# Neutrosophic soft sets with applications in decision making

Faruk Karaaslan

Department of Mathematics, Faculty of Science, Çankırı Karatekin University, 18100 Çankırı, Turkey

fkaraaslan@karatekin.edu.tr

**Abstract**

We firstly present definitions and properties in study of Maji [11] on neutrosophic soft sets. We then give a few notes on his study. Next, based on Çağman [4], we redefine the notion of neutrosophic soft set and neutrosophic soft set operations to make more functional. By using these new definitions we construct a decision making method and a group decision making method which selects a set of optimum elements from the alternatives. We finally present examples which shows that the methods can be successfully applied to many problems that contain uncertainties.

**Keyword 0.1** *Neutrosophic set; Soft set; Neutrosophic soft set; decision making.*

## 1 Introduction

Many problems including uncertainties are a major issue in many fields of real life such as economics, engineering, environment, social sciences, medical sciences and business management. Uncertain data in these fields could be caused by complexities and difficulties in classical mathematical modeling. To avoid difficulties in dealing with uncertainties, many tools have been studied by researchers. Some of these tools are fuzzy sets [18], rough sets [16] and intuitionistic fuzzy sets [1]. Fuzzy sets and intuitionistic fuzzy sets are characterized by membership functions, membership and non-membership functions, respectively. In some real life problems for proper description of an object in uncertain and ambiguous environment, we need to handle the

indeterminate and incomplete information. But fuzzy sets and intuitionistic fuzzy sets don't handle the indeterminant and inconsistent information. Samarandache [13] defined the notion of neutrosophic set which is a mathematical tool for dealing with problems involving imprecise and indeterminant data.

Molodtsov introduced concept of soft sets [9] to solve complicated problems and various types of uncertainties. In [10], Maji et al. introduced several operators for soft set theory: equality of two soft sets, subsets and superset of soft sets, complement of soft set, null soft sets and absolute soft sets. But some of these definitions and their properties have few gaps, which have been pointed out by Ali et al.[12] and Yang [17]. In 2010, Çağman and Enginoğlu [5] made some modifications the operations of soft sets and filled in these gap. In 2014, Çağman [4] redefined soft sets using the single parameter set and compared definitions with those defined before.

Maji [11] combined the concept of soft set and neutrosophic set together by introducing a new concept called neutrosophic soft set and gave an application of neutrosophic soft set in decision making problem. Recently, the properties and applications on the neutrosophic sets have been studied increasingly [2, 3, 7, 8].The propose of this paper is to fill the gaps of the Maji's neutrosophic soft set [11] definition and operations redefining concept of neutrosophic soft set and operations between neutrosophic soft sets. First, we present Maji's definitions and operations and we verify that some propositions are incorrect by a counterexample. Then based on Çağman's [4] study we redefine neutrosophic soft sets and their operations. Also, we investigate properties of neutrosophic soft sets operations. Finally we present an application of a neutrosophic soft set in decision making.

# 2 Preliminaries

In this section, we will recall the notions of neutrosophic sets [15] and soft sets [9]. Then, we will give some properties of these notions. Throughout this paper $X$, $E$ and $P(X)$ denote initial universe, set of parameters and power set of $X$, respectively.

**Definition 2.1** *[15] A neutrosophic set $A$ on the universe of discourse $X$ is defined as*

$$A = \left\{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \right\}$$

*where $T_A, I_A, F_A : X \rightarrow ]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application*

*in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of* $]^{-}0, 1^{+}[$. *Hence we consider the neutrosophic set which takes the value from the subset of* $[0, 1]$.

**Definition 2.2** *[9] Let consider a nonempty set* $A$, $A \subseteq E$. *A pair* $(F, A)$ *is called a soft set over* $X$, *where* $F$ *is a mapping given by* $F : A \to P(X)$.

**Example 2.3** *Let* $X = \{x_l, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ *be the universe which are eight houses and* $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ *be the set of parameters. Here,* $e_i$ $(i = 1, 2, 3, 4, 5, 6)$ *stand for the parameters "*modern*", "*with parking*", "*expensive*", "*cheap*", "*large*" and "*near to city*" respectively. Then, following soft sets are described respectively Mr. A and Mr. B who are going to buy*

$$\begin{aligned} F &= \big\{(e_1, \{x_1, x_3, x_4\}), (e_2, \{x_1, x_4, x_7, x_8\}), (e_3, \{x_1, x_2, x_3, x_8\})\big\} \\ G &= \big\{(e_2\{x_1, x_3, x_6\}), (e_3, X), (e_5, \{x_2, x_4, x_4, x_6\})\big\}. \end{aligned}$$

From now on, we will use definitions and operations of soft sets which are more suitable for pure mathematics based on study of Çağman [4].

**Definition 2.4** *[4] A soft set* $F$ *over* $X$ *is a set valued function from* $E$ *to* $P(X)$. *It can be written a set of ordered pairs*

$$F = \big\{(e, F(e)) : e \in E\big\}.$$

*Note that if* $F(e) = \emptyset$, *then the element* $(e, F(e))$ *is not appeared in* $F$. *Set of all soft sets over* $X$ *is denoted by* $\mathbb{S}$.

**Definition 2.5** *[4] Let* $F, G \in \mathbb{S}$. *Then,*

*i.* *If* $F(e) = \emptyset$ *for all* $e \in E$, $F$ *is said to be a null soft set, denoted by* $\Phi$.

*ii.* *If* $F(e) = X$ *for all* $e \in E$, $F$ *is said to be absolute soft set, denoted by* $\hat{X}$.

*iii.* $F$ *is soft subset of* $G$, *denoted by* $F \tilde{\subseteq} G$, *if* $F(e) \subseteq G(e)$ *for all* $e \in E$.

*iv.* $F = G$, *if* $F \tilde{\subseteq} G$ *and* $G \tilde{\subseteq} F$.

*v.* *Soft union of* $F$ *and* $G$, *denoted by* $F \tilde{\cup} G$, *is a soft set over* $X$ *and defined by* $F \tilde{\cup} G : E \to P(X)$ *such that* $(F \tilde{\cup} G)(e) = F(e) \cup G(e)$ *for all* $e \in E$.

*vi.* *Soft intersection of* $F$ *and* $G$, *denoted by* $F \tilde{\cap} G$, *is a soft set over* $X$ *and defined by* $F \tilde{\cap} G : E \to P(X)$ *such that* $(F \tilde{\cap} G)(e) = F(e) \cap G(e)$ *for all* $e \in E$.

*vii. Soft complement of $F$ is denoted by $F^{\tilde{c}}$ and defined by $F^{\tilde{c}} : E \to P(X)$ such that $F^{\tilde{c}}(e) = X \setminus F(e)$ for all $e \in E$.*

**Example 2.6** *Let us consider soft sets $F$,$G$ in the Example 2.3. Then, we have*

$$\begin{aligned} F\tilde{\cup}G &= \big\{(e_1, \{x_1, x_3, x_4\}), (e_2, \{x_1, x_3, x_4, x_6, x_7, x_8\}), \\ &\quad (e_3, X), (e_5, \{x_2, x_4, x_4, x_6\})\big\} \\ F\tilde{\cap}G &= \big\{(e_2\{x_1\}), (e_3, \{x_1, x_2, x_3, x_8\})\big\} \\ F^{\tilde{c}} &= \big\{(e_1, \{x_2, x_5, x_6, x_7, x_8\}), (e_2, \{x_2, x_3, x_5, x_6\}), \\ &\quad (e_3, \{x_4, x_5, x_6, x_7\}), (e_4, X), (e_5, X), (e_6, X)\big\}. \end{aligned}$$

**Definition 2.7** *[11] Let $X$ be an initial universe set and $E$ be a set of parameters. Consider $A \subset E$. Let $P(X)$ denotes the set of all neutrosophic sets of $X$. The collection $(F, A)$ is termed to be the soft neutrosophic set over $X$, where $F$ is a mapping given by $F : A \to P(X)$.*

For illustration we consider an example.

