Smarandache bisubgroupoids?
vi) Does P contain S-bizero divisors?

49. Let $M = M_1 \cup M_2 = \{[0, aI] \mid a \in Z_{25}, \times, +\} \cup \{[0, aI] \mid a \in Z^+ \cup \{0\}, +, \times\}$ be a pure neutrosophic interval ring - semiring.
i) Is M a Smarandache bistructure?
ii) Can M have S-bizero divisors?
iii) Can M have S-biidempotents?
iv) Can M have S-subring - subsemiring?
v) Can M be a quasi Smarandache bistructure?
vi) Find any other property associated with M.

50. Let $T = T_1 \cup T_2 = [[0, a+bI] \mid a, b \in Z_{20}, \times, +\} \cup \{[0, a+bI] \mid a, b \in Z_{24}, \times, +\}$ be a neutrosophic interval biring.
i) Find biorder of T.
ii) Is T a S-biring?

iii) Can T be a quasi bifield?
iv) Find bizero divisors and S-bizero divisors in T.
v) Can T have S-biidempotents?
vi) Find S-biunits in T.
vii) Find a biideal $I = I_1 \cup I_2$ and find T/I.

51. Let $V = V_1 \cup V_2 = \{[0, a+bI] \mid a, b \in Z_{43}, +\} \cup \left\{\begin{bmatrix}[0,aI]\\ [0,bI]\\ [0,cI]\end{bmatrix} \middle| a,b,c \in Z_{43}, +\right\}$ be a neutrosophic interval bivector space over the field $F = Z_{43}$.
i) What is the bidimension of V?
ii) Find bivector subspaces of V.
iii) Can V be a Smarandache bivector space?
iv) Write V as a direct sum.
v) Find linear bioperator T on V such that T^{-1} exists.

52. Let $V = V_1 \cup V_2 = \begin{bmatrix}[0,5I] & 0 & [0,I] & 0\\ 0 & [0,2I] & 0 & 0\\ 0 & [0,I] & 0 & [0,5I]\\ [0,2I] & 0 & [0,I] & 0\end{bmatrix} \cup \begin{bmatrix}[0,I] & [0,2I] & 0\\ 0 & [0,I] & 0\\ [0,I] & 0 & [0,3I]\end{bmatrix}$ be a pure neutrosophic interval bimatrix with entries from Z_{11}.
i) Find the characteristic bivalues.
ii) Find the characteristic bivector.
iii) Is V bidiagonalizable?

53. Let $V = V_1 \cup V_2$ be a pure neutrosophic bivector space where V_1 =

$$\left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_7I] & [0,a_8I] & [0,a_9I] \\ [0,a_{10}I] & [0,a_{11}I] & [0,a_{12}I] \end{bmatrix} \right| a_i \in Z_{29}, 1 \le i \le 12 \right\} \cup$$

$\{([0, a+bI], [0, c+dI], [0, e+fI]) \mid a, b, c, d, e, f, \in Z_{29}\}$;

a) Find a bibasis for V.
b) What is the bidimension of V?
c) Find a linear bioperator T on V so that T^{-1} does not exist.
d) Write V as a bidirect sum.
e) Define a biprojection on V.
f) Is V a Smarandache bivector space?
g) Does V contain subbivector spaces which are not Smarandache?

54. Let V = $V_1 \cup V_2$ = $\left\{\sum_{i=0}^{8}[0,aI]x^i \,\middle|\, a \in Z_{43}, +\right\}$ $\cup$ $\{([0, aI], [0, bI], [0, cI]) \mid a, b, c \in Z_{43}, +\}$ be a neutrosophic interval bivector space over the field Z_{43}.

i) Find a bibasis for V.
ii) What is the bidimesnion of V?
iii) Is V Smarandache?
iv) Write V as a direct sum.
v) Find a Smarandache subbivector space if any in V.
vi) Find T a linear bioperator with nontrivial bikernel.

55. Let V = $V_1 \cup V_2$ = $\left\{\begin{bmatrix} [0,a+bI] \\ [0,c+dI] \\ \vdots \\ [0,r+sI] \end{bmatrix}\right.$ be the collection of all 9 × 1 neutrosophic interval matrices with entries from $Z_7\}$ $\cup$ $\{([0, aI], [0, bI], [0, cI], [0, dI]) \mid a, b, c, d \in Z_7\}$ be a neutrosophic interval bimatrix.

i) Find the number of elements in V.
ii) What is the bidimension of V?
iii) Find a bibasis for V.

56. Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 = \left\{\sum_{i=0}^{5}[0, aI]x^i \middle| a \in Z_7, +\right\} \cup$ $\{([0, a_1I], [0, a_2I], \ldots, [0, a_9I) \mid a_i \in Z_9, \times\} \cup$ $\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \\ \vdots & \vdots \\ [0,a_{11}I] & [0,a_{12}I] \end{bmatrix} \middle| a_I \in Z_{10}, +\right\} \cup \{([0, a+bI] \mid a, b \in$ $Z_{12}, \times\}$ be a neutrosophic interval 4-semigroup.

i) Find the 4-order S.
ii) Find 4-subsemigroups.
iii) Can S have 4-ideals?
iv) Can S have 4-zero divisors?
v) Can S have 4-units?
vi) Is S a Smarandache 4-semigroup?

57. Let $V = V_1 \cup V_2 = \{[0, aI] \mid a \in Z^+ \cup \{0\}\} \cup \{[0, aI] \mid a \in Z_7\}$ be a neutrosophic interval semivector space - vector space over the semifield - field $(Z^+ \cup \{0\}) \cup Z_7 = S$.

i) Find a bibasis of V over S.
ii) Can V have neutrosophic subsemivector space - vector subspace of V over S?
iii) What the bidimension of V over S?

58. Let $W = W_1 \cup W_2 = \{[0, a+bI] \mid a, b \in Z_{19}\} \cup \{[0, a + bI] \mid a, b \in Z_{23}\}$ be a special interval bivector space over the bifield $F = F_1 \cup F_2 = Z_{19} \cup Z_{23}$.

i) Find a bibasis of W.
ii) Is W a finite bidimensional?
iii) Find bivector subspace of W.
iv) Find a special linear bioperator of W, which is invertible.
v) Prove W has both.
   a) Pure special neutrosophic interval subbivector space of W.
   b) Find special interval bivector subspace of W.

59. Let $W = W_1 \cup W_2 = \{([0, a_1I], [0, a_2I], [0, a_3I], [0, a_4I]) \mid a_i \in Z_{19}, 1 \le i \le 4\} \cup \left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_7I] & [0,a_8I] & [0,a_9I] \end{bmatrix}\right.$ where $a_i \in Z_{19}; 1 \le i \le 9\}$ be a neutrosophic interval bivector space over the field $S = Z_{19}$.

i) Find a bibasis of W.
ii) What is a bidimension of W?
iii) Find a subbivector space of W.
iv) Define a linear bioperator T on W so that T^{-1} exists.
v) Define $T : W \to W$ so that bikernel is nontrivial.

60. Let $W = W_1 \cup W_2 = \{[0, a+bI] \mid a, b \in Z_7\} \cup \{[0, a+bI] \mid a, b \in Z_{11}\}$ be a special neutrosophic interval bivector space over the bifield $F = F_1 \cup F_2 = Z_7 \cup Z_{11}$.

i) Find bidimension of W.
ii) Find a bibasis of W.
iii) Find a special neutrosophic interval bivector subspace of V.
iv) Write W as a bidirect sum.
v) Find a bioperator T so that T^{-1} does not exist.
vi) Write W as a pseudo direct sum.

61. Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 = \left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ \vdots & \vdots \\ [0,a_9I] & [0,a_{10}I] \end{bmatrix} \middle| a_i \in Z_{43}, 1 \le i \le 10\right\} \cup \{([0, a_1I], [0, a_2I], [0, a_3I]) \mid a_i \in Z_7; 1 \le i \le 3\} \cup \left\{\sum_{i=0}^{7}[0,aI]x^i \middle| a \in Z_{11}\right\} \cup \left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \end{bmatrix} \middle| a_i \in Z_{17},+\right\}$ be a neutrosophic interval 4-group.

i) Find the 4-order of V.
ii) Find neutrosophic interval four subgroups.

62. Let $V = V_1 \cup V_2 \cup V_3 \cup V_4$ be a neutrosophic interval 4-semigroup where $V_1 = \left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ \vdots & \vdots & \vdots \\ [0,a_{28}I] & [0,a_{29}I] & [0,a_{30}I] \end{bmatrix}\right.$ where $a_i \in Z_{20}$, $1 \leq i \leq 30$, $+\}$, $V_2 = \{[0, a+bI] \mid a, b \in Z_{40}, \times\}$, $V_3 = \left\{\sum_{i=0}^{6}[0,aI] \middle| a \in Z_{14},+\right\}$ and $V_4 = \left\{\begin{bmatrix} [0,aI] & [0,bI] \\ [0,cI] & [0,dI] \end{bmatrix} \middle| a,b,c,d \in Z_{12}\right\}$ be a neutrosophic interval 4-semigroup.

i) Is V a Smarandache 4-interval semigroup?
ii) Prove V is of finite order.
iii) Find neutrosophic interval 4-subsemigroup.
iv) Does V contain 4-zero divisor?
v) Does V contain Smarandache 4-units?
vi) Is V a S-weakly Lagrange 4-semigroup?
vii) Can V have S-4-ideals?
viii) Does V contain S-4-subsemigroups which are not S-4-ideals?

