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# ALGEBRAIC STRUCTURES ON FUZZY UNIT SQUARE AND NEUTROSOPHIC UNIT SQAURE

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# PREFACE

In this book authors build algebraic structures on fuzzy unit semi open square $U_F = \{(a, b) \mid a, b \in [0, 1)\}$ and on the fuzzy neutrosophic unit semi open square $U_N = \{a + bI \mid a, b \in [0, 1)\}$.

This study is new and we define, develop and describe several interesting and innovative theories about them. We cannot build ring on $U_N$ or $U_F$. We have only pseudo rings of infinite order.

We also build pseudo semirings using these semi open unit squares. We construct vector spaces, S-vector spaces and strong pseudo special vector space using $U_F$ and $U_N$. As distributive laws are not true we are not in a position to develop several properties of rings, semirings and linear algebras. Several open conjectures are proposed.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

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**Chapter One**

# ALGEBRAIC STRUCTURES ON THE FUZZY UNIT SQUARE $U_F = \{(a, b) \mid a, b \in [0, 1)\}$

In this chapter we for the first time study the algebraic structures related with the fuzzy unit square.
$U_F = \{(a, b) \mid a, b \in [0, 1)\}$ is defined as the fuzzy unit semi open square.

$U_N = \{a + bI \mid a, b \in [0, 1); I^2 = I$; I the indeterminate$\}$ is defined as the fuzzy neutrosophic unit square. This chapter is devoted to the study of algebraic structures only using the fuzzy unit semi open square.

Throughout this chapter $U_F = \{(a, b) \mid a, b \in [0, 1)\}$ is defined as the half open fuzzy unit square or semi open fuzzy unit square. For $(1, 1) \notin U_F$ also $(a, 1)$ for $a \in [0, 1)$ and $(1, b)$ for $b \in [0, 1)$ does not belong to $U_F$. That is $(a, 1)$ and $(1, b) \notin U_F$. We build algebraic structures on $U_F$.

**DEFINITION 1.1:** *Let $U_F = \{(a, b) \mid a, b \in [0, 1)\}$ be the fuzzy unit semi open square. Define $\times$ on $U_F$ as follows for $(a, b)$ and $(c, d) \in U_F$, $(a, b) \times (c, d) = (ac, bd) \in U_F$; $\{U_F, \times\}$ is a semigroup called as the unit fuzzy semi open square semigroup.*

Clearly $o(U_F) = \infty$. Further $U_F$ is a commutative semigroup. $U_F$ has infinite number of zero divisors. $U_F$ can never be made into a monoid by adjoining (1, 1) to $U_F$ for by very operation $(1, 1) \notin U_F$. $U_F$ has subsemigroups and $U_F$ has ideals.

We will illustrate how operations are performed on the fuzzy unit semi open square $U_F$.

Let x = (0.3, 0.7) and y = (0.115, 0.871) $\in U_F$.
$x \times y = (0.3, 0.7) \times (0.115, 0.871) = (0.0345, 0.6097) \in U_F$.

Let x = (0.7785, 0) and y = (0, 0.113) $\in U_F$
we see $x \times y = (0, 0)$.

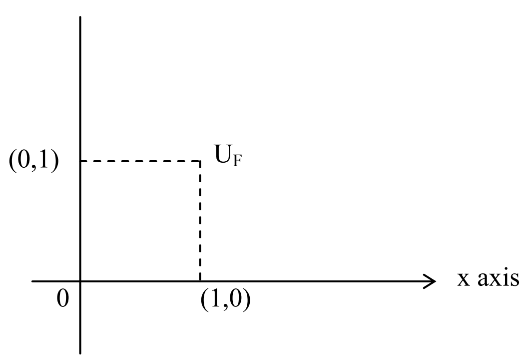
Thus it is easily verified $U_F$ has infinite number of zero divisors.

Let $I = \{(0, x) \mid x \in [0, 1)\} \subseteq U_F$ ; clearly I is an ideal of $U_F$. Consider $J = \{(y, 0) \mid y \in [0, 1)\} \subseteq U_F$ ; we see J is an ideal of $U_F$.

$$J.I = I.J = \{(0, 0)\}.$$

We call I and J as a annihilating pair of ideals.

Let $P = \{(x, y) \mid x, y \in [0, 0.5)\} \subseteq U_F$ ; P is a subsemigroup of $U_F$ and is not an ideal of $U_F$. Infact $U_F$ has infinite number of subsemigroups which are not ideals of $U_F$.



Dotted lines show they do not belong to $U_F$.

Further [1, a) and (b, 1] $\notin U_F$ for all a, b $\in$ [0, 1). Also the element (1, 1) $\notin U_F$. [1, 0) $\notin U_F$. (0, 1] $\notin U_F$. $U_F$ is defined as the half open fuzzy unit square (or semi open unit square).

We can using $U_F$ build more algebraic structures.

Let $U_F$ be the half open fuzzy unit square. We define on $U_F$ the operation max so that $\{U_F, \max\}$ is a semigroup infact a semilattice.

We see $\{U_F, \max\}$ is a semigroup of infinite order. Every element is an idempotent. Infact every singleton element is a subsemigroup of $\{U_F, \max\}$.

We see every pair of elements in $U_F$ need not be a subsemigroup.

Take x = (0.3, 0.71) and y = (0.8, 0.2107) $\in U_F$.

$$\begin{aligned}\max\{x, y\} &= \max\{(0.3, 0.71), (0.8, 0.2107)\}\\ &= (\max\{0.3, 0.8\}, \max\{0.71, 0.2107\})\\ &= (0.8, 0.71) \neq x \text{ or } y.\end{aligned}$$

Thus every pair in $U_F$ need not be a subsemigroup under the max operation. However P = {x, y, (0.8, 0.71)} $\subseteq U_F$ is a subsemigroup of $U_F$ under max operation.

Let M = {x = (0.9, 0.3), y = (0.7, 0.4), z = (0.69, 0.59), u = (0.8, 0.7)} $\subseteq U_F$.

We see M is not a subsemigroup of $U_F$. M is only a subset of $U_F$.

Now max {x, y} = max {(0.9, 0.3), (0.7, 0.4)} = {(0.9, 0.4)} $\notin$ M.

max {x, z} = max {(0.9, 0.3), (0.69, 0.59)} = {0.9, 0.59)} $\notin$ M.
max {x, u} = max {(0.9, 0.3), (0.8, 0.7)} = {(0.9, 0.7)} $\notin$ M.

Now max {y, z} = max {(0.7, 0.4), (0.69, 0.59)} = {(0.7, 0.59)} $\notin$ M.

max {y, u} = max {(0.7, 0.4), (0.8, 0.7)}
= {(0.8, 0.7)} $\in$ M
max {z, u} = max {(0.69, 0.59), (0.8, 0.7)}
= {(0.8, 0.7)} $\in$ M

Thus
$M_c$ = {x, y, z, u, (0.7, 0.59), (0.9, 0.7), (0.9, 0.59), (0.9, 0.4)} is a subsemigroup defined as the completed subsemigroup of the set M $\subseteq$ $U_F$. Any subset can be completed to form a subsemigroup under max operation.

Every subset of $U_F$ can be completed to form a subsemigroup and however all these completed subsemigroups cannot be ideals of $U_F$.

We will now proceed onto give other algebraic structures using $U_F$.

Now let $U_F$ = {(a, b) | a, b $\in$ [0, 1)} be the fuzzy unit square set.

Define on $U_F$ the min operation on it {$U_F$, min} is the semigroup under min operation.

Let x = (0.7, 0.2) and y = (0.5, 0.8) $\in$ $U_F$ .
min {x, y} = min {(0.7, 0.2), (0.5, 0.8)}
= (min {0.7, 0.5), min {0.2, 0.8})
= (0.5, 0.2) $\in$ $U_F$ .

This is the way min operation is performed on $U_F$.

We see if x = (0, 0.9) and y = (0.8, 0) $\in$ $U_F$.
Now min {x, y} = min {(0. 0.9), (0.8, 0)}
= (min {0, 0.8}, min {0.9, 0})
= (0,0)$\in$ $U_F$.

Thus we see $\{U_F , \min\}$ has zero divisors under min operation. However $\{U_F , \max\}$ under max operation the fuzzy set semigroup has no zero divisors. Only $\{U_F , \min\}$ be the fuzzy set semigroup under min operation. We see $\{U_F , \min\}$ has zero divisors.

Infact $U_F$ has infinite number of zero divisors. However all the zero divisors are of the form x = (0, a) and y = (b, 0);

min (x, y) = (0, 0) ∈ [0, 1)) be in $U_F$ .

min {x, y} = (0, 0).

Let P = {x, y} where x = (0.3, 0.9) and y = (0.6, 0.21) ∈ P

min {x, y} = min {(0.3, 0.9), (0.6, 0.21)}

= (min {0.3, 0.6} min {0.9, 0.21})

= (0.3, 0.21) ∉ P.

$P_c$ = {x = (0.3, 0.9), y = (0.6, 0.21), min {x, y} = (0.3, 0.21)} ⊆ $U_F$

$P_c$ is the extended subsemigroup of the subset P.

Now if P is any set of cardinality two then the completion of P, $P_c$ is a subsemigroup of order three.

Let

P = {x = (0.2, 0.94), y = (0.5, 0.26) and z = (0.3, 0.9)} ⊆ $U_F$.

Clearly P under min operation in $U_F$ is not a subsemigroup only a subset.

min {x, y} = min {(0.2, 0.94), (0.5, 0.26)}

= (min {0.2, 0.5}, min {0.94, 0.26})

= (0.2, 0.26) ∉ P.

min {x, z} = min {(0.2, 0.94), (0.3, 0.9)}

= (min {0.2, 0.3}, min {0.94, 0.9})

= (0.2, 0.9) ∉ P

min {y, z} = min {(0.5, 0.26), (0.3, 0.9)}
= (min {0.5, 0.3}, min {0.26, 0.9})
= (0.3, 0.26) $\notin$ P.

Now $P_c$ = {x = (0.2, 0.94), y = (0.5, 0.26), z = (0.3, 0.9), (2, 0.26), (0.2, 0.9), (0.3, 0.26)} $\subseteq U_F$ .

$P_c$ is a completed subsemigroup of the subset P of $U_F$ .

This is the way completion of subset in $U_F$ is performed in order to get a subsemigroup of $U_F$.

We can find ideals in {$U_F$, min}.

Let R = {[0, 0.3), min} $\subseteq U_F$.
R is a subsemigroup as well as an ideal of $U_F$.
Let M = {(0.7, 1), min} $\subseteq U_F$.

We see M is only a subsemigroup under min operation and M is not an ideal for if

x = (0.3, 0.8) $\in U_F$ and y = (0.5, 0.9) $\in$ M we see
min {x, y} = min {(0.3, 0.8), (0.5, 0.9)}
= (min {0.3, 0.5}, min {0.8, 0.9})
= (0.3, 0.8) $\notin$ M.

So M is only a subsemigroup under min and is not an ideal of M.

We can have infinite number of subsemigroups which are not ideals; similarly we see $U_F$ under min operation can have infinite number of subsemigroups which are ideals.

We will describe this by the following theorems.

**THEOREM 1.1:** *Let {$U_F$ , min} be a semigroup. P = {[a, 1), min where 0 < a} $\subseteq U_F$ ; P is a subsemigroup of $U_F$. P is not an ideal of $U_F$.*

Proof is direct so it is left as an exercise to the reader. Now in case of $\{U_F, \max\}$ the semigroup we have subsemigroups which are not ideals. This is described by the following theorem.

**THEOREM 1.2:** *Let $\{U_F, \max\}$ be the fuzzy unit semi open square semigroup under max operation.*
*$M = \{(x, y) \in [0, a) \mid a < b < 1\ b \neq a\} \subseteq U_F$ is only a subsemigroup under max operation and is not an ideal of $\{U_F, \max\}$.*

Proof is direct and hence left as an exercise to the reader.

**THEOREM 1.3:** *Let $\{U_F, \times\}$ be fuzzy unit square the semigroup under product $\times$. $A = \{(x, y) \in [a, 1);\ 0 < a, \times\} \subseteq \{U_F, \times\}$ is not a subsemigroup of $\{U_F, \times\}$.*

Proof follows from the simple fact that even $a \times a \notin A$.

Hence the claim.

Let $A = \{(x, y) \mid x, y \in [0.5, 1), \times\} \subseteq \{U_F, \times\}$ we see $(0.5, 0.6) \times (0.5, 0.6) = (0.25, 0.36) \notin A$.

We now describe other algebraic structures by some examples.

***Example 1.1:*** Let $M = \{(a_1, a_2, a_3) \mid a_i \in U_F;\ 1 \leq i \leq 3, \times\}$ be the unit fuzzy semi open square semigroup under $\times$. M has subsemigroups of infinite order.

Let $x = ((0.3, 0.2), (0.9, 0.4), (0.7, 0.8))$ and
$y = ((0.4, 0.7), (0.3, 0.2), (0.6, 0.5)) \in M$

$$x \times y = ((0.3, 0.2), (0.9, 0.4), (0.7, 0.8)) \times ((0.4, 0.7), (0.3, 0.2), (0.6, 0.5))$$

$$= ((0.3, 0.2) \times (0.4, 0.7), (0.9, 0.4) \times (0.3, 0.2), (0.7, 0.8) \times (0.6, 0.5))$$

$$= ((0.12, 0.14), (0.27, 0.08), (0.42, 0.40)) \in M.$$

This is the way operation on M is defined.
M has zero divisors.

For take x = ((0, 0.2), (0.7, 0) (0.5, 0.2)) and
y = ((0.7, 0), (0, 0.9), (0, 0)) ∈ M.

x × y = ((0, 0.2), (0.7, 0), (0.5, 0.2)) ×((0.7, 0), (0, 0.9), (0, 0))
= ((0, 0.2), (0.7, 0), (0.7, 0), (0, 0.9), (0.5, 0.2), (0, 0))
= ((0, 0), (0, 0), (0, 0)) ∈ M.

Thus M has infinite number of zero divisors.

Let P = {((a, b), (0, 0), (0, 0)) | a, b ∈ [0, 1), ×} ⊆ M; P is a subsemigroup and P is an ideal of M.

Let T = {((0, 0), (a, b), (c, d)) | a, b, c, d ∈ [0, 1), ×} ⊆ M; T is a subsemigroup and also an ideal; M is infinite order.

Let P = {((a, b), (c, d), (e, f)) | a, b, c, d, e, f ∈ [0, 0.8), ×} ⊆ M; P is a subsemigroup of M as well as an ideal of M.

Thus this M has subsemigroups as well as ideals of infinite order.

***Example 1.2:*** Let

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_9 \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \times_n\} \right.$$

be the semigroup under the natural product $\times_n$.

T has infinite number of zero divisors. T has infinite order subsemigroups and ideals.

***Example 1.3:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \end{bmatrix} \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1)\};$$

$$1 \leq i \leq 20, \times_n\}$$

be the fuzzy unit semi open square matrix semigroup. M is of infinite order.

M has several zero divisors. M has subsemigroups and ideals of infinite order.

***Example 1.4:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \times_n\}$$

be the fuzzy unit semi open square matrix semigroup under product $\times_n$.

P has infinite number of zero divisors. P has subsemigroups and ideals of infinite order.

***Example 1.5:*** Let $M = \{(a_1 \mid a_2\ a_3\ a_4\ a_5 \mid a_6\ a_7\ a_8 \mid a_9\ a_{10} \mid a_{11}) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \times, 1 \leq i \leq 11\}$ be the fuzzy unit semi open square row supermatrix semigroup under product. M has infinite number of zero divisors.

***Example 1.6:*** Let

$$T = \left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \\ a_{13} \\ \hline a_{14} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \times_n, 1 \leq i \leq 14\} \right.$$

be the fuzzy unit semi open square super column matrix semigroup of infinite order. M has ideals and subsemigroups of infinite order.

***Example 1.7:*** Let

$$T = \left\{ \left[ \begin{array}{cc|cc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \hline a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ \hline a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \right.$$

$$\times_n, 1 \leq i \leq 28\}$$

be the fuzzy unit semi open square super matrix semigroup under natural product $\times_n$. T has infinite number of zero divisors.

***Example 1.8:*** Let P = $\{(a_1, a_2, a_3, a_4) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 4, \min\}$ be the fuzzy unit square row matrix semigroup under the min operation.

Every singleton set is a subsemigroup under min operation. Every element x in P is an idempotent. P is a semilattice.

P has zero divisors and they are infinite in number and P has subsemigroups which are not ideals.

Every ideal in P is of infinite order.

We have subsemigroups of order one, two, three and so on. P has subsemigroups of infinite order also which are not ideals.

Take A = $\{(a_1, a_2, a_3, a_4) \mid a_i = (c_i, d_i); c_i, d_i \in [0.3, 1), \min, 1 \leq i \leq 4\} \subseteq P$ to be a subsemigroup of infinite order.

It is easily verified A is not an ideal only a subsemigroup.

B = $\{(a_1, a_2, 0, 0) \mid a_1 = (c, d)\ a_2 = (b, e)$ where $b, c, d, e \in [0, 0.4), \min\} \subseteq P$ is a subsemigroup of infinite order, B is also an ideal of infinite order.

Thus P has infinite number of ideals all of which are of infinite order. P also has subsemigroups of infinite order which are not ideals.

P has subsemigroups of finite order which are not ideals.

T = {(0.3, 0.2), (0, 0.7), (0.9, 0.2), (0.7, (0.111)} $\subseteq$ P is a subsemigroup of order four and is not an ideal of P.

***Example 1.9*** Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \min, 1 \leq i \leq 8\} \right.$$

be the fuzzy unit square column matrix semigroup under min operation.

S has infinite number of zero divisors and every element in S is an idempotent. S has subsemigroups which are not ideals.

Take

$$P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_8 \end{bmatrix} \middle| \; a_i \in [0.7, 1), 1 \leq i \leq 8, \min \right\} \subseteq S,$$

P is only a subsemigroup and not an ideal of S.

$$B = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_8 \end{bmatrix} \middle| \; a_i \in [0, 0.5), 1 \leq i \leq 8, \min \right\} \subseteq S$$

is a subsemigroup which is an ideal of S.

Infact S has infinite number of ideals and infinite number of subsemigroups which are not ideals.

***Example 1.10:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \right.$$

$$\min, 1 \leq i \leq 16\}$$

be the special fuzzy unit square matrix semigroup of infinite order. M has infinite number of zero divisors. Every element in M is an idempotent.

M has ideals and M has subsemigroups which are not ideals. M has both finite and infinite ordered subsemigroups. All ideals of M are of infinite order.

***Example 1.11:*** Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in \right.$$

$$[0, 1)\}, \min, 1 \leq i \leq 65\}$$

be the fuzzy unit square semigroup of infinite order. S has infinite number of zero divisors. Every element in S is an idempotent.

S has infinite number of ideals all of which are of infinite order.

S has infinite number of subsemigroups which are not ideals some are of finite order and some of them are of infinite order.

***Example 1.12:*** Let V = {($a_1$ | $a_2$ | $a_3$ $a_4$ $a_5$ $a_6$ | $a_7$ $a_8$ $a_9$ | $a_{10}$ $a_{11}$ | $a_{12}$) | $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1)}, min, 1 ≤ i ≤ 12} be the semigroup of super row matrices built using the fuzzy unit half open square $U_F$.

V too has infinite number of zero divisors. Every element in V is an idempotent.

V has ideals all of which are of infinite order.

V also has subsemigroups of finite order which are infinite in number.

***Example 1.13:*** Let

$$T = \left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 10, \min\} \right.$$

be the fuzzy unit half open square super column matrix semigroup of infinite order.

T has infinite number of zero divisors.

***Example 1.14:*** Let

$$P = \left\{ \left[ \begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$$

$$1 \leq i \leq 36, \min\}$$

be the special fuzzy unit semi open square super column matrix semigroup of infinite order. P has subsemigroup and ideals.

***Example 1.15:*** Let

$$N = \left\{ \left( \begin{array}{c|cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \end{array} \right) \right| a_i \in U_F = \{(a, b) \mid$$

$$a, b \in [0, 1), 1 \leq i \leq 14, \min\}$$

be the special fuzzy unit semi open super row matrix semigroup.

N enjoys all properties as that of any row matrix built using $U_F$.

***Example 1.16:*** Let

$$S = \left\{ \left[ \begin{array}{c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 45, \min\} \right.$$

be the special fuzzy unit semi open super matrix semigroup of infinite order. Almost all properties mentioned for matrices hold good for this S.

We now give some theorems which will describe the properties of a matrix semigroup built using fuzzy unit semi open square.

**THEOREM 1.4:** *Let M = {m × n matrices with entries from $U_F$ = {(a, b) | a, b ∈ [0, 1)}, min} be the special fuzzy semi open unit matrix semigroup.*

*(i) o (M) = ∞.*
*(ii) M has infinite number of zero divisors.*
*(iii) M has infinite number of idempotents.*
*(iv) All singleton sets are subsemigroups of M.*
*(v) All ideals in M are of infinite order.*
*(vi) All subsemigroups of finite order are not ideals.*
*(vii) All subsemigroups built using*
*$A = \{(a, b) \mid a, b \in [0, a); a < b < 1\} \subseteq U_F$ are ideals of M.*

(viii) *All subsemigroups P built using elements from $B = \{(a, b) \mid a, b \in [a, 1), 0 < b < a\} \subseteq U_F$ are never ideals of M*

(ix) *All subsets in M can be completed to form subsemigroups which in general are not ideals of M.*

Proof follows from simple deductions and hence left as an exercise to the reader.

We can define additive group using the unit fuzzy square $U_F$.

$U_F = \{(a, b) \mid a, b \in [0, 1)\}$. Define '+' on $U_F$ modulo 1 as $(a, b) + (d, c) = (a + d, b + c)$ for $a, b, c, d \in U_F$.

(0, 0) acts as the additive identity of $U_F$.

Let $x = (0.06, 0.74)$ and $y = (0.9, 0.1) \in U_F$;
$x + y = (0.06, 0.74) + (0.9, 0.1)$
$= (0.96, 0.84) \in U_F$.

Let $x = (0.7, 0.81)$ then we have unique $y = (0.3, 0.19) \in U_F$ such that

$$x + y = (0.7, 0.81) + (0.3, 0.19)$$
$$= (0, 0) \in U_F.$$

We define $(U_F, +)$ to be the special fuzzy unit semi open square group.

Having seen such group we now proceed onto use $(U_F, +)$ to construct such groups which is illustrated by examples.

***Example 1.17:*** Let $W = \{(a_1, a_2, a_3, a_4, a_5, a_6) \mid a_i \in \{U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 6, +\}$ be a special fuzzy unit semi open square row matrix group.

W has infinite order subgroups and W is abelian.

***Example 1.18:*** Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \le i \le 12, +\} \right.$$

be the group of column matrix of the fuzzy unit semi open square built using $U_F$.

M has several subgroups of infinite order.

***Example 1.19:*** Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \\ \vdots & \vdots & & \vdots \\ a_{91} & a_{92} & \dots & a_{100} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \right.$$

$$1 \le i \le 100, +\}$$

be the fuzzy unit semi open unique square additive group of infinite order. W has several subgroups.

Take

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \\ \vdots & \vdots & & \vdots \\ a_{91} & a_{92} & \dots & a_{100} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in \right.$$

$$\{0, 0.5\}\}), 1 \le i \le 100, +\} \subseteq W$$

to be the subgroup of W. W is of finite order.

We have infinite number of subgroups of finite order also.

***Example 1.20:*** Let S = $\{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6 \mid a_7) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, +, 1 \leq i \leq 7, +\}$ be the super row matrix group of fuzzy unit semi open square.

P has subgroups of finite and infinite order.

***Example 1.21:*** Let

$$B = \left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ \hline a_{12} \end{array} \right] \middle|\ a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), +, 1 \leq i \leq 12\} \right.$$

be the special semi open unit fuzzy square super column matrix group.

B has subgroups of finite order. B has subgroups of infinite order.

***Example 1.22:*** Let

$$M = \left\{ \left[\begin{array}{c|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ \hline a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & a_{30} \\ \hline a_{31} & \cdots & \cdots & \cdots & \cdots & a_{36} \\ a_{37} & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & a_{48} \end{array}\right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), +, 1 \leq i \leq 48\}$$

be the special semi open unit square super matrix group under +.

***Example 1.23:*** Let

$$M = \left\{ \left(\begin{array}{cc|c|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \end{array}\right) \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), +, 1 \leq i \leq 24\}$$

be the fuzzy semi open unit square super row matrix group under +.

We see M has both finite and infinite order subgroups.

***Example 1.24:*** Let

$$T = \left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ \hline a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$$

$$1 \leq i \leq 52, +\}$$

be the fuzzy unit semi open super column matrix square group of infinite order.

**THEOREM 1.5*:*** *Let M = {m × n matrix with entries from $U_F$} be the unit fuzzy semi open unit square group under +.*

- *(i) M has subgroups of finite order.*
- *(ii) M has subgroups of infinite order.*

Proof follows from simple calculation.

Now we proceed onto describe on $U_F$ an algebraic structure using two binary operations.

Let $U_F = \{(a, b) \mid a, b \in [0, 1), \min, \max\}$ be the special fuzzy unit square semiring of infinite order.

$U_F$ has zero divisors. For if x = (0, 0.7) and y = (0.9, 0) ∈ $U_F$ then min {x, y} = {(0, 0)}.

Infact $U_F$ has infinite number of zero divisors.

Every element under both max and min is an idempotent. We see all pairs of the form B = {(0, 0), (x, y) | x, y ∈ [0, 1)} is a subsemiring of $U_F$.

We can as in case of other algebraic structures complete any subset into a subsemiring.

Let M = {(0, 0), (0.3, 0.5), (0.7, 0.01)} ⊆ $U_F$. Clearly M is not a subsemiring.

$$\min \{(0.3, 0.5), (0.7, 0.01)\} = \{(0.03, 0.01)\}.$$

max {(0.3, 0.5), (0.7, 0.01)} = {(0.7, 0.5)}.
$M_c$ = {(0, 0), (0.3, 0.5), (0.7, 0.01), (0.3, 0.01), (0.7, 0.5)} ⊆ $U_F$ is the completed subsemiring of the set M.

In this way one can easily find the completion of a subset of $U_F$.

We can using {$U_F$, max, min} build several semirings which are illustrated by the following examples.

***Example 1.25:*** Let M = {($a_1$, $a_2$, $a_3$, $a_4$, $a_5$) | $a_i$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1)}, 1 ≤ i ≤ 5, min, max} be the special fuzzy square unit semi open row matrix semiring.

Let x = (0, 0, 0, $a_1$, $a_2$) and y = ($a_1$, $a_2$, 0, 0, 0) ∈ M,
min {x, y} = (0, 0, 0, 0, 0) is a zero divisor.

Let x = (0.2, 0.7, 0.1, 0.3, 0.01) and
y = (0.1, 0.8, 0.5, 0.2, 0.4) ∈ M;

min {x, y} = (0.1, 0.7, 0.1, 0.2, 0.01) and
max {x, y} = (0.2, 0.8, 0.5, 0.3, 0.4).

P = {(0, 0, 0, 0, 0), x, y, min {x, y}, max {x, y}} $\subseteq$ M is a subsemiring of M.

Let T = {0, 0.7, 0.4, 0.5, 0.2), (0.3, 0.9, 0.6, 0.8, 0.4), (0.2, 0.6, 0.7, 0.6, 0.3)}.

min {(0, 0.7, 0.4, 0.5, 0.2), (0.3, 0.9, 0.6, 0.8, 0.4)}

= (0, 0.7, 0.4, 0.5, 0.2) = $t_1$

min {(0, 0.7, 0.4, 0.5, 0.2), (0.2, 0.6, 0.7, 0.6, 0.3)}

= (0.2, 0.6, 0.4, 0.5, 0.2) = $t_2$

min {(0.3, 0.9, 0.6, 0.8, 0.4), (0.2, 0.6, 0.7, 0.6, 0.3)}

= (0.2, 0.6, 0.6, 0.6, 0.3) = $t_3$

max {(0, 0.7, 0.4, 0.5, 0.2), (0.3, 0.9, 0.6, 0.8, 0.4)}

= {(0.3, 0.9, 0.6, 0.8, 0.4)} = $t_4$

max {(0, 0.7, 0.4, 0.5, 0.2) (0.2, 0.6, 0.7, 0.6, 0.3)}

= (0.2, 0.7, 0.7, 0.6, 0.3) = $t_5$

max {(0.3, 0.9, 0.6, 0.8, 0.4), (0.2, 0.6, 0.7, 0.6, 0.3)}

= (0.3, 0.9, 0.7, 0.8, 0.4) = $t_6$

Hence $T_c$ = {(0, 0.7, 0.4, 0.5, 0.2), $t_1$, $t_2$, $t_3$, $t_4$, $t_5$, $t_6$, (0.2, 0.6, 0.7, 0.6, 0.3), (0.3, 0.9, 0.6, 0.8, 0.4)} $\subseteq$ M.

$T_c$ is the completed subsemiring of the set T of M.

***Example 1.26:*** Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 12, \max, \min\} \right.$$

be the special fuzzy unit semi open square semiring.