**Example 2.8** *Let $X$ be the set of houses under consideration and $E$ is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider $E = \{$beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive$\}$. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe $X$ given by, $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$, where $e_1$ stands for the parameter 'beautiful', $e_2$ stands for the parameter 'wooden', $e_3$ stands for the parameter 'costly' and the parameter $e_4$ stands for 'moderate'. Suppose that,*

$$\begin{aligned} F(beautiful) &= \{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle, \langle h_3, 0.6, 0.2, 0.3\rangle, \\ &\quad \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}, \\ F(wooden) &= \{\langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_2, 0.7, 0.4, 0.3\rangle, \langle h_3, 0.8, 0.1, 0.2\rangle, \\ &\quad \langle h_4, 0.7, 0.1, 0.3\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\}, \\ F(costly) &= \{\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.7, 0.2\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \\ &\quad \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\}, \\ F(moderate) &= \{\langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.9, 0.6\rangle, \langle h_3, 0.7, 0.6, 0.4\rangle, \\ &\quad \langle h_4, 0.7, 0.8, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\}. \end{aligned}$$

*The neutrosophic soft set $(NSS)$ $(F, E)$ is a parameterized family $\{F(e_i); i = 1, 2, ..., 10\}$ of all neutrosophic sets of $X$ and describes a collection of approximation of an object.*

*Thus we can view the neutrosophic soft set $(NSS)$ $(F, A)$ as a collection of approximation as below:*

$$\begin{aligned}(F,A) \;=\; & \{beautiful\ houses = \{\langle h_1, 0.5, 0.6, 0.3\rangle, \langle h_2, 0.4, 0.7, 0.6\rangle,\\ & \langle h_3, 0.6, 0.2, 0.3\rangle, \langle h_4, 0.7, 0.3, 0.2\rangle, \langle h5, 0.8, 0.2, 0.3\rangle\},\\ & wooden\ houses = \{\langle h_1, 0.6, 0.3, 0.5\rangle, \langle h_2, 0.7, 0.4, 0.3\rangle,\\ & \langle h_3, 0.8, 0.1, 0.2\rangle, \langle h_4, 0.7, 0.1, 0.3\rangle, \langle h_5, 0.8, 0.3, 0.6\rangle\},\\ & costly\ houses = f\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.7, 0.2\rangle,\\ & \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\},\\ & moderate\ houses = \langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.9, 0.6\rangle,\\ & \langle h_3, 0.7, 0.6, 0.4\rangle, \langle h_4, 0.7, 0.8, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\}\}.\end{aligned}$$

**Definition 2.9** *[11] Let $(F, A)$ and $(G, B)$ be two neutrosophic sets over the common universe $X$. $(F, A)$ is said to be neutrosophic soft subset of $(G, B)$ is $A \subset B$, and $T_{F(e)}(x) \leq T_{G(e)}(x)$, $I_{F(e)}(x) \leq I_{G(e)}(x)$ $F_{F(e)}(x) \geq F_{G(e)}(x)$, $\forall e \in A$, $\forall x \in U$. We denote it by $(F, A) \subseteq (G, B)$. $(F, A)$ is said to be neutrosophic soft super set of $(G, B)$ if $(G, B)$ is a neutrosophic soft subset of $(F, A)$. We denote it by $(F, A) \supseteq (G, B)$.*

*if $(F, A)$ is neutrosophic soft subset of $(G, B)$ and $(G, B)$ is neutrosophic soft subset of $(F, A)$. We denote it $(F, A) = (G, B)$.*

**Definition 2.10** *[11] NOT set of a parameters. Let $E = \{e_1, e_2, ...e_n\}$ be a set of parameters. The NOT set of $E$, denoted by $\rceil E$ is defined by $\rceil E = \{\neg e_1, \neg e_2, ...\neg e_n\}$, where $\neg e_i$ =not $e_i$ $\forall i$(it may be noted that $\rceil$ and $\neg$ are different operators).*

**Definition 2.11** *[11] Complement of a neutrosophic soft set $(F, A)$ denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \rceil A)$, where $F^c.\rceil A \to P(X)$ is mapping given by $F^c(\alpha)$ = neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}$, $I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.*

**Definition 2.12** *[11] Empty or null neutrosophic soft set with respect to a parameter. A neutrosophic soft set $(H, A)$ over the universe $X$ is termed to be empty or null neutrosophic soft set with respect to the parameter $e$ if $T_{H(e)}(m) = 0, F_{H(e)} = 0$ and $I_{H(e)}(m) = 0$ $\forall m \in X$, $\forall e \in A$*

*In this case the null neutrosophic soft set $(NNSS)$ is denoted by $\Phi_A$*

**Definition 2.13** *[11] Union of two neutrosophic soft sets. Let $(H, A)$ and $(G, B)$ be two $NSSs$ over the common universe $X$. Then the union of $(H, A)$ and $(G, B)$ is defined by $(H, A) \cup (G, B) = (K, C)$, where $C = A \cup B$ and*

*the truth-membership, indeterminacy-membership and falsity-membership of* $(K, C)$ *are as follow.*

$$\begin{aligned}
T_{K(e)}(m) &= T_{H(e)}(m), if e \in A - B \\
&= T_{G(e)}(m), if e \in B - A \\
&= max(T_{H(e)}(m), T_{G(e)}(m)), if e \in A \cap B \\
I_{K(e)}(m) &= I_{H(e)}(m), if e \in A - B \\
&= I_{G(e)}(m), if e \in B - A \\
&= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}, if e \in A \cap B. \\
F_{K(e)}(m) &= F_{H(e)}(m), if e \in A - B \\
&= F_{G(e)}(m), if e \in B - A \\
&= min(F_{H(e)}(m), F_{G(e)}(m)), if e \in A \cap B
\end{aligned}$$

**Definition 2.14** *[11] Let* $(H, A)$ *and* $(G, B)$ *be two* $NSSs$ *over the common universe* $X$*. Then, intersection of* $(H, A)$ *and* $(G, B)$ *is defined by* $(H, A) \cap (G, B) = (K, C)$*, where* $C = A \cap B$ *and the truth-membership, indeterminacy-membership and falsity-membership of* $(K, C)$ *are as follow.*

$$\begin{aligned}
T_{K(e)}(m) &= min(T_{H(e)}(m), T_{G(e)}(m)),\ if\ e \in A \cap B \\
I_{K(e)}(m) &= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2},\ if\ e \in A \cap B. \\
F_{K(e)}(m) &= max(F_{H(e)}(m), F_{G(e)}(m)),\ if\ e \in A \cap B
\end{aligned}$$

For any two $NSSs$ $(H, A)$ and $(G, B)$ over the same universe $X$ and on the basis of the operations defined above, we have the following propositions.

