Neutrosophic Soft Multi-Attribute Group Decision Making Based On Grey Relational Analysis Method

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Abstract - The objective of the paper is to present neutrosophic soft multi attribute group decision making based on grey relational analysis involving multiple decision makers. The concept of neutrosophic soft sets is derived from the hybridization of the concepts of neutrosophic set and soft set. In the decision making process, the decision makers offer the rating of alternatives with respect to the parameters in terms of single valued neutrosophic set. We utilize AND operator of neutrosophic soft sets in order to aggregate the individual decision maker's opinion into a common opinion based on choice parameters of the evaluator. Then, information entropy method is employed in order to attain the weights of the choice parameters. We determine the order of the alternatives and identify the most suitable alternative based on grey relational analysis. Finally, in order to show the effectiveness of the proposed approach, a numerical example is solved.

Keywords: *Neutrosophic set*, *neutrosophic soft set*, *grey relational analysis*, *multi-attribute group decision making*.

1. Introduction

Multi attribute group decision making (MAGDM) is one of the significant topics in modern society, where it is necessary to select the best alternative from a list of feasible alternatives with respect to some predefined attribute values provided by the multiple decision makers (DMs). However, a DM's preferences for alternatives may not be expressed precisely due to the fact that the information about attribute values may be vague, incomplete or indeterminate. Zadeh [45] proposed fuzzy set theory by incorporating degree of membership (acceptance) in order to deal with different types of uncertainties. Atanassov [3] extended the concept of Zadeh [45] and defined intuitionistic fuzzy sets by introducing degree of non-membership (rejection) such that the sum of degree of membership and degree of non-membership is less than one. Smarandache [34, 35, 36, 37] initiated neutrosophic sets (NSs) by introducing degree of indeterminacy as independent component for dealing with uncertain, incomplete, imprecise, inconsistent information. However, in order to cope with practical engineering and scientific problems, Wang et al. [39] proposed a subclass of NSs called single valued neutrosophic sets (SVNSs) such that the sum of degree of membership, degree of non-membership and degree of indeterminacy is less than or equal to 3.

Molodtsov [28] developed soft set theory in 1999 as a general mathematical apparatus for dealing with uncertainty and vagueness which is free from parameterization insufficiency syndrome of fuzzy set theory, rough set theory and probability theory. Maji et al. [25] applied soft set theory to solve a decision making problem by using rough technique of Pawlak [31]. Maji et al. [22] also provided theoretical studies on soft set theory initiated by Molodtsov [28] in details. Thereafter, many researchers have discussed diverse mathematical hybrid structures such as fuzzy soft sets [10, 11, 24], intuitionistic fuzzy soft sets [8, 9, 23], possibility fuzzy soft sets [2], generalized fuzzy soft sets [27, 44], generalized intuitionistic fuzzy soft sets [4], possibility intuitionistic fuzzy soft sets [5], vague soft sets [43], possibility vague soft sets [1], etc by generalizing and extending the pioneering work of Molodtsov [28]. Recently, Maji [21] initiated a hybrid structure called neutrosophic soft sets (NSSs) where the parameters considered are neutrosophic in nature. Maji [20] incorporated weighted NSSs by imposing weights on the parameters (may be in a particular parameter) and also defined some operations and verified some propositions. Maji [19] applied WNSSs approach to solve a decision making problem.

Deng [13] developed the concept of grey relational analysis (GRA) method and it has been applied widely for different practical problems such as corrosion failure of oil tubes [14], vendor selection [38], watermarking scheme [18], teacher selection [32] comprehensive evaluation [12], advanced manufacturing systems [15], optimal welding parameter selection [33], etc. GRA has been recognized as an appropriate multi-attribute decision making device for solving problems with complicated interrelationships between numerous factors and variables [17, 41, 42]. Biswas et al. [7] studied entropy based GRA method for solving multi-attribute decision making

(MADM) problem under neutrosophic environment. Biswas et al. [6] also presented a procedure for solving MADM problem with incompletely known or completely unknown attribute weight information based on modified GRA method under single-valued neutrosophic assessments. Mondal and Pramanik [29] presented a GRA method for neutrosophic MADM problem with interval weight information for selecting the best school for the children. Mondal and Pramanik [30] also introduced rough neutrosophic MADM based on modified GRA.

In the paper, an attempt has been made to develop neutrosophic soft MAGDM based on GRA. Firstly, the multiple DMs assign their preference values on the alternatives with respect to the specified parameters in terms of SVNSs. Then, AND operation of NSSs is applied to aggregate the DMs opinion into a common opinion based on the choice parameters of the evaluator in the decision making situation. Thereafter, ideal neutrosophic estimates reliability solution (INERS) and ideal neutrosophic estimates un-reliability solution (INEURS) are identified and grey relational coefficient between each alternative from INERS and INEURS are calculated. Finally, best alternative is selected based on biggest value of grey relational degree.