N has zero divisors, subsemirings of order two, order three and so on can be found.

We can also complete subsets to form a subsemiring. N has subsemirings which are not ideals. Also N has subsemirings which are ideals.

Take

$$A_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1)\} \right\} \subseteq N.$$

$A_1$ is a subsemiring and an ideal of $U_F$. However $A_1$ is not a filter of $U_F$.

Let

$$A_2 = \left\{ \begin{bmatrix} 0 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1)\} \right\} \subseteq N;$$

$A_2$ is a subsemiring and an ideal of $U_F$.

On similar lines we can have

$$A_{12} = \left\{ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_{12} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1)\} \right\} \subseteq N$$

is again a subsemiring which is an ideal of $U_F$ and not a filter of $U_F$.

In this way we can find ideals.

Now if we take

$$B = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{12} \end{bmatrix} \middle| \; a_i = (c_i, d_i) \text{ where } c_i, d_i \in [0.5, 1); 1 \leq i \leq 12 \right\}$$

$\subseteq N;$

then B is a subsemiring of N and is not an ideal of N. However B is a filter of N.

We have infinite number of filters in N. Also we have infinite number of finite subsemirings in N which are neither ideals nor filters of N.

***Example 1.27:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \le i \le 30, \text{max, min}\} \right.$$

be the special fuzzy unit square semiring.

This M also has infinite number of subsets which can be completed to form a subsemiring. M also has infinite order subsemirings which are ideals and not filters and has infinite order subsemirings which are filters and not ideals.

We also have zero divisors and all subsemirings which has zero entry in any of the places in the matrices can never be ideals.

***Example 1.28:*** Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{46} & a_{47} & a_{48} & a_{49} & a_{50} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \le i \le 50, \text{max, min}\} \right.$$

be the special fuzzy unit semi open square semiring. V has ideals of the form

$$M_1 = \left\{\left.\begin{bmatrix} a_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}\right| a_1 \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, \right.$$

$$\text{max, min}\} \subseteq V$$

is a subsemiring which is also an ideal.

$$M_2 = \left\{\left.\begin{bmatrix} 0 & a_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right| a_2 \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, \right.$$

$$\text{max, min}\} \subseteq V$$

is a subsemiring which is also an ideal.

$$M_6 = \left\{\left.\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ a_6 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\right| a_6 \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, \right.$$

$$\text{max, min}\} \subseteq V,$$

and so on;

$$M_{50} = \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & a_{50} \end{bmatrix} \right| a_{50} \in U_F = \{(a, b) \mid a, b \in [0, 1)\},$$

$$\text{max, min}\} \subseteq V$$

is again a subsemiring which are also ideals of V.

Take

$$B_{0.4} = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{46} & a_{47} & a_{48} & a_{49} & a_{50} \end{bmatrix} \right| a_i = (c_i, d_i) \text{ where } c_i, d_i$$

$$\in [0.4, 1);\ 1 \leq i \leq 50\} \subseteq V$$

and

$$B_{0.72} = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{46} & a_{47} & a_{48} & a_{49} & a_{50} \end{bmatrix} \right| a_i = (c_i, d_i) \text{ where } c_i, d_i \in$$

$$[0.72, 1);\ 1 \leq i \leq 50\} \subseteq V$$

are subsemirings which are also filters of V.

Clearly $B_{0.4}$ and $B_{0.72}$ are not ideals of V.

$$B_{0.113} = \left\{\left[\begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{46} & a_{47} & a_{48} & a_{49} & a_{50} \end{array}\right] \right| a_i = (c_i, d_i) \text{ where } c_i, d_i$$

$$\in [0.113, 1);\ 1 \leq i \leq 50\} \subseteq V$$

is again a subsemiring which is also a filter. However $B_{0.113}$ is not an ideal of V.

***Example 1.29:*** Let M = $\{(a_1\ a_2 \mid a_3\ a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9\ a_{10} \mid a_{11}\ a_{12}\ a_{13} \mid a_{14}) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 14, \max, \min\}$ be the special fuzzy unit semi open square super row matrix semiring. M is of infinite order.

$V_{0.3} = \{(a_1\ a_2 \mid a_3\ a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9\ a_{10} \mid a_{11}\ a_{12}\ a_{13} \mid a_{14}) \mid a_i = (c_i, d_i)$ where $c_i, d_i \in [0.3, 1);\ 1 \leq i \leq 14\} \subseteq M$ is subsemiring as well as a filter of M however $V_{0.3}$ is not a ideal of M.

$V_{0.42} = \{(a_1\ a_2 \mid a_3\ a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9\ a_{10} \mid a_{11}\ a_{12}\ a_{13} \mid a_{14}) \mid a_i = (c_i, d_i)$ where $c_i, d_i \in [0.42, 1);\ 1 \leq i \leq 14\} \subseteq M$ is a subsemiring which is not an ideal and but is a filter. Infact M has infinite number of filters which are not ideals.

Let $T_1 = \{(a_1\ 0 \mid 0\ 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0) \mid a_1 \in U_F = \{(a, b) \mid a, b \in [0, 1)\} \subseteq M$,

$T_2 = \{(0\ a_2 \mid 0\ 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0) \mid a_2 \in U_F = \{(a, b) \mid a, b \in [0, 1)\} \subseteq M$ and so on.

$T_{14} = \{(0\ 0 \mid 0\ 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid a_{14}) \mid a_1 \in U_F = \{(a, b) \mid a, b \in [0, 1)\} \subseteq M$ are all subsemirings which are not ideals of M. None of these ideals of filters of M.

Let A = $\{(0\ 0 \mid 0\ 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0), (a_1\ a_2 \mid a_3\ a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9\ a_{10} \mid a_{11}\ a_{12}\ a_{13} \mid a_{14}) \mid a_i \in U_F$ are fixed $1 \leq i \leq 14\} \subseteq M$. A is a subsemiring of order two which is not an ideal or filter of M.

***Example 1.30:*** Let

$$V = \left\{ \left( \begin{array}{c|cc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \end{array} \right) \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 18, \max, \min\} \right.$$

be the special fuzzy unit square matrix super row matrix semiring. N has subsemirings of finite order which are not ideals and which are not filters. N has infinite number of filters which are not ideals.

***Example 1.31:*** Let

$$V = \left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ \hline a_{45} & a_{46} & a_{47} & a_{48} \\ \hline a_{49} & a_{50} & a_{51} & a_{52} \\ a_{53} & a_{54} & a_{55} & a_{56} \\ a_{57} & a_{58} & a_{59} & a_{60} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 60, \max, \min\} \right.$$

be the special fuzzy unit semi open square semiring of infinite order.

W has infinite number of finite subsemirings, infinite number of subsemirings which are ideals and not filters and some infinite number of subsemirings which are filters are not ideals.

***Example 1.32:*** Let

$$V = \left\{ \left[ \begin{array}{cc|c|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ \hline a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \\ a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} & a_{53} & a_{54} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 54, \max, \min\}$$

be the special fuzzy unit semi open square semiring. V has infinite number of zero divisors. Every element is an idempotent with respect to max and min operation.

V has infinite number of finite subsemirings.

Now we give a theorem.

**THEOREM 1.6:** *Let S = {Collection of all m × n matrices with entries from* $U_F = \{(a, b) \mid a, b \in [0, 1)\}$*; min, max} be the semiring.*

*(i)* $o(S) = \infty$.
*(ii) V has infinite number of zero divisors.*
*(iii) S has infinite number of finite subsemirings.*

(iv) *S has infinite number of infinite order ideals which are not filters.*

(v) *S has infinite number of infinite order filters which are not ideals.*

(vi) *S has subsets P of finite or infinite order which can be completed to $P_c$ to get a subsemiring of finite or infinite order respectively.*

(vii) *Every x ∈ S is an idempotent with respect to min or max operation.*

The proof is direct, hence left as an exercise to the reader.

Now we proceed onto describe pseudo semirings built using the fuzzy unit square $U_F$.

Let S = {$U_F$ = {(a, b) | a, b ∈ [0, 1)}, min, ×} be the semiring we see the operation min is not distributive over product so only we define S to be a pseudo semiring of $U_F$.

We give the properties enjoyed by S.

Let x = (0.3, 0.2) and y = (0.5, 0.13) ∈ S

$$\min\{x, y\} = \min\{(0.3, 0.2), (0.5, 0.13)\} = (0.3, 0.13) \in S$$

$$x \times y = \{(0.3, 0.2) \times (0.5, 0.13)\} = (0.15, 0.026) \in S.$$

x × min {y, z} ≠ min {x × y, x × z} in general for x, y, z ∈ S.
Take x = (0.7, 0.2) , y = (0.5, 0.7) and z = (0.6, 0.5) ∈ S.

$$x \times \min\{y, z\} = (0.7, 0.2) \times \min\{(0.5, 0.7), (0.6, 0.5)\} = (0.7, 0.2) \times (0.5, 0.5) = (0.35, 0.10) \quad \ldots \quad \text{I}$$

$$\min\{x \times y, x \times z\} = \min\{(0.7, 0.2) \times (0.5, 0.7), (0.7, 0.2) \times (0.6, 0.5)\} = \min\{(0.35, 0.14), (0.42, 0.10)\} = (0.35, 0.10) \quad \ldots \quad \text{II}$$

For this pair distributive law is true.

Let x = (0.14, 0.3), y = (0.2, 0.15) and z = (0.21, 0.4) $\in$ S.

x $\times$ min {y, z} = (0.14, 0.3) $\times$ min {(0.2, 0.15), (0.21, 0.4)}
= (0.14, 0.3) $\times$ (0.2, 0.15)
= (0.028, 0.045) … I

min {xy, xz} = min {(0.14, 0.3) $\times$ (0.2, 0.15), (0.14, 0.3) $\times$ (0.21, 0.4)}
= min {(0.028, 0.045), (0.0294, 0.12)}
= (0.0294, 0.045) … II

Clearly I and II are distinct, hence we call S to be a pseudo semiring.

We see S has infinite number of zero divisors with respect to min and $\times$.

All x = (0.3, 0) and y = (0, 0.9) $\in$ S then min {x, y} = (0, 0) and x $\times$ y = (0.3, 0), (0, 0.9) = (0, 0).

Thus it is a zero divisor with $\times$ and min.

However we show that $\times$ and min are distinct.

For if x = (0.7, 0.92) and y = (0.3, 0.95) $\in$ S
x $\times$ y = (0.7, 0.92) $\times$ (0.3, 0.95)
= (0.14, 0.9740) … I

min {x, y} = min {(0.7, 0.92), (0.3, 0.95)}
= (0.3, 0.92) … II

I and II are distinct so the operations on S are distinct S = {$U_F$, $\times$, min} is a pseudo semiring of infinite order.

P = {(0, a) | a $\in$ [0, 1)} $\subseteq$ S is a pseudo subsemiring of S.

P is also an pseudo ideal of S but P is not a filter of S.

R = {(a, 0) | a ∈[0, 1)} ⊆ S is a pseudo subsemiring of S. R is also a pseudo ideal but R is not a filter of S.

Let M = {(a, b) | a, b ∈ [0, 1)} has infinite number of pseudo ideals and pseudo subsemirings each of infinite order.

It is left as an open conjecture whether S has finite pseudo subsemirings.

Now using this pseudo subsemiring M = {$U_F$, ×, min} we can build matrix pseudo semiring of the fuzzy unit square.

***Example 1.33:*** Let M = {($a_1$, $a_2$, …, $a_{12}$) | $a_i$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1)}; $1 \leq i \leq 12$, ×, min} be the special fuzzy unit square pseudo semiring of infinite order. M has pseudo subsemiring of finite and infinite order.

M has no filters however M has ideals.

$P_1$ = {($a_1$, 0, …, 0) | $a_1$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1)}, min, ×} ⊆ M be the pseudo subsemiring which is a pseudo ideal of M.

$P_2$ = {(0, $a_2$, 0, …, 0) | $a_2$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1)}, min, ×} ⊆ M be the pseudo subsemiring which is also a pseudo ideal of M and so on.

$P_{12}$ = {(0, 0, …, $a_{12}$) | $a_{12}$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1)}, min, ×} ⊆ M be the pseudo subsemiring which is also a pseudo ideal of M.

$P_{1,2}$ = {($a_1$, $a_2$, 0, …, 0) | $a_1$, $a_2$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1)}, min, ×} ⊆ M be the pseudo subsemiring which is also a pseudo ideal of M.

$P_{1,3}$ = {($a_1$, 0, $a_3$, 0, …, 0) | $a_1$, $a_3$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1)}, min, ×} be the pseudo subsemiring which is a pseudo ideal of M.

We have several such pseudo subsemirings which are not pseudo ideals.

Thus we have at least ${}_{12}C_1 + {}_{12}C_2 + {}_{12}C_3 + {}_{12}C_4 + {}_{12}C_5 + \ldots + {}_{12}C_{11}$ number of pseudo subsemirings which are also pseudo ideals.

We are not in a position to know whether we can have finite pseudo subsemirings or finite pseudo ideals.

Consider $M_{0.5} = \{(a_1, a_2, a_3, a_4, a_5, \ldots,. a_{12}) \mid a_i \in \{(a, b) \mid a, b \in [0, 0.5)\}, 1 \leq i \leq 12; \times, \min\}$ be the pseudo subsemiring which is both an pseudo ideal and pseudo filter.

It is only in the pseudo semirings we have got both to be a pseudo ideals and a pseudo filter.

Let $M_{0.3} = \{(a_1, a_2, \ldots., a_{15}) \mid a_i \in \{(a, b) \mid a, b \in [0, 0.3), 1 \leq i \leq 15; \min, \times\} \subseteq M$ be the pseudo subsemiring which is both a pseudo ideal and a pseudo filter. Thus M has infinite number of subsemirings which are ideals and filters of M.

***Example 1.34:*** Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{20} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 20, \times, \min\} \right.$$

be the pseudo semiring of infinite order.

N has infinite number subsemirings which are pseudo filters as well as pseudo ideals.

This N has infinite number of zero divisors and ideals.

***Example 1.35:*** Let

$$M = \left\{ \begin{pmatrix} a_1 & a_2 & ... & a_5 \\ a_6 & a_7 & ... & a_{10} \\ a_{11} & a_{12} & ... & a_{15} \end{pmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \leq i \leq 15, \times, \min\} \right.$$

be the special fuzzy unit square semiring of infinite order.

M has atleast ${}_{15}C_1 + {}_{15}C_2 + {}_{15}C_3 + \ldots + {}_{15}C_{14}$ number of pseudo ideals which are not pseudo ideals which are not pseudo filters.

Let

$$M_{0.2} = \left\{ \begin{pmatrix} a_1 & a_2 & ... & a_5 \\ a_6 & a_7 & ... & a_{10} \\ a_{11} & a_{12} & ... & a_{15} \end{pmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 0.2), \; 1 \leq i \leq 15\} \subseteq M \right.$$

be a subsemiring which is a pseudo filter of M.

Likewise we have infinite number of pseudo ideals and pseudo filters all of which are of infinite order.

***Example 1.36:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & ... & ... & ... & a_{10} \\ a_{11} & ... & ... & ... & a_{15} \\ a_{16} & ... & ... & ... & a_{20} \\ a_{21} & ... & ... & ... & a_{25} \\ a_{26} & ... & ... & ... & a_{30} \\ a_{31} & ... & ... & ... & a_{36} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \leq i \leq 36, \times, \min\} \right.$$

be the special fuzzy unit semi open square matrix pseudo semiring of infinite order.

P has pseudo subsemiring of infinite order. P has also infinite number of pseudo ideals which are also pseudo filters. This study is important.

***Example 1.37:*** Let R = $\{(a_1 \mid a_2 \mid a_3\ a_4\ a_5\ a_6 \mid a_7\ a_8\ a_9 \mid a_{10}\ a_{11} \mid a_{12}) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \le i \le 12, \times, \min\}$ be the special fuzzy unit semi open super row matrix pseudo semiring.

We see R has infinite number of pseudo subsemirings which are pseudo ideals and pseudo filters.

***Example 1.38*** Let

$$N = \left\{ \left[\begin{array}{c} a_1 \\ a_2 \\ \hline a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \end{array}\right] \middle|\ a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \le i \le 11, \times, \min\} \right\}$$

be the special fuzzy unit semi open square super column matrix pseudo semiring.

M has infinite number of zero divisors and idempotents.

***Example 1.39:*** Let

$$V = \left\{ \left[ \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \ldots & \ldots & \ldots & a_{10} \\ \hline a_{11} & \ldots & \ldots & \ldots & a_{15} \\ a_{16} & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & a_{25} \\ \hline a_{26} & \ldots & \ldots & \ldots & a_{30} \\ a_{31} & \ldots & \ldots & \ldots & a_{35} \\ a_{36} & \ldots & \ldots & \ldots & a_{40} \\ a_{41} & \ldots & \ldots & \ldots & a_{45} \\ \hline a_{46} & \ldots & \ldots & \ldots & a_{50} \\ a_{51} & \ldots & \ldots & \ldots & a_{55} \\ a_{56} & \ldots & \ldots & \ldots & a_{60} \\ a_{61} & \ldots & \ldots & \ldots & a_{65} \\ a_{66} & \ldots & \ldots & \ldots & a_{70} \\ \hline a_{71} & \ldots & \ldots & \ldots & a_{75} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$$

$$1 \leq i \leq 75, \times, \min\}$$

be the special fuzzy unit square super column matrix pseudo semiring.

M has infinite number of zero divisors.

***Example 1.40:*** Let

$$V = \left\{\left[\begin{array}{ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \cdots & \cdots & \cdots & a_{10} \\ a_{11} & \cdots & \cdots & \cdots & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & a_{20} \\ \hline a_{21} & \cdots & \cdots & \cdots & a_{25} \\ a_{26} & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & a_{40} \end{array}\right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \right.$$

$$1 \leq i \leq 40, \times, \min\}$$

be the special fuzzy unit semi open square super matrix pseudo semiring.

We see W has pseudo subsemirings which are pseudo filters and pseudo ideals.

Now we proceed onto describe fuzzy unit square pseudo rings with examples.

***Example 1.41:*** Let W = $\{U_F, +, \times\}$ be the fuzzy unit semi open square pseudo ring. We call W only as pseudo ring as + and × are distributive over each other.

Let x = (0.3, 0.7), y = (0.2, 0.1) and z = (0.8, 0.4) ∈ W.

$$\begin{aligned} \text{Consider } x \times (y + z) &= (0.3, 0.7) \times ((0.2, 0.1) + (0.8, 0.4)) \\ &= (0.3, 0.7) \times (0, 0.5) \\ &= (0, 0.35) \qquad \ldots \text{ I} \end{aligned}$$

$$\begin{aligned} x \times y + x \times z &= (0.3, 0.7) \times (0.2, 0.1) + (0.3, 0.7)\,(0.8, 0.4) \\ &= (0.06, 0.07) + (0.24, 0.28) \\ &= (0.3, 0.35) \qquad \ldots \text{ II} \end{aligned}$$

I and II are different hence we call W only a pseudo ring. Pseudo ring has zero divisors.

Let x = (0, 0.74) and y = (0.64, 0) $\in$ W

x $\times$ y = (0, 0) hence it is a zero divisor.

x + y = (0, 0.74) + (0.64, 0)
= (0.64, 0.74) $\in$ W.

One of the open problems is that can W have finite pseudo subrings and finite pseudo ideals?

***Example 1.42:*** Let
M = $\{(a_1, a_2, a_3) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 3, \times, +\}$ be the pseudo subring built on the fuzzy unit semi open square.

Let $P_1 = \{(a_1, 0, 0) \mid a_1 \in U_F = \{(a, b) \mid a, b \in [0, 1), \times, +\} \subseteq$ M, is a pseudo subring as well as pseudo ideal of M.

$P_2 = \{(0, a_2, 0) \mid a_2 \in U_F = \{(a, b) \mid a, b \in [0, 1), \times, +\} \subseteq$ M is a pseudo subring as well as pseudo ideal of M.

$P_3 = \{(0, 0, a_3) \mid a_3 \in U_F = \{(a, b) \mid a, b \in [0, 1), \times, +\} \subseteq$ M is the special fuzzy unit square pseudo subring which is also a pseudo ideal.

Can we have any other pseudo ideal?

$P_{1,2} = \{(a_1, a_2, 0) \mid a_1, a_2 \in U_F = \{(a, b) \mid a, b \in [0, 1), \times, +\} \subseteq$ M,

$P_{1,3} = \{(a_1, 0, a_3) \mid a_1, a_3 \in U_F = \{(a, b) \mid a, b \in [0, 1), \times, +\} \subseteq$ M,

$P_{23} = \{(0, a_2, a_3) \mid a_2, a_3 \in U_F = \{(a, b) \mid a, b \in [0, 1), \times, +\} \subseteq$ M are all pseudo subring which are also pseudo ideals of M.

***Example 1.43:*** Let

$$V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \le i \le 9, \times, +\} \right.$$

be the fuzzy unit semi open square pseudo ring of infinite order.

V has atleast ${}_9C_1 + {}_9C_2 + {}_9C_3 + {}_9C_4 + {}_9C_5 + {}_9C_6 + {}_9C_7 + {}_9C_8$ number of fuzzy unit square unit pseudo subrings which are ideals.

***Example 1.44:*** Let

$$V = \left\{ \begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \end{pmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \le i \le 20, \times, +\} \right.$$

be the fuzzy unit square pseudo ring of infinite order.

V has atleast ${}_{20}C_1 + {}_{20}C_2 + {}_{20}C_3 + \ldots + {}_{20}C_{19}$ number of pseudo subrings which are pseudo ideals of M.

M has infinite number of zero divisors.

***Example 1.45:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \\ a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} & a_{53} & a_{54} \end{bmatrix} \right| a_i \in U_F = \{(a, b) \mid$$

$$a, b \in [0, 1), 1 \leq i \leq 54, +, \times\}$$

be the fuzzy unit semi open square pseudo ring. M has atleast ${}_{54}C_1 + {}_{54}C_2 + \ldots + {}_{54}C_{53}$ number of pseudo subrings which are pseudo ideals of M. M has infinite number of zero divisors.

***Example 1.46:*** Let M = $\{(a_1\ a_2\ a_3 \mid a_4\ a_5 \mid a_6) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 6, +, \times\}$ be the fuzzy unit semi open square super row matrix pseudo ring of infinite order.

M has pseudo subrings and pseudo ideals. M also has infinite number of zero divisors.

***Example 1.47:*** Let

$$M = \left\{ \left( \begin{array}{c|cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \ldots & \ldots & \ldots & \ldots & \ldots & a_{14} \\ a_{15} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{21} \\ a_{22} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{28} \end{array} \right) \right| a_i \in U_F = \{(a, b) \mid a, b$$

$$\in [0, 1), 1 \leq i \leq 28, +, \times\}$$

be the fuzzy unit semi open square super row matrix pseudo ring of infinite order.

M has pseudo subrings and pseudo ideals of infinite order.

***Example 1.48:*** Let

$$P = \left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 12, +, \times\} \right\}$$

be the special fuzzy unit semi open super column matrix pseudo ring of infinite order.

P has atleast ${}_{12}C_1 + {}_{12}C_2 + \ldots + {}_{12}C_{11}$ number of pseudo subrings which are pseudo ideals of P.

***Example 1.49:*** Let

$$P = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ \hline a_3 & a_4 \\ a_5 & a_6 \\ \hline a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ \hline a_{13} & a_{14} \\ a_{15} & a_{16} \\ a_{17} & a_{18} \\ a_{19} & a_{20} \\ \hline a_{21} & a_{22} \\ \hline a_{23} & a_{24} \\ a_{25} & a_{26} \\ a_{27} & a_{28} \end{array}\right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 28, +, \times \right\}$$

be the fuzzy unit semi open square super column matrix pseudo ring of infinite order.

Thus M has atleast ${}_{28}C_1 + {}_{28}C_2 + \ldots + {}_{28}C_{27}$ number of pseudo subrings which are pseudo ideals of M.

***Example 1.50:*** Let

$$P = \left\{ \left[ \begin{array}{cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ \hline a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \\ a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} & a_{53} & a_{54} \\ \hline a_{55} & a_{56} & a_{57} & a_{58} & a_{59} & a_{60} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 60, +, \times\} \right.$$

be the special fuzzy unit semi open square super matrix pseudo ring of infinite order.

This P has atleast ${}_{60}C_1 + {}_{60}C_2 + \ldots + {}_{60}C_{59}$ number of distinct pseudo subrings which are pseudo ideals of P.

Now having seen some of the properties we give the following theorem.

**THEOREM 1.7:** *Let*
*S = {m × n matrices with entries from $U_F$, +, ×} be the special fuzzy unit square matrix pseudo ring.*

- *(i)* *$o(S) = \infty$.*
- *(ii)* *S has atleast ${}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{(m\times n-1)}$ number of pseudo subrings which are pseudo ideals.*
- *(iii)* *S has infinite number of zero divisors.*

The proof is direct and hence left as an exercise to the reader.

Now we proceed onto define pseudo linear algebras of fuzzy unit semi open square over the pseudo ring [0, 1) or over the pseudo ring $U_F = \{(a, b) \mid a, b \in [0, 1), +, \times\}$.

**DEFINITION 1.2:** *Let $V = \{U_F, +\}$ be an additive abelian group using the fuzzy unit semi open square. $R = \{[0, 1), +, \times\}$ be the pseudo ring. V is a pseudo vector space over the pseudo ring R.*

Let $x = (0.3, 0.75) \in V$ and $a = 0.7 \in R$,
$ax = 0.7 \times (0.3, 0.75) = (0.21, 0.525) \in V$.

We see in general;

$a(x + y) \neq ax + ay$ for all $a \in R$ and $x, y \in V$.

Also $(a + b)x \neq ax + bx$ for all $a, b \in R$ and $x \in V$. That is why we call V only as a pseudo vector space over the pseudo ring.

Let $a = 0.3$, $x = (0.7, 0.1)$ and $y = (0.31, 0.25) \in V$

$$\begin{aligned} a \times (x + y) &= 0.3 \times [(0.7, 0.1) + (0.31, 0.25)] \\ &= 0.3\,[(0.01, 0.35)] \\ &= (0.003, 0.105) \qquad \ldots \text{ I} \end{aligned}$$

$$\begin{aligned} \text{Now } a \times x + a \times y &= 0.3 \times (0.7, 0.1) + 0.3 \times (0.31, 0.25) \\ &= (0.21, 0.03) + (0.093, 0.075) \\ &= (0.303, 0.105) \qquad \ldots \text{ II} \end{aligned}$$

Clearly I and II are distinct and are in V; hence the claim.

***Example 1.51:*** Let
$W = \{(a_1, a_2, a_3) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 3\}$ be the fuzzy unit square pseudo vector space over the pseudo ring $R = \{[0, 1), +, \times\}$. W has subspaces like;

$V_1 = \{(a_1, 0, 0) \mid a_1 \in U_F\} \subseteq W$,
$V_2 = \{(0, a_2, 0) \mid a_2 \in U_F\} \subseteq W$

and $V_3 = \{(0, 0, a_3) \mid a_3 \in U_F\} \subseteq W$ are pseudo subspaces of W over R.

We have $W = V_1 + V_2 + V_3$ is a direct sum and
$V_i \cap V_j = \{(0, 0, 0)\}$ if $i \neq j$, $1 \leq i, j \leq 3$.

Apart from this take

$$P_1 = \{((a, 0), 0, 0) \mid a \in [0, 1)\} \subseteq W$$
$$P_2 = \{((0, a), 0, 0) \mid a \in [0, 1)\} \subseteq W$$
$$P_3 = \{(0, (a, 0), 0) \mid a \in [0, 1)\} \subseteq W$$
$$P_4 = \{(0, (0, a), 0) \mid a \in [0, 1)\} \subseteq W$$
$$P_5 = \{(0, 0, (a, 0)) \mid a \in [0, 1)\} \subseteq W$$

and $P_6 = \{(0, 0, (0, a)) \mid a \in [0, 1)\} \subseteq W$ are six pseudo vector subspaces of W and $P_i \cap P_j = (0, 0, 0)$ if $i \neq j$, $1 \leq i \leq 6$ and $W = P_1 + P_2 + \ldots + P_6$ is again a direct sum of pseudo subspaces of W.

We can also say the pseudo space $P_i$ is orthogonal with $P_j$ with $i \neq j$, $1 \leq i, j \leq 6$. We see $P_1$ is orthogonal with $P_2$ but $P_1 \oplus P_2 \neq W$.

Now we can have several such pseudo subspaces of W.

Let $M_1 = \{(a_1, a_2, 0) \mid a_1, a_2, \in U_F\} \subseteq W$ and

$M_2 = \{(0, 0, a_3) \mid a_3 \in U_F\} \subseteq W$; we see $M_1 + M_2 = W$ and $M_1 \cap M_2 = \{(0, 0, 0)\}$.

That is $M_1$ is the orthogonal pseudo subspace of $M_2$ and vice versa.

***Example 1.52:*** Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{15} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 15, +\} \right.$$

be the pseudo vector space over the pseudo ring
$R = \{a / a \in [0, 1), +, \times\}$

M has several pseudo subspaces over R. M also has orthogonal pseudo subspaces.