63. Let $R = R_1 \cup R_2 \cup R_3 \cup R_4 \cup R_5 = \{[0, a+bI] \mid a, b \in Z_8, \times, +\} \cup \left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_7I] & [0,a_8I] & [0,a_9I] \end{bmatrix} \middle| a_i \in Z_{27}, 1 \leq i \leq 9, +, \times\right\} \cup \{([0, a_1I], [0, a_2I], \ldots, [0, a_8I]) \mid a_i \in Z_{20}, 1 \leq i \leq 8, +, \times\} \cup \left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \end{bmatrix} \middle| a_i \in Z_{42}; 1 \leq i \leq 4\right\} \cup \left\{\sum_{i=0}^{\infty}[0,a_i + b_iI]x^i \middle| a_i, b_i \in Z_{40}\right\}$ be a neutrosophic interval 5-ring.

i) Find S-5-subring of R.

ii) Is R a S-5-ring?
iii) Is R a Smarandache commutative 5-ring?
iv) Does R contain S-zero divisors?
v) Can R have zero divisors which are not S-zero divisors?
vi) Can R have ideals which are not S-ideals?
vii) Can R have a principal 5-ideal?

64. Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 = \{[0, a+bI] \mid a, b \in Z_8, * (2, 6)\} \cup \{[0, a+bI] \mid a, b \in Z_8, * (3, 5)\} \cup \{[0, a+bI] \mid a, b \in Z_8, * (1, 7)\} \cup \{[0, a+bI] \mid a, b \in Z_8, * (4, 4)\}$ be a neutrosophic interval 4-groupoid.
i) Is S a S-4-groupoid?
ii) Can S have Smarandache 4-subgroupoid?
iii) Does S satisfy any of the Smarandache identities?
iv) Is S a S-P-4-groupoid?

65. Let $T = T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5 = \{[0, a+bI] \mid a, b \in Z_7, * (3, 4)\} \cup \{[0, a+bI] \mid a, b \in Z_9, * (2, 7)\} \cup \{[0, a+bI] \mid a, b \in Z_8, * (1, 7)\} \cup \{[0, a+bI] \mid a, b \in Z_{11}, * (3, 8)\} \cup \{[0, a+bI] \mid a, b \in Z_{13}, * (12, 1)\}$ be a neutrosophic interval 5-groupoid.
i) Is T a S- neutrosophic interval 5-groupoid?
ii) Can T have S-sub 5 - groupoid?
iii) What is the order of T?
iv) Is T a S-strong P-5-groupoid?
v) Is T a S-Bol 5-groupoid?
vi) Find 5-zero divisors and S-zero divisors if any in T.

66. Let $M = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 = \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 33\}, 14, *\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 29\}, 20, *\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 31\}, 15, *\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 53\}, 27, *\} \cup \{[0, a+bI] \mid a, b \in \{e, 1, 2, \ldots, 47\}, 23, *\}$ be a neutrosophic interval 5-loop.
i) Find the order of M.
ii) Find S-5 subloops of M.
iii) Is M a S-5 loop?
iv) Is M a S-strong Bol 5-loop?

v) Is M a S-Moufang 5-loop?
vi) Is M a S-strong alternative 5-loop?
vii) Find S-strong P-5-loop.
viii) Obtain some interesting properties enjoyed by M.
ix) Prove $x = x_1 \cup x_2 \cup x_3 \cup x_4 \cup x_5$ in M is such that x^2 = identity.

67. Let $P = P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5 \cup P_6 = \{[0, a+bI] \mid a, b \in Z_7, \times\} \cup \{[0, aI] \mid a \in Z_{15}, +\} \cup \{[0, a+bI] \mid a, b \in Z_{20}, (3, 8), *\} \cup \{[0, aI] \mid a \in Z_{19}, +\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 19\}\ 9, *\} \cup \{[0, a+bI] \mid a, b \in Z_{12}, *. (3, 9)\}$ be a mixed neutrosophic interval (1, 2, 2, 1) - (semigroup - group - groupoid - loop).
i) Find order of P.
ii) Find substructures in P.
iii) Is every element in P of finite order?

68. Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 = \{[0, aI] \mid a \in Z_9, +\} \cup \{[0, aI] \mid a \in Z_9, \times\} \cup \{[0, aI] \mid a \in Z_9, (2, 7), *\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 9\}, 5, *\} \cup \{[0, aI] \mid a \in Z_9, (0, 4), *\}$ be a mixed neutrosophic interval (1, 1, 2, 1) - group - semigroup - groupoid - loop.
i) Find order of S.
ii) Obtain substructures in S.
iii) Determine some interesting properties enjoyed by S.
iv) Does the order of 5-substructures divide the order of S?
v) Let $x = [0, 3I] \cup [0, 2I] \cup [0, 4I] \cup [0, 5I] \cup [0, 7I] \in S$. What is the order of the element x in S?

69. Let $S = S_1 \cup S_2 \cup S_3 = \{[0, aI]$ where $a \in Z_{24}, +, \times\} \cup \{[0, aI] \mid a \in Z^+ \cup \{0\}, \times, +\} \cup \{([0, a_1+b_1I], [0, a_2 + b_2I], [0, a_3 + b_3I]) \mid a_i, b_i \in Z_{10}, 1 \le i \le 3, +, \times\}$ be a mixed neutrosophic interval (2, 1) ring - semiring.
i) Is S a Smarandache structure?
ii) Can S have 3-ideals?
iii) Find substructures in S.
iv) Can S have 3-zero divisors?

v) Can S have S-3 units?
vi) Is S a commutative 3-structure?

70. Let $M = M_1 \cup M_2 \cup M_3 \cup M_4 = \begin{bmatrix} [0,6I] & 0 & [0,2I] \\ 0 & [0,3I] & 0 \\ [0,7I] & 0 & 0 \end{bmatrix}$

$$\cup \begin{bmatrix} [0,4I] & 0 \\ 0 & [0,2I] \end{bmatrix} \cup \begin{bmatrix} [0,2I] & 0 & [0,3I] & 0 \\ 0 & [0,4I] & [0,I] & 0 \\ [0,I] & 0 & 0 & [0,7I] \\ 0 & [0,8I] & 0 & [0,I] \end{bmatrix}$$

$$\cup \begin{bmatrix} [0,2I] & [0,3I] & 0 & [0,I] & [0,7I] \\ 0 & [0,I] & [0,2I] & 0 & 0 \\ 0 & 0 & [0,4I] & [0,I] & [0,2I] \\ 0 & 0 & 0 & [0,5I] & 0 \\ 0 & 0 & 0 & 0 & [0,7I] \end{bmatrix}$$ be a

neutrosophic interval 4-matrix with entries from $R^+ \cup \{0\}$.

i) Find characteristic 4-values associated with M.
ii) Is the characteristic 4-values associated with M-neutrosophic intervals?
iii) Can the characteristic 4-values be in $R^+ \cup \{0\}$?
iv) Obtain some interesting results associated with M.
v) Does M^{-1} exist?

71. Obtain some interesting properties related with neutrosophic interval n-vector spaces.

72. Find some properties enjoyed by special neutrosophic interval n-vector spaces.

73. Study the difference between the structures described problems (71) and (72).

74. Let $M = M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 = \{([0, a_1I], [0, a_2I], \ldots, [0, a_{12}I]) \mid a_i \in Z_7; 1 \leq i \leq 7\} \cup$

$$\left\{\sum_{i=0}^{12}[0, a_i + b_iI]x^i \,\middle|\, a_i, b_i \in Z_{13}, +\right\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_7I] & [0,a_8I] & [0,a_9I] \end{bmatrix} \,\middle|\, a_i \in Z_{17},\ 1 \le i \le 9, +\right\} \cup$$

$$\left\{\begin{bmatrix} [0,a_1+b_1I] & [0,a_2+b_2I] \\ [0,a_3+b_3I] & [0,a_4+b_4I] \\ \vdots & \vdots \\ [0,a_{13}+b_{13}I] & [0,a_{14}+b_{14}I] \end{bmatrix} \,\middle|\, a_i, b_i \in Z_3, 1 \le i \le 14\right\}$$

$$\cup \left\{\begin{pmatrix} [0,a_1I] & [0,a_2I] & \ldots & [0,a_8I] \\ [0,a_9I] & [0,a_{10}I] & \ldots & [0,a_{16}I] \end{pmatrix} \,\middle|\, a_i \in Z_5, 1 \le i \le 16, +\right\}$$ be

a special neutrosophic interval 5-vector space over the 5-field $F = F_1 \cup F_2 \cup F_3 \cup F_4 \cup F_5 = Z_7 \cup Z_{13} \cup Z_{17} \cup Z_3 \cup Z_5$.

i) Find a special 5-basis of M over F.
ii) What is the special 5-dimension of M over F?
iii) Can M be written as a direct union of special neutrosophic interval 5-subspaces over F?
iv) Find a special linear 5-operator $T = T_1 \cup T_2 \cup T_3 \cup T_4 \cup T_5$ so that T^{-1} exists.