The remaining of the paper is structured as follows: Section 2 presents some preliminaries regarding NSs, SVNSs, soft sets, and NSSs. Section 3 is devoted to present GRA method for solving neutrosophic soft MAGDM problem. A numerical example is solved to demonstrate the effectiveness of the proposed method in Section 4. Finally, the last Section concludes the paper.

2. Preliminaries

In this Section, we provide some basic definitions concerning NSs, SVNSs, soft sets, and NSSs.

2.1. Neutrosophic sets

Definition 2.1.1 [34-37] A neutrosophic set S on the universal space X is represented as follows:

$$\mathrm{S} = \{x, \langle \mathrm{T_S}(x), \mathrm{I_S}(x), \mathrm{F_S}(x) \rangle \mid x \in X\}$$

where, $\mathrm{T_S}(x)$, $\mathrm{I_S}(x)$, $\mathrm{F_S}(x) : X \to]^-0, 1^+[$ and $^-0 \le \mathrm{T_S}(x) + \mathrm{I_S}(x) + \mathrm{F_S}(x) \le 3^+$. Here, $\mathrm{T_S}(x)$, $\mathrm{I_S}(x)$, $\mathrm{F_S}(x)$ are the truth-membership, indeterminacy-membership, and falsity-membership functions, respectively of a point $x \in X$.

Definition 2.1.2. [39] Let X be a universal space of points, then a SVNS is defined as follows:

$$\mathrm{N} = \{x, \langle \mathrm{t_N}(x), \mathrm{u_N}(x), \mathrm{v_N}(x) \rangle \mid x \in X\}$$

where, $\mathrm{t_N}(x)$, $\mathrm{u_N}(x)$, $\mathrm{v_N}(x) : X \to [0, 1]$ and $0 \le \mathrm{t_N}(x) + \mathrm{u_N}(x) + \mathrm{v_N}(x) \le 3$ for each point $x \in X$. We will represent the set of all SVNSs in the universal space X by Q and for convenience, a single – valued neutrosophic number (SVNN) is expressed as $\tilde{\mathrm{q}} = \langle \mathrm{t}, \mathrm{u}, \mathrm{v} \rangle$.

Definition 2.1.3. [39] The Hamming distance between two NSSs $\mathrm{N_C}$ == $\{x_i, \langle \mathrm{t}_{\mathrm{N_C}}(x_i), \mathrm{u}_{\mathrm{N_C}}(x_i), \mathrm{v}_{\mathrm{N_C}}(x_i) \rangle \mid x_i \in X\}$ and $\mathrm{N_D}$ = $\{x_i, \langle \mathrm{t}_{\mathrm{N_D}}(x_i), \mathrm{u}_{\mathrm{N_D}}(x_i), \mathrm{v}_{\tilde{\mathrm{N}}_\mathrm{D}}(x_i) \rangle \mid x_i \in X\}$ is defined as follows:

$$\mathrm{H}(\mathrm{N_C}, \mathrm{N_D}) = \frac{1}{3} \sum_{i=1}^{n} \mid \mathrm{t}_{\mathrm{N_C}}(x_i) - \mathrm{t}_{\mathrm{N_D}}(x_i) \mid + \mid \mathrm{u}_{\mathrm{N_C}}(x_i) - \mathrm{u}_{\mathrm{N_D}}(x_i) \mid + \mid \mathrm{v}_{\mathrm{N_C}}(x_i) - \mathrm{v}_{\mathrm{N_D}}(x_i) \mid \qquad (2.1)$$

with the property: $0 \le \mathrm{H}(\mathrm{N_C}, \mathrm{N_D}) \le 1$.

2.2. Soft set and neutrosophic soft sets

Definition 2.2.1. [28] Assume that U is a universal set, E is a set of parameters and P (U) represents a power set of U. Let A be a non-empty set, where A $\subset$ E. Then, a pair (Φ, A) is called a soft set over U, where Φ is a mapping given by Φ : A$\to$ P (U).

Definition 2.2.2. [21] Consider U be a universal set. Suppose E be a set of parameters and A be a non-empty set such that A$\subset$ E. P ($\mathrm{U_E}$) denotes the set of all neutrosophic subsets of U. A pair (Φ, A) is termed to be a NSSs over U, where Φ is a mapping given by Φ : A$\to$ P ($\mathrm{U_E}$).