Finally M can be written as a direct sum of pseudo subspaces. Infact dimension of M over R is infinite.

***Example 1.53:*** Let

$$T = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \leq i \leq 9, +\} \right.$$

be the special fuzzy unit semi open square vector space over the pseudo ring R = {a | a ∈ [0, 1), +, ×}.

This T also has pseudo subspaces. However dimension of T over R is infinite.

***Example 1.54:*** Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{19} & a_{20} & a_{21} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \leq i \leq 21, +\} \right.$$

be the fuzzy unit semi open square pseudo vector space over the pseudo ring R = {[0, 1), +, ×}.

W has an infinite basis and has several pseudo subspaces.

***Example 1.55:*** Let

$$N = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{array}\right] \right| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 12, +\}$$

be the special unit fuzzy unit semi open square super matrix pseudo vector space over the pseudo ring $R = \{[0, 1), +, \times\}$.

S has pseudo vector subspaces and S is of infinite dimension over R.

***Example 1.56:*** Let $V = \{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6 \mid a_7\ a_8 \mid a_9\ a_{10}\ a_{11} \mid a_{12}) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 12, +\}$ be the special fuzzy unit semi open square super matrix pseudo vector space over $R = \{a \mid a \in [0, 1), +, \times\}$; the pseudo ring.

V is infinite dimensional over the pseudo field R. V has infinite number of pseudo subspaces.

$P_1 = \{(a_1 \mid 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0)$ where $a_1 \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, +\} \subseteq V$;

$P_2 = \{(\ 0 \mid a_2\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0)$ where $a_2 \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, +\} \subseteq V$;

$P_3 = \{(0 \mid 0\ a_3\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0)$ where $a_3 \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, +\} \subseteq V, \ldots,$

$P_{11} = \{(0 \mid 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ a_{11} \mid 0)$ where $a_{11} \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, +\} \subseteq V$ and

$P_{12} = \{(0 \mid 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid a_{12})$ where $a_{12} \in U_F = \{(a, b) \mid a, b \in [0, 1)\}, +\} \subseteq V$ be 12 pseudo subspaces of V.

$P_i \cap P_j = \{(0 \mid 0\ 0\ 0 \mid 0\ 0 \mid 0\ 0 \mid 0\ 0\ 0 \mid 0)\}, i \neq j, 1 \leq i, j \leq 12.$

$V = P_1 + P_2 + \ldots + P_{12}$ is a direct sum of pseudo subspaces.

V is infinite dimensional over R.

***Example 1.57:*** Let

$$M = \left\{ \left[ \begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} \\ a_{40} & a_{41} & a_{42} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$$

$1 \leq i \leq 42, +\}$

be the fuzzy unit semi open square special super column matrix pseudo space over the pseudo ring R = {a | a ∈ [0, 0), +, ×}.

Thus M has several pseudo subspaces. We see M has an infinite basis over R.

M can also be written as a direct sum of pseudo subspaces.

We can for any pseudo subspace V of M find $V^{\perp}$ such that $V \oplus V^{\perp} = M$.

For instance let

$$V = \left\{ \left[ \begin{array}{ccc} a_1 & 0 & 0 \\ \hline a_2 & 0 & 0 \\ a_3 & 0 & 0 \\ a_4 & 0 & 0 \\ \hline a_5 & 0 & 0 \\ a_6 & 0 & 0 \\ a_7 & 0 & 0 \\ a_8 & 0 & 0 \\ \hline a_9 & 0 & 0 \\ a_{10} & 0 & 0 \\ \hline a_{11} & 0 & 0 \\ a_{12} & 0 & 0 \\ a_{13} & 0 & 0 \\ a_{14} & 0 & 0 \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 14, +\} \subseteq M$$

be a pseudo subspace of V over R.

$$V^{\perp} = \left\{ \left[ \begin{array}{ccc} 0 & a_1 & a_2 \\ \hline 0 & a_3 & a_4 \\ 0 & a_5 & a_6 \\ 0 & a_7 & a_8 \\ \hline 0 & a_9 & a_{10} \\ 0 & a_{11} & a_{12} \\ 0 & a_{13} & a_{14} \\ 0 & a_{15} & a_{16} \\ \hline 0 & a_{17} & a_{18} \\ 0 & a_{19} & a_{20} \\ \hline 0 & a_{21} & a_{22} \\ 0 & a_{23} & a_{24} \\ 0 & a_{25} & a_{26} \\ 0 & a_{27} & a_{28} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$$

$$1 \leq i \leq 28, +\} \subseteq M,$$

$V^{\perp}$ is the pseudo subspace which is the orthogonal subspace of M.

Thus $V \oplus V^{\perp} = M$ so $V \cap V^{\perp} = \left\{ \left[\begin{array}{ccc} 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \right\}$.

***Example 1.58:*** Let

$$M = \left\{ \left( \begin{array}{cc|ccc|c|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \\ a_{25} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{32} \\ a_{33} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{40} \end{array} \right) \middle| \; a_i \in U_F = \{(a, b) \right.$$

$$| \; a, b \in [0, 1), 1 \leq i \leq 40, +\}$$

be the special fuzzy unit semi open square super row matrix pseudo vector space over the pseudo ring

$$R = \{a \mid a \in [0, 1), +, \times\}.$$

The dimension of M over R is infinite and M has pseudo subspaces so that M can be written as a direct sum of subspaces. Also we can build orthogonal pseudo subspaces for appropriate pseudo subspaces of M.

***Example 1.59:*** Let

$$T = \left\{ \left[\begin{array}{cc|cccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ \hline a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \\ \hline a_{49} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ \hline a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{64} \\ a_{65} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{72} \end{array}\right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 72, +\} \right.$$

be the special fuzzy unit semi open square pseudo vector space over the pseudo ring $R = \{a \mid a \in [0, 1), +, \times\}$ be the pseudo ring.

Now we proceed onto define pseudo linear transformation and pseudo linear operator on pseudo vector spaces.

***Example 1.60:*** Let

$V = \{(a_1\ a_2 \ldots a_8) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 8, +\}$ and

$$W = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 9, \right.$$

$+\}$ be two special fuzzy unit semi open square pseudo vector spaces over the pseudo ring $R = \{a \mid a \in [0, 1), +, \times\}$.

Define T : V → W

by $T((a_1, a_2, \ldots, a_8)) = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & 0 \end{pmatrix}$

For every $(a_1, a_2, \ldots, a_8) \in V$.

Clearly T is a pseudo linear transformation from V to W.

Now S : W → V can also be defined as

$$S\left(\begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix}\right) = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$$

For every $\begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix} \in W$.

It is easily verified S is also a pseudo linear transformation from W to V.

Now we can also define pseudo linear operators on V (or W) T : V → V be a map such that

$$T((a_1, a_2, \ldots, a_8)) = (a_1, 0, a_2, 0, a_3, 0, a_4, 0);$$

T is a pseudo linear operator on V.

Let S : W → W be a map such that $S\left\{\begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix}\right\}$

$$= \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix}.$$

S is a pseudo linear operator on W.

***Example 1.61:*** Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{15} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 15, +\} \right.$$

be a pseudo vector space over the pseudo ring $R = \{a \mid a \in [0, 1), +, \times\}$.

Define $T : M \to M$ by

$$T \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{15} \end{bmatrix} \right\} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_1 \\ a_2 \end{bmatrix};$$

T is a pseudo linear operator on M. One can study $Hom_R(M, M)$ and its algebraic structure.

Now we proceed onto define strong pseudo vector space built over the pseudo ring $R = \{U_F = \{(a, b) \mid a, b \in [0, 1)\}, +, \times\}$

**DEFINITION 1.3:** *V is defined as the special fuzzy unit square strong pseudo vector space defined over the pseudo fuzzy unit semi open square pseudo ring $R = \{U_F, +, \times\}$ only if V is an additive abelian group and for all $a \in R$ and $v \in V$, $av = va \in V$.*

We will illustrate this by some examples.

***Example 1.62:*** Let W = $\{(a_1, a_2, a_3, a_4) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 4, +\}$ be the special fuzzy unit square strong pseudo vector space over the pseudo ring
$R = \{(a, b) \mid a, b \in [0, 1), +, \times\} = \{U_F, +, \times\}$.

W is of infinite dimension over R. W has strong pseudo vector subspaces all of which are of infinite dimension over R.

$P_1 = \{(a_1, 0, 0, 0) \mid a_1 \in U_F = \{(a, b) \mid a, b \in [0, 1), +\} \subseteq W$ is a pseudo strong vector subspace of W over R.

Clearly $P_1 \cong R$. Likewise $P_2$ is also a pseudo strong vector subspace of W over R given by

$P_2 = \{(0, a_2, 0, 0) \mid a_2 \in U_F = \{(a, b) \mid a, b \in [0, 1), +\} \subseteq W$,
$P_3 = \{(0, 0, a_3, 0) \mid a_3 \in U_F = \{(a, b) \mid a, b \in [0, 1), +\} \subseteq W$
and
$P_4 = \{(0, 0, 0, a_4) \mid a_1 \in U_F = \{(a, b) \mid a, b \in [0, 1), +\} \subseteq W$
are all pseudo strong vector subspace of W over R.

We see $W = P_1 + P_2 + P_3 + P_4$ and $P_i \cap P_j = \{(0, 0, 0, 0)\}$; $i \neq j$, $1 \leq i, j \leq 4$.

Thus W is a direct sum of pseudo strong subspaces.

Interested reader can find other pseudo strong subspaces of W.

***Example 1.63:*** Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 10, +\} \right.$$

be the special fuzzy unit square strong pseudo vector space over the fuzzy unit square pseudo ring $R = \{U_F = \{(a, b) \mid a, b \in [0, 1)\}$.

S has several subspaces. Also S has subspaces which has orthogonal complements.

For instance if

$$P_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 5, +\} \subseteq S$$

then

$$P_1^{\perp} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 5, +\} \subseteq S.$$

We see $P_1^{\perp} + P_1 = S$ and $P_1^{\perp}$ is the orthogonal complement of $P_1$.

We can also write S as a direct sum of special strong pseudo subspaces.

***Example 1.64:*** Let

$$T = \left\{ \left( \begin{array}{cccc} a_1 & a_2 & \ldots & a_7 \\ a_8 & a_9 & \ldots & a_{14} \\ a_{15} & a_{16} & \ldots & a_{21} \end{array} \right) \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \leq i \leq 21, +\} \right.$$

be the special strong pseudo vector space over $\{U_F, +, \times\} = R$, the pseudo ring.

***Example 1.65:*** Let

$$T = \left\{ \left[ \begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{31} & a_{32} & a_{33} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \leq i \leq 33, +\} \right.$$

be the special strong pseudo vector space over the pseudo ring $R = \{U_F, +, \times\}$.

We see T has atleast ${}_{33}C_1 + {}_{33}C_2 + \ldots + {}_{33}C_{32}$ number of pseudo strong subspaces.

***Example 1.66:*** Let

$$T = \left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{array} \right] \middle| \; a_i \in U_F, 1 \leq i \leq 12, +\right\}$$

and

$$N = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in U_F, 1 \leq i \leq 15, +\right\}$$

be two special fuzzy unit square strong pseudo vector spaces over the pseudo ring $R = \{(a, b) \mid a, b \in [0, 1), +, \times\}$.

$T : M \to N$ be a map such that

$$T \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix}\right\} = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ 0 & 0 & 0 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \end{bmatrix}\right\}.$$

Then T is a special strong pseudo linear transformation from M to N.

We can also find special strong pseudo linear operators on M and N.

Finally we can define strong special pseudo linear functional f from M to R which is as follows:

f is a map from M to R ie. $f : M \to R$ is such that

$$f\left(\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix}\right) = a_4 + a_8 + a_{12} \in R.$$

That is if $a_4 = (0.3, 0.75)$
$a_8 = (0.7, 0.42)$

and $a_{12}$ = (0.115, 0.25) then $a_4 + a_8 + a_{12}$ = (0.115, 0.42) ∈ R.

Thus f is a pseudo linear functional.

***Example 1.67:*** Let M = {($a_1$, $a_2$, $a_3$) | $a_i \in U_F$, 1 ≤ i ≤ 3} be the special strong fuzzy square unit pseudo vector space over R = {$U_F$, +, ×}.

f : M → R given by

f($a_1$, $a_2$, $a_3$) = $a_1 + a_2 + a_3$

f((0.7, 0.2) (0, 0.7), (0.35, 0.8))

= (0.7, 0.2) + (0, 0.7) + (0.35, 0.8)

= (0.05, 0.7) ∈ R.

f is a pseudo linear functional on M.

Thus f {($a_1$, 0, 0)} = $a_1$
f {(0, $a_2$, 0)} = $a_2$
and f {(0, 0, $a_3$) = $a_3$

***Example 1.68:*** Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \right.$$

$$1 \leq i \leq 20\}$$

be the special fuzzy unit square strong pseudo vector space over R = {$U_F$, +, ×} be the pseudo ring.

Let f : T → R where

$$f\left\{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \end{pmatrix}\right\} = a_1 + a_6 + a_{11} + a_{16} + a_{17}$$

f is a pseudo linear functional on T.

$$f\left\{\begin{pmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right\} = a_1 \text{ and so}$$

$$f\left\{\begin{pmatrix} 0 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\right\} = 0.$$

***Example 1.69*** Let

$$M = \left\{\left(\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix}\right) \middle|\ a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \le i \le 10, +\}\right.$$

be the special fuzzy unit square strong pseudo special vector space over the pseudo ring R = {(a, b) | a, b ∈ [0, 1), +, ×}.

Define f : M → R

$$f\left\{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix}\right\} = a_1 + a_2 + \ldots + a_{10}$$

$$f\left\{\begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\} = a_1 \text{ and so on.}$$

f is a pseudo linear functional on M.

***Example 1.70:*** Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \; 1 \leq i \leq 12\}\right.$$

be the special fuzzy unit semi open square strong pseudo vector space over the pseudo ring R = {(a, b) | a, b ∈ [0, 1), +, ×}.

Define f : S → R

$$\text{by } f\left\{\left(\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix}\right)\right\} = a_2 + a_6 + a_{10}.$$

f is a pseudo linear functional on S.

Now we can define the notion of pseudo inner product on pseudo vector space over the pseudo ring.

***Example 1.71:*** Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \;\middle|\; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 4\}\right.$$

be the pseudo strong vector space over the pseudo unit fuzzy ring R = $\{(a, b) \mid a, b \in [0, 1), +, \times\}$.

Let x, y $\in$ V we define the pseudo inner product

$$\langle x, y\rangle = \sum_{i=1}^{4} a_i b_i \text{ and } y = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} \in V.$$

$$x = \begin{pmatrix} (0.3, 0.2) & (0.8, 0.7) \\ (0.6, 0.1) & (0, 0.9) \end{pmatrix} \text{ and } y = \begin{pmatrix} (0.8, 0) & (0, 0.8) \\ (0, 0.9) & (0.1, 0) \end{pmatrix} \in V$$

$\langle x, y\rangle = (0.3, 0.2)\,(0.8, 0) + (0.8, 0.7)\,(0, 0.8) +$
$(0.6, 0.1)\,(0, 0.9) + (0, 0.9)\,(0.1, 0)$
$= (0.24, 0) + (0, 0.56) + (0, 0.09) + (0, 0)$
$= (0.24, 0.65).$

$\langle \quad \rangle$ the pseudo inner product.

The study in this direction is innovative and interesting.

Next we proceed onto define special fuzzy unit square semivector space.

Let V = {($a_1$, $a_2$, $a_3$) | $a_i$ ∈ {$U_F$, max}, 1 ≤ i ≤ 3} be the pseudo semivector space over the semiring S = {$U_F$, min, max}.

Let x = {(0.3, 0.7), (0.1, 0.02), (0.4, 0.1)} ∈ V;
for a = {(0.1, 0.2)} ∈ S
min {(0.1, 0.2) × {(0.3, 0.7), (0.1, 0.02), (0.4, 0.1)}}
= (min {(0.1, 0.2), (0.3, 0.7)}, min {(0.1, 0.2), (0.1, 0.02)}
= {(0.1, 0.2), (0.1, 0.02), (0.1, 0.1)}.

This is the way min {a, x} ∈ V.

Likewise we can use in V instead of max operation take a semigroup under min. Also we can have max {x, a} or min {x, a}.

Thus for a semigroup under min we can have max or min as operation of multiplying by the scalar from the semiring {$U_F$, min, max}. Similarly for the semigroup under max we can have max or min as operation of multiplying by a scalar.

Thus we get four types of semivector spaces over the fuzzy unit square semiring S = {$U_F$, min, max}.

We will first illustrate this situation by some examples.

***Example 1.72:*** Let

$M_1$ = {($a_1$, $a_2$, $a_3$, $a_4$) | $a_i$ ∈ {$U_F$}, min, 1 ≤ i ≤ 4}

$M_2$ = {($a_1$, $a_2$, $a_3$, $a_4$) | $a_i$ ∈ {$U_F$}, max, 1 ≤ i ≤ 4}

$M_3$ = {($a_1$, $a_2$, $a_3$, $a_4$) | $a_i$ ∈ {$U_F$}, max, 1 ≤ i ≤ 4}

and $M_4$ = {($a_1$, $a_2$, $a_3$, $a_4$) | $a_i$ ∈ {$U_F$}, min, 1 ≤ i ≤ 4} be four semivector over the semiring S = {$U_F$, min, max}.

For $M_1$ the scalar product is max.
for $M_2$ the scalar product in min.
for $M_3$ the scalar product is max.
and for $M_4$ the scalar product is min.

Let x = ((0.2, 0.71), (0.21, 0.1), (0, 0.5), (0.7, 0)),
y = ((0.5, 0), (0.3, 0.21), (0.8, 0.3), (0, 0.5) $\in M_i$, $1 \leq i \leq 4$ and a = (0.8, 0.3) $\in$ S.

min {a, min(x, y)}
= min {a, ((0.2, 0), (0.21, 0.1), (0, 0.3), (0, 0)}
= (min {(0.8, 0.3), (0.2, 0)}, min {(0.8, 0.3), (0.21, 0.1)} min {(0.8, 0.3), (0, 0.3)}, min {(0.8, 0.3), (0, 0)}
= ((0.2, 0), (0.21, 0.1), (0, 0.3), (0, 0) $\in M_1$ … I

max {a, min(x, y)}
= ((0.8, 0.3), (0.8, 0.3), (0.8, 0.3), (0.8, 0.3)) $\in M_2$ … II

min {a, max{x, y}}
= ((0.5, 0.3), (0.3, 0.21), (0.8, 0.3), (0.7, 0.3)) $\in M_3$ … III

max {a, max{x, y}}
= ((0.8, 0.71), (0.8, 0.3), (0.8, 0.5), (0.8, 0.5)) $\in M_4$ … IV

We see all the four operations for the same set of elements yield different elements in $M_i$; $1 \leq i \leq 4$.

Hence the claim.

Thus we have four types of semivector spaces using the special fuzzy unit square.

We will give examples of these four types of semivector spaces over the fuzzy unit square semiring ($U_F$, min, max) under min or max operation.

***Example 1.73:*** Let

$$N = \left\{ \left[ \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \end{array} \right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 10, \max\} \right\}$$

be the special unit fuzzy semi open square semivector space over the semirings = $\{U_F, \max, \min\}$ under min operation.

This has several subsemivector spaces over S.

We can also write M as a direct sum of subsemivector spaces.

Now we can proceed onto describe in few words the notion of algebraic structures built using the fuzzy neutrosophic semi open unit square.

We see $N_s = \{a + bI \mid a, b \in [0, 1), I^2 = I\}$ is the fuzzy neutrosophic semi open unit square. Only elements of the form $x = 0.5 + 0.5I$ and $y = I$ in $N_s$ we have $x \times y = 0$.

So $a + bI$ where $a + b \equiv 0 \pmod 1$ pave way for zero divisors when multiplied by I.

In the following chapters we will built algebraic structures using $N_s$, the fuzzy neutrosophic semi open unit square.

However in case of fuzzy unit semi open square $U_F$ we have if x = (0, a) and y = (b, 0) then x × y = (0, 0).

However we see this is not true in case of $N_s$. This is the marked difference between these two new structures.

We present the following problems for this chapter.

**Problems**

1. Find some special features enjoyed by fuzzy semi open unit square.

2. What is the difference between fuzzy special interval [0, 1) and $U_F$ = {(a, b) | a, b ∈ [0, 1)} as semigroups under product?

3. Let P = {($a_1$, $a_2$, …, $a_{10}$) | $a_i$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 10, ×} be the semigroup.

   (i) Prove P has infinite number of zero divisors.

   (ii) Can P have finite subsemigroups?

   (iii) Can P have units?

   (iv) Can P have idempotents?

   (v) Can P have subsemigroups which are not ideals?

   (vi) Can P have ideals of finite order?

4. Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \right| \left. a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 9, \times\} \right.$$

be the special fuzzy unit semi open square semigroup of infinite order.

Study questions (i) to (vi) of problem (3) for this M.

5. Let $B = \left\{ \begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \end{pmatrix} \right| a_i \in U_F = \{(a, b) \mid a, b \in$

$[0, 1), 1 \leq i \leq 20, \times_n\}$ be the fuzzy unit semi open semigroup.

Study questions (i) to (vi) of problem (3) for this B.

6. Let $M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{55} & a_{56} & a_{57} \\ a_{58} & a_{59} & a_{60} \end{bmatrix} \right| a_i \in U_F = \{(a, b) \mid a, b \in [0,1),$

$1 \leq i \leq 60, \times_n\}$ be the fuzzy unit semi open semigroup.

Study questions (i) to (vi) of problem (3) for this M.

7. Let M = $\left\{\begin{bmatrix} a_1 & a_2 & \ldots & a_7 \\ a_8 & a_9 & \ldots & a_{14} \\ \vdots & \vdots & & \vdots \\ a_{43} & a_{44} & \ldots & a_{49} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in\right.$ $[0, 1), 1 \leq i \leq 49, \times_n\}$ be the fuzzy unit semi open square matrix semigroup.

Study questions (i) to (vi) of problem (3) for this M.

8. Let M = $\{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6 \mid a_7 \mid a_8) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 8, \times_n\}$ be the fuzzy unit semi open square matrix semigroup.

Study questions (i) to (vi) of problem (3) for this M.

9. Let B = $\left\{\begin{bmatrix} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \\ \hline a_{12} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),\right.$ $1 \leq i \leq 12, \times_n\}$ be the fuzzy unit semi open square matrix semigroup.

Study questions (i) to (vi) of problem (3) for this B.

10. Let $T = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} \\ a_{53} & a_{54} & a_{55} & a_{56} \\ a_{57} & a_{58} & a_{59} & a_{60} \end{array}\right] \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in$ $[0, 1), 1 \leq i \leq 60, \times_n\}$ be the super column fuzzy matrix semigroup.

Study questions (i) to (vi) of problem (3) for this T.

11. Let $P = \left\{ \left(\begin{array}{c|ccc|cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{18} \\ a_{19} & \dots & \dots & \dots & \dots & \dots & \dots & \dots & a_{27} \end{array}\right) \right|$ $a_i$ $\in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 27, \times_n\}$ be the super column fuzzy matrix semigroup.

Study questions (i) to (vi) of problem (3) for this P.

12. Let P = $\left\{ \left( \begin{array}{cc|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ \hline a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ \hline a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \end{array} \right) \middle| \; a_i \in U_F \right.$

= {(a, b) | a, b ∈ [0, 1), $1 \leq i \leq 48$, ×} be the super column fuzzy matrix semigroup.

Study questions (i) to (vi) of problem (3) for this P.

13. Study the semigroup B = {$U_F$, min} of the fuzzy semi open unit square.

(i) Show B has infinite number of ideals.

(ii) Show B has infinite number of infinite order subsemigroups which are not ideals.

(iii) Show B has no ideals of finite order.

(iv) Can B have zero divisors?

(v) Prove B has idempotents of all orders.

(vi) Show B has no units.

14. Let M = {($a_1, a_2, \ldots, a_{10}$) | $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1), $1 \leq i \leq 10$, ×} be the special fuzzy unit semi open square row matrix semigroup under min operation.

Study questions (i) to (vi) of problem 13 for this M.

15. Let T = $\left\{\begin{bmatrix} a_1 & a_2 & \ldots & a_{15} \\ a_{16} & a_{17} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{45} \\ a_{46} & a_{47} & \ldots & a_{60} \end{bmatrix} \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in$ $[0, 1), 1 \leq i \leq 60, \times_n\}$ be the special fuzzy unit semi open square row matrix of infinite order semigroup.

Study questions (i) to (vi) of problem 13 for this M.

16. Let T = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{43} & a_{44} & a_{45} \end{bmatrix} \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in [0,$ $1), 1 \leq i \leq 45, \times_n\}$ be the special fuzzy unit semi open square column matrix semigroup of infinite order.

Study questions (i) to (vi) of problem 13 for this T.

17. Let T = $\left\{\begin{bmatrix} a_1 & a_2 & \ldots & \ldots & a_{15} \\ a_{16} & a_{17} & \ldots & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & \ldots & a_{45} \\ a_{46} & a_{47} & \ldots & \ldots & a_{60} \\ a_{61} & a_{62} & \ldots & \ldots & a_{75} \\ a_{76} & a_{77} & \ldots & \ldots & a_{90} \end{bmatrix} \right|$ $a_i \in U_F = \{(a, b) \mid a, b$ $\in [0, 1), 1 \leq i \leq 90, \min\}$ be the special fuzzy unit semi open square matrix semigroup under min.

Study questions (i) to (vi) of problem 13 for this T.

18. Let W = {($a_1$ $a_2$ | $a_3$ | $a_4$ $a_5$ $a_6$ | $a_7$ $a_8$ | $a_9$) | $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 9, min} be the special fuzzy unit semi open super matrix semigroup under min operation.

Study questions (i) to (vi) of problem 13 for this W.

19. Let P = $\left\{\left.\left[\begin{array}{c} a_1 \\ a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ \hline a_{12} \end{array}\right]\right|\right.$ $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1),

1 ≤ i ≤ 12, min} be the special unit semi open square column super matrix semigroup of infinite order.

Study questions (i) to (vi) of problem 13 for this P.

20. Let W = $\left\{\left.\left(\begin{array}{cc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \end{array}\right)\right|\right.$ $a_i \in U_F$ =

{(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 14, min} be the special unit semi open square column super matrix semigroup of infinite order.

Study questions (i) to (vi) of problem 13 for this W.

21. Let S = 
$$\left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ \hline a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ \hline a_{45} & a_{46} & a_{47} & a_{48} \end{array} \right] \right|$$
$a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 48, \min\}$ be the special unit semi open square column matrix semigroup of infinite order.

Study questions (i) to (vi) of problem 13 for this S.

Prove S has infinite number of zero divisors.

22. Let M = 
$$\left\{ \left[ \begin{array}{c|cc|ccccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ \hline a_{10} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{27} \\ \hline a_{28} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{36} \\ a_{37} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{45} \\ a_{46} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{54} \\ a_{55} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{63} \\ a_{64} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{72} \\ \hline a_{73} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{81} \end{array} \right] \right| a_i$$
$\in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 81, \min\}$ be the special unit semi open square column matrix semigroup of infinite order.

Study questions (i) to (vi) of problem 13 for this M.

Prove M has infinite number of zero divisors.

23. Let W = $\{U_F, \max\}$ be the special fuzzy unit square semigroup under max operation.

(i) Can W have zero divisor?

(ii) Prove every element in W is an idempotent.

(iii) Study the subsemirings and ideals of W.

(iv) Can W have finite ideals?

(v) Is it possible for W to have subsemigroups which are not ideals of infinite order.

(vi) Find infinite order subsemigroups of W.

(vii) Prove every subset of W can be completed into a subsemigroup.

24. Let P = $\{(a_1, a_2, \ldots, a_8) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$ $1 \leq i \leq 8, \max\}$ be the special fuzzy unit semi open square semigroup of infinite order under max operation.

Study questions (i) to (vii) of problem 23 for this P.

25. Let $M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{15} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), \right.$

$1 \leq i \leq 15, \max\}$ be the fuzzy unit semi open square column matrix semigroup under max operation.

Study questions (i) to (vii) of problem 23 for this M.

26. Let T = $\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \\ a_{31} & a_{32} & \dots & a_{40} \end{bmatrix}\right|$ $a_i \in U_F = \{(a, b) \mid a, b \in$ $[0, 1), 1 \leq i \leq 40, \times\}$ be the special fuzzy unit semi open square row matrix of infinite order semigroup.

Study questions (i) to (vii) of problem 23 for this T.

27. Let W = $\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ \vdots & \vdots & \dots & \vdots \\ a_{71} & a_{72} & \dots & a_{80} \end{bmatrix}\right|$ $a_i \in U_F = \{(a, b) \mid a, b \in$ $[0, 1), 1 \leq i \leq 8, \max\}$ be the special fuzzy unit semi open square row matrix of infinite order semigroup.

Study questions (i) to (vii) of problem 23 for this W.