**Proposition 2.15** *[11]*

*(1)* $(H, A) \cup (H, A) = (H, A)$

*(2)* $(H, A) \cup (G, B) = (G, B) \cup (H, A)$

*(3)* $(H, A) \cap (H, A) = (H, A)$

*(4)* $(H, A) \cap (G, B) = (G, B) \cap (H, A)$

*(5)* $(H, A) \cup \Phi = (H, A)$

*(6)* $(H, A) \cap \Phi = \Phi$

*(7)* $[(H, A)^c]^c = (H, A)$

For any two $NSSs$ $(H, A)$, $(G, B)$ and $(K, C)$over the same universe $X$, we have the following propositions.

**Proposition 2.16** *[11]*

*(1)* $(H, A) \cup [(G, B) \cup (K, C)] = [(H, A) \cup (G, B)] \cup (K, C)$.

*(2)* $(H, A) \cap [(G, B) \cap (K, C)] = [(H, A) \cap (G, B)] \cap (K, C)$.

*(3)* $(H, A) \cup [(G, B) \cap (K, C)] = [(H, A) \cup (G, B)] \cap [(H, A) \cup (K, C)]$.

*(4)* $(H, A) \cap [(G, B) \cup (K, C)] = [(H, A) \cap (G, B)] \cup [(H, A) \cap (K, C)]$.

**Definition 2.17** *[11] Let $(H, A)$ and $(G, B)$ be two $NSSs$ over the common universe $X$. Then 'AND' operation on them is denoted by '$(H, A) \bigwedge (G, B)$' and is defined by $(H, A) \bigwedge (G, B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follow.*

$$\begin{aligned} T_{K(\alpha,\beta)}(m) &= min(T_{H(e)}(m), T_{G(e)}(m)) \\ I_{K(\alpha,\beta))}(m) &= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2} \\ F_{K(\alpha,\beta))}(m) &= max(F_{H(e)}(m), F_{G(e)}(m)), \forall \alpha \in A, \forall b \in B \end{aligned}$$

**Definition 2.18** *[11] Let $(H, A)$ and $(G, B)$ be two $NSSs$ over the common universe $X$. Then 'OR' operation on them is denoted by '$(H, A) \bigvee (G, B)$' and is defined by $(H, A) \bigvee (G, B) = (O, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(O, A \times B)$ are as follow.*

$$\begin{aligned} T_{O(\alpha,\beta))}(m) &= max(T_{H(e)}(m), T_{G(e)}(m)), \\ I_{O(\alpha,\beta))}(m) &= \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}, \\ F_{O(\alpha,\beta))}(m) &= min(F_{H(e)}(m), F_{G(e)}(m)), \forall \alpha \in A, \forall b \in B \end{aligned}$$

## Notes on neutrosophic soft sets [11]

In this section, we verify that some propositions in the study of Maji [11] are incorrect by counterexamples.

1. If Definition (2.9) is true, then Definition (3.3) is incorrect.

2. Proposition (2.15)-(5) and (6), $(F,A)\cap\Phi=\Phi$ and $(F,A)\cup\Phi=(F,A)$ are incorrect.

We verify these notes by counterexamples.

**Example 2.19** *Let us consider neutrosophic soft set* $(F,A)$ *in Example* (2.8) *and null neutrosophic soft set* $\Phi$*. If Definition* (2.9) *is true, it is required that null soft set is neutrosophic soft subset of all neutrosophic soft sets. But, since* $T_{\Phi(beautiful)}(h_1)\leq T_{F(beautiful)}(h_1)$ *and* $I_{\Phi(beautiful)}(h_1)\leq I_{F(beautiful)}(h_1)$ *but* $F_{\Phi(beautiful)}(h_1)\not\geq F_{F(beautiful)}(h_1)$*,* $\Phi\not\subseteq(F,A)$*.*

**Example 2.20** *Let us consider neutrosophic soft set* $(F,A)$ *in Example* (2.8) *and null neutrosophic soft set* $\Phi$*. Then,*

$$\begin{aligned}(F,A)\cap\Phi &= \{e_1=\{\langle h_1,0,0.3,0.3\rangle,\langle h_2,0,0.35,0.6\rangle,\\ &\quad \langle h_3,0,0.1,0.3\rangle,\langle h_4,0,0.15,0.2\rangle,\langle h_5,0,0.1,0.3\rangle\},\\ &\quad e_2=\{\langle h_1,0,0.15,0.5\rangle,\langle h_2,0,0.2,0.3\rangle,\langle h_3,0,0.05,0.2\rangle,\\ &\quad \langle h_4,0,0.05,0.3\rangle,\langle h_5,0,0.15,0.6\rangle\},\\ &\quad e_3=\{\langle h_1,0,0.2,0.3\rangle,\langle h_2,0,0.35,0.2\rangle,\langle h_3,0,0.1,0.5\rangle,\\ &\quad \langle h_4,0,0.1,0.6\rangle,\langle h_5,0,0.15,0.4\rangle\},\\ &\quad e_5=\{\langle h_1,0,0.3,0.4\rangle,\langle h_2,0,0.45,0.6\rangle,\langle h_3,0,0.3,0.4\rangle,\\ &\quad \langle h_4,0,0.4,0.6\rangle,\langle h_5,0,0.25,0.7\rangle\}\}.\\ &\neq \Phi\end{aligned}$$

*and*

$$\begin{aligned}(F,A)\cup\Phi &= \{e_1=\{\langle h_1,0.5,0.3,0\rangle,\langle h_2,0.40.35,0\rangle,\\ &\quad \langle h_3,0.6,0.1,0\rangle,\langle h_4,0.7,0.15,0\rangle,\langle h_5,0.8,0.1,0\rangle\},\\ &\quad e_2=\{\langle h_1,0.6,0.15,0\rangle,\langle h_2,0.7,0.2,0\rangle,\langle h_3,0.8,0.05,0\rangle,\\ &\quad \langle h_4,0.7,0.05,0\rangle,\langle h_5,0.8,0.15,0\rangle\},\\ &\quad e_3=\{\langle h_1,0.7,0.2,0\rangle,\langle h_2,0.6,0.35,0\rangle,\langle h_3,0.7,0.1,0\rangle,\\ &\quad \langle h_4,0.5,0.1,0\rangle,\langle h_5,0.7,0.15,0\rangle\},\\ &\quad e_5=\{\langle h_1,0.8,0.3,0\rangle,\langle h_2,0.7,0.45,0\rangle,\langle h_3,0.7,0.3,0\rangle,\\ &\quad \langle h_4,0.7,0.4,0\rangle,\langle h_5,0.9,0.25,0\rangle\}\}.\\ &\neq (F,A)\end{aligned}$$

# 3 Neutrosophic soft sets

In this section, we will redefine the neutrosophic soft set based on paper of Çağman [4].