Example: Suppose U be the universal set of objects and E = {very large, large, low, attractive, cheap, expensive, beautiful} be the set of parameters. Here, each parameter is a neutrosophic word or sentence regarding neutrosophic word. To describe NSS means to point out very large objects, large objects, low objects, attractive objects, cheap objects, etc. Consider five objects in the universe U given by U = (u_1, u_2, u_3, u_4, u_5) and E = {e_1, e_2, e_3, e_4, e_5} be a set of parameters. Here, e_1, e_2, e_3, e_4, e_5 denote the parameters 'very large', 'low', 'attractive', 'cheap', 'beautiful' respectively. Suppose that,

Φ (very large) = {< u_1, 0.9, 0.3, 0.4>, < u_2, 0.8, 0.3, 0.4>, < u_3, 0.7, 0.2, 0.3>, < u_4, 0.6, 0.3, 0.5>, < u_5, 0.9, 0.1, 0.3>},

Φ (low) = {< u_1, 0.5, 0.3, 0.3>, < u_2, 0.6, 0.3, 0.3>, < u_3, 0.7, 0.2, 0.4>, < u_4, 0.6, 0.4, 0.2>, < u_5, 0.5, 0.4, 0.4>},

Φ (attractive) = {< u_1, 0.9, 0.1, 0.2>, < u_2, 0.8, 0.2, 0.2>, < u_3, 0.9, 0.2, 0.2>, < u_4, 0.9, 0.3, 0.2>, < u_5, 0.8, 0.4, 0.3>},

Φ (cheap) = {< u_1, 0.5, 0.6, 0.8>, < u_2, 0.4, 0.7, 0.7>, < u_3, 0.6, 0.7, 0.6>, < u_4, 0.5, 0.5, 0.7>, < u_5, 0.3, 0.8, 0.8>}

Φ (beautiful) = {< u_1, 0.8, 0.2, 0.3>, < u_2, 0.9, 0.3, 0.3>, < u_3, 0.8, 0.4, 0.3>, < u_4, 0.7, 0.2, 0.3>, < u_5, 0.9, 0.1, 0.2>}

Consequently, Φ (large) stands for large objects, Φ (cheap) stands for cheap objects, Φ (beautiful) stands for beautiful objects, etc. The tabular representation of NSS (Φ, A) is presented in the table 1.

Insert table 1

Definition 2.2.3. [21]: Let (Φ_1, A) and (Φ_2, B) be two NSSs over a common universe U. The union (Φ_1, A) and (Φ_2, B) is defined by (Φ_1, A) $\cup$ (Φ_2, B) = (Φ_3, C), where C = A $\cup$ B. The truth-membership, indeterminacy-membership and falsity-membership functions are presented as follows:

$$T_{\Phi_3(e)}(m) = T_{\Phi_1(e)}(m), \text{ if } e \in \Phi_1 - \Phi_2,$$

$$= T_{\Phi_2(e)}(m), \text{ if } f \in \Phi_2 - \Phi_1,$$

$$= \text{Max}\,(T_{\Phi_1(e)}(m), T_{\Phi_2(e)}(m)), \text{ if } e \in \Phi_1 \cap \Phi_2 .$$

$\mathrm{I}_{\Phi_3(\mathrm{e})}(m) = \mathrm{I}_{\Phi_1(\mathrm{e})}(m)$, if $e \in \Phi_1$ - Φ_2,

$= \mathrm{I}_{\Phi_2(\mathrm{e})}(m)$, if $e \in \Phi_2 - \Phi_1$,

$= \dfrac{\mathrm{I}_{\Phi_1(\mathrm{e})}(m) + \mathrm{I}_{\Phi_2(\mathrm{e})}(m)}{2}$ if $e \in \Phi_1 \cap \Phi_2$.

$\mathrm{F}_{\Phi_3(\mathrm{e})}(x) = \mathrm{F}_{\Phi_1(\mathrm{e})}(m)$, if $e \in \Phi_1$ - Φ_2,

$= \mathrm{F}_{\Phi_2(\mathrm{e})}(m)$, if $e \in \Phi_2 - \Phi_1$,

$=$ Min $(\mathrm{F}_{\Phi_1(\mathrm{e})}(m),\ \mathrm{F}_{\Phi_2(\mathrm{e})}(m))$, if $e \in \Phi_1 \cap \Phi_2$.

Definition 2.2.4. [21]: Suppose $(\Phi_1$, A) and $(\Phi_2$, B) are two NSSs over the same universe U. The intersection $(\Phi_1$, A) and $(\Phi_2$, B) is defined by $(\Phi_1$, A) $\cap$ $(\Phi_2$, B) = $(\Phi_4$, D), where D = $\mathrm{A} \cap \mathrm{B}$ and the truth-membership, indeterminacy-membership and falsity-membership functions of $(\Phi_4$, D) are defined as follows:

$\mathrm{T}_{\Phi_4(e)}(x) =$ Min $(\mathrm{T}_{\Phi_1(e)}(m),\ \mathrm{T}_{\Phi_2(e)}(m))$, $\mathrm{I}_{\Phi_4(e)}(m) = \dfrac{\mathrm{I}_{\Phi_1(e)}(m) + \mathrm{I}_{\Phi_2(e)}(m)}{2}$, $\mathrm{F}_{\Phi_4(e)}(m) =$ Max $(\mathrm{F}_{\Phi_1(e)}(m),\ \mathrm{F}_{\Phi_2(e)}(m))$, $\forall\, e \in$ D.