75. Let $V = V_1 \cup V_2 \cup V_3 \cup V_4 = \{([0, a_1+b_1I], \ldots, [0, a_8 + b_8I]) \mid a_i, b_i \in Z^+ \cup \{0\}, +\} \cup \left\{\begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ \vdots \\ [0,a_{10}I] \end{bmatrix}\right.$ where $a_i \in Z^+ \cup \{0\};\ 1 \le i \le 10\} \cup \left\{\sum_{i=0}^{9}[0, a+bI]x^i \,\middle|\, a, b \in Z^+ \cup \{0\}\right\} \cup$

$$\left\{\begin{pmatrix} [0,a+bI] & [0,c+dI] \\ [0,c+dI] & [0,a+bI] \end{pmatrix} \,\middle|\, a,b,c,d \in Z^+ \cup \{0\}, +\right\}$$ be a

neutrosophic interval 4-semivector space over the semifield $S = Z^+ \cup \{0\}$.

i) Find a 4-basis of V over $Z^+ \cup \{0\}$.
ii) What is the 4-dimesnion of V over $Z^+ \cup \{0\}$
iii) Find 4-subspaces of V over $S = Z^+ \cup \{0\}$
iv) Is V a Smarandache 4-semivector space over $Z^+ \cup \{0\}$?
v) Find a linear 4-operator $T = T_1 \cup T_2 \cup T_3 \cup T_4$ on V so that T^{-1} does not exist.
vi) Is 4-kerT = ker $T_1 \cup$ ker $T_2 \cup$ ker $T_3 \cup$ ker T_4 a 4-subspace?

76. Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5 =$

$$\left\{ \sum_{i=0}^{\infty} [0, aI]x^i \,\middle|\, a \in Z^+ \cup \{0\}, +, \times \right\} \cup \{([0, a_1I], [0,a_2I], \ldots, [0, a_8I]) \mid a_i \in Z^+ \cup \{0\}, +, \times\} \cup$$

$$\left\{ \begin{bmatrix} [0,a_1I] & [0,a_2I] \\ [0,a_3I] & [0,a_4I] \end{bmatrix} \,\middle|\, a_1, a_2, a_3, a_4 \in Z^+ \cup \{0\}, +, \times \right\} \cup$$

$$\left\{ \begin{pmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ 0 & [0,a_4I] & [0,a_5I] \\ 0 & 0 & [0,a_6I] \end{pmatrix} \,\middle|\, a_i \in Z^+ \cup \{0\}, 1 \le i \le 6, +, \times \right\}$$

$\cup$

$$\left\{ \begin{pmatrix} [0,a_1I] & 0 & 0 & 0 \\ [0,a_2I] & [0,a_3I] & 0 & 0 \\ 0 & [0,a_4I] & [0,a_5I] & 0 \\ 0 & 0 & 0 & [0,a_6I] \end{pmatrix} \,\middle|\, a_i \in Z^+ \cup \{0\}, 1 \le i \le 6, +, \times \right\}$$

be a neutrosophic interval 5-semiring.

i) Is S-a S-5-semiring?
ii) Does S contain a S-5-subsemiring which is not Smarandache?
iii) Can S have 5-zero divisors?
iv) Obtain some stricking properties about S.
v) Can S have S-5-ideals?

77. Give some special properties about quasi neutrosophic quasi interval n-groups ($n \geq 3$).

78. Prove Lagrange's theorem is true for quasi neutrosophic quasi interval n-group of finite n-order.

79. Let $G = G_1 \cup G_2 \cup G_3 \cup G_4 \cup G_5 = \{([0, a_1I], [0, a_2I], [0, a_3I]) \mid a_i \in Z_{11} \setminus \{0\}\} \cup$

$$\left\{\left.\begin{bmatrix} [0,a_1I] & [0,a_2I] & [0,a_3I] \\ [0,a_4I] & [0,a_5I] & [0,a_6I] \\ [0,a_7I] & [0,a_8I] & [0,a_9I] \\ [0,a_{10}I] & [0,a_{11}I] & [0,a_{12}I] \\ [0,a_{13}I] & [0,a_{14}I] & [0,a_{15}I] \end{bmatrix}\right| a_i \in Z_{12}, +, 1 \leq i \leq 15\right\} \cup$$

$$\left\{\left.\begin{pmatrix} [0,a_1+b_1I] & \ldots & [0,a_7+b_7I] \\ [0,a_8+b_8I] & \ldots & [0,a_{14}+b_{14}I] \end{pmatrix}\right| a_i, b_i \in Z_{24}, +; 1 \leq i \leq 14\right\}$$