**Definition 3.1** *A neutrosophic soft set (or namely* ns*-set)* $f$ *over* $X$ *is a neutrosophic set valued function from* $E$ *to* $N(X)$*. It can be written as*

$$f = \Big\{\big(e, \{\langle x, T_{f(e)}(x), I_{f(e)}(x), F_{f(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\}$$

*where,* $N(X)$ *denotes all neutrosophic sets over* $X$*. Note that if* $f(e) = \big\{\langle x, 0, 1, 1\rangle : x \in X\big\}$*, the element* $(e, f(e))$ *is not appeared in the neutrosophic soft set* $f$*.Set of all* ns*-sets over* $X$ *is denoted by* $\mathbb{NS}$*.*

**Definition 3.2** *Let* $f, g \in \mathbb{NS}$*.* $f$ *is said to be neutrosophic soft subset of* $g$*, if* $T_{f(e)}(x) \leq T_{g(e)}(x)$*,* $I_{f(e)}(x) \geq I_{g(e)}(x)$ $F_{f(e)}(x) \geq F_{g(e)}(x)$*,* $\forall e \in E$*,* $\forall x \in U$*. We denote it by* $f \sqsubseteq g$*.* $f$ *is said to be neutrosophic soft super set of* $g$ *if* $g$ *is a neutrosophic soft subset of* $f$*. We denote it by* $f \sqsupseteq g$*.*

*If* $f$ *is neutrosophic soft subset of* $g$ *and* $g$ *is neutrosophic soft subset of* $f$*. We denote it* $f = g$

**Definition 3.3** *Let* $f \in \mathbb{NS}$*. If* $T_{f(e)}(x) = 0$ *and* $I_{f(e)}(x) = F_{f(e)}(x) = 1$ *for all* $e \in E$ *and for all* $x \in X$*, then* $f$ *is called null* ns*-set and denoted by* $\tilde{\Phi}$*.*

**Definition 3.4** *Let* $f \in \mathbb{NS}$*.If* $T_{f(e)}(x) = 1$ *and* $I_{f(e)}(x) = F_{f(e)}(x) = 0$ *for all* $e \in E$ *and for all* $x \in X$*, then* $f$ *is called universal* ns*-set and denoted by* $\tilde{X}$*.*

**Definition 3.5** *Let* $f, g \in \mathbb{NS}$*. Then union and intersection of ns-sets* $f$ *and* $g$ *denoted by* $f \sqcup g$ *and* $f \sqcap g$ *respectively, are defined by as follow*

$$\begin{aligned} f \sqcup g &= \Big\{\big(e, \{\langle x, T_{f(e)}(x) \vee T_{g(e)}(x), I_{f(e)}(x) \wedge I_{g(e)}(x), \\ &\qquad F_{f(e)}(x) \wedge F_{g(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\}. \end{aligned}$$

*and* ns*-intersection of* $f$ *and* $g$ *is defined as*

$$\begin{aligned} f \sqcap g &= \Big\{\big(e, \{\langle x, T_{f(e)}(x) \wedge T_{g(e)}(x), I_{f(e)}(x) \vee I_{g(e)}(x), \\ &\qquad F_{f(e)}(x) \vee F_{g(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\}. \end{aligned}$$

**Definition 3.6** *Let* $f, g \in \mathbb{NS}$*. Then complement of ns-set* $f$*, denoted by* $f^{\tilde{c}}$*, is defined as follow*

$$f^{\tilde{c}} = \Big\{\big(e, \{\langle x, F_{f(e)}(x), 1 - I_{f(e)}(x), T_{f(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\}.$$

**Proposition 3.7** *Let* $f, g, h \in \mathbb{NS}$*. Then,*

*i.* $\tilde{\Phi} \sqsubseteq f$

*ii.* $f \sqsubseteq \tilde{X}$

*iii.* $f \sqsubseteq f$

*iv.* $f \sqsubseteq g$ *and* $g \sqsubseteq h \Rightarrow f \sqsubseteq h$

*Proof.* The proof is obvious from Definition (3.2), (3.3) and Definition (3.4).

**Proposition 3.8** *Let* $f \in \mathbb{NS}$. *Then*

*i.* $\tilde{\Phi}^{\tilde{c}} = \tilde{X}$

*ii.* $\tilde{X}^{\tilde{c}} = \tilde{\Phi}$

*iii.* $(f^{\tilde{c}})^{\tilde{c}} = f$.

*Proof.* The proof is clear from Definition (3.3), (3.4) and (3.6).

**Theorem 3.9** *Let* $f, g, h \in \mathbb{NS}$. *Then,*

*i.* $f \sqcap f = f$ *and* $f \sqcup f = f$

*ii.* $f \sqcap g = g \sqcap f$ *and* $f \sqcup g = g \sqcup f$

*iii.* $f \sqcap \tilde{\Phi} = \tilde{\Phi}$ *and* $f \sqcap \tilde{X} = f$

*iv.* $f \sqcup \tilde{\Phi} = f$ *and* $f \sqcup \tilde{X} = \tilde{X}$

*v.* $f \sqcap (g \sqcap h) = (f \sqcap g) \sqcap h$ *and* $f \sqcup (g \sqcup h) = (f \sqcup g) \sqcup h$

*vi.* $f \sqcap (g \sqcup h) = (f \sqcap g) \sqcup (f \sqcap h)$ *and* $f \sqcup (g \sqcap h) = (f \sqcup g) \sqcap (f \sqcup h)$.

*Proof.* The proof is clear from definition and operations of neutrosophic soft sets.

**Theorem 3.10** *Let* $f, g \in \mathbb{NS}$. *Then, De Morgan's law is valid.*

*i.* $(f \sqcup g)^{\tilde{c}} = f^{\tilde{c}} \sqcap g^{\tilde{c}}$

*ii.* $(f \sqcup g)^{\tilde{c}} = f^{\tilde{c}} \sqcap g^{\tilde{c}}$

*Proof.* $f, g \in \mathbb{NS}$ is given.

*i.* From Definition 3.6, we have

$$\begin{aligned}
(f \sqcup g)^{\tilde{c}} &= \Big\{\big(e, \{\langle x, T_{f(e)}(x) \vee T_{g(e)}(x), I_{f(e)}(x) \wedge I_{f(e)}(x), \\
&\quad F_{f(e)}(x) \wedge F_{f(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\}^{\tilde{c}} \\
&= \Big\{\big(e, \{\langle x, F_{f(e)}(x) \wedge F_{f(e)}(x), 1 - (I_{f(e)}(x) \wedge I_{f(e)}(x)), \\
&\quad T_{f(e)}(x) \vee T_{g(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\} \\
&\quad \Big\{\langle x, F_{f(e)}(x), 1 - I_{f(e)}(x), T_{f(e)}(x)\rangle : e \in E\Big\} \\
&= \Big\{\big(e, \{X\}\big) : e \in E\Big\} \\
&\sqcap \Big\{\big(e, \{\langle x, F_{g(e)}(x), 1 - I_{g(e)}(x), T_{g(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\} \\
&= f^{\tilde{c}} \sqcap g^{\tilde{c}}.
\end{aligned}$$

*ii.* It can be proved similar way (*i.*)

**Definition 3.11** *Let $f, g \in \mathbb{NS}$. Then, difference of $f$ and $g$, denoted by $f \setminus g$ is defined by the set of ordered pairs*

$$f \setminus g = \Big\{\big(e, \{\langle x, T_{f\setminus g(e)}(x), I_{f\setminus g(e)}(x), F_{f\setminus g(e)}(x)\rangle : x \in X\}\big) : e \in E\Big\}$$

*here, $T_{f\setminus g(e)}(x)$, $I_{f\setminus g(e)}(x)$ and $F_{f\setminus g(e)}(x)$ are defined by*

$$T_{f\setminus g(e)}(x) = \begin{cases} T_{f(e)}(x) - T_{g(e)}(x), & T_{f(e)}(x) > T_{g(e)}(x) \\ 0, & otherwise \end{cases}$$

$$I_{f\setminus g(e)}(x) = \begin{cases} I_{g(e)}(x) - I_{f(e)}(x), & I_{f(e)}(x) < I_{g(e)}(x) \\ 0, & otherwise \end{cases}$$

$$F_{f\setminus g(e)}(x) = \begin{cases} F_{g(e)}(x) - F_{f(e)}(x), & G_{f(e)}(x) < G_{g(e)}(x) \\ 0, & otherwise \end{cases}$$