28. Let M = $\{(a_1\ a_2 \mid a_3 \mid a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 9, \max\}$ be the special fuzzy unit semi open square row super matrix semigroup under max operation.

Study questions (i) to (vii) of problem 23 for this M.

29. Let $V = \left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ \hline a_8 \\ a_9 \\ \hline a_{10} \end{array} \right] \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$

$1 \leq i \leq 10$, max} be the special fuzzy unit semi open square column super row matrix semigroup under max operation.

Study questions (i) to (vii) of problem 23 for this V.

30. Let $M = \left\{ \left[ \begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{array} \right] \right|$ $a_i \in U_F = \{(a, b) \mid$

$a, b \in [0, 1)$, $1 \leq i \leq 25\}$ be the special fuzzy unit semi open square super matrix semigroup under max operation.

Study questions (i) to (vii) of problem 23 for this M.

31. Let M = 

$$\left\{\left.\left[\begin{array}{c|ccc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ \hline a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ \hline a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \\ a_{49} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \end{array}\right] \right| a_i \in U_F\right.$$

= {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 56, max} be the special fuzzy unit semi open square super matrix semigroup under max operation.

Study questions (i) to (vii) of problem 23 for this M.

32. Let W = 

$$\left\{\left.\left[\begin{array}{c|cc|ccc|c|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \end{array}\right] \right| a_i \in U_F\right.$$

= {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 32, max} be the special fuzzy unit semi open square super matrix semigroup under max operation.

Study questions (i) to (vii) of problem 23 for this W.

33. Let W = $\left\{\left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ \hline a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} \\ a_{40} & a_{41} & a_{42} \end{array}\right] \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in$ $[0, 1), 1 \leq i \leq 42\}$ be the special fuzzy unit semi open square super column matrix of infinite order under max operation.

Study questions (i) to (vii) of problem 23 for this W.

34. Let $\{U_F, +\}$ = G be the unit semi open fuzzy square group under +.

(i) Find all subgroups of finite order.
(ii) Prove G has subgroups of infinite order.

35. Let M = $\{(a_1, a_2, a_3, a_4, a_5) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 5, +\}$ be the special fuzzy unit semi open square row matrix group under +.

(i) Find subgroups of finite order.
(ii) Can M have infinite number of finite subgroups?
(iii) Can M have infinite number of infinite subgroups?

36. Let $M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{17} \end{bmatrix} \middle| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),\right.$

$1 \leq i \leq 17, +\}$ be the unit fuzzy square column matrix group under +.

Study questions (i) to (iii) of problem 35 for this M.

37. Let $S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \end{bmatrix} \middle| a_i \in U_F = \right.$

$\{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 42, +\}$ be the fuzzy unit semi open fuzzy matrix group of infinite order.

Study questions (i) to (iii) of problem 35 for this S.

38. Let $M = \{(a_1, a_2 \mid a_3 \mid a_4 \mid a_5\ a_6\ a_7\ a_8) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 8, +\}$ be the fuzzy unit semi open square super row matrix group of infinite order.

Study questions (i) to (iii) of problem 35 for this M.

39. Let $B = \left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ \hline a_{12} \\ a_{13} \\ a_{14} \\ \hline a_{15} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$

$1 \leq i \leq 15, +\}$ be the fuzzy unit semi open square super column matrix group of infinite order.

Study questions (i) to (vii) of problem 35 for this B.

40. Let $M = \left\{ \left( \begin{array}{cc|c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \dots & \dots & \dots & \dots & \dots & \dots & a_{16} \\ a_{17} & \dots & \dots & \dots & \dots & \dots & \dots & a_{24} \end{array} \right) \right| a_i \in$

$U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 24, +\}$ be the fuzzy unit semi open square super row matrix group of infinite order.

Study questions (i) to (iii) of problem 35 for this M.

41. Let T = $\left\{\left[\begin{array}{cc|ccccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ \hline a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ \hline a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \end{array}\right] \middle| \; a_i \in U_F \right.$

= {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 48, +} be the super matrix fuzzy unit semi open square group.

Study questions (i) to (iii) of problem 35 for this T.

42. Let T = $\left\{\left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} \\ \hline a_{53} & a_{54} & a_{55} & a_{56} \end{array}\right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in \right.$

[0, 1), 1 ≤ i ≤ 56, +} be the super column matrix fuzzy unit semi open square group.

Study questions (i) to (iii) of problem 35 for this T.

43. Let T = $\left\{\left[\begin{array}{cc|ccc|c|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \\ \hline a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \\ \hline a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} & a_{49} \\ a_{50} & a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ \hline a_{57} & a_{58} & a_{59} & a_{60} & a_{61} & a_{62} & a_{63} \end{array}\right] \middle| \; a_i \in U_F\right.$

= {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 63, +} be the fuzzy unit semi open square group.

Study questions (i) to (iii) of problem 35 for this T.

44. {$U_F$, min, max} be the semiring of fuzzy unit semi open square.

(i) Prove $U_F$ has zero divisors with respect to min.
(ii) Every element x ∈ $U_F$ with {0, 0} is a subsemiring of {$U_F$, max, min}.
(iii) Prove every element in {$U_F$, max, min} is an idempotent with respect to max and min.
(iv) Prove {$U_F$, max, min} has finite subsemirings.
(v) Prove {$U_F$, max, min} has ideals only of infinite order.
(vi) Prove subsets in {$U_F$, max, min} can be completed to form subsemirings and the completed subsemiring in general is not an ideal.
(vii) Prove {$U_F$, max, min} has infinite order subsemirings which are not ideals of $U_F$.

45. Let P = {($a_1$, $a_2$, …, $a_{10}$) | $a_i$ ∈ $U_F$ = {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 10, min, max} be the semiring on the fuzzy unit semi open square.

(i) Study questions (i) to (vii) of problem 44 for this P.
(ii) Find all filters of P.
(iii) Can a subsemiring of P be both filter and an ideal?
(iv) Can filters in P be a finite order?
(v) Obtain some special properties enjoyed by P.
(vi) Can P be a S-semiring?

46. Let $M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ \vdots \\ a_{19} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),\right.$

$1 \leq i \leq 19$, min, max} be the special fuzzy unit semi open square semiring.

Study questions (i) to (vi) of problem 45 for this M.

47. Let $P = \left\{ \begin{pmatrix} a_1 & a_2 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{24} \\ a_{25} & a_{26} & \dots & a_{36} \end{pmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in\right.$

$[0, 1)$, $1 \leq i \leq 36$, min, max} be the special fuzzy unit semi open square semiring.

Study questions (i) to (vi) of problem 45 for this P.

48. Let $P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in\right.$

$[0, 1)$, $1 \leq i \leq 33$, min, max}

be the special fuzzy unit semi open square column matrix semiring.

Study questions (i) to (vi) of problem 45 for this P.

49. Let M = $\{(a_1\ a_2\ a_3 \mid a_4\ a_5\ a_6\ a_7 \mid a_8\ a_9 \mid a_{10}) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 10$, min, max$\}$ be the unit fuzzy semi open square super row matrix semiring of infinite order.

Study questions (i) to (vi) of problem 45 for this M.

50. Let T = $$\left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ \hline a_{13} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$$

$1 \leq i \leq 13$, min, max$\}$

be the unit fuzzy semi open square semiring of infinite order.

Study questions (i) to (iii) of problem 35 for this T.

51. Let $M = \left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ \hline a_{45} & a_{46} & a_{47} & a_{48} \\ \hline a_{49} & a_{50} & a_{51} & a_{52} \\ a_{53} & a_{54} & a_{55} & a_{56} \\ a_{57} & a_{58} & a_{59} & a_{60} \end{array} \right] \right| a_i \in U_F = \{(a, b) \mid a, b \in$

$[0, 1), 1 \leq i \leq 72, \min, \max\}$

be the special fuzzy unit semi open square semiring.

Study questions (i) to (vii) of problem 44 for this M.

52. Let $T = \left\{ \left( \begin{array}{cc|cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \end{array} \right) \right| a_i \in U_F$

$= \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 24, \min, \max\}$

be the special fuzzy unit semi open square super row matrix semiring.

Study questions (i) to (vi) of problem 45 for this T.

53. Let W = $\left\{ \left( \begin{array}{cc|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ \hline a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ \hline a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \\ a_{49} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ \hline a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{64} \end{array} \right) \middle| \; a_i \in \right.$

$U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 64, \min, \max\}$

be the special fuzzy unit semi open square super row matrix semiring.

Study questions (i) to (vii) of problem 44 for this W.

54. Let M = $\{U_F, \times, \min\}$ be the pseudo semiring built using the unit fuzzy square.

Obtain the special features enjoyed by the pseudo semiring.

55. Let T = $\{(a_1, a_2, a_3, a_4, a_5, a_6) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 6, \min, \times\}$ be the pseudo semiring.

(i) Find pseudo subsemiring if any of finite order.
(ii) Can T have zero divisors?
(iii) Can T have idempotents?
(iv) Can T have pseudo ideals?
(v) Can T have pseudo filters?
(vi) Can T have a pseudo subsemiring which is both a pseudo filter and pseudo ideal?
(vii) Can T have pseudo ideals of finite order?

56. Let M = $\left\{\left.\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{25} \end{bmatrix}\right| a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),\right.$ $1 \le i \le 25, \min, \times\}$ be the special unit semi open fuzzy square column matrix pseudo semiring of infinite order.

Study questions (i) to (vii) of problem 55 for this M.

57. Let V = $\left\{\left.\begin{pmatrix} a_1 & a_2 & \dots & a_7 \\ a_8 & a_9 & \dots & a_{14} \\ a_{15} & a_{16} & \dots & a_{21} \\ a_{22} & a_{23} & \dots & a_{28} \end{pmatrix}\right| a_i \in U_F = \{(a, b) \mid a, b \in\right.$ $[0, 1), 1 \le i \le 28, \min, \times\}$ be the special unit semi open fuzzy square pseudo semiring of infinite order.

Study questions (i) to (vii) of problem 55 for this V.

58. Let V = $\left\{\left.\begin{pmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ \vdots & \vdots & \dots & \vdots \\ a_{91} & a_{92} & \dots & a_{100} \end{pmatrix}\right| a_i \in U_F = \{(a, b) \mid a, b \in\right.$ $[0, 1), 1 \le i \le 100, \min, \times\}$ be the fuzzy unit semi open square pseudo semiring of infinite order.

Study questions (i) to (vii) of problem 55 for this V.

59. Let W = $\{(a_1 \mid a_2\ a_3 \mid a_4\ a_5\ a_6\ a_7 \mid a_8\ a_9\ a_{10} \mid a_{11}) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \le i \le 11, \min, \times\}$ be the special fuzzy unit square pseudo semiring of infinite order.

Study questions (i) to (vii) of problem 55 for this W.

60. Let V =

$$\left\{\left(\begin{array}{c|ccc|cc|cccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{30} \\ a_{31} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{40} \\ a_{41} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{50} \end{array}\right) \middle| \; a_i \in U_F \right.$$

= {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 50, min, ×} be the special fuzzy unit semi open square pseudo semiring of infinite order.

Study questions (i) to (vii) of problem 55 for this V.

61. Let N = $\left\{\left[\begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} \\ a_{40} & a_{41} & a_{42} \\ a_{43} & a_{44} & a_{45} \end{array}\right] \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in \right.$

[0, 1), $1 \leq i \leq 45$, min, ×} be the pseudo semiring.

Study questions (i) to (vii) of problem 55 for this N.

62. Can a pseudo ring M = {$U_F$, +, ×} have finite pseudo subrings and finite pseudo ideals?

Enumerate the difference between a ring and a pseudo ring.

63. Let W = {($a_1$ $a_2$ $a_3$ $a_4$ $a_5$ ) | $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1), $1 \leq i \leq 5$, +, ×} be the fuzzy unit semi open square row matrix pseudo ring of infinite order.

(i) Can W have finite pseudo subrings?
(ii) Can W have finite pseudo ideals?
(iii) Can W have zero divisors?
(iv) Can W have S-zero divisors?
(v) Find those pseudo subrings which are not pseudo ideals?
(vi) Obtain some conditions on zero divisors which are not S-zero divisors.
(vii) Can we have finite S-subrings?
(viii) Is W a S-pseudo ring?

64. Let M = $\left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{15} \end{bmatrix} \right|$ $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1),

$1 \leq i \leq 15$, +, ×} be the pseudo ring.

Study questions (i) to (viii) of problem 63 for this M.

65. Let $M_1 = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & \ldots & a_8 \\ a_9 & a_{10} & \ldots & a_{16} \\ a_{17} & a_{18} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{32} \\ a_{33} & a_{34} & \ldots & a_{40} \\ a_{41} & a_{42} & \ldots & a_{48} \\ a_{49} & a_{50} & \ldots & a_{56} \end{array}\right] \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in$ $[0, 1), 1 \leq i \leq 56, +, \times\}$ be the pseudo ring.

Study questions (i) to (viii) of problem 63 for this $M_1$.

66. Let $T = \{(a_1 \mid a_2\ a_3\ \ a_4\ a_5 \mid a_6\ a_7\ \ a_8 \mid a_9\ \ a_{10} \mid a_{11} \mid a_{12}) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 12, +, \times\}$ be the unit fuzzy square super row matrix semiring of infinite order.

Study questions (i) to (viii) of problem 63 for this T.

67. Let $D = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ \hline a_{13} \end{array}\right] \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$

$1 \leq i \leq 13, +, \times\}$

be the super column matrix pseudo ring.

Study questions (i) to (viii) of problem 63 for this D.

68. Let B = $\left\{\left(\begin{array}{c|cc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \cdots & \cdots & \cdots & \cdots & \cdots & a_{14} \\ a_{15} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{21} \\ a_{22} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{28} \end{array}\right) \middle| \right.$ $a_i \in U_F =$ {(a, b) | a, b ∈ [0, 1), $1 \leq i \leq 28$, +, ×} be the fuzzy unit semi open square super row matrix pseudo ring.

Study questions (i) to (viii) of problem 63 for this B.

69. Let W = $\left\{\left(\begin{array}{cc|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ \hline a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ \hline a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \\ a_{49} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \\ a_{57} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{64} \\ \hline a_{65} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{72} \end{array}\right) \middle| \right.$ $a_i \in$ $U_F$ = {(a, b) | a, b ∈ [0, 1), $1 \leq i \leq 72$, +, ×} be the special fuzzy unit semi open square super row matrix semiring.

Study questions (i) to (viii) of problem 63 for this W.

70. Let T = {($a_1$ $a_2$ $a_3$ $a_4$) | $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1), $1 \leq i \leq 4$, +, ×} be the fuzzy unit square pseudo vector space over the pseudo ring; R = {[0, 1), +, ×}.

(i) Can T have finite dimensional pseudo subspace?

(ii) Can T have finite dimensional pseudo vector spaces?
(iii) Find atleast ${}_4C_1 + {}_4C_2 + {}_4C_3$ pseudo subspaces.
(iv) Can T have more than ${}_4C_1 + {}_4C_2 + {}_4C_3$ number of pseudo subspaces?

71. Let $V = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 8, \right.$

$+, \times\}$ be the special fuzzy unit semi open square pseudo vector space over the pseudo ring $R = \{[0, 1), +, \times\}$.

(i) Can V be finite dimensional over R?
(ii) Can V have a pseudo subspace of finite dimensional?
(iii) How many pseudo vector subspaces V can have?
(iv) Can every pseudo vector subspace W have an orthogonal pseudo vector subspace $W^{\perp}$ so that $W + W^{\perp} = V$?

72. Let $M = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_8 \\ a_9 & a_{10} & \ldots & a_{16} \\ \vdots & \vdots & \ldots & \vdots \\ a_{57} & a_{58} & \ldots & a_{64} \end{bmatrix} \middle| \; a_i \in U_F = \{(a, b) \mid a, b \in \right.$

$[0, 1), 1 \leq i \leq 64, +\}$ be the special fuzzy unit semi open square pseudo vector space over the pseudo ring $R = \{[0, 1), +, \times\}$.

Study questions (i) to (iv) of problem 71 for this M.

73. Let $V = \{(a_1\ a_2\ a_3 \mid a_4\ a_5 \mid a_6\ a_7\ a_8\ a_9 \mid a_{10}) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 10, +\}$ be the special fuzzy unit semi open square pseudo vector space over pseudo ring $R = \{[0, 1), +, \times\}$.

Study questions (i) to (iv) of problem 71 for this V.

74. Let $W = \left\{\left(\begin{array}{c|cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ a_{17} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \end{array}\right) \middle|\ a_i \in\right.$

$U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 32, +, \times\}$ be the fuzzy unit semi open square pseudo vector space over the pseudo ring $R = \{[0, 1), +, \times\}$.

Study questions (i) to (iv) of problem 71 for this W.

75. Let $T = \left\{\left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ a_{45} & a_{46} & a_{47} & a_{48} \end{array}\right] \middle|\ a_i \in U_F = \{(a, b) \mid a, b \in\right.$

[0, 1), $1 \leq i \leq 48$, +} be the special fuzzy unit semi open square pseudo vector space over the pseudo ring R = {[0, 1), +, ×}.

Study questions (i) to (iv) of problem 71 for this T.

76. Let $S = \left\{ \left[ \begin{array}{c|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \\ a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \\ \hline a_{49} & a_{50} & a_{51} & a_{52} & a_{53} & a_{54} \\ a_{55} & a_{56} & a_{57} & a_{58} & a_{59} & a_{60} \\ \hline a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{array} \right] \right| a_i \in U_F =$

{(a, b) | a, b ∈ [0, 1), $1 \leq i \leq 66$, +} be the unit fuzzy semi open square pseudo vector space over the pseudo ring R = {[0, 1), +, ×}.

Study questions (i) to (iv) of problem 71 for this S.

77. Let V = {($a_1$ $a_2$ $a_3$ $a_4$ $a_5$) | $a_i \in U_F$, $1 \leq i \leq 5$, +} and

$W = \left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{array} \right] \right| a_i \in U_F, 1 \leq i \leq 8, +\}$ be the

pseudo vector spaces of over the pseudo ring R = {[0, 1), +, ×}.

(i) Find the algebraic structure enjoyed by $Hom_R$ (V, W) = {T : V → W where T is a pseudo linear transformation from V to W}.

(ii) Find $Hom_R$ (W, V) = {T : W → V, the collection of all pseudo linear transformations from W to V.

(iii) Compare $Hom_R$ (W, V) with $Hom_R$ (V, W).

(iv) Find $Hom_R$ (V, V) and $Hom_R$ (W, W) and compare them and find the algebraic structure enjoyed by them.

78. Let V = {m × n matrices with entries from $U_F$} be the special fuzzy unit square pseudo vector space over the pseudo ring R = {[0, 1), +, ×}.

(i) Can linear functionals be defined on V?

(ii) Find $Hom_R(V,V)$.

(iii) Can V have the dual space?

(iv) Can we define on V an inner product?

(v) Can V ever be an inner product space?

79. Let V = $\left\{\left[\begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_{15} \end{array}\right] \middle| \; a_i \in U_F, 1 \le i \le 15, +\right\}$ be the pseudo vector space over the pseudo ring R = {[0, 1), +, ×}.

Study questions (i) to (v) of problem 78 for this V.

80. Let V = $\{(a_1 \mid a_2\ a_3 \mid a_4\ a_5\ a_6 \mid a_7\ a_8\ a_9\ a_{10}) \mid a_i \in U_F, 1 \le i \le 10, +\}$ be the special pseudo vector space over the pseudo ring R = {[0, 1), +, ×}.

Study questions (i) to (v) of problem 78 for this V.

81. Let $M = \left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \end{array} \right] \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in [0, 1)$, $1 \leq i \leq 10, +\}$ be the pseudo vector space over the pseudo ring $R = \{[0, 1), +, \times\}$.

Study questions (i) to (v) of problem 78 for this M.

82. Let $S = \left\{ \left[ \begin{array}{c|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \\ \hline a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} \end{array} \right] \right|$ $a_i \in U_F$, $1 \leq i \leq 48, +\}$ be the special fuzzy unit semi open square super matrix pseudo vector space over the pseudo ring $R = \{[0, 1), +, \times\}$.

Study questions (i) to (v) of problem 78 for this S.

83. Obtain any special properties enjoyed by strong special pseudo vector space over the unit square pseudo ring $R = \{U_F, +, \times\}$.

84. Let $S = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_6 \\ a_7 & a_8 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{18} \\ a_{19} & a_{20} & \dots & a_{24} \\ a_{25} & a_{26} & \dots & a_{30} \end{bmatrix} \middle| \, a_i \in U_F, 1 \leq i \leq 30, + \right\}$ be

the special fuzzy unit semi open square pseudo strong vector space over the pseudo ring $R = \{(a, b) \mid a, b \in [0, 1), +, \times\}$.

(i) What is dimension of S over R?

(ii) Write S as a direct sum of subspaces.

(iii) Prove S has atleast ${}_{30}C_1 + {}_{30}C_2 + \dots + {}_{30}C_{29}$ number of strong special pseudo subspaces.

(iv) Let $M = \left\{ \begin{bmatrix} a_1 & a_2 & \dots & a_6 \\ a_7 & a_8 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{18} \\ a_{19} & a_{20} & \dots & a_{24} \\ a_{25} & a_{26} & \dots & a_{30} \end{bmatrix} \middle| \, a_i \in U_F = \{(a, b) \mid a, b \in [0, 0.5), 1 \leq i \leq 30\} \right\} \subseteq S$; will M be a strong pseudo subspace of S?

$$\text{(v) } N = \left\{ \begin{bmatrix} (0.3,0.8) & (0.1,0.9) & (0.4,0.1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \right\} \subseteq$$

S be a subset. Find $N^{\perp}$. Is $N^{\perp}$ a pseudo strong subspace of S?

(vi) Can S have a finite basis?

(vii) Can a strong pseudo vector space of S be finite dimensional?

85. Let $V = \{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8) \mid a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 8, +\}$ be the special fuzzy unit semi open square strong special pseudo vector space over the pseudo ring $R = \{U_F = \{\{(a, b) \mid a, b \in [0, 1), +, \times\}$.

Study questions (i) to (vii) of problem 84 for this V.

86. Let $T = \left\{ \begin{bmatrix} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \end{bmatrix} \right|$ $a_i \in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 10, +\}$ be the special pseudo strong vector space over the pseudo ring $R = \{U_F, +, \times\}$.

Study questions (i) to (vii) of problem 84 for this T.

87. Let V = {(a, b, c, d, e) | a, b, c, d, e ∈ (x, y) | x, y ∈ [0, 1), +} be the pseudo strong vector space over the pseudo ring R = {$U_F$, +, ×}.

Study questions (i) to (vii) of problem 84 for this V.

88. Let M = $\left\{\begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \\ a_{31} & a_{32} & \ldots & a_{40} \end{bmatrix} \right|$ $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 40, +} be the pseudo vector space over the pseudo ring R = {$U_F$, +, ×}.

Study questions (i) to (vii) of problem 84 for this M.

89. Let $W_1$ = {($a_1$, $a_2$, $a_3$, $a_4$, $a_5$) | $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 5, min} be the fuzzy unit semi open square semivector space over the semiring S = {$U_F$, min, max} with product min.

(i) Can W be finite dimensional of $W_1$ over S?
(ii) Can $W_1$ have subspaces which are orthogonal subspaces?

90. Let $W_2$ = {($a_1$, $a_2$, $a_3$, $a_4$, $a_5$) | $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 5, min} be the fuzzy unit semi open square semivector space over the semiring S = {$U_F$, min, max} with product min.

(i) Study questions (i) to (ii) of problem 89 for this $W_2$.
(ii) Compare $W_1$ and $W_2$ of problems 88 and 89.

91. Let $W_3$ = {($a_1$, $a_2$, $a_3$, $a_4$, $a_5$) | $a_i \in U_F$ = {(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 5, min} be the fuzzy unit square semivector

space over the semiring S = {$U_F$, min, max} with product min.

(i) Study questions (i) to (ii) of problem 89 for this $W_3$.
(ii) Compare $W_1$ and $W_2$ with $W_3$ given in problems 88, 89 and 90 respectively.

92. Let T = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{14} \\ a_{15} \end{bmatrix} \middle|\right.$ $a_i \in U_F = \{(a, b) \mid a, b \in [0, 1),$

$1 \leq i \leq 15$, max} be the special fuzzy unit square semivector space over the semiring {$U_F$ , min, max}.

(i) Study questions (i) to (ii) of problem 90 for this T.
(ii) Construct four types of semivector spaces over S = {$U_F$, min, max} and compare them.

93. Let W = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \end{bmatrix} \middle|\right.$ $a_i \in U_F = \{(a, b) \mid$

$a, b \in [0, 1)$, $1 \leq i \leq 25$, min} be the semivector space over the semivector space {$U_F$ , min, max} the semiring under min.

Study questions (i) to (iii) of problem 84 for this W.

94. Let W = $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{bmatrix} \right|\right.$ $a_i \in U_F = \{(a, b) \mid$

a, b ∈ [0, 1), 1 ≤ i ≤ 35, max} be the special unit fuzzy square semivector space over the semiring {$U_F$, min, max} under the max operation.

Study questions (i) to (iii) of problem 84 for this W.

95. Let in W of problem 92 be replaced by min operation; Study questions (i) to (iii) of problem 92 for this new semi vector space.

96. Let M = $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \dots & \dots & \dots & \dots & \dots & a_{14} \end{bmatrix} \right|\right.$ $a_i \in U_F =$

{(a, b) | a, b ∈ [0, 1), 1 ≤ i ≤ 14, max} be the special unit fuzzy square semivector space over the semiring {$U_F$, min, max} under the max operation.

Study questions (i) to (iii) of problem 84 for this M.

97. Let T = $\left\{\left[\begin{array}{c|cccc|cc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ \hline a_{10} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{18} \\ a_{19} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{27} \\ a_{28} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{36} \\ a_{37} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{45} \\ \hline a_{46} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{54} \\ a_{55} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{63} \end{array}\right] \middle| \; a_i\right.$

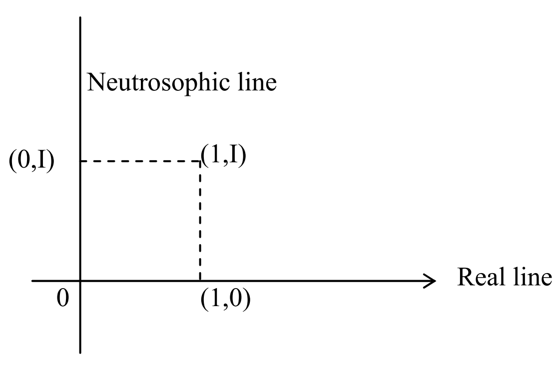
$\in U_F = \{(a, b) \mid a, b \in [0, 1), 1 \leq i \leq 63, \max\}$ be the special unit fuzzy square semivector space over the semiring $S = \{U_F, \min, \max\}$ under the max operation.

Study questions (i) to (iii) of problem 87 for this T.

Chapter Two

# FUZZY NEUTROSOPHIC SEMIGROUPS AND GROUPS USING $U_N = \{(a+bI) \mid a,b \in [0,1)\}$

In this chapter we for the first time introduce single binary operation on the unit fuzzy neutrosophic semi open square given by the following diagram.




This plane will also be known as neutrosophic plane.

$U_N = \{a + bI \mid a, b \in [0, 1)\}$ denotes the fuzzy neutrosophic semi open square.

We perform algebraic operations on $U_N$. Other types of operations also have been performed on $U_N$.

Clearly $U_N = \{a + bI \mid a, b \in [0, 1); I^2 = I$ is the indeterminate$\}$. This $U_N$ will be known as fuzzy neutrosophic semi open unit square.

**DEFINITION 2.1:** *Let $U_N = \{a + bI \mid a, b \in [0, 1), I^2 = I\}$ be the unit fuzzy neutrosophic semi open square.*

*Define product $\times$ on $U_N$ as follows for $x = a + bI$ and $y = c + dI$ in $U_N$.*

$x \times y = (a + bI) \times (c + dI)$

$= ac + (bc + ad + bd) I.$

*If $ad + bd + bc = t$ and if $t \geq 1$ put*

$ad + bd + bc = t - 1;$

*if $t < 1$ then $ad + bd + bc = t$; $\{U_N, \times\}$ is defined as the unit semi open square fuzzy neutrosophic semigroup.*

We will illustrate this situation by some examples.

***Example 2.1:*** Let $S = \{U_N, \times\}$ be the unit semi open fuzzy neutrosophic semigroup. $S = \{U_N, \times\}$ is of infinite order and has zero divisors or not is not known.

Let $x = 0.31 + 0.23I$ and $y = 0.2 + 0.167I \in U_N$.

$x\times y = (0.31 + 0.23I) \times (0.2 + 0.16I)$

$= (0.31 \times 0.2 + 0.23 \times .2I + 0.31 \times 0.16I + 0.23I \times 0.16I)$

$= (0.062 + 0.046I + 0.0496I + 0.0368I)$

$= 0.062 + (0.0460 + 0.0496 + 0.0368)I$

$= 0.062 + (0.1324)I \in U_N$

Consider $x = (0.09\, I + 0.01)$ and $y = (0.1I + 0.9I) \in U_N$

$x \times y = (0.09I + 0.01) \times (0.1I + 0.9)$
$= (0.009I + 0.001I + 0.081I + 0.009)$
$= (0.009) + (0.091I) \in U_N$
$I \notin S = (U_N, \times)$.