$\cup \{[0, a+bI] \mid a, b \in Z_{25}, +\}$

$$\cup \left\{\left.\begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ [0,a_3I] \\ [0,a_4I] \end{bmatrix}\right| a_i \in Z_{15}, 1 \leq i \leq 14; +\right\}$$ be a neutrosophic interval 5-group.

i) Find the 5-order of G.
ii) Prove Lagrange's theorem is true for G.
iii) Find all p-sylow 5-subgroups of G.
iv) Prove Cauchy theorem is true for G.
v) If $x \in G$; does there exists an integer such that

$$x^n = x_1^{n_1} \cup x_2^{n_2} \cup x_3^{n_3} \cup x_4^{n_4} \cup x_5^{n_5}$$

$$= ([0, I], [0, I], [0, I]) \cup (0) \cup \begin{pmatrix} 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \end{pmatrix} \cup$$

$$[0, 0] \cup \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

80. Let S = $S_1 \cup S_2 \cup S_3 \cup S_4$ = $\left\{ \sum_{i=0}^{20} [0, a_i I] x^i \middle| a_i \in Z^+ \cup \{0\} \right\}$

$$\cup \left\{ \begin{bmatrix} [0,a_1 I] & [0,a_2 I] & [0,a_3 I] \\ [0,a_4 I] & [0,a_5 I] & [0,a_6 I] \\ [0,a_7 I] & [0,a_8 I] & [0,a_9 I] \end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}, 1 \le i \le 9 \right\} \cup$$

$$\left\{ \begin{bmatrix} [0,a_1 + b_1 I] & [0,a_2 + b_2 I] \\ \vdots & \vdots \\ [0,a_7 + b_7 I] & [0,a_8 + b_8 I] \end{bmatrix} \middle| a_i, b_i \in Z^+ \cup \{0\}, 1 \le i \le 8 \right\}$$

$\cup$ {([0, $a_1 + b_1 I$], …, [0, $a_{11} + b_{11} I$]) | a_i, $b_i \in Z^+ \cup \{0\}$; $1 \le i \le 11$} be a neutrosophic interval 4- semivector space over the semifield = $Z^+ \cup \{0\}$.

i) What is the 4-dimension of S over F?
ii) Find a 4-basis of S over F.
iii) Find atleast 3 4-subsemivector spaces of S over F.
iv) Write S as a direct sum.
v) Is S a 4-semilinear algebra over $Z^+ \cup \{0\}$?
vi) Is S a Smarandache 4-semivector space over F?
vii) Define a 4-linear operator T on S so that T^{-1} exists.

81. Give an example of a neutrosophic fuzzy interval 4-semigroup of infinite order.

82. Give an example of a fuzzy neutrosophic interval 8-semiring which is not Smarandache.

83. Derive any interesting property about fuzzy neutrosophic interval n-rings.

84. Can the concept of principal ideal domain be extended to the fuzzy neutrosophic interval n-rings?

85. Let P = $P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5$ = {[0, a+bI] | a, b ∈ Z_{12}, ×} ∪ $\left\{\begin{bmatrix}[0,a_1I]\\ [0,a_2I]\\ \vdots\\ [0,a_9I]\end{bmatrix} \middle| a_i \in Z_{12}, \times, 1 \le i \le 9\right\}$ ∪ {([0, a_1I], …, [0, a_{10}I]) | a_i ∈ Z^+ ∪ {0}; 1 ≤ i ≤ 10, ×} ∪ $\left\{\sum_{i=0}^{25}[0,a_i+b_iI]x^i \middle| a_i, b_i \in Z^+ \cup \{0\}, 0 \le i \le 25, +\right\}$ ∪ $\left\{\begin{bmatrix}[0,a_1I] & [0,a_2I]\\ [0,a_3I] & [0,a_4I]\end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}, \times\right\}$ be a neutrosophic interval 5-semigroup.

i) Define a map η : P = $P_1 \cup P_2 \cup P_3 \cup P_4 \cup P_5$ → ⟨[0, I] ∪ [0, 1]⟩ ∪ ⟨[0, I] ∪ [0, 1]⟩ ∪ ⟨[0, I] ∪ [0, 1]⟩ ⟨[0, I], [0, 1]⟩ ∪ ⟨[0, I], [0, 1]⟩, so that (p, η) is a special neutrosophic fuzzy interval fuzzy 5-semigroup.

ii) How many such special fuzzy 5-semigroup can be constructed?