Definition 2.2.5. [21]: Let $(\Phi_1$, A) and $(\Phi_2$, B) be two NSSs over the identical universe U. Then 'AND' operation on $(\Phi_1$, A) and $(\Phi_2$, B) is defined by $(\Phi_1$, A) $\wedge$ $(\Phi_2$, B) = $(\Phi_5$, H), where H = $\mathrm{A} \times \mathrm{B}$ and the truth-membership, indeterminacy-membership and falsity-membership functions of $(\Phi_5, \mathrm{A} \times \mathrm{B})$ are defined as follows:

$\mathrm{T}_{\Phi_5(\gamma,\delta)}(m) =$ Min $(\mathrm{T}_{\Phi_1(\gamma)}(m),\ \mathrm{T}_{\Phi_2(\delta)}(m))$, $\mathrm{I}_{\Phi_5(\gamma,\delta)}(m) = \dfrac{\mathrm{I}_{\Phi_1(\gamma)}(m) + \mathrm{I}_{\Phi_2(\delta)}(m)}{2}$, $\mathrm{F}_{\Phi_5(\gamma,\delta)}(m) =$ Max $(\mathrm{F}_{\Phi_1(\gamma)}(m),\ \mathrm{F}_{\Phi_2(\delta)}(m))$, $\forall\ \gamma \in \mathrm{A}$, $\forall\ \delta \in \mathrm{B}$, $m \in$ U.

3. A neutrosophic soft MAGDM based on grey relational analysis

Suppose G = {g_1, g_2, …, g_p}, ($p \geq 2$) be a discrete set of alternatives under consideration in a MAGDM problem with k DMs. Let, q be the total number of parameters under the assessment of DMs. Also, let q_1, q_2, …, q_k be the number of parameters under the consideration of DM_1, DM_2, …, DM_k respectively such that $q = q_1 + q_2 + \ldots + q_k$. The rating of performance value of alternative g_i, (i = 1, 2, …, p) with respect to the choice parameters is provided by the DMs and they can be expressed in terms of SVNSs. Therefore, the steps for solving neutrosophic soft MAGDM based on GRA method is presented as follows:

Step 1. Formulation of criterion matrix with SVNSs

Selection of key parameters is one of the important issues in a MAGDM problem. The key parameters are either identified by the evaluator or by some other methods that are technically useful. Suppose that the rating of alternative g_i (i = 1, 2, …, p) with respect to the parameters provided by the s-th (s = 1, 2, …, k) DM is represented by NSSs (Φ_s, H_s), (s = 1, 2, …, k) and they can be presented in matrix form $d^s_{N_{ij}}$ (i = 1, 2, …, p; j = 1, 2, …, q_s; s = 1, 2, …, k). Therefore, criterion matrix for s-th DM can be explicitly constructed as follows:

$$D^s_N = \left\langle d^s_{ij} \right\rangle_{p \times q_s} = \begin{bmatrix} d^s_{11} & d^s_{12} & \cdots & d^s_{1q_s} \\ d^s_{21} & d^s_{22} & \cdots & d^s_{2q_s} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ d^s_{p1} & d^s_{p2} & \cdots & d^s_{pq_s} \end{bmatrix}$$

Here, $d^s_{ij} = (t^s_{ij}, u^s_{ij}, v^s_{ij})$ where $t^s_{ij}, u^s_{ij}, v^s_{ij} \in [0, 1]$ and $0 \leq t^s_{ij} + u^s_{ij} + v^s_{ij} \leq 3$, i = 1, 2, …, p; j = 1, 2, …, q_s; s =1, 2, …, k.

Step 2. Construction of aggregated criterion matrix with SVNSs

In the group decision making situation, all the individual assessments require to be combined into a group opinion on the basis of the choice parameters of the evaluator. Suppose the evaluator considers *r* number of choice parameters in the decision making situation. The resultant NSS can be obtained by using ‘AND’ operator of NSSs proposed by Maji [21] and is placed in a criterion matrix as follows:

$$D_N = \langle d_{ij} \rangle_{p\times r} = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1r} \\ d_{21} & d_{22} & \dots & d_{2r} \\ . & . & \dots & . \\ . & . & \dots & . \\ d_{p1} & d_{p2} & \dots & d_{pr} \end{bmatrix}$$

Here, $d_{ij} = (t_{ij}, u_{ij}, v_{ij})$ where $t_{ij}, u_{ij}, v_{ij} \in [0, 1]$ and $0 \le t_{ij} + u_{ij} + v_{ij} \le 3$, i = 1, 2, …, p; j = 1, 2, …, r.

Step 3. Determination of weights of the choice parameters

In general, the weights of the choice parameters are different. In this paper, we use information entropy method in order to obtain the weights of the choice parameters. The entropy measure [26] can be used when weights of the choice parameters are different and completely unknown to the evaluator. The entropy measure of a SVNS $\Re$ = $\{x, \langle t_{\Re}(x), u_{\Re}(x), v_{\Re}(x) \rangle$ is defined as follows:

$$G_i(\Re) = 1 - \frac{1}{r}\sum_{i=1}^{p}(t_{\Re}(x_i) + v_{\Re}(x_i))\left|u_{\Re}(x_i) - u_{\Re}^{C}(x_i)\right| \tag{3.1}$$

which has the following properties:

(i) $G_i(\Re) = 0$ if $\Re$ is a crisp set and $u_{\Re}(x_i) = 0, \forall\, x \in X$.

