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On Bipolar Single Valued Neutrosophic Graphs

Abstract

In this article, we combine the concept of bipolar neutrosophic set and graph theory. We introduce the notions of bipolar single valued neutrosophic graphs, strong bipolar single valued neutrosophic graphs, complete bipolar single valued neutrosophic graphs, regular bipolar single valued neutrosophic graphs and investigate some of their related properties.

Keywords

Bipolar neutrosophic sets, bipolar single valued neutrosophic graph, strong bipolar single valued neutrosophic graph, complete bipolar single valued neutrosophic graph.

1. Introduction

Zadeh [32] coined the term 'degree of membership' and defined the concept of fuzzy set in order to deal with uncertainty. Atanassov [29, 31] incorporated the degree of non-membership in the concept of fuzzy set as an independent component and defined the concept of intuitionistic fuzzy set. Smarandache [12, 13] grounded the term 'degree of indeterminacy as an independent component and defined the concept of neutrosophic set from the philosophical point of view to deal with incomplete, indeterminate and inconsistent information in real world. The concept of neutrosophic sets is a generalization of the theory of fuzzy sets, intuitionistic fuzzy sets. Each element of a neutrosophic sets has three membership degrees including a truth membership degree, an indeterminacy membership degree, and a falsity membership degree which are within the real standard or nonstandard unit interval]−0, 1+[. Therefore, if their range is restrained within the real standard unit interval [0, 1], the neutrosophic set is easily applied to engineering problems. For this purpose, Wang et al. [17] introduced the concept of a single valued neutrosophic set (SVNS) as a subclass of the neutrosophic set. Recently, Deli et al. [23] defined the concept of bipolar neutrosophic as an extension of the fuzzy sets, bipolar fuzzy sets, intuitionistic fuzzy sets and neutrosophic sets studied some of their related properties including the score, certainty and accuracy functions to compare the bipolar neutrosophic sets. The neutrosophic sets theory of and their extensions have been applied in various part [1, 2, 3, 16, 18, 19, 20, 21, 25, 26, 27, 41, 42, 50, 51, 53].

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and the relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to designe a fuzzy graph Model. The extension of fuzzy graph theory [4, 6, 11] have been developed by several researchers including intuitionistic fuzzy graphs [5, 35, 44] considered the vertex sets and edge sets as intuitionistic fuzzy sets. Interval valued fuzzy graphs [32, 34] considered the vertex sets and edge sets as interval valued fuzzy sets. Interval valued intuitionistic fuzzy graphs [8, 52] considered the vertex sets and edge sets as interval valued intuitionstic fuzzy sets. Bipolar fuzzy graphs [6, 7, 40] considered the vertex sets and edge sets as bipolar fuzzy sets. M-polar fuzzy graphs [39] considered the vertex sets and edge sets as m-polar fuzzy sets. Bipolar intuitionistic fuzzy graphs [9] considered the vertex sets and edge sets as bipolar intuitionistic fuzzy sets. But, when the relations between nodes (or vertices) in problems are indeterminate, the fuzzy graphs and their extensions are failed. For this purpose, Samarandache [10, 11] have defined four main categories of neutrosophic graphs, two based on literal indeterminacy (I), which called them; I-edge neutrosophic graph and I-vertex neutrosophic graph, these concepts are studied deeply and has gained popularity among the researchers due to its applications via real world problems [7, 14, 15, 54, 55, 56]. The two others graphs are based on (t, i, f) components and called them; The (t, i, f)-Edge neutrosophic graph and the (t, i, f)-vertex neutrosophic graph, these concepts are not developed at all. Later on, Broumi et al. [46] introduced a third neutrosophic graph model. This model allows the attachment of truth-membership (t), indeterminacy–membership (i) and falsity-membership degrees (f) both to vertices and edges, and investigated some of their properties. The third neutrosophic graph model is called single valued neutrosophic graph (SVNG for short). The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionistic fuzzy graph. Also the same authors [45] introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. Also, Broumi et al. [47] introduced the concept of interval valued neutrosophic graph as a generalization fuzzy graph, intuitionistic fuzzy graph, interval valued fuzzy graph, interval valued intuitionistic fuzzy graph and single valued neutrosophic graph and have discussed some of their properties with proof and examples. In addition Broumi et al [48] have introduced some operations such as cartesian product, composition, union and join on interval valued neutrosophic graphs and investigate some their properties. On the other hand, Broumi et al [49] have discussed a sub class of interval valued neutrosophic graph called strong interval valued neutrosophic graph, and have introduced some operations such as, cartesian product, composition and join of two strong interval valued neutrosophic graph with proofs. In the literature the study of bipolar single valued neutrosophic graphs (BSVN-graph) is still blank, we shall focus on the study of bipolar single valued neutrosophic graphs in this paper. In the present paper, bipolar neutrosophic sets are employed to study graphs and give rise to a new class of graphs called bipolar single valued neutrosophic graphs. We introduce the notions of bipolar single valued neutrosophic graphs, strong bipolar single valued neutrosophic graphs, complete bipolar single valued neutrosophic graphs, regular bipolar single valued neutrosophic graphs and investigate some of their related properties. This paper is organized as follows;