**Definition 3.12** *Let $f, g \in \mathbb{NS}$. Then 'OR' product of ns-sets $f$ and $g$ denoted by $f \wedge g$, is defined as follow*

$$\begin{aligned}
f \bigvee g &= \Big\{\big((e, e'), \{\langle x, T_{f(e)}(x) \vee T_{g(e)}(x), I_{f(e)}(x) \wedge I_{g(e)}(x), \\
&\quad F_{f(e)}(x) \wedge F_{g(e)}(x)\rangle : x \in X\}\big) : (e, e') \in E \times E\Big\}.
\end{aligned}$$

**Definition 3.13** *Let $f, g \in \mathbb{NS}$. Then 'AND' product of ns-sets $f$ and $g$ denoted by $f \vee g$, is defined as follow*

$$f \bigwedge g = \Big\{\big((e,e'), \{\langle x, T_{f(e)}(x) \wedge T_{g(e)}(x), I_{f(e)}(x) \vee I_{g(e)}(x), F_{f(e)}(x) \vee F_{g(e)}(x)\rangle : x \in X\}\big) : (e,e') \in E \times E\Big\}.$$

**Proposition 3.14** *Let $f, g \in \mathbb{NS}$. Then,*

1. $(f \bigvee g)^{\tilde{c}} = f^{\tilde{c}} \bigwedge g^{\tilde{c}}$
2. $(f \bigwedge g)^{\tilde{c}} = f^{\tilde{c}} \bigvee g^{\tilde{c}}$

*Proof.* The proof is clear from Definition (3.12) and (3.13).

# 4 Decision making method

In this section we will construct a decision making method over the neutrosophic soft set. Firstly, we will define some notions that necessary to construct algorithm of decision making method.

**Definition 4.1** *Let $X = \{x_1, x_2, ...x_m\}$ be an initial universe, $E = \{e_1, e_2, ...e_n\}$ be a parameter set and $f$ be a neutrosophic soft set over $X$. Then, according to the Table of "Saaty Rating Scale" relative parameter matrix $d_E$ is defined as follow*

$$d_E = \begin{bmatrix} 1 & d_E(e_1,e_2) & \dots & d_E(e_1,e_n) \\ d_E(e_2,e_1) & 1 & \dots & d_E(e_2,e_n) \\ \vdots & \vdots & \vdots & \vdots \\ d_E(e_n,e_1) & d_E(e_n,e_2) & \dots & 1 \end{bmatrix}$$

*If $d_E(e_i, e_j) = d_{12}$, we can write matrix*

$$d_E = \begin{bmatrix} 1 & d_{11} & \dots & d_{1n} \\ d_{21} & 1 & \dots & d_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \dots & 1 \end{bmatrix}$$

*Here, $d_{12}$ means that how much important $e_1$ by $e_2$. For example, if $e_1$ is much more important by $e_2$, then we can write $d_{12} = 5$ from Table 1.*

| *Intensity importance* | *Definition* | *Explanation* |
|---|---|---|
| 1 | *Equal importance* | *Two factors contribute equally to the objective* |
| 3 | *Somewhat more important* | *Experience and judgement slightly favour one over the other* |
| 5 | *Much more important* | *Experience and judgement strongly favour one over the other* |
| 7 | *Very much more important* | *Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice* |
| 9 | *Absolutely more important* | *The evidence favouring one over the other is of the highest possible validity.* |
| $2, 4, 6, 8$ | *Intermediate values* | *When compromise is needed* |

*Table 1.* The Saaty Rating Scale

**Definition 4.2** *Let $f$ be a neutrosophic soft set and $d_E$ be a relative parameter matrix of $f$. Then, score of parameter $e_i$, denoted by $c_i$ and is calculated as follows*

$$c_i = \sum_{j=1}^{n} d_{ij}$$

**Definition 4.3** *Normalized relative parameter matrix ($nd_E$ for short) of relative parameter matrix $d_E$, denoted by $\hat{d}$, is defined as follow,*

$$nd_E = \begin{bmatrix} \frac{1}{c_1} & \frac{d_{12}}{c_1} & \dots & \frac{d_{1n}}{c_1} \\ \frac{d_{21}}{c_2} & \frac{1}{c_2} & \dots & \frac{d_{2n}}{c_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{d_{n1}}{c_n} & \frac{d_{n2}}{c_n} & \dots & \frac{1}{c_n} \end{bmatrix}$$

*if $\frac{d_{ij}}{c_i} = \hat{d}_{ij}$, we can write matrix $nd_E$*

$$\hat{d} = \begin{bmatrix} \hat{d}_{11} & \hat{d}_{12} & \dots & \hat{d}_{1n} \\ \hat{d}_{21} & \hat{d}_{22} & \dots & \hat{d}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{d}_{n1} & \hat{d}_{n2} & \dots & \hat{d}_{nn} \end{bmatrix}$$

**Definition 4.4** *Let $f$ be a neutrosophic soft set and $\hat{d}$ be a normalized parameter matrix of $f$. Then, weight of parameter $e_j \in E$, denoted by $w(e_j)$ and is formulated as follows.*

$$w(e_j) = \frac{1}{|E|} \sum_{i=1}^{n} \hat{d}_{ij}$$

Now, we construct compare matrices of elements of $X$ in neutrosophic sets $f(e)$, $\forall e \in E$.

**Definition 4.5** *Let $E$ be a parameter set and $f$ be a neutrosophic soft set over $X$. Then, for all $e \in E$, compression matrices of $f$, denoted $X_{f(e)}$ is defined as follow*

$$X_{f(e)} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mm} \end{bmatrix}$$

$$x_{ij} = \frac{\Delta_{T(e)}(x_{ij}) + \Delta_{I(e)}(x_{ij}) + \Delta_{F(e)}(x_{ij}) + 1}{2}$$

*such that*

$$\begin{aligned} \Delta_{T(e)}(x_{ij}) &= T(e)(x_i) - T(e)(x_j) \\ \Delta_{I(e)}(x_{ij}) &= I(e)(x_j) - I(e)(x_i) \\ \Delta_{F(e)}(x_{ij}) &= F(e)(x_j) - F(e)(x_i) \end{aligned}$$

**Definition 4.6** *Let $X_{f(e)}$ be compare matrix for $e \in E$. Then, weight of $x_j \in X$ related to parameter $e \in E$, denoted by $W_{f(e)}(x_j)$ is defined as follow,*

$$W_{f(e)}(x_j) = \frac{1}{|X|} \sum_{i=1}^{m} x_{ij}$$

**Definition 4.7** *Let $E$ be a parameter set, $X$ be an initial universe and $w(e)$ and $W_{f(e)}(x_j)$ be weight of parameter $e$ and membership degree of $x_j$ which related to $e_j \in E$, respectively. Then, decision set, denoted $D_E$, is defined by the set of ordered pairs*

$$D_E = \{(x_j, F(x_j)) : x_j \in X\}$$

*where*

$$F(x_j) = \frac{1}{|E|} \sum_{j=1}^{n} w(e_j) \cdot W_{f(e)}(x_j)$$

*Note that, $F$ is a fuzzy set over $X$.*

Now, we construct a neutrosophic soft set decision making method by the following algorithm;

# Algorithm 1

***Step 1:*** Input the neutrosophic soft set $f$,
***Step 2:*** Construct the normalized parameter matrix,
***Step 3:*** Compute the weight of each parameters,
***Step 4:*** Construct the compare matrix for each parameter,
***Step 5:*** Compute membership degree, for all $x_j \in X$
***Step 6:*** Construct decision set $D_E$
***Step 7:*** The optimal decision is to select $x_k = maxF(x_j)$.