(ii) $G_i(\Re) = 0$ if $\langle t_{\Re}(x_i), u_{\Re}(x_i), v_{\Re}(x_i) \rangle$ = <0.5, 0.5, 0.5>, $\forall\, x \in X$.

(iii) $G_i(\Re_1) \le G_i(\Re_2)$ if $\Re_1$ is more uncertain than $\Re_2$ i.e.

$$t_{\Re_1}(x_i) + v_{\Re_1}(x_i) \le t_{\Re_2}(x_i) + v_{\Re_2}(x_i) \text{ and } \left|v_{\Re_1}(x_i) - v_{\Re_1}^{C}(x_i)\right| \le \left|v_{\Re_2}(x_i) - v_{\Re_2}^{C}(x_i)\right|.$$

(iv) $G_i(\Re) = G_i(\Re^C), \forall\, x \in X$.

Therefore, the entropy value G_j of the j-th attribute can be defined as follows:

$$G_j = 1 - \frac{1}{r}\sum_{i=1}^{p}(t_{ij}(x_i) + v_{ij}(x_i))\left|u_{ij}(x_i) - u_{ij}^{C}(x_i)\right|, \ i = 1, 2, \ldots, p; j = 1, 2, \ldots, r. \tag{3.2}$$

Here, $0 \le G_j \le 1$ and the entropy weight owing to Hwang and Yoon [16] and Wang and Zhang [40] of the j-th attribute is presented as follows:

$$w_j = \frac{1 - G_j}{\sum_{j=1}^{r} 1 - G_j}, \text{ with } 0 \le w_j \le 1 \text{ and } \sum_{j=1}^{r} w_j = 1 \tag{3.3}$$

Step 4. Determination of INERS and INEURS based on SVNNs

Generally two types of attributes arise in practical decision making problems namely benefit type attribute (J_1) and cost type attribute (J_2). Let R_N^+ and R_N^+ be INERS and INEURS respectively. Then, R_N^+ and R_N^+ can be defined as follows:

$$R_N^+ = (\langle t_1^{'+}, u_1^{'+}, v_1^{'+}\rangle, \langle t_2^{'+}, u_2^{'+}, v_2^{'+}\rangle, \ldots, \langle t_r^{'+}, u_r^{'+}, v_r^{'+}\rangle)$$

$$R_N^- = (\langle t_1^{'-}, u_1^{'-}, v_1^{'-}\rangle, \langle t_2^{'-}, u_2^{'-}, v_2^{'-}\rangle, \ldots, \langle t_r^{'-}, u_r^{'-}, v_r^{'-}\rangle)$$

where

$\langle t_j^{'+}, u_j^{'+}, v_j^{'+}\rangle = <$ [{$\underset{i}{\text{Max}}(t_{ij}^{'})$ | j∈ J_1}; {$\underset{i}{\text{Min}}(t_{ij}^{'})$ | j∈ J_2}], [{$\underset{i}{\text{Min}}(u_{ij}^{'})$ | j∈ J_1}; {$\underset{i}{\text{Max}}(u_{ij}^{'})$ | j∈ J_2}], [{$\underset{i}{\text{Min}}(v_{ij}^{'})$ | j∈ J_1}; {$\underset{i}{\text{Max}}(v_{ij}^{'})$ | j∈ J_2}] >, j = 1, 2, …, r,

$\langle t_j^{'-}, u_j^{'-}, v_j^{'-}\rangle = <$ [{$\underset{i}{\text{Min}}(t_{ij}^{'})$ | j∈ J_1}; {$\underset{i}{\text{Max}}(t_{ij}^{'})$ | j∈ J_2}], [{$\underset{i}{\text{Max}}(u_{ij}^{'})$ | j∈ J_1}; {$\underset{i}{\text{Min}}(u_{ij}^{'})$ | j∈ J_2}], [{$\underset{i}{\text{Max}}(v_{ij}^{'})$ | j∈ J_1}; {$\underset{i}{\text{Min}}(v_{ij}^{'})$ | j∈ J_2}] >, j = 1, 2, …, r.

Step 5. Calculation of grey relational coefficient

The grey relational coefficient of each alternative from INERS is defined as follows:

$$\eta_{ij}^+ = \left(\underset{1\le i\le p}{\text{Min}}\,\underset{1\le j\le r}{\text{Min}}\,\sigma_{ij}^+ + \tau\,\underset{1\le i\le p}{\text{Max}}\,\underset{1\le j\le r}{\text{Max}}\,\sigma_{ij}^+\right) \times \left(\sigma_{ij}^+ + \tau\,\underset{1\le i\le p}{\text{Max}}\,\underset{1\le j\le r}{\text{Max}}\,\sigma_{ij}^+\right)^{-} \tag{3.4}$$

Where σ_{ij}^{+} = H (d_{ij}, $R_{N_j}^{+}$), for i = 1, 2, …, p; j = 1, 2, …, r.