In section 2, we give all the basic definitions related bipolar fuzzy set, neutrosophic sets, bipolar neutrosophic set, fuzzy graph, intuitionistic fuzzy graph, bipolar fuzzy graph, N-graph and single valued neutrosophic graph which will be employed in later sections. In section 3, we introduce certain notions including bipolar single valued neutrosophic graphs, strong bipolar single valued neutrosophic graphs, complete bipolar single valued neutrosophic graphs, the complement of strong bipolar single valued neutrosophic graphs, regular bipolar single valued neutrosophic graphs and illustrate these notions by several examples, also we described degree of a vertex, order, size of bipolar single valued neutrosophic graphs. In section 4, we give the conclusion.

2. Preliminaries

In this section, we mainly recall some notions related to bipolar fuzzy set, neutrosophic sets, bipolar neutrosophic set, fuzzy graph, intuitionistic fuzzy graph, bipolar fuzzy graph, ***N***-graph and single valued neutrosophic graph relevant to the present work. The readers are referred to [9, 12, 17, 35, 36, 38, 43, 46, 57] for further details and background.

Definition 2.1 [12]. Let U be an universe of discourse; then the neutrosophic set A is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ U}, where the functions T, I, F: U→]$^-$0,1$^+$[define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ U to the set A with the condition:

$$^-0 \leq T_A(x)+ I_A(x)+ F_A(x) \leq 3^+. \qquad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]−0,1+[.

Since it is difficult to apply NSs to practical problems, Wang et al. [16] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2.2 [17]. Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1]. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\} \qquad (2)$$

Definition 2.3 [9]. A bipolar neutrosophic set A in X is defined as an object of the form

A= {<x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x)>: x ∈ X}, where

T^P, I^P, F^P:X→ [1, 0] and T^N, I^N, F^N: X→ [-1, 0] .The Positive membership degree T^P(x), I^P(x), F^P(x) denotes the truth membership, indeterminate membership and false membership of an element ∈ X corresponding to a bipolar neutrosophic set A and the negative membership degree T^N (x), I^N (x), F^N (x) denotes the truth membership, indeterminate membership and false membership of an element ∈ X to some implicit counter-property corresponding to a bipolar neutrosophic set A.

Example 2.4 Let X = $\{x_1, x_2, x_3\}$

$$A=\begin{Bmatrix} <x_1,0.5,0.3,0.1,-0.6,-0.4,-0.05> \\ <x_2,0.3,0.2,0.7,-0.02,-0.3,-0.02> \\ <x_3,0.8,0.05,0.4,-0.6,-0.6,-0.03> \end{Bmatrix}$$

is a bipolar neutrosophic subset of X

Definition 2.5[9]. Let A_1={<x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x)>} and A_2={<x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >} be two bipolar neutrosophic sets . Then $A_1 \subseteq A_2$ if and only if

T_1^P(x) $\leq T_2^P$(x) , I_1^P(x) $\leq I_2^P$(x), F_1^P(x) $\geq F_2^P$(x) and T_1^N(x) $\geq T_2^N$(x) , I_1^N(x) $\geq I_2^N$(x) , F_1^N(x) $\leq F_2^N$(x) for all x ∈ X.

Definition 2.6[9]. Let A_1={<x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x)>} and A_2={<x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >} be two bipolar neutrosophic sets . Then $A_1 = A_2$ if and only if

T_1^P(x) $= T_2^P$(x) , I_1^P(x) $= I_2^P$(x), F_1^P(x) $= F_2^P$(x) and T_1^N(x) $= T_2^N$(x) , I_1^N(x) $= I_2^N$(x) , F_1^N(x) $= F_2^N$(x) for all x ∈ X

Definition 2.7 [9]. Let A_1={<x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x)>} and A_2={<x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >} be two bipolar neutrosophic sets . Then their union is defined as:

$$(A_1 \cup A_2)(\mathrm{x}) = \begin{pmatrix} \max(T_1^P(\mathrm{x}),T_2^p(\mathrm{x})), \frac{I_1^P(\mathrm{x})+ I_2^P(\mathrm{x})}{2}, \min(T_1^P(\mathrm{x}),T_2^P(\mathrm{x})) \\ \min(T_1^N(\mathrm{x}),T_2^N(\mathrm{x})), \frac{I_1^N(\mathrm{x})+ I_2^N(\mathrm{x})}{2}, \max(T_1^N(\mathrm{x}),T_2^N(\mathrm{x})) \end{pmatrix} \text{ for all x} \in \mathrm{X}.$$