Study of the unit fuzzy neutrosophic semi open square is interesting.

Using this unit fuzzy neutrosophic semi open square semigroup under product we build more algebraic structures which are illustrated by examples.

***Example 2.2:*** Let
$M = \{(a_1, a_2, a_3, a_4) \mid a_i \in U_N = \{a + bI; 1 \leq i \leq 4\}$ be the unit fuzzy neutrosophic semigroup of row matrices under product.

Clearly M has zero divisors.
If $x = (0.3 + 10.4I, 0.2 + 0.3I, 0, 0.5I)$
and $y = (0.2I, 0.4I, 0.8I, 0.3) \in M$.

$x \times y = (0.3 + 0.4I, 0.2 + 0.3I, 0, 0.5I) \times (0.2I, 0.4I, 0.8I, 0.3)$
$= (0.06I + 0.08I, 0.08I + 0.12I, 0, 0.15I)$
$= (0.14I, 0.2I, 0, 0.15I) \in M$.

This is the way product is performed on M. Let $x = (0, 0, 0.4I, 0.2 + 0.8I)$ and $y = (0.3I, 0.8 + 0.5I, 0, 0) \in M$.

We see $x \times y = (0, 0, 0, 0)$.

***Example 2.3:*** Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| a_i \in \{U_N, \times\}, \times_n, \; 1 \leq i \leq 6\} \text{ be the unit fuzzy} \right.$$

neutrosophic semi open square semigroup of column matrices. N has zero divisors.

N is of infinite order. N has subsemigroups of infinite order.

$$P_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| \; a_1 \in U_N \right\} \subseteq N \text{ is a subsemigroup of } N.$$

Infact $P_1 \cong U_N$.

$$\text{Likewise } P_2 = \left\{ \begin{bmatrix} 0 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| \; a_2 \in U_N \right\} \subseteq N,$$

$$P_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ a_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| \; a_3 \in U_N \right\} \subseteq N,$$

$$P_4 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_4 \\ 0 \\ 0 \end{bmatrix} \middle| \; a_4 \in U_N \right\} \subseteq N,$$

$$P_5 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ a_5 \\ 0 \end{bmatrix} \middle| \; a_5 \in U_N \right\} \subseteq N \text{ and}$$

$$P_6 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_6 \end{bmatrix} \middle| \; a_6 \in U_N \right\} \subseteq U_N$$

be the subsemigroups of $U_N$ each of them are of infinite order and each $P_i \cong U_N$ for $1 \leq i \leq 6$.

Further $P_i \cap P_j = \{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$; $i \neq j$, $1 \leq i, j \leq 6$. $P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = U_N$ is the direct sum of subsemigroup of $U_N$.

$$\text{Let } W_1 = \left\{\begin{bmatrix} a_1 \\ a_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_1, a_2 \in U_N, \times_n\right\} \subseteq N,$$

$$W_2 = \left\{\begin{bmatrix} 0 \\ 0 \\ a_3 \\ a_4 \\ 0 \\ 0 \end{bmatrix} \middle| a_3, a_4 \in U_N, \times_n\right\} \subseteq N \text{ and}$$

$$W_3 = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ a_5 \\ a_6 \end{bmatrix} \middle| a_5, a_6 \in U_N, \times_n\right\} \subseteq N$$

are subsemigroups of $U_N$.

We see $W_i \cap W_j = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$, $i \neq j$, $1 \leq i, j \leq 3$.

However $N = W_1 + W_2 + W_3$. This semigroup have pure neutrosophic subsemigroup and fuzzy subsemigroups which is as follows:

$$S_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} \middle| a_i \in [0, I);\ 1 \leq i \leq 6 \right\} \subseteq N \text{ and}$$

$$S_2 = \left\{ \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix} \middle| b_i \in [0, 1);\ 1 \leq i \leq 6 \right\} \subseteq N$$

are subsemigroups of infinite order.

$S_1$ is a neutrosophic semigroup and $S_2$ is a pure real semigroup.

Infact $S_2$ is not an ideal of N however $S_1$ is an ideal of N. $W_1$, $W_2$ and $W_3$ are all ideals of N and $P_1$, $P_2$, $P_3$, $P_4$, $P_5$ and $P_6$ are all ideals of N.

***Example 2.4:*** Let

$$T = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \middle| \; a_i \in \{U_N, \times\}, \times_n, \; 1 \leq i \leq 12\right\}$$

be a fuzzy neutrosophic semi open unit square semigroup under product. T is of infinite order and is commutative.

T has several subsemigroups and ideals of infinite order.

***Example 2.5:*** Let

$$S = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ \vdots & \vdots & \vdots & \vdots \\ a_{29} & a_{30} & a_{31} & a_{32} \end{bmatrix} \middle| \; a_i \in U_N, \times_n, \; 1 \leq i \leq 32\right\}$$

be the special unit neutrosophic fuzzy semi open unit square semigroup of infinite order.

S has number of subsemigroups and ideals all of them are of infinite order. S has infinite number of zero divisors and no idempotents.

S has no unit.

***Example 2.6:*** Let

$$A = \left\{\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 9, \times_n\right\}$$

be the fuzzy neutrosophic unit square semigroup of infinite order. A has no unit elements. A has no idempotents. A has subsemigroups and ideals of infinite order.

Inview of all these study we have the following theorem the proof of which is direct.

**THEOREM 2.1:** *Let $U_N = \{a + bI \mid a, b \in [0, 1), I^2 = I, \times\}$ be the special fuzzy neutrosophic unit square semigroup.*

- *(i)* $o(U_N) = \infty$.
- *(ii)* *$U_N$ is commutative.*
- *(iii)* *$U_N$ is of infinite order.*
- *(iv)* *$U_N$ has no unit.*
- *(v)* *$U_N$ has no zero divisors.*
- *(vi)* *$U_N$ has no idempotents.*
- *(vii)* *$U_N$ has pure neutrosophic fuzzy subsemigroup which is an ideal.*
- *(viii)* *$U_N$ has pure fuzzy subsemigroup which is not an ideal.*
- *(ix)* *1 and I* $\notin U_N$.

Proof is direct and hence left as an exercise to the reader.

**THEOREM 2.2:** *Let $M = \{m \times n$ matrices with entries from $U_N\}$; $\{M, \times_n\}$ is a fuzzy neutrosophic semi open unit semigroup.*

- *(i)* $|M| = \infty$.
- *(ii)* *M is commutative.*
- *(iii)* *M has infinite number of zero divisors.*
- *(iv)* *M has no idempotents.*
- *(v)* *M has no units.*
- *(vi)* *M has subsemigroups which are not ideals.*

The proof is direct hence left as an exercise to the reader.

We leave the following as an open conjectures.

**Conjecture 2.1:** Can $U_N$ have subsemigroups of finite order?
**Conjecture 2.2:** Can M have subsemigroups of finite order?
**Conjecture 2.3:** Can M have ideals of finite order?
**Conjecture 2.4:** Can $U_N$ have ideals of finite order?
**Conjecture 2.5:** Can $U_N$ have units under ×?

We now proceed onto define other types of operation max or min.

**DEFINITION 2.2:** *Let $U_N$ be the special neutrosophic fuzzy semi open unit square. Define max operation on $U_N$. $\{U_N, max\}$ is a semigroup, which is also a semilattice of infinite order.*

We will show how operations on $\{U_N, \max\}$ are performed.

Let x = 0.3 + 0.8I and y = 0.35 + 0.25I $\in U_N$.
max {x, y} = 0.35 + 0.8I $\in U_N$.

This is the way max operation is performed on $U_N$.

We see max {x, x} = x for all x $\in U_N$. max {x, 0} = x. Thus $U_N$ has no zero divisors and every element is an idempotent.

Further $U_N$ has subsemigroups of all orders from one to ∞.

Every x $\in U_N$ is a subsemigroup under max operation.

Every {x, 0} where x $\in U_N$ is a subsemigroup under max operation.

If P = {x, y} then if max {x, y} ≠ x or y and if max {x, y} = z and z ≠ x and y ≠ z then P ∪ {z} is a subsemigroup of order three. We call P ∪ {z} as the completion of the set P or the completed subsemigroup of the set P and it is denoted by $P_C$.

Let x = 0.3I + 0.45 and y = 0.35 + 0.35I $\in U_N$.
max {x, y} = {0.3I + 0.45}
= max {0.3I + 0.45, 0.35I + 0.35}

= max {0.35, 0.45} + max {0.3I, 0.35I}
= 0.45 + 0.35I.

Thus T = {x, y, 0.45 + 0.35I} is a subsemigroup of order three and T is called the completed subsemigroup of the set {x, y} of $U_N$.

Let P = {0.02 + 0.3I, 0.4 + 0.0I, 0.6 + 0.09I} $\subseteq U_N$.

Now max {0.02 + 0.3I, 0.4 + 0.0I}
= {0.4 + 0.3I}

max {0.02 + 0.3I, 0.6 + 0.009I}
= {0.6 + 0.3I}

max {0.4 + 0.0I, 0.6 + 0.009I}
= {0.6 + 0.0I}

Thus P $\cup$ {0.4 + 0.3I, 0.6 + 0.3I, 0.6 + 0.0I} is a subsemigroup of order six.

We can have subsemigroups of all orders under the max operation.

Also if we have a subset of $U_N$ which is not a subsemigroup then it can be completed to form a subsemigroup.

Inview of all these we have the following theorem.

**THEOREM 2.3:** *Let S = $\{U_N, \max\}$ be the special fuzzy neutrosophic unit square semigroup under max operation. If P = $\{x_1, \ldots, x_n \mid x_i \in U_N, 1 \leq i \leq n\} \subseteq U_N$ be only a subset then $\{P \cup \{\max \{x_i, x_j\}; i \neq j, 1 \leq i, j \leq n\} \subseteq U_N$ is the completed subsemigroup of the set P.*

Proof is direct and hence left as an exercise to the reader.

Now we give more number of semigroups using $\{U_N, \max\}$.

***Example 2.7:*** Let
M = {($a_1$, $a_2$, $a_3$, $a_4$, $a_5$) | $a_i \in U_N$, $1 \leq i \leq 5$, max} be the fuzzy neutrosophic unit square semigroup under max operation. M has no zero divisors. M has infinite number of subsemigroups. M has also ideals.

Let P = {(0.3I + 0.2, 0.4I + 0.03, 0.331 + 0.23I, 0.315 + 0.3I, 0.2015 + 0.3001I)} ∈ M. P is a subsemigroup of order one. Clearly M is also an idempotent fuzzy neutrosophic unit square subsemigroup of infinite order.

***Example 2.8:*** Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 8, \max \right\}$$

be the special fuzzy neutrosophic semi open unit square column matrix semigroup of infinite order.

***Example 2.9:*** Let

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{46} & a_{47} & a_{48} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 48, \max \right\}$$

be the semigroup matrix of infinite order. P has several subsemigroups.

Infact any subsemigroup

$$M = \left\{ \left[ \begin{matrix} a_1 & a_2 & a_3 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{matrix} \right] \middle| \; a_i \in U_N, 1 \leq i \leq 3, \max \right\}$$

is not an ideal if it contains even one zero entry in its matrix.

Thus P has several subsemigroups which are not ideals.

Let

$$S = \left\{ \left[ \begin{matrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{46} & a_{47} & a_{48} \end{matrix} \right] \middle| \; a_i \in T = \{a + bI \mid a, b \in [0.4, 1)\}, \max, \right.$$

$$\left. 1 \leq i \leq 48 \right\} \subseteq P$$

is a subsemigroup. S is an ideal of P.

Let

$$V = \left\{ \left[ \begin{matrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{46} & a_{47} & a_{48} \end{matrix} \right] \middle| \; a_i \in B = \{a + bI \mid a, b \in [0, 0.3), \max\}, \right.$$

$$\left. 1 \leq i \leq 48 \right\} \subseteq P$$

be the subsemigroup of P. Clearly V is not an ideal of P.

***Example 2.10:*** Let
W = $\{(a_1\ a_2\ a_3 \mid a_4 \mid a_5\ a_6\ a_7 \mid a_8\ a_9 \mid a_{10}) \mid a_i \in U_N, 1 \leq i \leq 10,$ max$\}$ be the special fuzzy neutrosophic unit square row super matrix semigroup under max operation. W has several subsemigroups which are not ideals. Also W has subsemigroups which are ideals. We just give an one or two ideals of W.

Take $M_1 = \{(a_1\ a_2\ a_3 \mid a_4 \mid a_5\ a_6\ a_7 \mid a_8\ a_9 \mid a_{10}) \mid a_i = a_j + b_jI$ where $a_j, b_j \in [0.5, 0.932), 1 \leq j \leq 10$, and $1 \leq i \leq 10\} \subseteq W$.

We see $M_1$ is a subsemigroup which is also an ideal of W.

Consider $N_1 = \{(a_1\ a_2\ a_3 \mid a_4 \mid a_5\ a_6\ a_7 \mid a_8\ a_9 \mid a_{10}) \mid a_i = c_i + d_iI, c_i, d_i \in [0, 0.342), 1 \leq i \leq 10\} \subseteq W$, $N_1$ is a subsemigroup which is clearly not an ideal of W.

Thus W has infinite number of subsemigroups which are ideals and infinitely many subsemigroups which are not ideals.

Thus these infinite fuzzy neutrosophic unit square semigroup leads to both ideals as well as subsemigroups which are not ideals.

***Example 2.11:*** Let

$$M = \left\{ \left. \begin{bmatrix} a_1 \\ a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ \hline a_6 \\ \hline a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ a_{12} \end{bmatrix} \right| a_i \in U_N, 1 \leq i \leq 12 \right\}$$

be the fuzzy neutrosophic unit square matrix semigroup of super column row matrix under the max operation.

M is commutative.

M has infinitely many subsemigroups of finite order.

Let

$$X = \left\{ \left[ \begin{array}{c} 0.01 \\ 0.2I + 0.3 \\ \hline 0 \\ 0 \\ 0 \\ \hline 0.71 + 0.2I \\ \hline 0.25 + 0.8I \\ 0.47 + 0.74I \\ \hline 0 \\ 0 \\ 0 \\ 0 \end{array} \right] \right\} \subseteq M,$$

{X, max} is a subsemigroup of order one.

Infact M has infinitely many subsemigroups of order one.

Consider

$$T = \left\{ \begin{bmatrix} 0.3I + 0.2 \\ 0.4I + 0.8 \\ \hline 0.3 + 0.7I \\ 0.4 + 0.5I \\ 0 \\ \hline 0.4I + 0.2 \\ \hline 0 \\ 0 \\ \hline 0.3I \\ 0.2 \\ 0.8I + 0.7 \\ 0.9 + 0.23I \end{bmatrix}, \begin{bmatrix} 0.2I + 0.1 \\ 0.04I + 0.08 \\ \hline 0.03 + 0.07I \\ 0.04 + 0.05I \\ 0 \\ \hline 0.2I + 0.1 \\ \hline 0 \\ 0 \\ \hline 0.2I \\ 0.1 \\ 0.6I + 0.6 \\ 0.8 + 0.13I \end{bmatrix} \right\} \subseteq M,$$

T is a subsemigroup under the max operation.

Infact T is not an ideal. We have infinite number of subsemigroups of infinite order in M.

Clearly none of the subsemigroups of finite order in M are ideals of M.

***Example 2.12:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ \vdots & \vdots & \vdots \\ a_{13} & a_{14} & a_{15} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 15, \max \right\}$$

be the special fuzzy neutrosophic unit square matrix semigroup under max operation.

M has infinite number of subsemigroups and ideals.

***Example 2.13:*** Let

$$P = \left\{ \left[ \begin{array}{cccccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ \hline a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ \hline a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & \cdots & a_{36} \\ \hline a_{37} & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & a_{48} \\ a_{49} & \cdots & \cdots & \cdots & \cdots & a_{54} \\ \hline a_{55} & \cdots & \cdots & \cdots & \cdots & a_{60} \\ \hline a_{61} & \cdots & \cdots & \cdots & \cdots & a_{66} \end{array} \right] \middle| \; a_i \in U_N, 1 \leq i \leq 66, \max \right\}$$

be the special fuzzy neutrosophic semigroup of infinite order. P has infinite number of subsemigroups as well as ideals.

***Example 2.14:*** Let B = $\{U_N, \min\}$ be the special neutrosophic fuzzy unit square under the min operation. B is of infinite order. We see B has no zero divisors and every element is an idempotent. Every singleton is a subsemigroup and is not an ideal of B.

Let x = {0.34I + 0.2231} $\subseteq$ B, x is a subsemigroup of B.
Let x = 0.4 + 0.224I and y = 0.3 + 0.771I $\in$ B.
min {x, y} = min {0.4, 0.3} + min {0.224I, 0.771I}
= 0.3 + 0.224I $\in$ B.

Hence C = {x, y, 0.3 + 0.224I} is a subsemigroup of order three.

Let x = 0.4 + 0.74I and y = 0.3 + 0.58I $\in$ B.

min {x, y} = min {0.4, 0.3} + min {0.74I, 0.58I}

= 0.3 + 0.58I ∈ B
= y.

We see P = {x, y} ⊆ B is a subsemigroup of order two we see P is not an ideal. If X = $\{x_1, x_2, \ldots, x_n\} \subseteq U_N$ with min $\{x_i, x_j\} \neq x_k$ for any k; $1 \leq k \leq n$, $\{i \neq j\}$ then we can define extended or complete x to be a subsemigroup called the completed subsemigroup of the subset X.

We will illustrate this situation by some examples.

Let X = $\{x_1 = 0.1 + 0.2I, x_2 = 0.7 + 0.004I, x_3 = 0.6I + 0.5, x_4 = 0.9I + 0.3, x_5 = 0.94I + 0.227\} \subseteq U_N$ be the subset of $U_N$. Clearly {X, min} is not a subsemigroup.

But we complete X into a subsemigroup.

The proof is as follows:

min $\{x_1, x_2\}$ = min {0.01 + 0.2I, 0.7 + 0.004I}
= {0.004I + 0.01} = $Z_1$

min $\{x_1, x_3\}$ = min {0.1 + 0.2I, 0.6I + 0.5}
= {0.1 + 0.2I} = $Z_2$

min $\{x_1, x_4\}$ = min {0.1 + 0.2I, 0.3 + 0.9I}
= {0.1 + 0.2I} = $Z_3$

min $\{x_1, x_5\}$ = min {0.1 + 0.2I, 0.227 + 0.94I}
= {0.1 + 0.2I} = $Z_4$

min $\{x_2, x_3\}$ = min {0.7 + 0.004I, 0.6I + 0.5}
= {0.004I + 0.5} = $Z_5$

min $\{x_2, x_4\}$ = min {0.7 + 0.04I, 0.9I + 0.3}
= {0.3 + 0.004I} = $Z_6$

min $\{x_2, x_5\}$ = min {0.7 + 0.004I, 0.94I + 0.227}
= 0.004I + 0.227} = $Z_7$

min $\{x_3, x_5\}$ = min $\{0.6I + 0.5, 0.3 + 0.9I\}$
= $\{0.3 + 0.6I\} = Z_8$

min $\{x_3, x_5\}$ = min $\{0.5 + 0.6I, 0.94I + 0.227\}$ and
= $0.227 + 0.6I\} = Z_9$

min $\{x_4, x_5\}$ = min $\{0.3 + 0.9I, 0.94I + 0.227\}$
= $\{0.227 + 0.9I\} = Z_{10}$.

Thus $y = \{Z_1, Z_2, \ldots, Z_{10}, x_1, x_2, x_3, x_4, x_5\} \subseteq U_N$ is the completed subsemigroup of X. This is the way subsets are completed into subsemigroups.

***Example 2.15:*** Let
$V = \{(x_1, x_2, x_3, x_4, x_5)$ where $x_i \in U_N$ ; $1 \leq i \leq 5\}$ be the special fuzzy neutrosophic unit square semigroup under the min operation. V is a semigroup of infinite order which is idempotent and has zero divisors.

Every singleton element is a subsemigroup of V and is not an ideal of V. Subsets in V can be completed to get subsemigroups. If T are of finite order certainly T is not an ideal of V.

Let
$S = \{(x_1, x_2, x_3, x_4, x_5) \mid x_i \in [0, 0.5)$, min, $1 \leq i \leq 5\} \subseteq V$ be the subsemigroup of V. Certainly S is of infinite order. Further S is an ideal of V.

Let
$S_a = \{(x_1, x_2, x_3, x_4, x_5) \mid x_i \in [0, a), 0 < a < 0.7 < 1$, a, a fixed value $1 \leq i \leq 5$, min$\} \subseteq V$, clearly $S_a$ is a subsemigroup of infinite order which is also an ideal of V. We have infinitely many such ideals in V.

Consider $P = S = \{(x_1, x_2, x_3, x_4, x_5) \mid x_i \in [0.3, 0.5)$, min, $1 \leq i \leq 5\} \subseteq V$. P is only a subsemigroup of V and is not an ideal of V. Infact V has infinitely many subsemigroups which are not ideals of V.

***Example 2.16:*** Let

$$S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} \right| a_i \in U_N, 1 \leq i \leq 9, \min\}$$

be the special fuzzy neutrosophic unit square semigroup under min operation of infinite order.

{S, min} is a semilattice, that is {S, min} has zero divisors, ideals and subsemigroups which are not ideals.

$$\text{Let } x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.3+0.4I \\ 0 \\ 0.I \\ 0.7+0.6I \\ 0.5I+0.9 \\ 0 \\ 0.3I \end{bmatrix} \text{ and } y = \begin{bmatrix} 0.3I \\ 0.2I \\ 0.3+5I \\ 0.7+0.2I \\ 0 \\ 0 \\ 0 \\ 0.7I+0.9 \\ 0 \\ 0 \end{bmatrix} \text{ be in S.}$$

We see min {x, y} = $\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}$.

Thus S has infinitely many zero divisors.

Now every singleton element in S is a subsemigroup and is not an ideal of S.

Consider $P_1 = \left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_i \in U_N, 1 \leq i \leq 3, \min\right\} \subseteq S$;

$P_1$ is a subsemigroup of infinite order.

$$P_2 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_1 \\ a_2 \\ a_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| a_i \in U_N, 1 \le i \le 3, \min \right\} \subseteq S;$$

be a subsemigroup of infinite order.

$$P_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \middle| a_i \in U_N, 1 \le i \le 3, \min \right\} \subseteq S$$

be a subsemigroup of infinite order.

$$P_i \cap P_j = \{\begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}\}, 1 \leq i, j \leq 3.$$

For every $x \in P_1$ and $y \in P_2$ we see $\min\{x, y\} = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$.

For every $x \in P_1$ and $y \in P_3$ we see $\min\{x, y\} = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}$.

$$\min \{x, y\} = \{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\} \text{ for } x \in P_2 \text{ and } y \in P_2.$$

Thus we have zero divisors we call these types of subsemigroups or ideals as annihilating ideals of S.

We see $S = P_1 + P_2 + P_3$ is a direct sum.

Take $R_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \; a_i = b_i + c_iI;\ b_i, c_i \in [0.3, 1),\ 1 \leq i \leq 9 \right\} \subseteq S,$

$R_1$ is only a subsemigroup but is not an ideal of S.

We have infinite number of subsemigroups which are not ideals of S.

$$W_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_9 \end{bmatrix} \middle| \; a_i \in [0, 0.7),\ 1 \leq i \leq 9, \min \right\} \subseteq S;$$

$W_1$ is a subsemiring which is also an ideal of S.

We have infinite number of ideals of S each of infinite order.

$$V_1 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ 0 \\ 0 \\ 0 \\ 0 \\ a_4 \end{bmatrix} \middle| \; a_i = b_i + d_iI, b_i, d_i \in [0.3, 0.8), 1 \leq i \leq 4, \right.$$

$$\left. a_5 = a + bI, a, b \in [0, 0.4) \right\} \subseteq S;$$

$V_1$ is a subsemigroup under min operation.

Clearly $V_1$ is not an ideal of S.

$$V_2 = \left\{ \begin{bmatrix} 0 \\ 0 \\ a_1 \\ a_2 \\ a_3 \\ 0 \\ 0 \\ 0 \\ a_4 \end{bmatrix} \middle| \; a_i = b_i + d_iI, b_i, d_i \in [0.2, 0.7), 1 \leq i \leq 3, \right.$$

$a_4 = a + bI$, a, b $\in$ [0, 0.8I)} $\subseteq$ S; $V_2$ is a subsemigroup of S which is not an ideal of S.

We see S has infinite number of subsemigroups which are not ideals of S.

***Example 2.17:*** Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \, a_i \in U_N, 1 \leq i \leq 9, \min \right\}$$

be the special fuzzy neutrosophic semi open unit square semigroup under min operation.

S has infinite number of zero divisors. Has no units. Has several ideals and several subsemigroups which are not ideals.

For

$$A = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \, a_i = b_i + c_iI, b_i, c_i \in [0.5, 1), 1 \leq i \leq 9 \right\} \subseteq S$$

be a subsemigroup of S. A is only a subsemigroup and is not an ideal.

$$\text{For if } y = \begin{bmatrix} 0 & 0 & 0 \\ 0.1 & 0.2+0.3I & 0.1+0.I \\ 0.3+0.4I & 0.3I & 0.2 \end{bmatrix} \in S.$$

$$\text{and } x = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \in A.$$

min $\{x, y\} = y \notin A$.

So A is not an ideal of S only a subsemigroup of S.

Let

$$B = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \middle| \; a_i = c_i + d_iI, \; c_i, d_i \in [0, 0.6), \right.$$

$$\left. 1 \leq i \leq 9, \min \right\} \subseteq S.$$

Clearly B is a subsemigroup as well as an ideal of S.

Infact S has infinite number of ideals.

***Example 2.18:*** Let

$$H = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \middle| \; a_i \in U_N, \; 1 \leq i \leq 33, \min \right\}$$

be the special fuzzy neutrosophic unit square matrix semigroup under min operation. H is of infinite order. H has infinite number of subsemigroups which are not ideals and also H has infinite number of ideals.

***Example 2.19:*** Let M = $\{(a_1 \mid a_2\ a_3 \mid a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9) \mid a_i \in U_N$, $1 \leq i \leq 9, \min\}$ be the special fuzzy neutrosophic semi open unit square super row matrix semigroup under the min operation. M is of infinite order. M has ideals and subsemigroups.

Infact all ideals of M are of infinite cardinality however M has subsemigroups of order 1 or 2 or 3 or 4 and so on.

***Example 2.20:*** Let

$$P = \left\{ \left[ \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ \hline a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \\ \hline a_{14} \end{array} \right] \right| a_i \in U_N, 1 \leq i \leq 14, \min\}$$

be the special fuzzy neutrosophic semi open unit square semigroup of super column matrices.

P is of infinite order. P is a semilattice.

P has infinite number of finite subsemigroups which are not ideals of P.

Infact all ideals of P are of infinite order. P has infinite number of zero divisors and idempotents.

***Example 2.21:*** Let

$$P = \left\{ \left[ \begin{array}{cc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ \hline a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ \hline a_{46} & a_{47} & a_{48} & a_{49} & a_{50} \end{array} \right] \middle| \; a_i \in U_N, 1 \leq i \leq 50, \min \right\}$$

be the special fuzzy neutrosophic unit semi open square super matrix semigroup of super matrices under min operation of infinite order.

M has infinite number of zero divisors and idempotents. Every ideal of M is of infinite order.

M has subsemigroups of finite order as well as infinite order.

Now having seen all types of special neutrosophic fuzzy unit square semigroups under different operations we now proceed onto define the notion of groups under addition.

We will also illustrate this situation by some examples.

Let $U_N = \{a + bI \mid a, b \in [0, 1)\}$ be the fuzzy neutrosophic unit square.

We define addition modulo 1 and I on $U_N$ as follows:

Let $x = 0.3 + 0.5I$ and $y = 0.7 + 0.5I \in U_N$.

$$x + y = (0.3 + 0.5I) + (0.7 + 0.5I)$$
$$= (0.3 + 0.7) + (0.5I + 0.5I)$$
$$= 1.0 + I.0 \text{ (mod 1 and I)}$$
$$= 0 + 0I \in U_N.$$

This is the way '+' operation is performed.

Let $x = 0.001 + 0.032I$ and $y = 0.216 + 0.601I \in U_N$

$$x + y = (0.001 + 0.032I) + (0.216 + 0.601I)$$
$$= 0.217 + 0.633I \in U_N.$$

Let $x = 0.6I + 0.884$

and $y = 0.734I + 0.652 \in U_N$

$$x + y = 0.884 + 0.6I + 0.652 + 0.734I$$
$$= (0.884 + 0.652) + (0.6I + 0.734I)$$
$$= 1.536 \text{ (mod 1)} + (1.334I) \text{ (mod I)}$$
$$= 0.536 + 0.334I \in U_N.$$

It is easily verified $U_N$ under + is closed and + is an associative operation on $U_N$.