Also, the grey relational coefficient of each alternative from INEURS is presented as follows:

$$\eta_{ij}^{-} = \left(\underset{1\le i\le p}{\mathrm{Min}}\, \underset{1\le j\le r}{\mathrm{Min}}\, \sigma_{ij}^{-} + \tau\, \underset{1\le i\le p}{\mathrm{Max}}\, \underset{1\le j\le r}{\mathrm{Max}}\, \sigma_{ij}^{-} \right) \times \left(\sigma_{ij}^{-} + \tau\, \underset{1\le i\le p}{\mathrm{Max}}\, \underset{1\le j\le r}{\mathrm{Max}}\, \sigma_{ij}^{-} \right)^{-} \tag{3.5}$$

Where σ_{ij}^{-} = H (d_{ij}, $R_{N_j}^{-}$), for i = 1, 2, …, p; j = 1, 2, …, r. Here, $\tau \in [0, 1]$ is called distinguishable coefficient which is used to control the level of difference of the relation coefficients. Generally, $\tau = 0.5$ is applied in the decision making circumstances.

Step 6. Computation of the degree of grey relational coefficient

Compute the degree of grey relational coefficient of each alternative from INERS and INEURS respectively as follows:

$$\eta_i^{+} = \sum_{j=1}^{r} w_j \eta_{ij}^{+}, \ i = 1, 2, \ldots p, \tag{3.6}$$

$$\eta_i^{-} = \sum_{j=1}^{r} w_j \eta_{ij}^{-}, \ i = 1, 2, \ldots p. \tag{3.7}$$

Step 7. Determination of the relative relational degree

We determine the relative relational degree of each alternative from INERS by using the Eq. as follows:

$$\eta_i = \frac{\eta_i^{+}}{\eta_i^{+} + \eta_i^{-}} \text{ for } i = 1, 2, \ldots, p. \tag{3.8}$$

Step 8. Rank the alternatives

We rank the alternatives according to the values of η_i, i = 1, 2, …, p and biggest value of η_i, i = 1, 2, …, p gives the most desirable alternative.

4. A numerical example

Let U = {g_1, g_2, g_3, g_4, g_5} be the set of objects characterized by different lengths, colors and surface textures and E = {blakish, dark brown, yellowish, reddish, large, small, very small, average, rough, very large, coarse, moderate, fine, smooth, extra fine} be the set of parameters [19]. Assume that E_1 = {very large, small, average}, E_2 = {reddish, yellowish, blakish}, E_3 = {smooth, rough, moderate} are three subsets of E. Let the NSSs (Φ_1, E_1), (Φ_2, E_2), (Φ_3, E_3) represent the items 'having diverse lengths', 'having diverse colours', 'surface structure features' respectively and they are computed by the three DMs namely Mr. X, Mr. Y, and Mr. Z, respectively. The criterion decision matrix of Mr. X, Mr. Y, and Mr. Z are presented respectively in tabular forms (see Table 2, Table 3, Table 4).

Insert Table 2, Table 3, Table 4

The proposed procedure is presented in the following steps.

Step 1: If the evaluator desires to perform the operation '(Φ_1, E_1) AND (Φ_2, E_2)' then we will get 3×3 parameters of the form ε_{ij}, where $\varepsilon_{ij}=\alpha_i \wedge \beta_j$, for i = 1, 2, 3; j = 1, 2, 3. Let S = {ε_{11}, ε_{21}, ε_{22}, ε_{31}, ε_{32}} be the set of choice parameters of the evaluator, where ε_{11} = (very large, reddish), ε_{21} = (small, reddish), ε_{22} = (small, yellowish), etc, (see Table 5).

Insert Table 5

Now the evaluator wants to compute (Φ_5, T) from (Φ_4, S) AND (Φ_3, E_3) for the specified parameters T = {$\varepsilon_{11} \wedge \lambda_1$, $\varepsilon_{21} \wedge \lambda_2$, $\varepsilon_{21} \wedge \lambda_3$, $\varepsilon_{31} \wedge \lambda_1$}, where $\varepsilon_{11} \wedge \lambda_1$ represents (very large, reddish, smooth), $\varepsilon_{21} \wedge \lambda_3$ represents (small, reddish, moderate), etc, (see Table 6).

Insert table 6

Step 2. Computation of the weights of the choice parameters

Entropy value G_j (j = 1, 2, …, 5) of the j-th choice parameter can be obtained from the decision matrix and Eq. 3.2 as follows:

$G_1 = 0.6932$, $G_2 = 0.7555$, $G_3 = 0.7338$, $G_4 = 0.865$.