Definition 2.8 [9]. Let A_1={<x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x)>} and A_2={<x, T_2^P(x), I_2^P(x), F_2^P(x), T_2^N(x), I_2^N(x), F_2^N(x) >} be two bipolar neutrosophic sets . Then their intersection is defined as:

$$(A_1 \cap A_2)(\mathrm{x}) = \begin{pmatrix} \min(T_1^P(\mathrm{x}),T_2^P(\mathrm{x})), \frac{I_1^P(\mathrm{x})+ I_2^P(\mathrm{x})}{2}, \max(T_1^P(\mathrm{x}),T_2^P(\mathrm{x})) \\ \max(T_1^N(\mathrm{x}),T_2^N(\mathrm{x})), \frac{I_1^N(\mathrm{x})+ I_2^N(\mathrm{x})}{2}, \min(T_1^N(\mathrm{x}),T_2^N(\mathrm{x})) \end{pmatrix} \text{ for all x} \in \mathrm{X}.$$

Definition 2.9 [9]. Let A_1= {<x, T_1^P(x), I_1^P(x), F_1^P(x), T_1^N(x), I_1^N(x), F_1^N(x)>: x ∈ X} be a bipolar neutrosophic set in X. Then the complement of A is denoted by A^c and is defined by

$T_{A^c}^P$(x)={1^P}-T_A^P(x), $I_{A^c}^P$(x)={1^P}-I_A^P(x), $F_{A^c}^P$(x)={1^P}-F_A^+(x)

And

$T_{A^c}^N$(x)={1^N}-T_A^N(x), $I_{A^c}^N$(x)={1^N}-I_A^N(x), $F_{A^c}^N$(x)={1^N}-F_A^N(x)

Definition 2.10 [43]. A fuzzy graph is a pair of functions G = (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. i.e σ: V → [0,1] and

μ: VxV→[0,1] such that μ(uv) ≤ σ(u) ∧ σ(v) for all u, v ∈ V where uv denotes the edge between u and v and σ(u) ∧ σ(v) denotes the minimum of σ(u) and σ(v). σ is called the fuzzy vertex set of V and μ is called the fuzzy edge set of E.

Definition 2.11[38]: By a *N*-graph G of a graph G^*, we mean a pair G= (μ_1, μ_2) where μ_1 is an *N*-function in V and μ_2 is an *N*-relation on E such that μ_2(u, v) ≥ max (μ_1(u), μ_1(v)) all u, v ∈ V.

Definition 2.12[35]: An Intuitionistic fuzzy graph is of the form G = (V, E) where

iii. V= {v_1, v_2,…., v_n} such that μ_1: V→ [0,1] and γ_1: V → [0,1] denote the degree of membership and non-membership of the element v_i ∈ V, respectively, and

0 ≤ $\mu_1(v_i) + \gamma_1(v_i)$) ≤ 1 for every v_i ∈ V, (i = 1, 2, ……. n),

iv. E ⊆ V x V where μ_2: VxV→[0,1] and γ_2: VxV→ [0,1] are such that

$\mu_2(v_i, v_j) \leq \min [\mu_1(v_i), \mu_1(v_j)]$ and $\gamma_2(v_i, v_j) \geq \max [\gamma_1(v_i), \gamma_1(v_j)]$

and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every (v_i, v_j) ∈ E, (i, j = 1,2, ……. n)

Definition 2.13 [57]. Let X be a non-empty set. A bipolar fuzzy set A in X is an object having the form A = {(x, μ_A^P(x), μ_A^N(x)) | x ∈ X}, where μ_A^P(x): X → [0, 1] and μ_A^N(x): X → [−1, 0] are mappings.

Definition 2.14[57] Let X be a non-empty set. Then we call a mapping A = (μ_A^P, μ_A^N): X × X → [−1, 0] × [0, 1] a bipolar fuzzy relation on X such that μ_A^P(x, y) ∈ [0, 1] and μ_A^N(x, y) ∈ [−1, 0].

Definition 2.15[36]. Let $A = (\mu_A^P,\ \mu_A^N)$ and $B = (\mu_B^P,\ \mu_B^N)$ be bipolar fuzzy sets on a set X. If $A = (\mu_A^P,\ \mu_A^N)$ is a bipolar fuzzy relation on a set X, then $A = (\mu_A^P,\ \mu_A^N)$ is called a bipolar fuzzy relation on $B = (\mu_B^P,\ \mu_B^N)$ if $\mu_B^P(x, y) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(x, y) \ge \max(\mu_A^N(x), \mu_A^N(y)$ for all $x, y \in X$.

A bipolar fuzzy relation A on X is called symmetric if $\mu_A^P(x, y) = \mu_A^P(y, x)$ and $\mu_A^N(x, y) = \mu_A^N(y, x)$ for all $x, y \in X$.

Definition 2.16[36]. A bipolar fuzzy graph of a graph $G^*=$ (V, E) is a pair G = (A,B), where A = (μ_A^P, μ_A^N) is a bipolar fuzzy set in V and B = (μ_B^P, μ_B^N) is a bipolar fuzzy set on E ⊆ V x V such that μ_B^P(xy) ≤ min{μ_A^P(x), μ_A^P(y)} for all xy ∈ E, μ_B^N(xy) ≥ min{μ_A^N(x), μ_A^N(y)} for all xy ∈ E and μ_B^P(xy) = μ_B^N(xy) = 0 for all xy ∈ $\tilde{V}^2$ –E. Here A is called bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E.