Further for every $x \in U_N$ we have a unique $y \in U_N$ such that $x + y = 0$. Thus every $x \in U_N$ has a unique inverse with respect to + modulo 1 and I.

$(U_N, +)$ is defined as the special fuzzy neutrosophic unit square group under addition.

$G_N = \{U_N, +\}$ is a commutative group of infinite order.

We see $0 = 0 + 0I$ acts as the additive identity.

It is an interesting problem to find finite order subgroups in $G_N$ for in our opinion all subgroups in $G_N$ are of infinite order as well as finite order.

We see $A = \{0.5 + 0.5I, 0\} \subseteq G_N$ is a subgroup of order two. $B = \{0.5, 0\} \subseteq G_N$ is subgroup of order two.

P = {0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9} $\subseteq G_N$ is again a subgroup of order 10.

R = {0, 0.01, 0.02, …, 0.99} $\subseteq G_N$ is again a subgroup of $G_N$ of order 100.

Thus $G_N$ has infinite number of subgroups of finite order. Take $S_1$ = {0.5I, 0} $\subseteq G_N$ is again a subgroup of order 2.

Let $S_2$ = {0, 0.I, 0.2I, 0.3I, 0.4I, 0.5I, 0.6I, 0.7I, 0.8I, 0.9I} $\subseteq$ $G_N$ is a subgroup of order 10.

$S_3$ = {0, 0.2I, 0.4I, 0.6I, 0.8I} $\subseteq G_N$ is a subgroup of $G_N$ of order 5 and so on.

Now having seen examples of finite subgroups of $G_N$ we proceed onto construct more groups using $G_N$.

***Example 2.22:*** Let S = $\{(a_1, a_2, a_3) \mid a_i \in U_N ; +, 1 \leq i \leq 3\}$ be the special fuzzy neutrosophic group of row matrices with (0, 0, 0) as the additive identity.

Let x = (0.3 + 0.5I, 0.8 + 0.7I, 0.11 + 0.37I) $\in$ S
x + x = (0.6 + 0, 0.6 + 0.4I, 0.22 + 74I} $\in$ S.

The inverse of x is y = (0.7 + 0.5I, 0.2 + 0.3I, 0.89 + 0.63I) $\in$ S.

We see x + y = (0, 0, 0).

Clearly every x in S has a unique inverse in S.

S has subgroups of infinite order as well as finite order.

Take $H_1 = \{(a_1, a_2, a_3) \mid a_i \in \{0.5, 0\}, 1 \leq i \leq 3\} \subseteq S$, $H_1$ is a subgroup of finite order.

$H_2 = \{(a_1, a_2, a_3) \mid a_i \in \{0.5, 0.5I, 0\}, 1 \leq i \leq 3\} \subseteq S$ is a subgroup of finite order.

$H_3 = \{(a_1, a_2, a_3) \mid a_i \in \{0, 0.2, 0.4, 0.6, 0.8\}, 1 \leq i \leq 3\} \subseteq S$ is a subgroup of finite order.

$H_4 = \{(a_1, a_2, a_3) \mid a_i \in \{0, 0.2 + 0.2I, 0.4 + 0.4I, 0.6 + 0.6I, 0.8 + 0.8I\}, 1 \leq i \leq 3\} \subseteq S$ is a subgroup S of finite order.

We have several such subgroups.

***Example 2.23:*** Let $P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} \middle| \, a_i \in U_N, 1 \leq i \leq 9, \min \right\}$ be the special fuzzy neutrosophic group of column matrices.

P is of infinite order; $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in P$ acts as the additive identity of P.

$$\text{Let } x = \begin{bmatrix} 0.3I+0.7 \\ 0 \\ 0.2+0.5I \\ 0.8I \\ 0.5 \\ 0.7I+0.1 \\ 0.85+0.16I \\ 0.12+0.07I \\ 0.9I \end{bmatrix} \in P.$$

We have a unique y ∈ P such that $x + y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ we see

$$y = \begin{bmatrix} 0.7I+0.3 \\ 0 \\ 0.8+0.5I \\ 0.2I \\ 0.5 \\ 0.3I+0.9 \\ 0.15+0.84I \\ 0.88+0.93I \\ 0.I \end{bmatrix} \in P.$$

Let

$$M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} \middle| \; a_i \in \{0.5, 0, 0.5I, 0.5+0.5I\}, 1 \leq i \leq 9, + \right\} \subseteq P$$

be the subgroup of P.

Cleary $|M| < \infty$.

Let

$$M_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \middle| \; a_1 \in U_N, + \right\} \subseteq P,$$

$M_1$ is a subgroup of infinite order.

$$M_2 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{bmatrix} \middle| \; a_i \in \{0, 0.2, 0.2I, 0.4, 0.4I, 0.6, 0.6I, 0.8, 0.8I, \right.$$

0.2+0.4I, 0.2+0.2I, 0.2 + 0.6I, 0.2 + 0.8I, …, 0.8 + 0.8I, $1 \leq i \leq 9, +\} \subseteq P$, $M_2$ is a subgroup of finite order in P.

We can have several such subgroups of finite order in P.

***Example 2.24:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 24, + \right\}$$

be the fuzzy neutrosophic unit square matrix group of infinite order.

M has subgroups of finite order as well as infinite order.

***Example 2.25:*** Let T = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right|$ $a_i \in U_N$,

$1 \leq i \leq 16, +\}$ be the group of infinite order.

$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is the additive identity in T.

T has infinite number of subgroups of both finite and infinite order.

***Example 2.26:*** Let

$$W = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \end{bmatrix} \right| a_i \in U_N, 1 \leq i \leq 12, +\}$$

be the group of infinite order.

$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ be the additive identity of W.

***Example 2.27:*** Let M = $\{(a_1\ a_2 \mid a_3\ a_4\ a_5\ a_6 \mid a_7\ a_8\ a_9 \mid a_{10}\ a_{11} \mid a_{12}) \mid a_i \in U_N, 1 \leq i \leq 12, +\}$ be the fuzzy neutrosophic unit square super row matrix group of infinite order.

(0 0 | 0 0 0 0 | 0 0 0 | 0 0 | 0) is the additive identity in M.

M has infinite number of subgroups of both finite order as well as infinite order.

***Example 2.28:*** Let

$$T = \left\{ \left[ \begin{array}{cc|c|cc} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ \hline a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{array} \right] \middle| a_i \in U_N, 1 \le i \le 35, + \right\}$$

be the special fuzzy neutrosophic super matrix group of infinite order.

$$\left[ \begin{array}{cc|c|cc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array} \right]$$ acts as the additive identity of M.

***Example 2.29:*** Let

$$W = \left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \end{array} \right] \middle| \; a_i \in U_N, 1 \leq i \leq 32, + \right\}$$

be a special fuzzy neutrosophic super column matrix group of infinite order.

W has subgroups of finite order given by

$$V = \left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \end{array} \right] \middle| \; a_i \in \{0.2, 0.4, 0.6, 0.8, 0\} \subseteq U_N, \right.$$

$1 \leq i \leq 32, +\} \subseteq W$ is a subgroup of infinite order.

$$B = \left\{ \left[ \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \right| a_i \in \{0, 0.5, 0.5I, 0.5 + 0.5I\}$$

$\subseteq U_N$, $1 \leq i \leq 16, +\} \subseteq W$ be the subgroup of W of finite order.

***Example 2.30:*** Let

$$M = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \end{pmatrix} \right| a_i \in U_N, 1 \leq i \leq 8, +\}$$

be the group of infinite order.

$$P = \left\{ \begin{pmatrix} 0 & 0 & a_1 & a_2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \right| a_1\ a_2 \in U_N, +\} \subseteq M$$

is a subgroup of infinite order.

**THEOREM 2.4**: *Let $G_N = (U_N, +)$ be the special fuzzy neutrosophic unit square group of infinite order.*

*(i) $G_N$ has finite order subgroups.*
*(ii) $G_N$ has infinite order subgroups.*

The proof is direct hence left as an exercise to the reader.

**THEOREM 2.5:** *Let*
*$M = \{n \times s$ matrices with entries from $U_N, +\}$ be the special fuzzy neutrosophic unit square matrix group.*

*(i)* *M has infinite number of subgroups of finite order.*
*(ii)* *M has subgroups of infinite order.*

Proof is direct hence left as an exercise to the reader. Group homomorphism can be defined for these groups as in case of usual groups.

As all these groups are of infinite order several properties of finite groups cannot be extended to these class of groups.

We present the following problems for this chapter.

**Problems**

1. Find some special and interesting properties enjoyed by the unit semi open fuzzy neutrosophic square $U_N$.

2. $(U_N, \times)$ is a semigroup; enumerate all the special properties associated with it.

3. Prove $(U_N, \times)$ the semigroup has all its subsemigroups to be of infinite order.

4. Prove $(U_N, \times)$ the semigroup has subsemigroups generated by a single element.

5. Let $(U_N, \times)$ be the semigroup. Prove $P = \{\langle 0.3 \rangle\}$ is a subsemigrousp of $(U_N, \times)$.

6. Is $\langle 0.31 + 0.2I \rangle = R \subseteq \{U_N, \times\}$ generate a subsemigroup of infinite order?

7. Is R in problem (6) cyclic?

8. Prove P in problem (5) is cyclic and is of infinite order?

9. Prove P in problem (5) is not an ideal of $\{U_N, \times\}$.

10. Is R in problem (6) an ideal of $\{U_N, \times\}$?

11. Prove $\{U_N, \times\}$ has infinite number of subsemigroups which are cyclic.

12. Can a cyclic subsemigroup in $(U_N, \times)$ be an ideal of $\{U_N, \times\}$?

13. Let A = $\langle 0.3 + 0.2I, 0.7 + 0.4I \rangle \subseteq \{U_N, \times\}$ be the subsemigroup.

    (i) Can A be a cyclic subsemigroup?
    (ii) Can A be an ideal of $\{U_N, \times\}$?

14. Can $\{U_N, \times\}$ have zero divisors?

15. Can $\{U_N, \times\}$ have idempotents?

16. Can $\{U_N, \times\}$ be a Smarandache semigroup?

17. Can $\{U_N, \times\}$ have S-ideals?

18. Can $\{U_N, \times\}$ have S-subsemigroups?

19. Obtain any other special or interesting property enjoyed by $\{U_N, \times\}$.

20. Can $\{U_N, \times\}$ have units?

21. Let M = $\{(a_1, a_2, \ldots, a_{10}) \mid a_i \in \{U_N, \times\}, 1 \leq i \leq 10\}$ be the fuzzy neutrosophic unit semi open square row matrix semigroup of infinite order.

    (i) Prove M has zero divisors.
    (ii) Can M have S-zero divisors?
    (iii) Can M have idempotents?
    (iv) Can M have S-idempotents?
    (v) Can M have units?
    (vi) Can M have S-units?

(vii) Can M have finite order subsemigroups?
(viii) Is every subsemigroup of M is of infinite order?
(ix) Can M have ideals of finite order?
(x) Is every subsemigroup of M an ideal of M?
(xi) Does there exists subsemigroup in M which are not ideals of M? Justify with examples.
(xii) If $U_N$ is replaced by the subsquare of the unit square $P = \{a + bI \mid a, b \in [0, 0.8)\} \subseteq U_N$, is P a subsemigroup?

22. Prove $M = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{15} \end{bmatrix} \middle| a_i \in U_N, 1 \le i \le 15 \right\}$ under $\times_n$ is a special fuzzy neutrosophic unit square semigroup.

Study questions (i) to (xii) of problem (21) for this M.

23. Let $T = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_8 \\ a_9 & a_{10} & \ldots & a_{16} \\ a_{17} & a_{18} & \ldots & a_{24} \end{bmatrix} \middle| a_i \in U_N, 1 \le i \le 24 \right\}$ be the special fuzzy neutrosophic unit square semigroup under product $\times_n$.

Study questions (i) to (xii) of problem (21) for this T.

24. Prove $U_N = \{a + bI \mid a, b \in [0, 1)\}$ can have infinite number of subsquares and subrectangles.

25. Let T = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & \dots & \dots & \dots & a_{10} \\ a_{11} & \dots & \dots & \dots & a_{15} \\ a_{16} & \dots & \dots & \dots & a_{20} \\ a_{21} & \dots & \dots & \dots & a_{25} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 25\right\}$ be

the special fuzzy neutrosophic unit square semigroup under product $\times_n$.

Study questions (i) to (xii) of problem (21) for this S.

26. Let V = $\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_8 \\ a_9 & a_{10} & \dots & a_{16} \\ a_{17} & a_{18} & \dots & a_{24} \\ a_{25} & a_{26} & \dots & a_{32} \\ a_{33} & a_{34} & \dots & a_{40} \\ a_{41} & a_{42} & \dots & a_{48} \\ a_{49} & a_{50} & \dots & a_{64} \\ a_{65} & a_{66} & \dots & a_{72} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 72\right\}$ be the

special fuzzy neutrosophic unit square semigroup under product ×.

Study questions (i) to (xii) of problem (21) for this V.

27. Let W = $\{(a_1 \mid a_2\ a_3\ a_4 \mid a_5\ a_6 \mid a_7\ a_8\ a_9) \mid a_i \in U_N, 1 \leq i \leq 72\}$ be the special fuzzy neutrosophic unit square semigroup under product ×.

Study questions (i) to (xii) of problem (21) for this V.

28. Let P = $\left\{ \begin{bmatrix} a_1 \\ a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{bmatrix} \right|$ $a_i \in U_N$, $1 \leq i \leq 12\}$ be the special fuzzy neutrosophic unit square semigroup under product $\times_n$.

Study questions (i) to (xii) of problem (21) for this P.

29. Let L = $\left\{ \left[ \begin{array}{c|cc|c|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & \dots & \dots & \dots & \dots & a_{14} \\ a_{15} & a_{16} & \dots & \dots & \dots & \dots & a_{21} \end{array} \right] \right|$ $a_i \in U_N$, $\times_n$, $1 \leq i \leq 21\}$ be the special fuzzy neutrosophic unit square semigroup under product $\times_n$.

Study questions (i) to (xii) of problem (21) for this L.

30. Let

$$Z = \left\{ \left[ \begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \end{array} \right] \middle| \; a_i \in U_N, \times_n, 1 \leq i \leq 36 \right\}$$

be the special fuzzy neutrosophic unit square super row matrix semigroup of infinite order.

Study questions (i) to (xii) of problem (21) for this Z.

31. Let

$$Y = \left\{ \left[ \begin{array}{cc|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} \\ \hline a_{15} & a_{16} & a_{17} & a_{18} & a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{36} & a_{37} & a_{38} & a_{39} & a_{40} & a_{41} & a_{42} \\ \hline a_{43} & a_{44} & a_{45} & a_{46} & a_{47} & a_{48} & a_{49} \\ a_{50} & a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ \hline a_{57} & a_{58} & a_{59} & a_{60} & a_{61} & a_{62} & a_{63} \\ a_{64} & a_{65} & a_{66} & a_{67} & a_{68} & a_{69} & a_{70} \end{array} \right] \middle| \; a_i \in U_N, 1 \leq i \leq 70 \right\}$$

be the special fuzzy neutrosophic unit square super row matrix semigroup of infinite order.

Study questions (i) to (xii) of problem (21) for this Y.

32. Let $\{U_N, \max\}$ be the fuzzy neutrosophic unit square semigroup under max operation.

    (i) Show every singleton element is a subsemigroup of $U_N$.
    (ii) Show every subsemigroup of finite order in $U_N$ is not an ideal of $U_N$.
    (iii) Prove for every integer n, n=1, 2, 3, …, m, we have subsemigroup of order n = 1, 2, 3, ….
    (iv) Prove every subset M of $U_N$ which is not a subsemigroup can be completed to get a subsemigroup of $U_N$.
    (v) Prove every ideal of $U_N$ is of infinite order.
    (vi) Prove $U_N$ has infinite number of ideals under max operation.
    (vii) If $I = \{a + bI$ in $U_N$ are such that $a, b \in [0.7, 1)\}$, then I is an ideal of $U_N$.
    (viii) Obtain any other interesting property associated with the semigroup $\{U_N, \max\}$.

33. Let $M = \{(a_1, a_2, \ldots, a_9) \mid a_i \in U_N, \max\}$ be the special fuzzy neutrosophic semigroup under max operation.

    (i) Study questions (i) to (viii) of problem (32) for this M.

    (ii) Compare the semigroup $\{U_N, \max\}$ with M.

34. Let $W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{18} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 18, \max \right\}$ be the special

    fuzzy neutrosophic unit square semigroup under max.

(i) Study questions (i) to (viii) of problem (32) for this W.

35. Let S = $\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_{10} \\ a_{11} & a_{12} & \dots & a_{20} \\ a_{21} & a_{22} & \dots & a_{30} \\ a_{31} & a_{32} & \dots & a_{40} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 40, \max\right\}$

be the special fuzzy neutrosophic unit square semigroup under max.

Study questions (i) to (viii) of problem (32) for this S.

36. Let P = $\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_9 \\ a_{10} & a_{11} & \dots & a_{18} \\ a_{19} & a_{20} & \dots & a_{27} \\ a_{28} & a_{29} & \dots & a_{36} \\ \vdots & \vdots & & \vdots \\ a_{73} & a_{74} & \dots & a_{81} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 81, \max\right\}$ be

the special fuzzy neutrosophic unit square semigroup under max.

Study questions (i) to (viii) of problem (32) for this P.

37. Let Z = $\left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{77} & a_{78} & a_{79} & a_{80} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 80, \max\right\}$

be the special fuzzy neutrosophic unit square semigroup under max.

Study questions (i) to (viii) of problem (32) for this Z.

38. Let $P_1 = \{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8) \mid a_i \in U_N, 1 \leq i \leq 8, \max\}$ be the special fuzzy neutrosophic unit square semigroup under max.

Study questions (i) to (viii) of problem (32) for this $P_1$.

39. Let $M_1 = \left\{\left[\begin{array}{c} a_1 \\ a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \end{array}\right] \middle| \; a_i \in U_N, 1 \leq i \leq 12, \max\right\}$ be the special fuzzy neutrosophic unit square semigroup under product $\times_n$.

Study questions (i) to (viii) of problem (32) for this $M_1$.

40. Let $S = \left\{\left[\begin{array}{cc|ccc|c|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & a_9 & \dots & \dots & \dots & \dots & a_{14} \\ a_{15} & a_{16} & \dots & \dots & \dots & \dots & a_{21} \\ a_{22} & a_{23} & \dots & \dots & \dots & \dots & a_{28} \end{array}\right] \middle| \; a_i \in U_N, \right.$

$1 \leq i \leq 28\}$ be the special fuzzy neutrosophic unit square super row matrix semigroup of infinite order.

Study questions (i) to (viii) of problem (32) for this S.

41. Let S = $$\left\{\left[\begin{array}{c|cc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ \hline a_7 & \cdots & \cdots & \cdots & \cdots & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ \hline a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & \cdots & a_{36} \\ \hline a_{37} & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & a_{48} \\ \hline a_{48} & \cdots & \cdots & \cdots & \cdots & a_{54} \end{array}\right] \middle|\ a_i \in U_N,\right.$$

$$1 \leq i \leq 54,\ \max\}$$

be the special fuzzy neutrosophic unit square super row matrix semigroup of infinite order.

Study questions (i) to (viii) of problem (32) for this S.

42. Let W = $\left\{\left[\begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & \ldots & \ldots & \ldots & a_{10} \\ \hline a_{11} & \ldots & \ldots & \ldots & a_{15} \\ a_{16} & \ldots & \ldots & \ldots & a_{20} \\ \hline a_{21} & \ldots & \ldots & \ldots & a_{25} \\ a_{26} & \ldots & \ldots & \ldots & a_{30} \\ a_{31} & \ldots & \ldots & \ldots & a_{35} \\ a_{36} & \ldots & \ldots & \ldots & a_{40} \\ \hline a_{41} & \ldots & \ldots & \ldots & a_{45} \\ a_{46} & \ldots & \ldots & \ldots & a_{50} \\ a_{51} & \ldots & \ldots & \ldots & a_{55} \\ \hline a_{56} & \ldots & \ldots & \ldots & a_{60} \\ a_{61} & \ldots & \ldots & \ldots & a_{65} \\ \hline a_{66} & \ldots & \ldots & \ldots & a_{70} \end{array}\right] \middle| \; a_i \in U_N, 1 \leq i \leq 70, \right.$

max} be the fuzzy neutrosophic special unit square super column matrix semigroup under max operation of infinite order.

Study questions (i) to (viii) of problem (32) for this W.

43. Let S = $\{U_N$, min$\}$ be the special fuzzy set neutrosophic unit square semigroup under min operation of infinite order.

(i) Prove S has infinite number of subsemigroups.
(ii) Find all ideals in S.
(iii) Can S have ideals of finite order?
(iv) Can S be a Smarandache semigroup?
(v) Can S have S-ideals?
(vi) Can S have S-subsemigroups?
(vii) Can S have S-units?
(viii) Prove every element in S is an idempotent.

44. Let V = $\{(a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7) \mid a_i \in U_N, 1 \leq i \leq 7, \min\}$ be the special fuzzy neutrosophic unit square row matrix semigroup under min of infinite order.

Study questions (i) to (viii) of problem (43) for this V.

45. Let M = $\left\{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{19} \end{bmatrix} \middle| a_i \in U_N, 1 \leq i \leq 19, \min\right\}$ be the fuzzy neutrosophic special unit square column matrix semigroup under min operation.

Study questions (i) to (viii) of problem (43) for this M.

46. Let R = $\left\{\begin{pmatrix} a_1 & a_2 & \ldots & a_{15} \\ a_{16} & a_{17} & \ldots & a_{30} \end{pmatrix} \middle| a_i \in U_N, 1 \leq i \leq 30, \min\right\}$ be the fuzzy neutrosophic special unit square matrix semigroup under min operation.

Study questions (i) to (viii) of problem (43) for this R.

47. Let T = $\left\{\begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{89} & a_{90} & a_{91} & a_{92} \end{pmatrix} \middle| a_i \in U_N, 1 \leq i \leq 92, \min\right\}$ be the special fuzzy neutrosophic unit square matrix semigroup under min operation.

Study questions (i) to (viii) of problem (43) for this T.

48. Let $(U_N, +)$ be the special fuzzy neutrosophic unit square group of infinite order.

Study the special features associated with this new structure.

49. Let M = $\{(a_1, a_2, \ldots, a_9) \mid a_i \in U_N, 1 \leq i \leq 9, +\}$ be the special fuzzy neutrosophic unit square row matrix group.

(i) Find at least 6 finite subgroups of M.
(ii) Find 8 infinite subgroups of M.
(iii) Find an automorphism $\eta : M \to M$ so that $\ker \eta \neq (0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0)$
(iv) Can M have a subgroup of order $2^9$?
(v) Can M have subgroup of prime order?

50. Let $P = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{12} \end{bmatrix} \middle| a_i \in U_N, 1 \leq i \leq 12, +\right\}$ be the special fuzzy neutrosophic unit square column matrix group.

Study questions (i) to (v) of problem (49) for this P.

51. Let $P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| a_i \in U_N, 1 \leq i \leq 16, +\right\}$

be the special fuzzy neutrosophic unit square matrix group under + modulo (1 and I).

Study questions (i) to (v) of problem (49) for this P.

52. Let $W = \left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{61} & a_{62} & a_{63} \end{bmatrix}\right| a_i \in U_N, 1 \leq i \leq 63, +\right\}$ be the special fuzzy neutrosophic unit square matrix group under addition modulo (1 and I).

Study questions (i) to (v) of problem (49) for this W.

53. Let T = $\{(a_1 \mid a_2\ a_3\ a_4 \mid a_5 \mid a_6\ a_7 \mid a_8) \mid a_i \in U_N, 1 \leq i \leq 8, +\}$ be the fuzzy neutrosophic unit square super row matrix under addition modulo (1 and I).

Study questions (i) to (v) of problem (49) for this T.

54. Let $S = \left\{\left.\begin{bmatrix} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ \hline a_8 \\ a_9 \\ a_{10} \\ \hline a_{11} \end{bmatrix}\right| a_i \in U_N, 1 \leq i \leq 11, +\right\}$ be the fuzzy unit square super column matrix group under addition modulo (1 and I).

Study questions (i) to (v) of problem (49) for this S.

55. Let $M = \left\{ \left( \begin{array}{cc|c|ccc|cc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{16} \\ a_{17} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{24} \\ a_{25} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & a_{32} \end{array} \right) \right| a_i \in U_N,$

$1 \leq i \leq 32, +\}$ be the fuzzy neutrosophic unit square super row matrix group under +.

Study questions (i) to (vii) of problem (49) for this M.

56. Let $V = \left\{ \left[ \begin{array}{cc|cccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \ldots & \ldots & \ldots & \ldots & \ldots & a_{14} \\ \hline a_{15} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{21} \\ a_{22} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{28} \\ a_{29} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{35} \\ a_{36} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{42} \\ \hline a_{43} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{49} \\ a_{50} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{56} \\ a_{57} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{63} \end{array} \right] \right| a_i \in U_N,$

$1 \leq i \leq 63, +\}$ be the fuzzy neutrosophic unit square super matrix group under +.

Study questions (i) to (v) of problem (49) for this V.

57. Let $M = \left\{ \left[ \begin{array}{ccccc} a_1 & a_2 & a_3 & a_4 & a_5 \\ \hline a_6 & \ldots & \ldots & \ldots & a_{10} \\ a_{11} & \ldots & \ldots & \ldots & a_{15} \\ \hline a_{16} & \ldots & \ldots & \ldots & a_{20} \\ a_{21} & \ldots & \ldots & \ldots & a_{25} \\ a_{26} & \ldots & \ldots & \ldots & a_{30} \\ \hline a_{31} & \ldots & \ldots & \ldots & a_{35} \\ a_{36} & \ldots & \ldots & \ldots & a_{40} \\ a_{41} & \ldots & \ldots & \ldots & a_{45} \\ a_{46} & \ldots & \ldots & \ldots & a_{50} \\ \hline a_{51} & \ldots & \ldots & \ldots & a_{55} \\ \hline a_{56} & \ldots & \ldots & \ldots & a_{60} \\ a_{61} & \ldots & \ldots & \ldots & a_{65} \end{array} \right] \middle| \; a_i \in U_N, 1 \leq i \leq 65, + \right\}$

be the special fuzzy neutrosophic unit square super column matrix under + modulo 1 and I.

Study questions (i) to (v) of problem (49) for this M.

58. Obtain some special and distinct features enjoyed by groups built using $(U_N, +)$.

59. Let $G = \{U_N, +\}$ be the group.

(i) Can any set $L = \{a + bI \mid a \in [0, 0.3) \text{ and } b \in [0, 0.5)\} \subseteq U_N$ be a group?

60. Prove P = { a | a $\in$ [0, 1)} $\subseteq$ $U_N$ is a subgroup of G given in problem 59 under +.

61. Prove T = {aI | a $\in$ [0, 1)} $\subseteq$ $U_N$ is also a subgroup of G given in problem 59 under +.

**Chapter Three**

# FUZZY NEUTROSOPHIC SEMIRINGS AND PSEUDO RINGS ON $U_N = \{(a + bI) \mid a, b \in [0,1)\}$

In this chapter we build semirings and pseudo rings using the fuzzy neutrosophic unit square $U_N$. We study some properties associated with them. This study is new and innovative. We define first fuzzy neutrosophic unit square semirings.

**DEFINITION 3.1:** *Let $S_N = \{a + Ib \mid a, b \in [0, 1)\ min, max\} = \{U_N, min, max\}$. Clearly $S_N$ is a semiring defined as the fuzzy neutrosophic unit square semiring.*

*$S_N$ is of infinite order. $S_N$ is not semifield as $1 \notin U_N$.*
*$S_N$ is a commutative semidomain.*

Let x = 0.3 + 81I and y = 0.7I + 0.8 $\in S_N$.
min {x, y} = min {0.3 + .81I, 0.7I + 0.8}
= min {0.3, 0.8} + min {0.7I, 0.81I}
= 0.3 + 0.7I $\in S_N$.

$$\begin{aligned}\max\{x, y\} &= \max\{0.3 + 0.81I, 0.7I + 0.8\}\\ &= \max\{0.3, 0.8\} + \max\{0.7I, 0.81I\}\\ &= 0.8 + 0.81I \in S_N.\end{aligned}$$

Thus $\{U_N, \max, \min\}$ is a semiring of infinite order. Every singleton set $A = \{x\}$ where $x \in S_N$ with $\{0\}$ is a semiring. That is $A \cup \{0\}$ is a subsemiring of order two.

We have infinitely many subsemirings of order two.

We can have subsemirings of order three and so on.

Let $P = \{0, 0.34 + 0.6I, 0.85 + 0.91I\} \subseteq S_N$. P is a subsemiring of order three.

Let $T = \{0, 0.34 + 0.62I, 0.28 + 0.31I, 0.16 + 0.16I\} \subseteq S_N$. T is a subsemiring order four.

Infact we have subsemirings of all possible orders. Further we can make a subset of $S_N$ which is not a subsemiring into a subsemiring by completing that subset.