**Example 4.8** *Let $X$ be the set of blouses under consideration and $E$ is the set of parameters. Each parameters is a neutrosophic word or sentence involving neutrosophic words. Consider $E = \{bright, cheap, colorful, cotton\}$. Suppose that, there are five blouses in the universe $X$ given by $X = \{x_1, x_2, x_3, x_4, x_5\}$. Suppose that,*

***Step 1:*** *Let us consider the decision making problem involving the neutrosophic soft set in [2]*

$$\begin{aligned}
F(Bright) &= \{\langle x_1, .5, .6, .3\rangle, \langle x_2, .4, .7, .2\rangle, \langle x_3, .6, .2, .3\rangle, \langle x_4, .7, .3, .2\rangle, \langle x_5, .8, .2, .3\rangle, \} \\
F(Cheap) &= \{\langle x_1, .6, .3, .5\rangle, \langle x_2, .7, .4, .3\rangle, \langle x_3, .8, .1, .2\rangle, \langle x_4, .7, .1, .3\rangle, \langle x_5, .8, .3, .4\rangle, \} \\
F(Colorful) &= \{\langle x_1, .7, .4, .3\rangle, \langle x_2, .6, .1, .2\rangle, \langle x_3, .7, .2, .5\rangle, \langle x_4, .5, .2, .6\rangle, \langle x_5, .7, .3, .2\rangle, \} \\
F(Cotton) &= \{\langle x_1, .4, .3, .7\rangle, \langle x_2, .5, .4, .2\rangle, \langle x_3, .7, .4, .3\rangle, \langle x_4, .2, .4, .5\rangle, \langle x_5, .6, .4, .4\rangle, \}
\end{aligned}$$

***Step 2:***

$$d_E = \begin{pmatrix} 1 & 1/3 & 5 & 1/3 \\ 3 & 1 & 2 & 3 \\ 1/5 & 1/2 & 1 & 2 \\ 3 & 1/3 & 1/2 & 1 \end{pmatrix}$$

$c_1 = 6.67$, $c_2 = 9$, $c_3 = 3.7$ *and* $c_4 = 4.88$ *and*

$$\hat{d}_E = \begin{pmatrix} .15 & .05 & .75 & .05 \\ .33 & .11 & .22 & .33 \\ .05 & .14 & .27 & .54 \\ .62 & .07 & .10 & .21 \end{pmatrix}$$

***Step 3:*** *From normalized matrix, weight of parameters are obtained as* $w(e_1) = .29$, $w(e_2) = .09$, $w(e_3) = .34$ *and* $w(e_4) = .28$.

***Step 4:*** *For each parameter, compare matrices and normalized compare matrices are constructed as follow*

*Let us consider parameter "bright". Then,*

$$X_{f(bright)} = \begin{bmatrix} .50 & .10 & .25 & .20 & .15 \\ .45 & .50 & .20 & .15 & .10 \\ .75 & .80 & .50 & .45 & .40 \\ .80 & .85 & .55 & .50 & .45 \\ .85 & .90 & .60 & .55 & .50 \end{bmatrix}, \quad X_{f(cheap)} = \begin{bmatrix} .50 & .40 & .15 & .25 & .35 \\ .50 & .50 & .30 & .35 & .45 \\ .85 & .75 & .50 & .60 & .70 \\ .75 & .65 & .40 & .50 & .60 \\ .65 & .55 & .30 & .40 & .50 \end{bmatrix}$$

*and*

$$X_{f(colorful)} = \begin{bmatrix} .50 & .35 & .55 & .65 & .40 \\ .65 & .50 & .65 & .18 & .55 \\ .50 & .35 & .50 & .65 & .40 \\ .35 & .30 & .15 & .50 & .25 \\ .40 & .45 & .60 & .75 & .50 \end{bmatrix}, \quad X_{f(cotton)} = \begin{bmatrix} .50 & .25 & .35 & .50 & .15 \\ .75 & .50 & .60 & .75 & .40 \\ .65 & .40 & .50 & .65 & .30 \\ .50 & .25 & .35 & .50 & .15 \\ .85 & .60 & .70 & .85 & .50 \end{bmatrix}$$

***Step 5:*** *For all $x_j \in X$ and $e \in E$,*

$W_{f(bright)}(x_1) = .67,\ W_{f(bright)}(x_2) = .63,\ W_{f(bright)}(x_3) = .42,$
$W_{f(bright)}(x_4) = .37,\ W_{f(bright)}(x_5) = .32$

$W_{f(cheap)}(x_1) = .80,\ W_{f(cheap)}(x_2) = .57,\ W_{f(cheap)}(x_3) = .33,$
$W_{f(cheap)}(x_4) = .42,\ W_{f(cheap)}(x_5) = .52$

$W_{f(colorful)}(x_1) = .48,\ W_{f(colorful)}(x_2) = .39,\ W_{f(colorful)}(x_3) = .49,$
$W_{f(colorful)}(x_4) = .55,\ W_{f(colorful)}(x_5) = .42$

$W_{f(cotton)}(x_1) = .65,\ W_{f(cotton)}(x_2) = .40,\ W_{f(cotton)}(x_3) = .50,$
$W_{f(cotton)}(x_4) = .65,\ W_{f(cotton)}(x_5) = .30$

***Step 6:*** *By using step 3 and step 5, $D_E$ is constructed as follow*

$$D_E = \{(x_1, 0.15), (x_2, 0.12), (x_3, 0.11), (x_4, 0.13), (x_5, 0.09)\}$$

***Step 7:*** *Note that, membership degree of $x_1$ is greater than the other. Therefore, optimal decision is $x_1$ for this decision making problem.*

# 5 Group decision making

In this section, we constructed a group decision making method using intersection of neutrosophic soft sets and Algorithm 1.

Let $X = \{x_1, x_2, ..., x_n\}$ be an initial universe and let $d = \{d^1, d^2, ..., d^m\}$ be a decision maker set and $E = \{e_1, e_2, ..., e_k\}$ be a set of parameters. Then, this method can be described by the following steps

# Algorithm 2

***Step 1:*** Each decision-maker $d^i$ construct own neutrosophic soft set, denoted by $f_{d_i}$, over $U$ and parameter set $E$.
***Step 2:*** Let for $p, r \leq k$, $[d^i_{pr}]$ a relative parameter matrix of decision-maker $d^i \in D$ based on the Saaty Rating Scale. Decision-maker $d^i$ gives his/her evaluations separately and independently according to his/her own preference based on Saaty Rating Scale. In this way, each decision-maker $d^i$ presents a relative parameter matrix.

$$[d^i_{pr}] = \begin{pmatrix} d^i_{11} & d^i_{12} & \cdots & d^i_{1k} \\ d^i_{21} & d^i_{22} & \cdots & d^i_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ d^i_{k1} & d^i_{k2} & \vdots & d^i_{kk} \end{pmatrix}$$

here $d^i_{pr}$ is equal $d_E(e_p, e_r)$ that in Definition (4.1).
***Step 3:*** Arithmetic mean matrix is constructed by using the the relative parameter matrix of each decision-maker $d^i$. It will be denoted by $[i_{pr}]$ and will be computed as in follow

$$i_{pr} = \frac{1}{|d|} \sum_{i=1}^{m} d^i_{pr}$$

***Step 4:*** Normalized parameter matrix, is constructed using the arithmetic mean matrix $[i_{pr}]$, it will be shown $[\hat{i}_{pr}]$ and weight of each parameter $e_i \in E$ $(w(e_i))$ is computed.
***Step 5:*** Intersection of neutrosophic soft sets (it will be denoted by $I_{f_{d_i}}$) which are constructed by decision makers is found.