Definition 2.17[46] A single valued neutrosophic graph (SVNG) of a graph G^*= (V, E) is a pair G = (A, B), where

1.V= $\{v_1, v_2, \ldots, v_n\}$ such that T_A:V→[0, 1], I_A:V→[0, 1] and F_A:V→[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element v_i ∈ V, respectively, and

0≤ $T_A(v_i)$ + $I_A(v_i)$ +$F_A(v_i)$ ≤3 for every v_i ∈ V (i=1, 2, …,n)

2. E ⊆ V x V where T_B:V x V →[0, 1], I_B:V x V →[0, 1] and F_B:V x V →[0, 1] are such that $T_B(v_i, v_j)$ ≤ min [$T_A(v_i)$, $T_A(v_j)$], $I_B(v_i, v_j)$ ≥ max [$I_A(v_i)$, $I_A(v_j)$] and $F_B(v_i, v_j)$ ≥ max [$F_A(v_i)$, $F_A(v_j)$] and

0≤ $T_B(v_i, v_j)$ + $I_B(v_i, v_j)$+ $F_B(v_i, v_j)$ ≤3 for every (v_i, v_j) ∈ E (i, j = 1, 2,…, n)

Definition 2.18[46]: Let G = (V, E) be a single valued neutrosophic graph. Then the degree of a vertex v is defined by d(v)= $(d_T(v), d_I(v), d_F(v))$ where

$d_T(v)=\sum_{u\neq v} T_B(u, v)$, $d_I(v)=\sum_{u\neq v} I_B(u, v)$ and $d_F(v)=\sum_{u\neq v} F_B(u, v)$

3. Bipolar Single Valued Neutrosophic Graph

Definition 3.1. Let X be a non-empty set. Then we call a mapping A = (x, T^P(x), I^P(x), F^P(x), T^N(x), I^N(x), F^N(x)):X × X → [−1, 0] × [0, 1] a bipolar single valued neutrosophic relation on X such that T_A^P(x, y) ∈ [0, 1], I_A^P(x, y) ∈ [0, 1], F_A^P(x, y) ∈ [0, 1], and T_A^N(x, y) ∈ [−1, 0], I_A^N(x, y) ∈ [−1, 0], F_A^N(x, y) ∈ [−1, 0].

Definition 3.2. Let $A = (T_A^P, I_A^P,\ F_A^P, T_A^N\ , I_A^N,\ F_A^N)$ and $B = (T_B^P, I_B^P, F_B^P, T_B^N\ , I_B^N,\ F_B^N)$ be bipolar single valued neutrosophic graph on a set X. If $B = (T_B^P, I_B^P,\ F_B^P, T_B^N, I_B^N,\ F_B^N)$ is a bipolar single valued neutrosophic relation on $A = (T_A^P, I_A^P, F_A^P, T_A^N\ , I_A^N,\ F_A^N)$ then

$T_B^P(x, y) \le \min(T_A^P(x), T_A^P(y))$, $T_B^N(x, y) \ge \max(T_A^N(x), T_A^N(y))$
$I_B^P(x, y) \ge \max(I_A^P(x), I_A^P(y))$, $I_B^N(x, y) \le \min(I_A^N(x), I_A^N(y))$
$F_B^P(x, y) \ge \max(F_A^P(x), F_A^P(y))$, $F_B^N(x, y) \le \min(F_A^N(x), F_A^N(y))$ for all $x, y \in X$.

A bipolar single valued neutrosophic relation B on X is called symmetric if $T_B^P(x, y) = T_B^P(y, x)$, $I_B^P(x, y) = I_B^P(y, x)$, $F_B^P(x, y) = F_B^P(y, x)$ and $T_B^N(x, y) = T_B^N(y, x)$, $I_B^N(x, y) = I_B^N(y, x)$, $F_B^N(x, y) = F_B^N(y, x)$, for all $x, y \in X$.

Definition 3.3. A bipolar single valued neutrosophic graph of a graph $G^{*}=$ (V, E) is a pair G = (A, B), where A = $(T_A^P, I_A^P, F_A^P, T_A^N, I_A^N, F_A^N)$ is a bipolar single valued neutrosophic set in V and B = $(T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N)$ is a bipolar single valued neutrosophic set in $\tilde{V}^2$ such that

$T_B^P(\boldsymbol{v_i}, \boldsymbol{v_j}) \leq \min(T_A^P(\boldsymbol{v_i}), T_A^P(\boldsymbol{v_j}))$

$I_B^P(\boldsymbol{v_i}, \boldsymbol{v_j}) \geq \max(I_A^P(\boldsymbol{v_i}), I_A^P(\boldsymbol{v_j}))$

$F_B^P(\boldsymbol{v_i}, \boldsymbol{v_j}) \geq \max(F_A^P(\boldsymbol{v_i}), F_A^P(\boldsymbol{v_j}))$

And

$T_B^N(\boldsymbol{v_i}, \boldsymbol{v_j}) \geq \max(T_A^N(\boldsymbol{v_i}), T_A^N(\boldsymbol{v_j}))$

$I_B^N(\boldsymbol{v_i}, \boldsymbol{v_j}) \leq \min(I_A^N(\boldsymbol{v_i}), I_A^N(\boldsymbol{v_j}))$

$F_B^N(\boldsymbol{v_i}, \boldsymbol{v_j}) \leq \min(F_A^N(\boldsymbol{v_i}), F_A^N(\boldsymbol{v_j}))$ for all $\boldsymbol{v_i v_j} \in \tilde{V}^2$.