Then the associated entropy weights are obtained with the help of Eq. 3.3 as follows:

$w_1 = 0.3297$, $w_2 = 0.2391$, $w_3 = 0.2861$, $w_4 = 0.1451$.

Step 3. Determination of INERS and INEURS

The INERS (R_N^+) and INEURS (R_N^-) from the decision matrix are presented as follows:

$$R_N^+ = < (0.7, 0.375, 0.7); (0.6, 0.35, 0.5); (0.8, 0.3, 0.5); (0.8, 0.475, 0.6) >$$

$$R_N^- = < (0.3, 0.675, 0.8); (0.3, 0.6, 0.8); (0.3, 0.575, 0.8); (0.3, 0.6, 0.8) >$$

Step 4. Determination the grey relational coefficient of each alternative from INERS and INEURS

Using Eq. 3.4, the grey relational coefficient of each alternative from INERS (R_N^+) can be obtained as follows:

$$\sigma_{ij}^+ = \begin{bmatrix} 0.167 & 0.117 & 0.183 & 0.142 \\ 0.133 & 0.208 & 0.292 & 0.125 \\ 0.242 & 0.067 & 0.033 & 0.183 \\ 0.000 & 0.192 & 0.325 & 0.100 \\ 0.067 & 0.200 & 0.317 & 0.083 \end{bmatrix}$$

Similarly, the grey relational coefficient of each alternative from INEURS (R_N^-) by using Eq. 3.5 is presented as follows:

$$\sigma_{ij}^{-}=\begin{bmatrix} 0.100 & 0.167 & 0.175 & 0.133 \\ 0.133 & 0.075 & 0.066 & 0.150 \\ 0.025 & 0.217 & 0.325 & 0.092 \\ 0.267 & 0.092 & 0.033 & 0.175 \\ 0.217 & 0.083 & 0.042 & 0.192 \end{bmatrix}$$

Step 5. Calculation of the degree of grey relational coefficient

Computation of the degree of grey relational coefficient of each alternative from INERS and INEURS can be determined from the Eq. 3.6 and 3.7 respectively as follows:

$\eta_1^{+} = 0.5134,\ \eta_2^{+} = 0.4705,\ \eta_3^{+} = 0.6078,\ \eta_4^{+} = 0.6243,\ \eta_5^{+} = 0.5336,$

$\eta_1^{-} = 0.6225,\ \eta_2^{-} = 0.7193,\ \eta_3^{-} = 0.6649,\ \eta_4^{-} = 0.675,\ \eta_5^{-} = 0.6846$

Step 6. Calculate the relative relational degree

We compute the relative relational degree of each alternative by using Eq. 3.8 and obtain values are as follows:

$\eta_1 = 0.452,\ \eta_2 = 0.3954,\ \eta_3 = 0.4776,\ \eta_4 = 0.4805,\ \eta_5 = 0.438.$

Step7. The ranking order of the objects can be obtained according to the value of grey relative relational degree. We observe that $\eta_4 > \eta_3 > \eta_1 > \eta_5 > \eta_2$ and so the largest value of grey relative relational degree is η_4. Therefore, the object g_4 is the most desirable object for the evaluator.

5. Conclusion

In the paper, we have presented a GRA method for solving MAGDM problem under neutrosophic soft environment. The problem comprises of multiple alternatives, several DMs, a set of parameters and our objective is to identify the best alternative based on the neutrosophic information provided by the DMs. The rating of performance values of the alternatives with respect to the parameters are specified by the multiple DMs and are expressed in NSSs. We use AND operator of NSSs to aggregate opinions of the DMs based on the choice parameters of the evaluator. We apply information entropy method to obtain weights of the choice parameters.

Then GRA method is employed to rank the alternatives and select the best one. We hope that the proposed concept will be helpful in dealing with different MAGDM problems such as pattern recognition, medical diagnosis, manufacturing systems, project evaluation and various practical decision making problems.

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Table 1. Tabular representation of NSS (Φ , A)

U	e_1 = very large	e_2 = low	e_3 = attractive	e_4 = cheap	e_5 = beautiful
u_1	(0.9, 0.3, 0.4)	(0.8, 0.3, 0.4)	(0.7, 0.2, 0.3)	(0.6, 0.3, 0.5)	(0.9, 0.1, 0.3)
u_2	(0.5, 0.3, 0.3)	(0.6, 0.3, 0.3)	(0.7, 0.2, 0.4)	(0.6, 0.4, 0.2)	(0.5, 0.4, 0.4)
u_3	(0.9, 0.1, 0.2)	(0.8, 0.2, 0.2)	(0.9, 0.2, 0.2)	(0.9, 0.3, 0.5)	(0.8, 0.4, 0.3)
u_4	(0.5, 0.6, 0.8)	(0.4, 0.7, 0.7)	(0.6, 0.7, 0.6)	(0.5, 0.5, 0.7)	(0.3, 0.8, 0.8)
u_5	(0.8, 0.2, 0.3)	(0.9, 0.3, 0.3)	(0.8, 0.4, 0.3)	(0.7, 0.2, 0.3)	(0.9, 0.1, 0.2)