This is illustrated in the following.

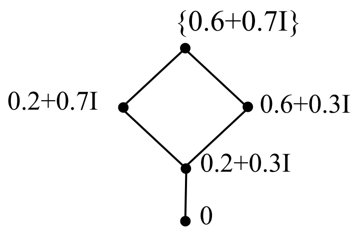
Let $P = \{0, 0.2 + 0.7I, 0.6 + 0.3I\} \subseteq S_N$. Clearly P is not a subsemiring so we have to complete P into subsemiring.

$$\begin{aligned}&\min\{0.2 + 0.7I, 0.6 + 0.3I\}\\ &= \min\{0.2, 0.6\} + \min\{0.7I, 0.3I\}\\ &= 0.2 + 0.3I \notin P.\end{aligned}$$

$$\begin{aligned}&\text{Now } \max\{0.2 + 0.7I, 0.6 + 0.3I\}\\ &= \max\{0.2 + 0.6\} + \max\{0.3I, 0.7I\}\\ &= 0.6 + 0.7I \notin P.\end{aligned}$$

Thus $P_c = \{0, 0.2 + 0.7I, 0.6 + 0.3I, 0.2 + 0.3I\ 0.6 + 0.7I\}$ is the completion of P is a subsemiring of $S_N$.

Infact $P_c$ is a distributive lattice given by the following Hasse diagram. $P_c$ is of order 5.




Let T = {0, 0.3 + 0.2I, 0.6 + 0.15I, 0.1 + 0.4I} $\subseteq S_N$. T is only a subset and is not a subsemiring of $S_N$. We now complete T into a subsemiring.

min {0.3 + 0.2I, 0.6 + 0.15I}
= min {0.3, 0.6} + min {0.2I, 0.15I}
= 0.3 + 0.15I $\notin$ T.

min {0.3 + 0.2I, 0.1 + 0.4I}
= min {0.3, 0.1} + min {0.2I, 0.4I}
= 0.1 + 0.2I $\notin$ T.

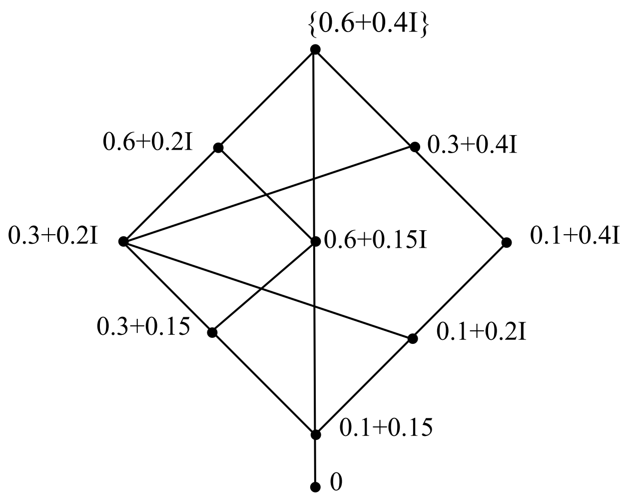
min {0.6 + 0.15I, 0.1 + 0.4I}
= min {0.6, 0.1} + min {0.15I, 0.4I}
= 0.1 + 0.15I $\notin$ T.

We find max {0.3 + 0.2I, 0.6 + 0.15I}
= max {0.3, 0.6} + max {0.2I, 0.15I}
= 0.6 + 0.2I $\notin$ T.

max {0.3 + 0.2I, 0.1 + 0.4I}
= max {0.3, 0.1} + max {0.2I, 0.4I}
= 0.3 + 0.4I $\notin$ T.

max {0.6 + 0.15I, 0.1 + 0.4I}
= max {0.6, 0.1} + max {0.15I, 0.4I}
= 0.6 + 0.4I $\notin$ T.

Now $T_c$ = {0, 0.3+2I, 0.2I, 0.6 + 0.15I, 0.1 + 0.4I, 0.3 + 0.15I, 0.1 + 0.2I, 0.1 + 0.15I, 0.6 + 0.2I, 0.3 + 0.4I, 0.6 + 0.4I} is the completed subsemiring of the subset T. Infact $T_c$ is a distributive lattice and the Hasse diagram of $T_c$ is as follows:




Clearly order of $T_c$ is 10. Thus we can construct any number of finite distributive lattices which are subsemirings of $S_N$.

Let P = {0, 0.1 + 0.2I, 0.2 + 0.I, 0.3 + 0.2I, 0.15 + 0.4I} $\subseteq$ $S_N$. Clearly P is only a subset of $S_N$.

max {0.1 + 0.2I, 0.2 + 0.I}
= 0.2 + 0.2I,

max {0.1 + 0.2I, 0.3 + 0.2I}
= 0.3 + 0.2I,

max {0.1 + 0.2I, 0.15 + 0.4I}
= 0.15 + 0.4I,

max {0.2 + 0.I, 0.3 + 0.2I}
= 0.3 + 0.2I $\in$ P,

max {0.2 + 0.I, 0.15 + 0.4I}
= 0.2 + 0.4I and

max {0.3 + 0.2I, 0.15 + 0.4I}
= 0.3 + 0.4I.

min {0.1 + 0.2I, 0.2 + 0.I}
= 0.1 + 0.I,

min {0.1 + 0.2I, 0.3 + 0.2I}
= 0.1 + 0.2I,

min {0.1 + 0.2I, 0.15 + 0.4I}
= 0.1 + 0.2I,

min {0.2 + 0.I, 0.3 + 0.2I}
= 0.2 + 0.I $\in$ P,

min {0.2 + 0.I, 0.15 + 0.4I}
= {0.15 + 0.I},

min {0.3 + 0.2I, 0.15 + 0.4I}
= 0.15 + 0.2I.

Now $P_c$ = {0, 0.1 + 0.2I, 0.2 + 0.I, 0.3 + 0.2I, 0.15 + 0.4I, 0.15 + 0.1I, 0.1 + 0.2I, 0.2 + 0.2I, 0.1 + 0.4I, 0.2 + 0.4I, 0.3 + 0.4I, 0.1 + 0.I, 0.15 + 0.2I} is a subsemiring.

Thus we can complete a subset even if two elements are comparable.

Here we use the term comparable in the following way.

Let x = a + Ib and y = c + Id, a, b, c, d $\in$ [0, 1) we say x is comparable with y if a < c and b < d or (or used in the mutually exclusive sense) c < a and d < b.

We will first illustrate this by the following example.

Let x = 0.31 + 0.7I and y = 0.84 + 0. 98I ∈ $S_N$ we see x < y as 0.31 < 0.84 and 0.7 < 0.98

Here if x = 0.81 + 0.5I and y = 0.98 + 0.4I ∈ $S_N$, we see x is not comparable with y as 0.81 < 0.98 and 0.5 $\nless$ 0.4 (0.4 < 0.5) so x and y are not comparable.

If x = 0.38 + 0.8I and y = 0.15 + 0.91I ∈ $S_N$ then also x and y are not comparable.

Thus even if a subset has some elements to be comparable and some other elements to be not comparable still we can complete the set to give us a subsemigroup.

If T is finite certainly the completion of T viz $T_c$ is also finite. If on the other hand T is infinite so is $T_c$.

Inview of this we have the following theorem.

**THEOREM 3.1:** *Let $S_N$ = {$U_N$, min, max} be the fuzzy neutrosophic unit semi open square semiring. If T is a subset of $S_N$ and if T is not a subsemiring T can be completed to $T_c$ to form a subsemiring.*

**Proof :** Consider T = {$x_1$, $x_2$, …, $x_n$} a proper subset of $S_N$ which is not a subsemiring of the semiring $S_N$.

Take $T_c$ = {T ∪ {max {$x_i$, $x_j$}} ∪ {min {$x_i$, $x_j$}}; i ≠ j, clearly $T_c$ is subsemiring {T ∩ max {$x_i$, $x_j$}} = ϕ if no $x_i$ is comparable with $x_j$.

Likewise {T ∩ min {$x_i$, $x_j$}} = ϕ if $x_i$ is not comparable with $x_j$. If some of $x_i$ is comparable with $x_j$ then {T ∩ {$x_i$, $x_j$}} ≠ ϕ likewise {T ∩ min {$x_i$, $x_j$}} ≠ ϕ.

Thus $T_c$ can be completed always to get a subsemiring. We can define ideals of the semirings $S_N$ = {$U_N$, max, min}.

Let A ⊆ $S_N$, A is a subsemiring of $S_N$. If for every x ∈ A and y ∈ $S_N$; min {x, y} ∈ A then we define A to be an ideal of

$S_N$. If on the other hand max $\{x, y\} \in A$ then A will be defined as the filter of $S_N$. To this end we will supply some examples.

Let $A = \{a + bI \mid a, b \in [0, 0.5)\} \subseteq S_N$, A is an ideal of $S_N$. However we see A is not a filter as $y = 0.9 + 0.8I \in S_N$ and $x = 0.4 + 0.3I \in A$ then $\max\{x, y\} = \max\{0.9 + 0.8I, 0.4 + 0.3I\} = 0.9 + 0.8I \notin A$.

Thus A is only an ideal of $S_N$ and is not a filter of $S_N$.

Let $B = \{a + bI \mid a, b \in [0.4, 1)\} \subseteq S_N$. We see B is a subsemiring of $S_N$.

However B is not ideal of $S_N$ for take $x = 0.2 + 0.15I \in S_N$ and $y = 0.6 + 0.4I \in B$.

We see $\min\{x, y\} = \min\{0.2 + 0.15I, 0.6 + 0.4I\} = \{\min\{0.2, 0.6\} + \min\{0.15I, 0.4I\} = 0.2 + 0.15I \notin B$.

Thus B is not an ideal of $S_N$. We see in $S_N$ an ideal in general is not a filter and a filter in general is not an ideal.

$S_N$ has infinite number of ideals and filters.

Now using $S_N = \{U_N, \max, \min\}$ we construct more and more semirings which is illustrated by examples.

***Example 3.1:*** Let $M = \{(a_1, a_2, a_3) \mid a_i = c_i + d_iI \in U_N; 1 \leq i \leq 3\}$ be the special fuzzy neutrosophic unit square semiring of infinite order under the max and min operation.

We see $P_1 = \{(a_1, 0, 0) \mid a_1 \in U_N\} \subseteq M$,
$P_2 = \{(0, a_1, 0) \mid a_1 \in U_N\} \subseteq M$ and
$P_3 = \{(0, 0, a_1) \mid a_1 \in U_N\} \subseteq M$

are special fuzzy neutrosophic unit square row matrix subsemirings of M. Clearly M has zero divisors and every element in M is an idempotent of M.

Let x = {(0.3I, 4 + 2I, 0)} and y = {(0, 0, 0.8 + 0.74I)} $\in$ M; min {x, y} = (0, 0, 0). Thus M has zero divisors. Infact M has infinite number of zero divisors and idempotents.

Every P = {(0,0, 0), (x, y, z)} where x, y, z $\in U_N$ is a subsemiring of order two.

We can as in case of usual semirings define the notion of ideals and subsemirings. $P_1$, $P_2$ and $P_3$ are also ideals of M. However $P_1$, $P_2$ and $P_3$ are not filters of M.

Let

$L = \{(a_1, a_2, a_3) \mid a_i = c_i + d_iI \text{ and } c_i, d_i \in [0.4, 1);\ 1 \leq i \leq 3\} \subseteq M$

be the subsemiring of M.

Clearly L is a subsemiring and not an ideal of M. But L is also a filter of M.

Now we see M has infinite number of filters which are not ideals and infinite number of ideals which are not filters. Further M has infinite number of subsemirings of finite order which are not ideals or filters.

***Example 3.2:*** Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \end{bmatrix} \middle| \; a_i \in U_N,\ 1 \leq i \leq 10, \max, \min \right\}$$

be the special fuzzy neutrosophic unit square column matrix semiring of infinite order.

This semiring has several ideals which are not filters and several filters which are not ideals. Let

$$P_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_1 \in U_N, \max, \min \right\} \subseteq N,$$

$$P_2 = \left\{ \begin{bmatrix} 0 \\ a_2 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_2 \in U_N, \max, \min \right\} \subseteq N,$$

$$P_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ a_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_3 \in U_N, \max, \min \right\} \subseteq N,$$

$$P_4 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_4 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \middle| \; a_4 \in U_N, \max, \min \right\} \subseteq N$$

and so on

$$P_9 = \left\{ \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_9 \\ 0 \end{bmatrix} \middle| \; a_9 \in U_N, \max, \min \right\} \subseteq N$$

and $P_{10} = \left\{ \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a_{10} \end{bmatrix} \middle| \; a_i \in U_N, \max, \min \right\} \subseteq N$ are 10 distinct subsemirings of N.

Clearly all these 10 subsemirings are also ideals of N and none of them is a filter of N.

Let

$$T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \end{bmatrix} \middle| \; a_i = c_i + d_iI \text{ where } d_i, c_i \in [0.2, 1); 1 \leq i \leq 10 \right\} \subseteq N$$

be a subsemiring which is also a filter of N.

We see

$$T_2 = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{10} \end{bmatrix} \middle| \; a_i = c_i + d_iI \text{ where } d_i, c_i \in [0.3,1); 1 \leq i \leq 10 \right\} \subseteq N$$

is a subsemiring of N which is also a filter of N. Clearly both $T_1$ and $T_2$ are not ideals of N.

We observe if any matrix has zeros even in one position then that subsemiring can never be a filter of N.

***Example 3.3:*** Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in U_N, \text{max, min}, 1 \leq i \leq 30 \right\}$$

be the special fuzzy neutrosophic unit square semiring.

W has subsemirings of finite order. Infact W has subsemirings of order two, three and so on. W has also subsemirings of infinite order which are ideals and some of them are not ideals.

$$N = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_1 = c_1 + d_1I, a_2 = c_2 + d_2I \text{ and} \right.$$

$a_3 = c_3 + d_3I$ where $c_1, c_2, d_1, d_2, c_3, d_3 \in [0.9, 1)$ and $a_j \in U_N$, $4 \leq j \leq 30$, max, min$\} \subseteq W$ is a subsemiring of infinite order.

It is easily verified N is not an ideal but N is a filter of N.

Now suppose we take

$$R = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_1 = c_1 + d_1 I, \text{ where } c_1, d_1 \in [0.3, 1) \text{ and } \right.$$

$$\left. a_j \in U_N, 2 \leq j \leq 30, \max, \min \right\} \subseteq N$$

to be a subsemiring of N.

Clearly R is not a filter of N. Further R is not even as ideal of N. This subsemiring R is of infinite order which is not an ideal and not a filter.

Infact N has infinitely many subsemirings of infinite order which are not ideals and not filters of N.

It is important to note that N has no subsemiring which is both an ideal as well as a filter of N.

***Example 3.4:*** Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & \ldots & a_9 \\ a_{10} & a_{11} & a_{12} & \ldots & a_{18} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 18, \max, \min \right\}$$

be the special fuzzy neutrosophic unit square semiring of infinite order.

T has ideals and filters of infinite order. T has infinite number of zero divisors. T has finite order subsemirings which are not ideals.

Infact T has also infinite order subsemirings which are not both ideals or filters.

Every element x ∈ T is an idempotent with respect to both max and min operation.

***Example 3.5:*** Let

$$T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| a_i \in U_N, 1 \leq i \leq 16, \max, \min \right\}$$

be the special fuzzy neutrosophic unit square matrix semiring of infinite order. S has ideals and filters of infinite order.

All infinite subsemirings of S can be given a Hasse diagram.

We see S has chain lattices if every element in S is comparable as subsemirings. Also S has finite distributive lattices and subsemirings.

Also for any finite or infinite subset T of S we can complete T to $T_c$ so that $T_c$ is a subsemiring.

Certainly if $T_c$ is of finite order then $T_c$ is not an ideal or a filter only a finite subsemiring.

$$P_1 = \left\{ \begin{bmatrix} a_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \middle| a_1 \in U_N, \max, \min \right\} \subseteq S$$

is a subsemiring which is an ideal but $P_1$ is not a filter.

***Example 3.6:*** Let

$$M = \left\{ \left. \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ \hline a_{10} \\ a_{11} \\ \hline a_{12} \end{array} \right] \right| a_i \in U_N, 1 \leq i \leq 12, \text{max, min} \right\}$$

be the special fuzzy neutrosophic unit square semiring of super column matrices. M has infinite number of subsemirings of finite order.

M also has infinite number of subsemirings which are ideals and every ideal of M is of infinite order.

We see M has filters of infinite order and none of them are ideals. M has infinite number of zero divisors. Every element in M is an idempotent with respect to both max and min operation.

***Example 3.7:*** Let W = $\{(a_1\ a_2 \mid a_3 \mid a_4\ a_5\ a_6 \mid a_7\ a_8 \mid a_9\ a_{10}\ a_{11} \mid a_{12}\ a_{13} \mid a_{14}) \mid a_i \in U_N, 1 \leq i \leq 14$, max, min$\}$ be the special fuzzy neutrosophic unit square super row matrix semiring under max, min operation.

W is of infinite order. W has infinite number of ideals. W has infinite number of subsemirings which are not ideals or filters of W. W has infinite number of zero divisors.

***Example 3.8:*** Let

$$W = \left\{ \left[ \begin{array}{cc} a_1 & a_2 \\ a_3 & a_4 \\ \hline a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ \hline a_{13} & a_{14} \\ a_{15} & a_{16} \\ a_{17} & a_{18} \\ a_{19} & a_{20} \\ a_{21} & a_{22} \\ \hline a_{23} & a_{24} \\ \hline a_{25} & a_{26} \end{array} \right] \middle| \; a_i \in U_N, 1 \leq i \leq 26, \max, \min \right\}$$

be the special fuzzy neutrosophic unit square super column matrix semiring. T is of infinite order. T has infinite number of subsemirings of finite order none of them are ideals.

T has infinite number subsemirings of infinite order which are ideals and not filters. T has infinite number of subsemirings of infinite order which are filters of T and not ideals of T. T has infinite number of idempotents and zero divisors.

***Example 3.9:*** Let

$$M = \left\{ \left( \begin{array}{cc|c|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \ldots & \ldots & \ldots & \ldots & \ldots & a_{14} \\ a_{15} & \ldots & \ldots & \ldots & \ldots & \ldots & a_{21} \end{array} \right) \middle| \; a_i \in U_N, 1 \leq i \leq 21, \right.$$

$$\left. \max, \min \right\}$$

be the special fuzzy neutrosophic unit square semiring of super row matrices.

M has infinite number of zero divisors and idempotents.

$$P_1 = \left\{ \left( \begin{array}{cc|c|ccc|c} a_1 & a_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \middle| \; a_1, a_2 \in U_N, \max, \min \right\} \subseteq M$$

be the subsemiring of infinite order. Clearly $P_1$ is also an ideal of M and is not a filter.

$$P_2 = \left\{ \left( \begin{array}{cc|c|ccc|c} 0 & 0 & a_1 & 0 & 0 & 0 & b_1 \\ 0 & 0 & a_2 & 0 & 0 & 0 & b_2 \\ 0 & 0 & a_3 & 0 & 0 & 0 & b_3 \end{array} \right) \middle| \; a_i \in U_N, 1 \leq i \leq 3, \right.$$

$b_j = d_j + c_jI$, $d_i$, $c_j \in [0.7, 9)$, $1 \leq j \leq 3$, max, min} be the subsemiring of infinite order. Clearly $P_2$ is not an ideal or filter of M.

$$P_3 = \left\{ \left( \begin{array}{cc|c|ccc|c} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_1 & a_2 & a_3 & 0 & 0 & 0 & a_4 \end{array} \right) \middle| \; a_i = c_i + d_iI, c_i, d_i \in \right.$$

$$[0.7, 1); 1 \leq i \leq 4\} \subseteq M$$

is a subsemiring of M of infinite order. $P_4$ is not a filter or ideal only a subsemiring.

$$P_4 = \left\{ \left( \begin{array}{cc|c|ccc|c} 0 & 0 & 0 & a_1 & a_2 & a_3 & 0 \\ 0 & 0 & 0 & a_4 & a_5 & a_6 & 0 \\ 0 & 0 & 0 & a_7 & a_8 & a_9 & 0 \end{array} \right) \middle| \; a_i = b_i + c_iI, b_i, c_i \in \right.$$

$$[0.5, 1); 1 \leq i \leq 9, \max, \min\} \subseteq M$$

be the subsemiring of infinite order. $P_4$ is not an ideal or a filter.

***Example 3.10:*** Let

$$S = \left\{ \left[\begin{array}{cc|ccc|c} a_1 & a_2 & a_3 & a_4 & & a_5 \\ a_6 & \cdots & \cdots & \cdots & & a_{10} \\ \hline a_{11} & \cdots & \cdots & \cdots & & a_{15} \\ a_{16} & \cdots & \cdots & \cdots & & a_{20} \\ a_{21} & \cdots & \cdots & \cdots & & a_{25} \\ \hline a_{26} & \cdots & \cdots & \cdots & & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & & a_{35} \\ a_{36} & \cdots & \cdots & \cdots & & a_{40} \\ \hline a_{41} & \cdots & \cdots & \cdots & & a_{45} \end{array}\right] \;\middle|\; a_i \in U_N, 1 \leq i \leq 45, \max, \min \right\}$$

be the special fuzzy neutrosophic unit semi open square super matrix semiring of infinite order.

$$T = \left\{ \left[\begin{array}{cc|cc|c} a_1 & a_2 & 0 & 0 & 0 \\ a_3 & a_4 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right] \;\middle|\; a_1, a_2, a_3, a_4 \in U_N, \max, \min \right\}$$

be the subsemiring of S. T is also an ideal of S. Clearly T is not a filter of S. S has infinite number of idempotents and zero divisors.

Next we proceed onto define special pseudo fuzzy neutrosophic semiring with operation min and $\times$.

Let $P_N = \{U_N, \min, \times\}$ be the special fuzzy neutrosophic unit square quasi semiring of infinite order.

Let x = a + bI and y = c + Id ∈ $P_N$
x × y = (a + bI) × (c + dI)
= ac + bcI + daI + bdI
= ac + (bc + ad + bd)I ∈ $P_N$.

Thus if x = 0.3 + 0.2I and y = 0.9 + 0.7I ∈ $P_N$
Then x × y = (0.3 + 0.2I) × (0.9 + 0.7I)
= 0.27 + 0.18I + 0.21I + 0.14I
= 0.27 + 0.53I ∈ $P_N$.

min {x, y} = min {0.3 + 0.2I, 0.9 + 0.7I}
= min {0.3, 0.9} + min {0.2I, 0.7I}
= 0.3 + 0.2I.

Let x = 0.7 + 0.4I
y = 0.6 + 0.5I and z = 0.4 + 0.9I ∈ $P_N$.

x × min {y, z} = x × min {0.6 + 0.5I, 0.4 + 0.9I}
= x × 0.4 + 0.5I
= (0.7 + 0.4I) (0.4 + 0.5I)
= 0.28 + 0.16I + 0.35I + 0.20I
= 0.28 + 0.71I … I

Consider min {x × y, x × z}
= min {0.7 + 0.4I × 0.6 + 0.5I, 0.7 + 0.4I × 0.4 + 0.9I}
= min [0.42 + 0.24I + 0.35I + 0.20I, 0.28 + 0.16I + 0.63I + 0.36I}
= min {0.42 + 0.79I, 0.28 + 0.15I}
= 0.28 + 0.15 … II

Clearly I and II are distinct so the operation × and min are not distributive.

That is why we have define $P_N$ to be a pseudo semiring.

This semiring has pseudo subsemirings, pseudo ideals and pseudo filters defined in a very special way.

Let $T_N = \{(0, 0.5), \times, \min\}$ be the pseudo subsemiring. Clearly $T_N$ is not a filter $P_N$ for any $x \in P_N$ and $y \in T_N$ we see $x \times y \notin T_N$.

$T_N$ is not an ideal for $\min\{x, y\} \notin T_N$ for all $x \in P_N$.

Thus only in this pseudo subsemiring we see it is not a filter.

Distributive laws in general are not true in $P_N = \{U_N, \min, \times\}$ the semiring that is why we use the term pseudo semiring.

Let $x = 0.9 + 0.4I$, $y = 0.6 + 0.6I$ and $z = 0.2 + 0.8I \in P_N$.

$x \times \min\{y, z\} = 0.9 + 0.4I \times \min\{0.6 + 0.6I, 0.2 + 0.8I\}$

$= 0.9 + 0.4I \times \{0.2 + 0.6I\}$

$= 0.18 + 0.54I + 0.08I + 0.24I$

$= 0.18 + 0.86I$ … I

$\min\{x \times y, x \times z\}$

$= \min\{0.9 + 0.4I \times 0.6 + 0.6I, 0.9 + 0.4I \times 0.2 + 0.8I\}$

$= \min\{0.54 + 0.24I + 54I + 0.24I,$

$0.18 + 0.08I + 0.72I + 0.32I\}$

$= \min\{0.54 + 0.02I, 0.18 + 0.12I\}$

$= 0.18 + 0.2I$ … II

I and II are distinct.

$x \times \min\{y, z\} \neq \min\{x \times y, x \times z\}$ in general.

So $P_N$ is a pseudo semiring. We construct several such pseudo semiring using $P_N$.

***Example 3.11:*** Let $M = \{(a_1, a_2, a_3, a_4)$ where $a_i \in P_N$, $1 \leq i \leq 4$, $\min, \times\}$ be the special fuzzy neutrosophic unit square row matrix pseudo semiring.

M has several pseudo subsemirings some of which are pseudo ideals.

We see M has no pseudo subsemiring of finite order say order two order three and so on.

$P_1 = \{(a_1, 0, 0, 0) \mid a_1 \in P_N; \min, \times\} \subseteq M$ is a special fuzzy pseudo subsemiring which is also an ideal of M.

***Example 3.12:*** Let

$$N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{14} \\ a_{15} \end{bmatrix} \middle| \, a_i \in P_N, 1 \leq i \leq 4, \min, \times \right\}$$

be the special fuzzy neutrosophic unit square pseudo semiring of infinite order. N has pseudo subsemirings which have no proper pseudo ideals.

$$\text{Take } B_1 = \left\{ \begin{bmatrix} a_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \middle| \, a_1 \in P_N, \min, \times \right\} \subseteq N,$$

$$B_2 = \left\{ \begin{bmatrix} 0 \\ a_2 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \middle| \, a_2 \in P_N, \min, \times \right\} \subseteq N,$$

$$B_3 = \left\{ \left[ \begin{array}{c} 0 \\ 0 \\ a_3 \\ \vdots \\ 0 \\ 0 \end{array} \right] \middle| \; a_3 \in P_N, \min, \times \right\} \subseteq N,$$

$$B_4 = \left\{ \left[ \begin{array}{c} 0 \\ 0 \\ 0 \\ a_4 \\ 0 \\ \vdots \\ 0 \end{array} \right] \middle| \; a_4 \in P_N, \min, \times \right\} \subseteq N, \ldots,$$

$$B_{15} = \left\{ \left[ \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ a_{15} \end{array} \right] \middle| \; a_{15} \in P_N, \min, \times \right\} \subseteq N$$

are all pseudo subsemirings which are also pseudo ideals of infinite order.

Take

$$T_1 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.2+0.3I \\ 0 \\ 0.4+0.5I \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.95I \\ 0.91 \\ 0.7+0.2I \\ 0 \end{bmatrix} \right\} \subseteq N;$$

$T_1$ is only a subset however if we generate $\langle T_1 \rangle$ we see $|T_1| = \infty$.

Now no finite subset can be an pseudo subsemiring of N.

***Example 3.13:*** Let

$$W = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{28} & a_{29} & a_{30} \end{bmatrix} \middle| \; a_i \in P_N = \{U_N, \times, \min\};\ 1 \leq i \leq 30 \right\}$$

be the special fuzzy neutrosophic pseudo semiring of infinite order.

We see W has no subsemirings of finite order. Inview of this we give the following theorem.

**THEOREM 3.2:** *Let $P_N$ = $\{U_N, \times, min\}$ be the special fuzzy neutrosophic unit semi open square pseudo semiring.*

*(1) All subsemirings of $P_N$ are of infinite order.*
*(2) $P_N$ has subsemirings which are not ideals.*

The proof is direct and hence left as an exercise to the reader.

***Example 3.14:*** Let

$$V = \left\{ \begin{bmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{bmatrix} \middle| a_i \in P_N = \{U_N, \times, \min\}; 1 \leq i \leq 30 \right\}$$

be the special fuzzy neutrosophic pseudo semiring.

V has infinitely many pseudo subsemirings and ideals. All of them are only of infinite order.

***Example 3.15:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ \vdots & \vdots & \vdots & \vdots \\ a_{61} & a_{62} & a_{63} & a_{64} \end{bmatrix} \middle| a_i \in P_N = \{U_N, \times, \min\}, 1 \leq i \leq 64 \right\}$$

be the special fuzzy neutrosophic matrix pseudo semiring.

This M has infinite number of zero divisors and idempotents.

M has several pseudo subsemrings of infinite order which are not ideals. M also has pseudo ideals of infinite order.