Notation: An edge of BSVNG is denoted by $e_{ij} \in$ E or $\boldsymbol{v_i v_j} \in$ E

Here the sextuple (v_i, $T_A^P(v_i)$, $I_A^P(v_i)$, $F_A^P(v_i)$, $T_A^N(v_i)$, $I_A^N(v_i)$, $F_A^N(v_i)$) denotes the positive degree of truth-membership, the positive degree of indeterminacy-membership, the positive degree of falsity-membership, the negative degree of truth-membership, the negative degree of indeterminacy-membership, the negative degree of falsity- membership of the vertex vi.

The sextuple (e_{ij}, $T_B^P, I_B^P, F_B^P, T_B^N, I_B^N, F_B^N$) denotes the positive degree of truth-membership, the positive degree of indeterminacy-membership, the positive degree of falsity-membership, the negative degree of truth-membership, the negative degree of indeterminacy-membership, the negative degree of falsity- membership of the edge relation $e_{ij} = (\boldsymbol{v_i}, \boldsymbol{v}_j)$ on V× V.

Note 1. (i) When $T_A^P = I_A^P = F_A^P = 0$ and $T_A^N = I_A^N = F_A^N = 0$ for some i and j, then there is no edge between v_i and v_j .

Otherwise there exists an edge between v_i and v_j .

(ii) If one of the inequalities is not satisfied in (1) and (2), then G is not an BSVNG

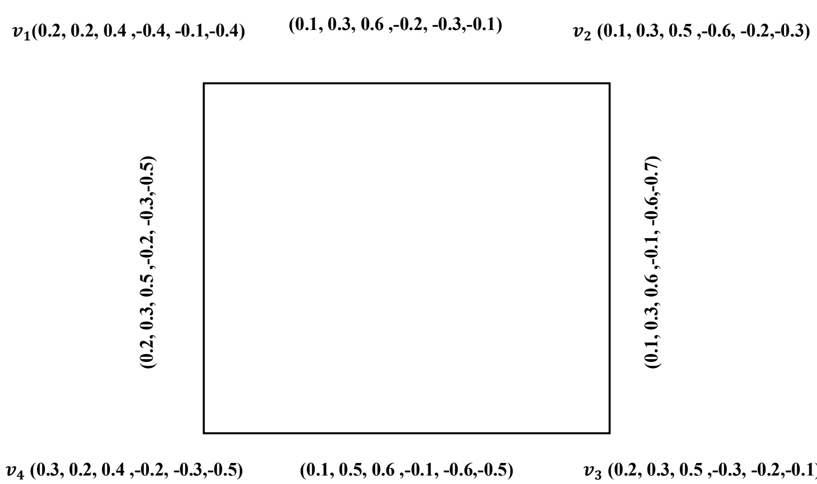

Fig. 1: *Bipolar single valued neutrosophic graph.*

Proposition 3.5: A bipolar single valued neutrosophic graph is the generalization of fuzzy graph

Proof: Suppose G= (A, B) be a bipolar single valued neutrosophic graph. Then by setting the positive indeterminacy-membership, positive falsity-membership and negative truth-membership, negative indeterminacy-membership, negative falsity-membership values of vertex set and edge set equals to zero reduces the bipolar single valued neutrosophic graph to fuzzy graph.

Example 3.6:

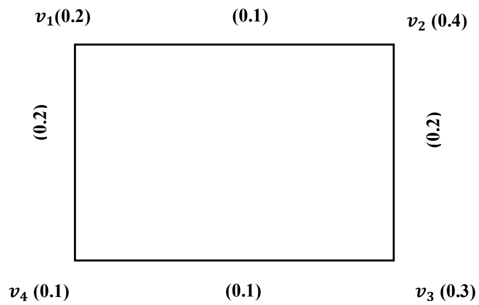

Fig.2**:** *Fuzzy graph*

Proposition 3.7: A bipolar single valued neutrosophic graph is the generalization of intuitionistic fuzzy graph

Proof: Suppose G= (A, B) be a bipolar single valued neutrosophic graph. Then by setting the positive indeterminacy-membership, negative truth-membership, negative indeterminacy-membership, negative falsity-membership values of vertex set and edge set equals to zero reduces the bipolar single valued neutrosophic graph to intuitionistic fuzzy graph.