Table 2: Tabular form of NSS (Φ_1, E_1)

U	α_1 = very large	α_2 = small	α_3 = average large
g_1	(0.5, 0.6, 0.8)	(0.7, 0.3, 0.5)	(0.6, 0.7, 0.3)
g_2	(0.6, 0.8, 0.7)	(0.3, 0.6, 0.4)	(0.8, 0.3, 0.5)
g_3	(0.3, 0.5, 0.8)	(0.8, 0.3, 0.2)	(0.3, 0.2, 0.6)
g_4	(0.8, 0.3, 0.5)	(0.3, 0.5, 0.3)	(0.6, 0.7, 0.3)
g_5	(0.7, 0.3, 0.6)	(0.4, 0.6, 0.8)	(0.8, 0.3, 0.8)

Table 3: Tabular form of NSS (Φ_2 , E_2)

U	β_1 = reddish	β_2 = yellowish	β_3 = blackish
g_1	(0.5, 0.7, 0.3)	(0.7, 0.8, 0.6)	(0.8, 0.3, 0.4)
g_2	(0.6, 0.7, 0.3)	(0.8, 0.5, 0.7)	(0.6, 0.7, 0.3)
g_3	(0.8, 0.5, 0.6)	(0.7, 0.3, 0.6)	(0.8, 0.3, 0.5)
g_4	(0.7, 0.2, 0.6)	(0.8, 0.6, 0.5)	(0.6, 0.7, 0.3)
g_5	(0.8, 0.4, 0.7)	(0.6, 0.5, 0.8)	(0.7, 0.4, 0.2)

Table 4: Tabular form of NSS (Φ_3 , E_3)

U	λ_1 = smooth	λ_2 = rough	λ_3 = moderate
g_1	(0.8, 0.5, 0.6)	(0.8, 0.7, 0.3)	(0.8, 0.6, 0.4)
g_2	(0.7, 0.6, 0.7)	(0.7, 0.5, 0.6)	(0.7, 0.5, 0.6)
g_3	(0.8, 0.7, 0.6)	(0.6, 0.3, 0.7)	(0.8, 0.2, 0.4)
g_4	(0.7, 0.5, 0.7)	(0.8, 0.7, 0.4)	(0.7, 0.8, 0.7)
g_5	(0.8, 0.7, 0.4)	(0.7, 0.4, 0.8)	(0.8, 0.6, 0.5)

Table 5: Tabular form of NSSs'(Φ_1, E_1) AND (Φ_2 , E_2)'

U	ε_{11}	ε_{21}	ε_{22}	ε_{31}	ε_{32}
g_1	(0.5, 0.65, 0.8)	(0.5, 0.5, 0.5)	(0.7, 0.55, 0.6)	(0.5, 0.7, 0.3)	(0.6, 0.75, 0.6)
g_2	(0.6, 0.75, 0.7)	(0.3, 0.65, 0.4)	(0.3, 0.55, 0.7)	(0.6, 0.5, 0.5)	(0.8, 0.4, 0.7)
g_3	(0.3, 0.5, 0.8)	(0.8, 0.4, 0.6)	(0.7, 0.3, 0.6)	(0.3, 0.35, 0.6)	(0.3, 0.25, 0.6)
g_4	(0.7, 0.25, 0.6)	(0.3, 0.35, 0.6)	(0.3, 0.55, 0.5)	(0.6, 0.45, 0.6)	(0.6, 0.65, 0.5)
g_5	(0.7, 0.35, 0.7)	(0.4, 0.5, 0.8)	(0.4, 0.55, 0.8)	(0.8, 0.35, 0.8)	(0.6, 0.4, 0.8)

Table 6: Tabular form of NSS '(Φ_5, T)'

U	$\varepsilon_{11} \wedge \lambda_1$	$\varepsilon_{21} \wedge \lambda_2$	$\varepsilon_{21} \wedge \lambda_3$	$\varepsilon_{31} \wedge \lambda_1$
g_1	(0.5, 0.575, 0.8)	(0.5, 0.6, 0.5)	(0.5, 0.55, 0.5)	(0.5, 0.6, 0.6)
g_2	(0.6, 0.675, 0.7)	(0.3, 0.575, 0.6)	(0.3, 0.575, 0.6)	(0.6, 0.55, 0.7)
g_3	(0.3, 0.6, 0.8)	(0.6, 0.35, 0.7)	(0.8, 0.3, 0.6)	(0.3, 0.525, 0.6)
g_4	(0.7, 0.375, 0.7)	(0.3, 0.525, 0.6)	(0.3, 0.575, 0.7)	(0.6, 0.475, 0.7)
g_5	(0.7, 0.525, 0.7)	(0.4, 0.45, 0.8)	(0.4, 0.55, 0.8)	(0.8, 0.525, 0.8)