***Example 3.16:*** Let
T = {$(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6)$ | $a_i \in P_N$ = {$U_N$, ×, min} $1 \leq i \leq 6$} be the special fuzzy neutrosophic unit open square super matrix pseudo semiring of infinite order.

We see $P_1$ = {$(a_1\ a_2 \mid 0\ 0\ 0 \mid 0)$ | $a_1, a_2 \in P_N$ = {$U_N$, ×, min}} $\subseteq$ T is a pseudo subsemiring which is also a pseudo ideal of T.

M = {$(a_1\ a_2 \mid 0\ 0\ 0 \mid 0)$ | $a_1 = c_1 + d_1I$, $a_2 = c_2 + d_2I$, $c_1, c_2, d_1, d_2 \in [0, 0.5)$} $\subseteq$ T is not a pseudo subsemiring of infinite order so naturally is not a pseudo ideal of T.

If (0.3 + 0.4I, 0.2 + 0.3I | 0 0 0 | 0) = x and y = (0.3 + 0.4I, 0.21 + 0.45I | 0 0 0 | 0) ∈ M, consider

x × y = (0.3 + 0.4I, 0.2 + 0.3I | 0 0 0 | 0) × (0.3 + 0.4I, 0.21 + 0.45I | 0 0 0 | 0)

= (0.09 + 0.12I + 0.12I + 0.16I, 0.42 + 0.09I + 0.063I + .125I | 0 0 0 | 0)

= (0.09 + 0.5I, 0.42 + 0.278I | 0 0 0 | 0)
∉ M so M is not even closed under ×.

***Example 3.17:*** Let

$$V = \left\{ \left[ \begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ \hline a_4 \\ a_5 \\ \hline a_6 \\ a_7 \\ a_8 \\ \hline a_9 \end{array} \right] \middle|\ a_i \in P_N = \{U_N, \times_n, \min\},\ 1 \leq i \leq 9 \right\}$$

be the special fuzzy neutrosophic semi open unit square super matrix pseudo semiring of infinite order.

V has infinite number of zero divisors and idempotents. V has several pseudo subsemirings and ideals all of which are of infinite order.

***Example 3.18:*** Let

$$V = \left\{ \left[ \begin{array}{ccc} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ \hline a_{22} & a_{23} & a_{24} \\ \hline a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \\ a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} \end{array} \right] \middle| \; a_i \in P_N = \{U_N, \times_n, \min\} \; 1 \leq i \leq 39 \right\}$$

be the special fuzzy neutrosophic unit semi open square super matrix pseudo semiring of infinite order.

V has infinite number of zero divisors and idempotents. V has infinite number of pseudo subsemirings of infinite order.

V has also pseudo ideals of infinite order. V has no pseudo subsemirings of finite order.

***Example 3.19:*** Let

$$T = \left\{ \left( \begin{array}{c|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ a_8 & \ldots & \ldots & \ldots & \ldots & \ldots & a_{14} \end{array} \right) \middle| \; a_i \in P_N = \{U_N, \times, \min\} \right.$$

$1 \leq i \leq 14\}$ be the special fuzzy neutrosophic unit semi open square pseudo semiring of infinite order.

T has infinite number of zero divisors and idempotents. T has infinite number of pseudo ideals of infinite order and pseudo subsemirings of infinite order which are not ideals.

Take

$$M = \left\{ \left( \begin{array}{c|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \middle| \; a_1 \in P_N = \{U_N, \times, \min\}\} \right.$$

and $a_2, a_3, a_4 \in [0, 1)\} \subseteq T$,

M is a pseudo subsemiring of infinite order which is not a pseudo ideal of T.

We now proceed onto describe pseudo ring using the fuzzy neutrosophic unit square.

***Example 3.20:*** Let $R_N = \{a + bI \mid a, b \in [0, 1), +, \times\}$ be the special fuzzy neutrosophic unit semi open square pseudo ring of infinite order.

$R_N$ has pseudo subrings of infinite order. $R_N$ has pseudo ideals of infinite order.

$P_n = \{aI \mid a \in [0, 1), +, \times\} \subseteq R_N$ is a pseudo ideal of $R_N$.

$T_n = \{a \mid a \in [0, 1), +, \times\}$ is a pseudo subsemiring of $R_N$ which is not an ideal of $R_N$.

Let x = 0.3 + 0.71I, y = 0.2 + 0.5I and z = 0.21 + 0.2I ∈ $R_N$.

Consider x × (y + z)
= (0.3 + 0.7I) × [0.2 + 0.5I + 0.21 + 0.2I]
= (0.3 + 0.7I) × (0.41 + 0.7I)
= 0.123 + 0.287I + 0.21I + 0.287I
= 0.123 + 0.784I … I

Consider x × y + x × z
= 0.3 + 0.7I × 0.2 + 0.5I + 0.3 + 0.7I × 0.21 + 0.2I
= (0.06 + 0.14I + 0.15I + 0.35I) +
(0.063 + 0.147I + 0.06I + 0.14I)
= 0.123 + 0.059I … II

Clearly I and II are distinct hence x ×(y + z) ≠ xy + xz in general for all x, y, z ∈ $R_N$.

That is why we call $R_N$ as the pseudo ring.

Let N = {a + bI | a, b ∈ [0.2), ×, +} ⊆ $R_N$. N is only a set and is not a pseudo ring.

For N is not even closed under + as if x = 0.1 + 0.12I and y = 0.15 + 0.18I in N

x + y = 0.1 + 0.12I + 0.15 + 0.18I
= 0.25 + 0.30I ∉ N.

Hence the claim.

Now we build other pseudo rings using the pseudo ring $R_N$ which is illustrated by the following examples.

***Example 3.21:*** Let
R = {($a_1$, $a_2$, $a_3$, $a_4$) | $a_i$ ∈ $U_N$, 1 ≤ i ≤ 4, ×, +} be the special fuzzy neutrosophic unit semi open square row matrix pseudo ring of infinite order.

R has infinite number of zero divisors. R has pseudo ideals of infinite order.

***Example 3.22:*** Let $S = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_9 \\ a_{10} \end{bmatrix} \middle| \; a_i \in U_N,\ 1 \leq i \leq 10,\ \times_n, + \right\}$

be the special fuzzy neutrosophic unit semi open square column matrix pseudo ring of infinite order under the natural product $\times_n$ of matrices.

S has atleast 10 pseudo subrings which are also pseudo ideals.

S has infinite number of zero divisors.

***Example 3.23:*** Let

$$S = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in U_N,\ 1 \leq i \leq 16,\ \times_n, + \right\}$$

be the special fuzzy neutrosophic unit semi open square pseudo ring.

S has infinite number of zero divisors, no idempotents and no units.

All pseudo subrings of S are of infinite order. All ideals in S are also of infinite order.

$$P_1 = \left\{ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \middle| \; a_i \in U_N, \times_n, + \right\} \subseteq S$$

is a pseudo subring which is also a pseudo ideal. Infact P has atleast ${}_{16}C_1 + {}_{16}C_2 + \ldots + {}_{16}C_{15}$ number of pseudo subrings which are pseudo ideals of S.

***Example 3.24:*** Let

$$M = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \end{bmatrix} \middle| \; a_i \in U_N, 1 \leq i \leq 16, \times_n, + \right\}$$

be the special fuzzy neutrosophic unit square matrix pseudo ring.

M has atleast ${}_{32}C_1 + {}_{32}C_2 + \ldots + {}_{32}C_{31}$ number of distinct pseudo subrings which are pseudo ideals of M.

Inview of all these we have the following theorem.

**THEOREM 3.3:** *Let M = {Collection of all m × n matrices with entries from $U_N = \{a + bI \mid a, b \in [0, 1)\}$, +, $\times_n$} be the special* *fuzzy neutrosophic unit semi open square m × n matrix pseudo ring.*

*M has atleast* ${}_{m\times n}C_1 + {}_{m\times n}C_2 + \ldots + {}_{m\times n}C_{m\times n-1}$ *number of distinct pseudo subrings which are pseudo ideals.*

The proof is direct hence left as an exercise to the reader.

However we leave the following open problem.

Can M in theorem have any other pseudo ideals?

***Example 3.25:*** Let
$S = (a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6) \mid a_i \in U_N, +, \times, 1 \leq i \leq 6\}$ be the special fuzzy neutrosophic unit semi open square super row matrix pseudo ring.

S has atleast ${}_6C_1 + {}_6C_2 + {}_6C_3 + {}_6C_4 + {}_6C_5$ pseudo subrings which are pseudo ideals.

$N = \{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6) \mid a_1\ a_2\ a_3 \in U_N = \{a + bI \mid a, b \in [0, 1), a_4, a_5, a_6 \in [0,1), +, \times\} \subseteq S$ be a special fuzzy neutrosophic unit semi open square pseudo subring.

Clearly N is not an ideal of S.

Let $M = \{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid 0) \mid a_1\ a_2\ a_3\ a_4 \in U_N = \{a + bI \mid a, b \in [0, 1), a_4, a_5, a_6 \in [0, 1); +, \times\} \subseteq S$ be a special fuzzy neutrosophic unit semi open square pseudo subring.

Clearly N is not an ideal of S.

Let $M = \{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid 0) \mid a_1\ a_2\ a_3\ a_4 \in U_N = \{a + bI \mid a, b \in [0, 1)\}, a_5 \in [0, I); +, \times) \subseteq S$ be the pseudo subring.

Clearly M is not a pseudo ideal of S.

***Example 3.26:*** Let

$$P = \left\{ \left[\begin{array}{c} a_1 \\ \hline a_2 \\ \hline a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ a_7 \\ a_8 \\ \hline a_9 \\ a_{10} \\ a_{11} \\ \hline a_{12} \\ a_{13} \\ \hline a_{14} \end{array}\right] \middle| \; a_i \in U_N = \{a + bI \mid a, b \in [0, 1)\}, \right.$$

$$\left. 1 \leq i \leq 14, \times, + \right\}$$

be the special fuzzy neutrosophic unit semi open square super column matrix pseudo ring.

P has several pseudo subrings which are pseudo ideals.

P also has pseudo subrings which are not pseudo ideals of P.

***Example 3.27:*** Let

$$M = \left\{ \left[ \begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ \hline a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} \\ \hline a_{19} & a_{20} & a_{21} \\ a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \end{array} \right] \right| a_i \in U_N = \{a + bI \mid a, b \in [0, 1)\},$$

$$1 \leq i \leq 30, \times, +\}$$

be the special fuzzy neutrosophic unit semi open square super column matrix pseudo ring.

Clearly M has infinite number of zero divisors. M has pseudo ideals and pseudo subrings.

We as in case of usual rings study several properties about pseudo rings.

The only problem in case of pseudo rings is that they do not in general obey the distributive law.

Homomorphism and other properties are defined for pseudo rings also.

We suggest several problems some are simple and some are really difficult.

We have given every type of pseudo ring in the exercise. However several properties can be derived provided they are not dependent on the distributive laws.

**Problems:**

1. Enumerate any of the special properties enjoyed by $S_N = \{U_N, \max, \min\}$ the special fuzzy neutrosophic unit semi open square semiring.

2. Prove $S_N$ has infinite number of finite subsemirings which are not ideals or filters of $S_N$.

3. Can $S_N$ have filters of finite order?

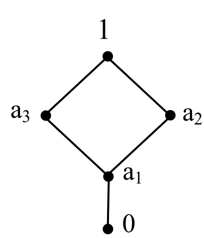
4. Can $S_N$ have ideals of finite order?

5. Prove $S_N$ has several distributive lattices.

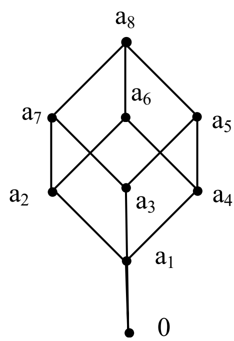
6. Can $S_N$ have as subsemirings which are isomorphic to Boolean algebras of all orders?

7. Prove $S_N$ cannot have subsemirings isomorphic to Boolean algebras of order greater than or equal to four.

8. Can we say $S_N$ has a subsemiring whose lattice diagram is given below? ($a_i \in S_N$; $1 \leq i \leq 4$).




9. Can $S_N$ have a subsemiring which is isomorphic to the distributive lattice; whose Hasse Diagram is given in the following?




(where $a_i \in S_N$; $1 \leq i \leq 8$)

10. Can $S_N$ have subsemiring of finite order which is isomorphic to a distributive lattice of order $2^n+1$?

11. Let
$M = \{(a_1, a_2, …, a_{10}) \mid a_i \in U_N, 1 \leq i \leq 10, \max, \min\}$ be the special fuzzy neutrosophic unit square semiring of infinite order.

(i) Show every element x with (0 0 0 0 0 0 0 0 0 0) is a subsemirings of order two.

(ii) Show M has subsemirings of every order.

(iii) Show M has subsemirings of infinite order.

(iv) Show no ideal of M can be of finite order.

(v) Show M has zero divisors under min operation.

(vi) Can a subsemiring in M be both an ideal and filter?

(vii) Show no filter of M can be of finite order?

(viii) Prove every subset of M can be completed to a subsemiring.

12. Let $N = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \end{bmatrix} \middle| \, a_i \in U_N, \max, \min, \ 1 \le i \le 8 \right\}$ be the special fuzzy neutrosophic unit square column matrix semiring of infinite order.

Study questions (i) to (viii) of problem 11 for this N.

13. Let

$$P = \left\{ \begin{bmatrix} a_1 & a_2 & \cdots & a_9 \\ a_{10} & a_{11} & \cdots & a_{18} \\ a_{19} & a_{20} & \cdots & a_{27} \\ a_{28} & a_{29} & \cdots & a_{36} \\ a_{37} & a_{38} & \cdots & a_{45} \\ a_{46} & a_{47} & \cdots & a_{54} \end{bmatrix} \middle| \, a_i \in U_N, \max, \min, \right.$$

$1 \le i \le 54\}$ be the special fuzzy neutrosophic unit semi open square column matrix semiring.

Study questions (i) to (viii) of problem 11 for this P.

14. Let $X = \left\{\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \right| a_i \in U_N, 1 \leq i \leq 16,$ min, max} be the special fuzzy neutrosophic unit semi open square matrix semiring.

    Study questions (i) to (viii) of problem 11 for this X.

15. Let Y = $\{(a_1\ a_2 \mid a_3\ a_4 \mid a_5\ a_6\ a_7 \mid a_8\ a_9 \mid a_{10}) \mid a_i \in U_N, 1 \leq i \leq 10$, min, max} be the special fuzzy neutrosophic unit semi open square semiring.

    Study questions (i) to (viii) of problem 11 for this Y.

16. Let

$$W = \left\{\left[\begin{array}{c} a_1 \\ \hline a_2 \\ a_3 \\ a_4 \\ \hline a_5 \\ a_6 \\ \hline a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \end{array}\right] \right| a_i \in U_N,\ 1 \leq i \leq 11, \text{min, max}\} \text{ be the}$$

special fuzzy neutrosophic unit semi open square semiring.

Study questions (i) to (viii) of problem 11 for this W.

17. Let

$$W = \left\{ \left[\begin{array}{c|cc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ \hline a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & \cdots & \cdots & \cdots & \cdots & a_{18} \\ \hline a_{19} & \cdots & \cdots & \cdots & \cdots & a_{24} \\ a_{25} & \cdots & \cdots & \cdots & \cdots & a_{30} \\ a_{31} & \cdots & \cdots & \cdots & \cdots & a_{36} \\ \hline a_{37} & \cdots & \cdots & \cdots & \cdots & a_{42} \\ a_{43} & \cdots & \cdots & \cdots & \cdots & a_{48} \end{array}\right] \right| a_i \in U_N,$$

$1 \leq i \leq 48$, min, max}
be the special fuzzy neutrosophic unit semi open square super matrix semiring.

Study questions (i) to (viii) of problem 11 for this W.

18. Let $V = \left\{ \left(\begin{array}{c|ccc|cc|c|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{16} \\ a_{17} & a_{18} & \cdots & \cdots & \cdots & \cdots & \cdots & a_{24} \end{array}\right) \right| a_i \in$

$U_N$, $1 \leq i \leq 24$, min, max} be the special fuzzy neutrosophic unit semi open square super matrix semiring.

Study questions (i) to (viii) of problem 11 for this V.

19. Distinguish between pseudo semiring
$P_N = \{U_N, \times, \min\}$ and semiring $S_N = \{U_N, \min, \max\}$.

20. Let $S = \left\{ \left[\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \hline a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ \hline a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} & a_{28} \\ a_{29} & a_{30} & a_{31} & a_{32} \\ \hline a_{33} & a_{34} & a_{35} & a_{36} \\ a_{37} & a_{38} & a_{39} & a_{40} \\ \hline a_{41} & a_{42} & a_{43} & a_{44} \\ a_{45} & a_{46} & a_{47} & a_{48} \\ a_{49} & a_{50} & a_{51} & a_{52} \end{array}\right] \right| a_i \in U_N, 1 \leq i \leq 52,$

min, ×, +} be the special fuzzy neutrosophic unit semi open square semiring.

Study questions (i) to (viii) of problem 11 for this V.

21. Distinguish between pseudo semiring $P_N = \{U_N, \times, \min\}$ and semiring pseudo ring $\{U_N, +, \times\}$.

22. Characterize those filters in $S_N = \{U_N, \min, \max\}$.

23. Can $P_N = \{U_N, \times, \min\}$ have pseudo filters?

24. Can the pseudo semiring $T_N$ have finite order pseudo subsemiring?

25. Can pseudo semiring $P_N$ has finite order pseudo filters?

26. Let $P_N = \{U_N, \min, \times\}$ be the pseudo fuzzy neutrosophic unit semi open square pseudo semiring.

(i) Can a pseudo filter be a pseudo ideal and vice versa?
(ii) Can $P_N$ have finite pseudo subsemirings?
(iii) Can $P_N$ have cyclic pseudo subsemirings?
(iv) Can a pseudo cyclic subsemiring be an ideal?
(v) Can $P_N$ have zero divisors?
(vi) Can $P_N$ have finite pseudo filters?

27. Let
$V = \{(a_1, a_2, a_3, a_4, a_5, a_6, a_7) \mid a_i \in U_N, \max, \times, 1 \leq i \leq 7\}$ be the special fuzzy neutrosophic unit square pseudo semiring.

Study questions (i) to (vi) of problem 26 for this V.

28. Let

$$W = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \\ a_{10} \\ a_{11} \\ a_{12} \\ a_{13} \end{bmatrix} \middle| \; a_i \in U_N, \min, \times, 1 \leq i \leq 13 \right\}$$ be the

special fuzzy neutrosophic unit square pseudo semiring.

Study questions (i) to (vi) of problem 26 for this W.

29. Let S= $\left\{\left.\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{16} & a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{26} & a_{27} & a_{28} & a_{29} & a_{30} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{bmatrix} \right|\right.$ $a_i \in U_N$, min, ×,

$1 \leq i \leq 35\}$

be the fuzzy neutrosophic unit square pseudo semiring.

Study questions (i) to (vi) of problem 26 for this S.

30. Let S= $\left\{\left.\begin{pmatrix} a_1 & a_2 & \ldots & a_{12} \\ a_{13} & a_{14} & \ldots & a_{24} \\ a_{25} & a_{26} & \ldots & a_{36} \end{pmatrix} \right|\right.$ $a_i \in U_N$, min, ×,

$1 \leq i \leq 36\}$

be the fuzzy neutrosophic unit square pseudo semiring.

Study questions (i) to (vi) of problem 26 for this S.

31. Let B = $\{(a_1\ a_2 \mid a_3\ a_4 \mid a_5\ a_6\ a_7 \mid a_8\ a_9 \mid a_{10}) \mid a_i \in U_N$, min, ×, $1 \leq i \leq 10\}$ be the fuzzy neutrosophic unit square pseudo semiring.

Study questions (i) to (vi) of problem 26 for this B.

32. Let $M = \left\{ \left[\begin{array}{cc} a_1 & a_2 \\ \hline a_3 & a_4 \\ a_5 & a_6 \\ \hline a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ \hline a_{13} & a_{14} \\ a_{15} & a_{16} \\ a_{17} & a_{18} \\ a_{19} & a_{20} \\ \hline a_{21} & a_{22} \\ a_{23} & a_{24} \\ a_{25} & a_{26} \\ \hline a_{27} & a_{28} \end{array}\right] \middle| \; a_i \in U_N, \min, \times, 1 \leq i \leq 28 \right\}$ be the special fuzzy neutrosophic unit square super column matrix pseudo semiring.

Study questions (i) to (vi) of problem 26 for this M.

33. Let $T = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix} \middle| \; a_i \in U_N, \times \right\}$ be a fuzzy neutrosophic pseudo semiring.

Study questions (i) to (vi) of problem 26 for this T for any special fuzzy neutrosophic unit square super square matrix pseudo semiring.

34. Study questions (i) to (vi) of problem 26 for any special fuzzy neutrosophic unit square rectangular super matrix semiring.

35. Obtain any special properties associated with special fuzzy neutrosophic unit square pseudo ring.

36. Can a pseudo ring $R_N = \{a + bI \mid a, b \in [0, 1), \times, +\}$ have idempotents?

37. Can the pseudo ring $R_N$ in problem 36 be a S-pseudo ring?

38. Can the pseudo ring $R_N$ in problem 36 have S-units?

39. Can the pseudo ring $R_N$ in problem 36 have finite pseudo subrings?

40. Can the pseudo ring $R_N$ in problem have pseudo ideals?

41. Can the pseudo ring $R_N$ in problem 36 have subring which satisfy the distributive law?

42. Can the pseudo ring $R_N$ in problem 36 have S-zero divisors?

43. Let $M = \{(a_1, a_2, \ldots, a_{15}) \mid a_i \in U_N, +, \times, 1 \leq i \leq 15\}$ be the special fuzzy neutrosophic unit semi open square row matrix pseudo ring of infinite order.

    (i) Can M have finite pseudo subrings?
    (ii) Can M have finite pseudo ideals?
    (iii) Can M have infinite number of pseudo ideals?
    (iv) Find those pseudo subrings which are not pseudo Ideal.
    (v) Prove M has infinite number of zero divisors.
    (vi) Prove M has no idempotents.
    (vii) Can M have units?
    (viii) Can M have S-zero divisors?

(ix) Can M be a pseudo Smarandache ring?

44. Let $T = \left\{ \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{15} \end{bmatrix} \middle| a_i \in U_N = \{a + bI \mid a, b \in [0, 1), \right.$

$1 \leq i \leq 15, +, \times\}$

be the special fuzzy neutrosophic unit square pseudo ring.

Study questions (i) to (ix) of problem 43 for this T.

45. Let $W = \left\{ \begin{pmatrix} a_1 & a_2 & \ldots & a_{10} \\ a_{11} & a_{12} & \ldots & a_{20} \\ a_{21} & a_{22} & \ldots & a_{30} \end{pmatrix} \middle| a_i \in U_N = \{a + bI \mid a, b \right.$

$\in [0, 1),\ 1 \leq i \leq 30, +, \times\}$

be the special fuzzy neutrosophic unit square pseudo ring.

Study questions (i) to (ix) of problem 43 for this W.

46. Let $P = \left\{ \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ \vdots & \vdots & \vdots \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \middle| a_i \in U_N = \{a + bI \mid a, b \in \right.$

$[0,1),\ 1 \leq i \leq 33, +, \times\}$

be the special fuzzy neutrosophic unit semi open square pseudo ring.

Study questions (i) to (ix) of problem 43 for this P.

47. Let M = $\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_6 \\ a_7 & a_8 & \dots & a_{12} \\ a_{13} & a_{14} & \dots & a_{18} \\ a_{19} & a_{20} & \dots & a_{24} \\ a_{25} & a_{26} & \dots & a_{30} \\ a_{31} & a_{32} & \dots & a_{36} \end{bmatrix} \right|$ $a_i \in U_N = \{a + bI \mid a, b$ $\in [0, 1),\ 1 \leq i \leq 36, +, \times\}$
be the special fuzzy neutrosophic unit semi open square pseudo ring.

Study questions (i) to (ix) of problem 43 for this M.

48. Let N = $\left\{\begin{bmatrix} a_1 & a_2 & \dots & a_{18} \\ a_{19} & a_{20} & \dots & a_{36} \\ a_{37} & a_{38} & \dots & a_{54} \end{bmatrix} \right|$ $a_i \in U_N = \{a + bI \mid a, b$

$\in [0, 1),\ 1 \leq i \leq 54, +, \times\}$

be the special fuzzy neutrosophic unit semi open square pseudo ring.

Study questions (i) to (ix) of problem 43 for this N.

49. Let W = $\{(a_1\ a_2 \mid a_3\ a_4\ a_5 \mid a_6\ a_7 \mid a_8) \mid a_i \in U_N = \{a + bI \mid a, b \in [0, 1),\ 1 \leq i \leq 8, +, \times\}$ be the special fuzzy neutrosophic unit semi open square pseudo ring.

Study questions (i) to (ix) of problem 43 for this W.

50. Let $P = \left\{ \left[ \begin{array}{cc|ccc|ccc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{18} \\ \hline a_{19} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{27} \\ a_{28} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{36} \\ a_{37} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{45} \\ \hline a_{46} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{54} \\ a_{55} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{63} \\ \hline a_{64} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{72} \end{array} \right] \right|$

$a_i \in U_N = \{a + bI \mid a, b \in [0, 1), \; 1 \leq i \leq 72, +, \times\}$ be the special fuzzy neutrosophic unit semi open square pseudo ring.

Study questions (i) to (ix) of problem 43 for this P.

51. Let T =

$\left\{ \left[ \begin{array}{c|cc|ccc|cc|c} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} & a_{18} \end{array} \right] \right| \; a_i \in$

$U_N = \{a + bI \mid a, b \in [0, 1), \; 1 \leq i \leq 17, +, \times\}$ be the special fuzzy neutrosophic unit semi open square pseudo ring.

Study questions (i) to (ix) of problem 43 for this T.

52. Let $L = \left\{ \left[ \begin{array}{cc|ccc|ccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} & a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{25} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{32} \\ a_{33} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{40} \\ \hline a_{41} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{48} \\ a_{49} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & a_{56} \end{array} \right] \right|$

$a_i \in U_N = \{a + bI \mid a, b \in [0, 1), \; 1 \leq i \leq 56, +, \times\}$ be the special fuzzy neutrosophic unit square pseudo ring.

Study questions (i) to (ix) of problem 43 for this L.

53. Let $M = \left\{ \left[ \begin{array}{ccc} a_1 & a_2 & a_3 \\ \hline a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \\ a_{10} & a_{11} & a_{12} \\ \hline a_{13} & a_{14} & a_{15} \\ \hline a_{16} & a_{17} & a_{18} \\ a_{19} & a_{20} & a_{21} \\ \hline a_{22} & a_{23} & a_{24} \\ a_{25} & a_{26} & a_{27} \\ a_{28} & a_{29} & a_{30} \\ \hline a_{31} & a_{32} & a_{33} \end{array} \right] \right|$ $a_i \in U_N = \{a + bI \mid a, b \in$ $[0,1), \; 1 \leq i \leq 33, +, \times_n\}$ be the special fuzzy neutrosophic unit semi open square pseudo ring.

Study questions (i) to (ix) of problem 43 for this M.

54. Let $W = (U_N \times U_N \times U_N) \mid U_N = \{a + bI \mid a, b \in [0, 1), +, \times\}$ be the special fuzzy neutrosophic unit semi open square pseudo ring.

Study questions (i) to (ix) of problem 43 for this W.

55. Obtain some special features enjoyed by fuzzy neutrosophic unit semi open square pseudo rings.

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# INDEX

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**Dr.W.B.Vasantha Kandasamy** is a Professor in the Department of Mathematics, Indian Institute of Technology Madras, Chennai. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 646 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 94th book.

On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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**Dr. Florentin Smarandache** is a Professor of Mathematics at the University of New Mexico in USA. He published over 75 books and 200 articles and notes in mathematics, physics, philosophy, psychology, rebus, literature. In mathematics his research is in number theory, non-Euclidean geometry, synthetic geometry, algebraic structures, statistics, neutrosophic logic and set (generalizations of fuzzy logic and set respectively), neutrosophic probability (generalization of classical and imprecise probability). Also, small contributions to nuclear and particle physics, information fusion, neutrosophy (a generalization of dialectics), law of sensations and stimuli, etc. He got the 2010 Telesio-Galilei Academy of Science Gold Medal, Adjunct Professor (equivalent to Doctor Honoris Causa) of Beijing Jiaotong University in 2011, and 2011 Romanian Academy Award for Technical Science (the highest in the country). Dr. W. B. Vasantha Kandasamy and Dr. Florentin Smarandache got the 2012 New Mexico-Arizona and 2011 New Mexico Book Award for Algebraic Structures. He can be contacted at smarand@unm.edu

**In this book authors build algebraic structures on fuzzy unit semi open square $U_F$ = {(a, b) | a, b in [0, 1)} and on the fuzzy neutrosophic unit semi open square $U_N$ = {a + bI | a, b in [0, 1)}. As distributive laws are not true we are not in a position to develop several properties of rings, semirings and linear algebras. Several open conjectures are proposed.**

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