Example 3.8

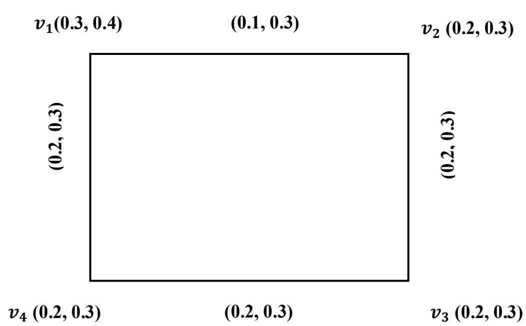

Fig.3: *Intuitionistic fuzzy graph*

Proposition 3.9: A bipolar single valued neutrosophic graph is the generalization of single valued neutrosophic graph

Proof: Suppose G= (A, B) be a bipolar single valued neutrosophic graph. Then by setting the negative truth-membership, negative indeterminacy-membership, negative falsity-membership values of vertex set and edge set equals to zero reduces the bipolar single valued neutrosophic graph to single valued neutrosophic graph.

Example 3.10

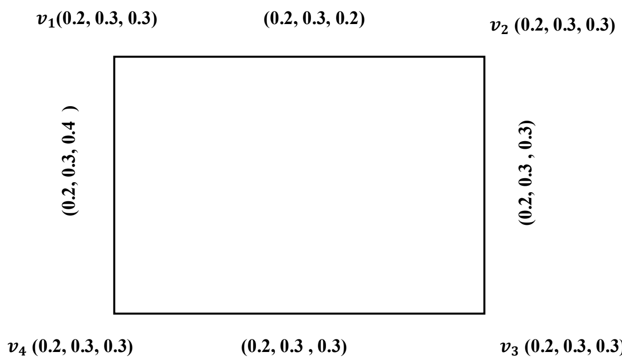

Fig.4: *Single valued neutrosophic graph*

Proposition **3.11:** A bipolar single valued neutrosophic graph is the generalization of bipolar intuitionstic fuzz graph

Proof: Suppose G= (A, B) be a bipolar single valued neutrosophic graph. Then by setting the positive indeterminacy-membership, negative indterminacy-membership values of vertex set and edge set equals to zero reduces the bipolar single valued neutrosophic graph to bipolar intuitionstic fuzzy graph

Example 3.12

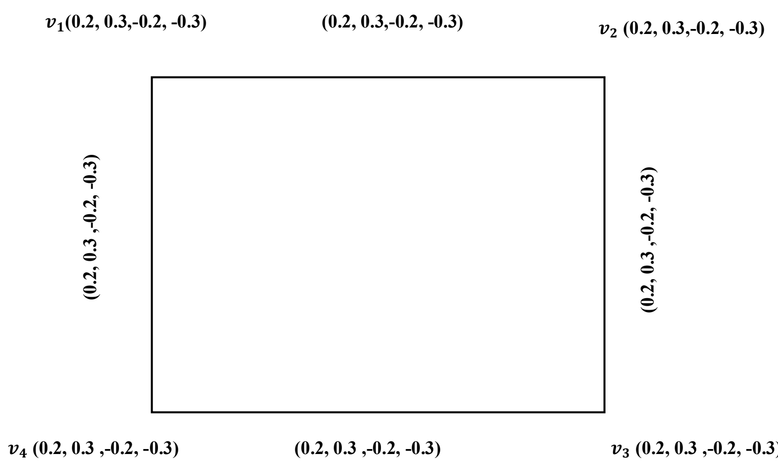

Fig. 5: *Bipolar intuitionistic fuzzy graph.*

Proposition 3.13: A bipolar single valued neutrosophic graph is the generalization of *N*-graph

Proof: Suppose G= (A, B) be a bipolar single valued neutrosophic graph. Then by setting the positive degree membership such truth-membership, indeterminacy- membership, falsity-membership and negative indeterminacy-membership, negative falsity-membership values of vertex set and edge set equals to zero reduces the single valued neutrosophic graph to *N*-graph.

Example 3.14:

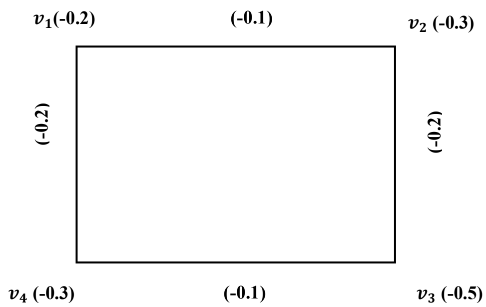

Fig. 6: *N*- graph

Definition 3.15. A bipolar single valued neutrosophic graph that has neither self loops nor parallel edge is called simple bipolar single valued neutrosophic graph.

Definition 3.16. A bipolar single valued neutrosophic graph is said to be connected if every pair of vertices has at least one bipolar single valued neutrosophic graph between them, otherwise it is disconnected.