$$I_{f_d} = \bigcap_{i=1}^{m} f_{d^i}$$

***Step 6:*** Based on the matrix $I_{f_d}$, for each element of $e \in E$ compare matrix, denoted by $I_{f_d(e)}$ is constructed.
***Step 7:*** By the $I_{f_d(e)}$, weight of each element of $X$ denoted by $W_{I_{f_d(e)}}(x_i)$,

are computed.
***Step 8:*** Decision set $D_E$ is constructed by using values of $w(e)$ and $W_{I_{f_{d^i}}}(x)$. Namely;

$$D_E = \{(x_i, F(x_i)) : x_i \in X\}$$

and

$$F(x_i) = \frac{1}{|E|} \sum_{j=1}^{n} w(e_j) \cdot W_{I_f(e)}(x_i)$$

***Step 9:*** From the decision set, $x_k = maxF(x_i)$ is selected as optimal decision.

**Example 5.1**

Assume that a company wants to fill a position. There are 6 candidates who fill in a form in order to apply formally for the position. There are three decision makers; one of them is from the department of human resources and the others is from the board of directors. They want to interview the candidates, but it is very difficult to make it all of them. Let $d = \{d_1, d_2, d_3\}$ be a decision makers set, $X = \{x_1, x_2, x_3, x_4, x_5\}$ be set of candidates and $E = \{e_1, e_2, e_3, e_4\}$ be a parameter set such that parameters $e_1, e_2, e_3$ and $e_4$ stand for "experience","computer knowledge, "higher education" and "good health", respectively.

***Step:1*** Let each decision maker construct neutrosophic soft sets over $X$ by own interview:

$$f_{d^1} = \left\{ \begin{array}{l} f_{d^1}(e_1) = \{\langle x_1, .4, .2, .7\rangle, \langle x_2, .5, .6, .2\rangle, \langle x_3, .7, .3, .3\rangle, \langle x_4, .6, .5, .4\rangle, \langle x_5, .3, .5, .5\rangle\}, \\ f_{d^1}(e_2) = \{\langle x_1, .3, .5, .2\rangle, \langle x_2, .4, .4, .3\rangle, \langle x_3, .5, .7, .8\rangle, \langle x_4, .7, .1, .3\rangle, \langle x_5, .6, .3, .2\rangle\}, \\ f_{d^1}(e_3) = \{\langle x_1, .7, .4, .3\rangle, \langle x_2, .6, .1, .5\rangle, \langle x_3, .5, .2, .4\rangle, \langle x_4, .2, .2, .6\rangle, \langle x_5, .3, .3, .6\rangle\}, \\ f_{d^1}(e_4) = \{\langle x_1, .7, .3, .5\rangle, \langle x_2, .3, .5, .3\rangle, \langle x_3, .2, .4, .3\rangle, \langle x_4, .4, .2, .5\rangle, \langle x_5, .5, .2, .6\rangle\} \end{array} \right\}$$

$$f_{d^2} = \left\{ \begin{array}{l} f_{d^2}(e_1) = \{\langle x_1, .5, .2, .3\rangle, \langle x_2, .3, .5, .6\rangle, \langle x_3, .4, .3, .3\rangle, \langle x_4, .2, .5, .4\rangle, \langle x_5, .5, .5, .5\rangle\}, \\ f_{d^2}(e_2) = \{\langle x_1, .5, .4, .6\rangle, \langle x_2, .7, .2, .5\rangle, \langle x_3, .6, .3, .5\rangle, \langle x_4, .7, .2, .3\rangle, \langle x_5, .6, .4, .2\rangle\}, \\ f_{d^2}(e_3) = \{\langle x_1, .6, .2, .5\rangle, \langle x_2, .4, .4, .6\rangle, \langle x_3, .2, .5, .4\rangle, \langle x_4, .3, .5, .4\rangle, \langle x_5, .3, .3, .6\rangle\}, \\ f_{d^2}(e_4) = \{\langle x_1, .3, .4, .5\rangle, \langle x_2, .4, .3, .2\rangle, \langle x_3, .4, .4, .3\rangle, \langle x_4, .4, .2, .5\rangle, \langle x_5, .2, .5, .6\rangle\} \end{array} \right\}$$

and

$$f_{d^3} = \left\{ \begin{array}{l} f_{d^3}(e_1) = \{\langle x_1, .4, .5, .7\rangle, \langle x_2, .5, .3, .4\rangle, \langle x_3, .7, .3, .5\rangle, \langle x_4, .4, .5, .3\rangle, \langle x_5, .7, .8, .6\rangle\}, \\ f_{d^3}(e_2) = \{\langle x_1, .6, .2, .6\rangle, \langle x_2, .4, .3, .5\rangle, \langle x_3, .5, .4, .7\rangle, \langle x_4, .3, .1, .5\rangle, \langle x_5, .4, .3, .1\rangle\}, \\ f_{d^3}(e_3) = \{\langle x_1, .4, .3, .2\rangle, \langle x_2, .6, .7, .2\rangle, \langle x_3, .3, .5, .2\rangle, \langle x_4, .6, .6, .4\rangle, \langle x_5, .6, .5, .5\rangle\}, \\ f_{d^3}(e_4) = \{\langle x_1, .5, .3, .1\rangle, \langle x_2, .2, .5, .2\rangle, \langle x_3, .5, .5, .4\rangle, \langle x_4, .5, .2, .5\rangle, \langle x_5, .5, .3, .6\rangle\} \end{array} \right\}$$

***Step 2:*** Relative parameter matrix of each decision maker are as in follow

$$[d^1_{pr}] = \begin{bmatrix} 1 & 3 & 1/5 & 2 \\ 1/3 & 1 & 3 & 6 \\ 5 & 1/3 & 1 & 1/5 \\ 1/2 & 1/6 & 5 & 1 \end{bmatrix} \quad [d^2_{pr}] = \begin{bmatrix} 1 & 5 & 1/7 & 2 \\ 1/5 & 1 & 1/2 & 6 \\ 7 & 2 & 1 & 1/3 \\ 1/2 & 1/6 & 3 & 1 \end{bmatrix}$$

and

$$[d^3_{pr}] = \begin{bmatrix} 1 & 3 & 1/3 & 4 \\ 1/3 & 1 & 1/3 & 1/6 \\ 3 & 3 & 1 & 1/2 \\ 1/4 & 6 & 2 & 1 \end{bmatrix}$$

***Step 3:*** $[i_{pr}]$ can be obtained as follow

$$[i_{pr}] = \begin{bmatrix} 1 & 3.67 & .23 & 2.67 \\ .29 & 1 & 1.28 & 4.06 \\ 5 & 1.78 & 1 & .34 \\ .42 & 4.06 & 3.33 & 1 \end{bmatrix}$$

***Step 4:*** $[\hat{i}_{pr}]$ and weight of each parameter can be obtained as follow

$$[\hat{i}_{pr}] = \begin{bmatrix} .13 & .49 & .03 & .35 \\ .04 & .15 & .19 & .61 \\ .62 & .22 & .12 & .04 \\ .05 & .46 & .38 & .11 \end{bmatrix}$$

and $w(e_1) = .21$, $w(e_2) = 0.33$, $w(e_3) = .18$ $w(e_4) = .28$.