86. Let S = $S_1 \cup S_2 \cup S_3 \cup S_4$ = {[0, aI] | a ∈ {e, 1, 2, …, 23}, 9, *} ∪ {[0, a+bI] | a, b ∈ {e, 1, 2, …, 29}, 20, *} ∪{([0, a_1I], [0, a_2I] … [0, a_7I]) | a ∈ {e, 1, 2, …, 33}, 14, *} ∪ {[0, a+bI] | a, b ∈ {e, 1, 2, …, 53}, 29, *} be a neutrosophic interval 4-loop.

i) Define a map η : S → ⟨[0, 1] ∪ [0, I]⟩ ∪ ⟨[0, 1] ∪ [0, I]⟩ ∪ ⟨[0, 1] ∪ [0, I]⟩ ∪ ⟨[0, 1] ∪ [0, I]⟩ so that (S, η) is a special neutrosophic fuzzy interval fuzzy 4-loop.

ii) Derive some of the properties enjoyed by (S, η).

87. Let G = $G_1 \cup G_2 \cup G_3 \cup G_4 \cup G_5$ = {[0, a+bI] | a, b ∈ Z_{45}, +} ∪ {[0, aI] | a ∈ Z_{220}, +} ∈ {[0, aI] | a ∈ Z_{53} \ {0}, ×} ∪ {([0, a_1I], …, [0, a_8I]) | a_i ∈ Z_{13} \ {0}, ×} ∪ {[0, a+bI] | a, b ∈ Z_{28}, +} be a neutrosophic interval 5-group.

i) Define η : G = $G_1 \cup G_2 \cup G_3 \cup G_4 \cup G_5 \to \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle$ so that (G, η) is a special fuzzy neutrosophic interval fuzzy 5-group.

ii) Find fuzzy 5-subgroups of (G, η).

88. Let V = $V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5 \cup V_6$ = {[0, aI] | a $\in Z_9$, *, (2, 7)} $\cup$ {[0, a] | a $\in Z_{48}$, *, (9, 0)} $\cup$ {[0, a+bI] | a,b $\in Z_{270}$, *, (2, 8)} $\cup$ {([0, a_1I], [0, a_2I], …, [0, a_{20}I]) | a $\in Z_{27}$, $1 \leq i \leq 20$, *, (11, 0)} $\cup$ {[0, aI] | a $\in Z_{48}$, *, (25, 23)} $\cup$ {[0, a+bI] | a,b $\in Z_{420}$, *, (29, 0)} be a quasi neutrosophic interval 6-groupoid.

i) Define η : V $\to \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle$ so that (V, η) is a special fuzzy quasi neutrosophic interval fuzzy 6-groupoid.

ii) Does (V, η) satisfy any of the special identities?

iii) How many (V, η)'s can be constructed using V?

89. Let M = $M_1 \cup M_2 \cup \ldots \cup M_5$ = {[0, aI] | a $\in Z_{40}$, +} $\cup$ {all 5 × 5 neutrosophic interval matrices with entries from $Z^+ \cup \{0\}$ under matrix multiplication} $\cup$ {([0, a_1I] , …, [0, a_8I]) | $a_i \in Z_{47}$, (8, 9), *, $1 \leq i \leq 8$} $\cup$ {[0, aI] | a $\in$ {e, 1, 2, …, 43}, *, 9} $\cup$ $\left\{\begin{bmatrix}[0,a_1I]\\[0,a_2I]\\[0,a_3I]\\[0,a_4I]\end{bmatrix} \middle| a_i \in Z^+ \cup \{0\}, 1 \leq i \leq 4, +\right\}$ be a mixed neutrosophic interval (1, 2, 1, 1) group semigroup - groupoid - loop.

i) Define η : M = $M_1 \cup M_2 \cup M_3 \cup M_4 \cup M_5 \to \langle [0, I] \cup [0, 1]\rangle \cup \langle [0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle \cup \langle [0, I] \cup [0, 1]\rangle \cup \langle[0, I] \cup [0, 1]\rangle$ so that (M, η) is a special mixed fuzzy neutrosophic interval (1, 2, 1, 1) fuzzy grouped - fuzzy semigroup - fuzzy groupoid - fuzzy loop.

ii) Find substructures of (M, η).
iii) Obtain some interesting results about (M, η).