Definition 3.17. When a vertex $\mathbf{v_i}$ is end vertex of some edges $(\mathbf{v_i}, \mathbf{v_j})$ of any BSVN-graph G= (A, B). Then $\mathbf{v_i}$ and $(\mathbf{v_i}, \mathbf{v_j})$ are said to be **incident** to each other.

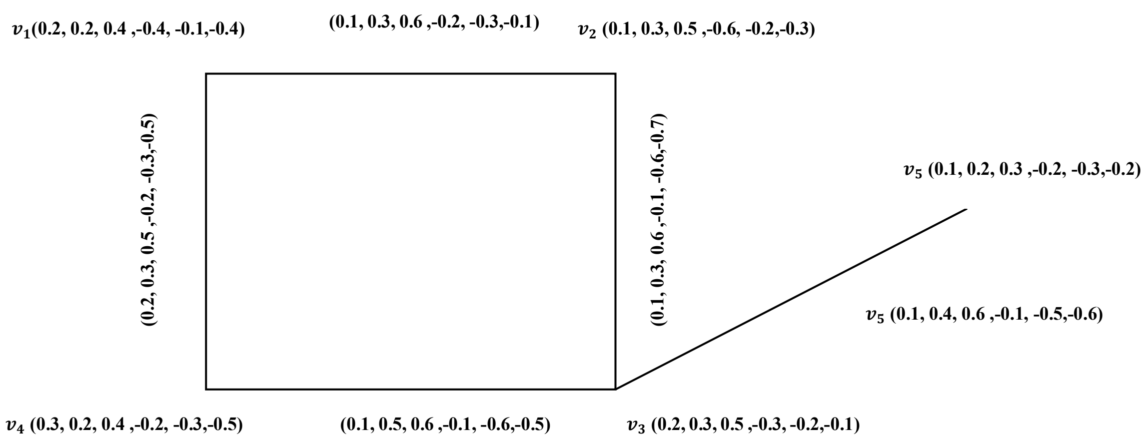

Fig. 7: *Incident BSVN-graph*

In this graph v_2v_3, v_3v_4 and v_3v_5 are incident on v_3.

Definition 3.18 Let G= (V, E) be a bipolar single valued neutrosophic graph. Then the degree of any vertex **v** is sum of positive degree of truth-membership, positive sum of degree of indeterminacy-membership, positive sum of degree of falsity-membership, negative degree of truth-membership, negative sum of degree of indeterminacy-membership, and negative sum of degree of falsity-membership of all those edges which are incident on vertex **v** denoted by d(v)= $(d_T^P(v), d_I^P(v), d_F^P(v), d_T^N(v), d_I^N(v), d_F^N(v))$ where

$d_T^P(v) = \sum_{u \neq v} T_B^P(u, v)$ denotes the positive T- degree of a vertex v,

$d_I^P(v) = \sum_{u \neq v} I_B^P(u, v)$ denotes the positive I- degree of a vertex v,

$d_F^P(v) = \sum_{u \neq v} F_B^P(u, v)$ denotes the positive F- degree of a vertex v,
$d_T^N(v) = \sum_{u \neq v} T_B^N(u, v)$ denotes the negative T- degree of a vertex v,
$d_I^N(v) = \sum_{u \neq v} I_B^N(u, v)$ denotes the negative I- degree of a vertex v,
$d_F^N(v) = \sum_{u \neq v} F_B^N(u, v)$ denotes the negative F- degree of a vertex v

Definition 3.19: The minimum degree of G is

$\delta(G) = (\delta_T^P(G), \delta_I^P(G), \delta_F^P(G), \delta_T^N(G), \delta_I^N(G), \delta_F^N(G))$, where

$\delta_T^P(G) = \wedge \{d_T^P(v) \mid v \in V\}$ denotes the minimum positive T- degree,

$\delta_I^P(G) = \wedge \{d_I^P(v) \mid v \in V\}$ denotes the minimum positive I- degree,

$\delta_F^P(G) = \wedge \{d_F^P(v) \mid v \in V\}$ denotes the minimum positive F- degree,

$\delta_T^N(G) = \wedge \{d_T^N(v) \mid v \in V\}$ denotes the minimum negative T- degree,

$\delta_I^N(G) = \wedge \{d_I^N(v) \mid v \in V\}$ denotes the minimum negative I- degree,

$\delta_F^N(G) = \wedge \{d_F^N(v) \mid v \in V\}$ denotes the minimum negative F- degree

Definition 3.20: The maximum degree of G is

$\Delta(G) = (\Delta_T^P(G), \Delta_I^P(G), \Delta_F^P(G), \Delta_T^N(G), \Delta_I^N(G), \Delta_F^N(G))$, where

$\Delta_T^P(G) = \vee \{d_T^P(v) \mid v \in V\}$ denotes the maximum positive T- degree,

$\Delta_I^P(G) = \vee \{d_I^P(v) \mid v \in V\}$ denotes the maximum positive I- degree,

$\Delta_F^P(G) = \vee \{d_F^P(v) \mid v \in V\}$ denotes the maximum positive F- degree,