***Step 5:*** Intersection of neutrosophic soft sets $f_{d^1}$, $f_{d^2}$ and $f_{d^3}$ is as follow;

$$I_{f_d} = \left\{ \begin{array}{l} I_{f_d}(e_1) = \{\langle x_1, .4, .5, .7\rangle, \langle x_2, .3, .6, .6\rangle, \langle x_3, .4, .3, .5\rangle, \langle x_4, .2, .5, .5\rangle, \langle x_5, .3, .8, .6\rangle\}, \\ I_{f_d}(e_2) = \{\langle x_1, .3, .5, .6\rangle, \langle x_2, .4, .4, .5\rangle, \langle x_3, .5, .7, .8\rangle, \langle x_4, .3, .2, .5\rangle, \langle x_5, .4, .4, .2\rangle\}, \\ I_{f_d}(e_3) = \{\langle x_1, .6, .5, .5\rangle, \langle x_2, .4, .7, .6\rangle, \langle x_3, .2, .5, .4\rangle, \langle x_4, .2, .6, .6\rangle, \langle x_5, .3, .5, .6\rangle\}, \\ I_{f_d}(e_4) = \{\langle x_1, .3, .4, .5\rangle, \langle x_2, .2, .5, .3\rangle, \langle x_3, .2, .5, .4\rangle, \langle x_4, .4, .2, .5\rangle, \langle x_5, .2, .5, .6\rangle\} \end{array} \right\}$$

***Step 6:*** For each parameter, compare matrices of elements of $X$ are obtained as in follow;

$$I_{f_d(e_1)} = \begin{bmatrix} .50 & .55 & .30 & .45 & .65 \\ .45 & .50 & .25 & .40 & .60 \\ .70 & .75 & .50 & .35 & .85 \\ .55 & .60 & .65 & .50 & .70 \\ .65 & .40 & .15 & .30 & .70 \end{bmatrix}, \quad I_{f_d(e_2)} = \begin{bmatrix} .50 & .35 & .60 & .30 & .20 \\ .65 & .50 & .75 & .35 & .35 \\ .40 & .25 & .50 & .20 & .10 \\ .70 & .65 & .80 & .50 & .40 \\ .80 & .65 & .90 & .60 & .50 \end{bmatrix}$$

and

$$I_{f_d(e_3)} = \begin{bmatrix} .50 & .75 & .65 & .80 & .70 \\ .25 & .50 & .40 & .55 & .45 \\ .35 & .60 & .50 & .65 & .55 \\ .20 & .45 & .35 & .50 & .45 \\ .30 & .55 & .45 & .55 & .50 \end{bmatrix}, \quad I_{f_d(e_4)} = \begin{bmatrix} .50 & .50 & .55 & .35 & .65 \\ .50 & .50 & .45 & .40 & .65 \\ .45 & .55 & .50 & .30 & .60 \\ .65 & .60 & .70 & .50 & .80 \\ .35 & .35 & .40 & .20 & .50 \end{bmatrix}$$

***Step 7:*** Membership degrees of elements of $X$ related to each parameter $e \in E$ are obtained as follow;

$W_{f_d(e_1)}(x_1) = .57$, $W_{f_d(e_1)}(x_2) = .56$, $W_{f_d(e_1)}(x_3) = .37$, $W_{f_d(e_1)}(x_4) = .40$ and $W_{f_d(e_1)}(x_5) = .66$

$W_{f_d(e_2)}(x_1) = .61$, $W_{f_d(e_2)}(x_2) = .48$, $W_{f_d(e_2)}(x_3) = .71$, $W_{f_d(e_2)}(x_4) = .39$ and $W_{f_d(e_2)}(x_5) = .31$

$W_{f_d(e_3)}(x_1) = .32$, $W_{f_d(e_3)}(x_2) = .57$, $W_{f_d(e_3)}(x_3) = .47$, $W_{f_d(e_3)}(x_4) = .61$ and $W_{f_d(e_3)}(x_5) = .53$

$W_{f_d(e_4)}(x_1) = .49$, $W_{f_d(e_4)}(x_2) = .50$, $W_{f_d(e_4)}(x_3) = .52$,$W_{f_d(e_4)}(x_4) = .35$ and $W_{f_d(e_4)}(x_5) = .64$

***Step 8:***

$$\begin{aligned} F(x_1) &= \frac{1}{|E|}\sum_{j=1}^{n} w(e_j)\cdot W_{f_d(e_j)}(x_1) \\ &= \frac{1}{4}(.21\cdot .57 + .33\cdot .61 + .18\cdot .32 + .28\cdot .49) \\ &= .126 \end{aligned}$$

similarly $F(x_2) = .130$, $F(x_3) = .136$, $F(x_4) = .105$ and $F(x_5) = .129$. Then, we get

$$D_E = \{(x_1, .126), (x_1, .130), (x_1, .136), (x_1, .105), (x_1, .129)\}$$

***Step 9:*** Note that, membership degree of $x_3$ is greater than membership degrees of the others. Therefore, optimal decision is $x_3$ for this decision making problem.

# 6 Conclusion

In this paper, we firstly investigate neutrosophic soft sets given paper of Maji [11] and then we redefine notion of neutrosophic soft set and neutrosophic soft set operations. Finally, we present two applications of neutrosophic soft sets in decision making problem.

# References

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986) 87-96.

[2] S. Broumi, Generalized Neutrosophic Soft Set International Journal of Computer Science, Engineering and Information Technology, 3/2 (2013) 17–30.

[3] S. Broumi, F. Smarandache, Intuitionistic Neutrosophic Soft Set, Journal of Information and Computing Science, 8/2 (2013) 130–140.

[4] N. Çağman, Contributions to the theory of soft sets, Journal of New Results in Science, 4 (2014), 33-41.

[5] N. Çağman and S. Enginoğlu, Soft set theory and uni-int decision making, Eur. J. Oper. Res. 207 (2010) 848-855.

[6] N. Çağman, Contributions to the Theory of Soft Sets, Journal of New Result in Science, 4 (2014) 33-41.

[7] . Deli,Interval-valued neutrosophic soft sets ant its decision making, arXiv:1402.3130

[8] . Deli, S. Broumi, Neutrosophic soft sets and neutrosophic soft matrices based on decision making, arXiv:1402.0673

[9] D. Molodtsov, Soft set theory first results, Computers and Mathematics with Applications, 37 (1999) 19-31.

[10] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555562.

[11] P.K. Maji, Neutrosophic soft set, Annals of Fuzzy Mathematics and Informatics, 5/ 1 (2013) 157-168.

[12] M.I. Ali, F. Feng, X. Liu, W.K. Min, On some new operations in soft set theory, Comput. Math. Appl. 57 (9) (2009) 15471553.

[13] P.K. Maji, Neutrosophis soft sets, Annals of Fuzzy Mathematics and Informatics, 5 (2013) 157-168.

[14] Saaty T L (1980) The Analytic Hierarchy Process, McGraw Hill International.

[15] F. Smarandache, Neutrosophic set - a generalization of the intuitionistic fuzzy set, International Journal of Pure and Applied Mathematics, 24/3 (2005) 287–297.

[16] Z. Pawlak, Rough sets, Int. J. Inform. Comput. Sci. 11 (1982) 341356.

[17] C.F. Yang, A note on Soft Set Theory [Comput. Math. Appl. 45 (45) (2003) 555562], Comput. Math. Appl., 56 (2008) 1899-1900.

[18] L.A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.