90. Determine some interesting properties about special fuzzy n-structures.

91. Compare the special fuzzy neutrosophic n-structures with fuzzy neutrosophic n-structures.

92. What is the advantage of defining directly fuzzy neutrosophic n-structures using the fuzzy neutrosophic interval ⟨[0, I]} ∪ [0, 1]⟩ = {[0, a+bI] | a, b ∈ [0, 1]}?

93. Give some applications of these new n-structures.

94. What is the advantage of using intervals / neutrosophic intervals instead of real values in mathematical models.

95. Find the uses and advantages of using neutrosophic interval n-matrices in stiffness n-matrices.

96. Describe some real mathematical models which function better if real values are replaced by intervals.

97. Let $S = S_1 \cup S_2 \cup S_3 \cup S_4 = \left\{ \sum_{i=0}^{28} [0, a_i] x^i \,\middle|\, a_i \in Z_7 \right\} \cup$

$\{([0, a_1+b_1I], \ldots, [0, a_8+b_8I]) \mid a_i, b_i \in Z_7, +\} \cup$

$$\left\{ \begin{bmatrix} [0,a_1I] \\ [0,a_2I] \\ \vdots \\ [0,a_{20}I] \end{bmatrix} \,\middle|\, a_i \in Z_7, 1 \leq i \leq 20 \right\} \cup$$

$$\left\{ \begin{bmatrix} [0,a_1I] & [0,a_2I] & \ldots & [0,a_{11}I] \\ [0,a_{12}I] & [0,a_{13}I] & \ldots & [0,a_{22}I] \end{bmatrix} \,\middle|\, a_i \in Z_7, 1 \leq i \leq 22 \right\}$$ be a

neutrosophic interval 4 - vector space over the field Z_7.

i) Find η so that (S, η) is a fuzzy neutrosophic interval 4-vector space.

ii) What is the 4-dimension of S?
iii) Obtain a linear 4-operator T on S so that T^{-1} does not exist.
iv) What is the advantage of studying fuzzy neutrosophic interval 4-vector space?
v) Find fuzzy neutrosophic interval 4-vector subspaces of S.

98. Let $L = L_1 \cup L_2 \cup L_3 \cup L_4 = \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 23\}, *, 8\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 29\}, *, 11\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 37\}, *, 12\} \cup \{[0, aI] \mid a \in \{e, 1, 2, \ldots, 43\}, *, 14\}$ be a neutrosophic interval 4-loop.
i) Is L a S-strong Bol 4-loop?
ii) Find (L, η) the fuzzy neutrosophic interval 4-loop.
iii) Is L S-commutative?

99. Give a class of S-commutative neutrosophic interval n-loop.

100. Does there exist a class of neutrosophic interval n-loop which is neutrosophic interval Moufang n-loop?

101. Find a class of S-strong neutrosophic interval Bol n-loop.

102. Find a class of S-strong neutrosophic interval alternative n-loop.

103. Find a class of S-strong neutrosophic interval Bol n-groupoid.

104. Does there exist a S-strong commutative neutrosophic interval n-groupoid?

105. Does there exist a S-strong Moufang neutrosophic fuzzy interval 8-groupoid?

106. Find a class of S-Moufang fuzzy neutrosophic interval 6-groupoid.

107. Find a class of S-strong neutrosophic fuzzy interval P-n-groupoid.

108. Give an example of a S-neutrosophic fuzzy interval idempotent 8-groupoid.

109. Give an example of a S-neutrosophic fuzzy interval Moufang 12-groupoid.

110. Give an example of a S-neutrosophic fuzzy interval Bol 3-groupoid which is not a S-strong neutrosophic fuzzy interval Bol 3-groupoid.

111. Give an example of a S-neutrosophic fuzzy interval alternative 5-groupoid.

112. Give an example of a S-neutrosophic fuzzy interval 5-semigroup which is a S-Lagrange neutrosophic fuzzy interval 5-semigroup (can such structure exist?)

113. Give an example of a S-neutrosophic fuzzy interval 9-loop, which is right alternative but not left alternative.

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The authors in this book introduce the notion of neutrosophic interval bialgebraic structures. Some research level problems are also given. Using these neutrosophic biintervals several new interval bialgebraic structures are introduced and studied. These concepts are generalized to neutrosophic n-interval algebraic structures.

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