$\Delta_T^N(G) = \vee \{d_T^N(v) \mid v \in V\}$ denotes the maximum negative T- degree,

$\Delta_I^N(G) = \vee \{d_I^N(v) \mid v \in V\}$ denotes the maximum negative I- degree,

$\Delta_F^N(G) = \vee \{d_F^N(v) \mid v \in V\}$ denotes the maximum negative F- degree

Example 3.21. Let us consider a bipolar single valued neutrosophic graph G= (A, B) of $G^* =$ (V, E), such that V = $\{v_1, v_2, v_3, v_4\}$, E = $\{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$

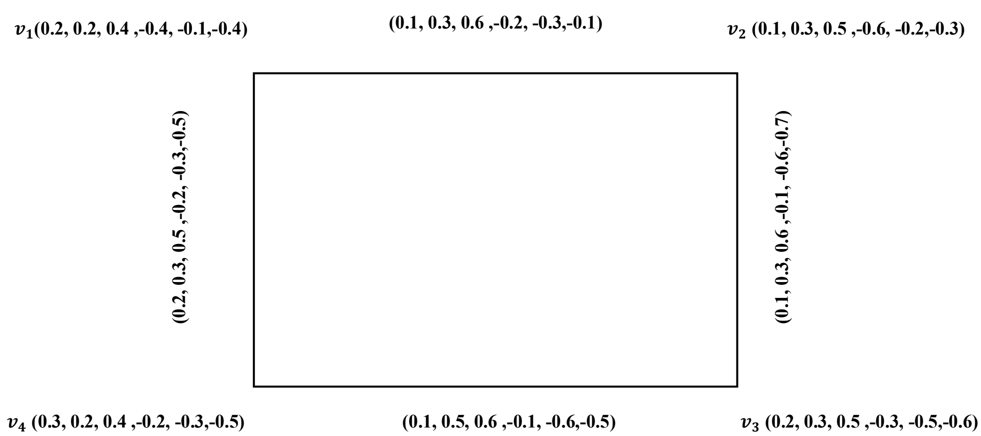

Figure 8: *Degree of a bipolar single valued neutrosophic graph G.*

In this example, the degree of v_1 is (0.3, 0.6, 1.1, -0.4, -0.6, -0.6). the degree of v_2 is (0.2, 0.6, 1.2, -0.3, -0.9, -0.8). the degree of v_3 is (0.2, 0.8, 1.2, -0.2, -1.2, -1.2). the degree of v_4 is (0.3, 0.8, 1.1, -0.3, -0.9, -1)

Order and size of a bipolar single valued neutrosophic graph is an important term in bipolar single valued neutrosophic graph theory. They are defined below.

Definition 3.22: Let G = (V, E) be a BSVNG. The order of G, denoted O(G) is defined as O(G)= $(O_T^p(G), O_I^p(G), O_F^p(G), O_T^N(G), O_I^N(G), O_F^N(G))$, where

$O_T^p(G)= \sum_{v\in V} T_1^p\,(v)$ denotes the positive T- order of a vertex v,

$O_I^p(G)= \sum_{v\in V} I_1^p\,(v)$ denotes the positive I- order of a vertex v,

$O_F^p(G)= \sum_{v\in V} F_1^p\,(v)$ denotes the positive F- order of a vertex v,

$O_T^N(G)= \sum_{v\in V} T_1^N\,(v)$ denotes the negative T- order of a vertex v,

$O_I^N(G)= \sum_{v\in V} I_1^N\,(v)$ denotes the negative I- order of a vertex v,

$O_F^N(G)= \sum_{v\in V} F_1^N\,(v)$ denotes the negative F- order of a vertex v.

Definition 3.23: Let G = (V, E) be a BSVNG. The size of G, denoted S(G) is defined as

S(G)= $(S_T^p(G), S_I^p(G), S_F^p(G), S_T^N(G), S_I^N(G), S_F^N(G))$, where

$S_T^p(G)= \sum_{u\neq v} T_2^p\,(u,v)$ denotes the positive T- size of a vertex v,

$S_I^p(G)= \sum_{u\neq v} I_2^p\,(u,v)$ denotes the positive I- size of a vertex v,

$S_F^p(G)= \sum_{u\neq v} F_2^p\,(u,v)$ denotes the positive F- size of a vertex v,

$S_T^N(G)= \sum_{u\neq v} T_2^N\,(u,v)$ denotes the negative T- size of a vertex v,

$S_I^N(G)= \sum_{u\neq v} I_2^N\,(u,v)$ denotes the negative I- size of a vertex v,

$S_F^N(G)= \sum_{u\neq v} F_2^N\,(u,v)$ denotes the negative F- size of a vertex v.

Definition 3.24 A bipolar single valued neutrosophic graph G = (V, E) is called constant if degree of each vertex is k = $(k_1, k_2, k_3, k_4, k_5, k_6)$. That is, d (v) = $(k_1, k_2, k_3, k_4, k_5, k_6)$ for all $v \in$ V.

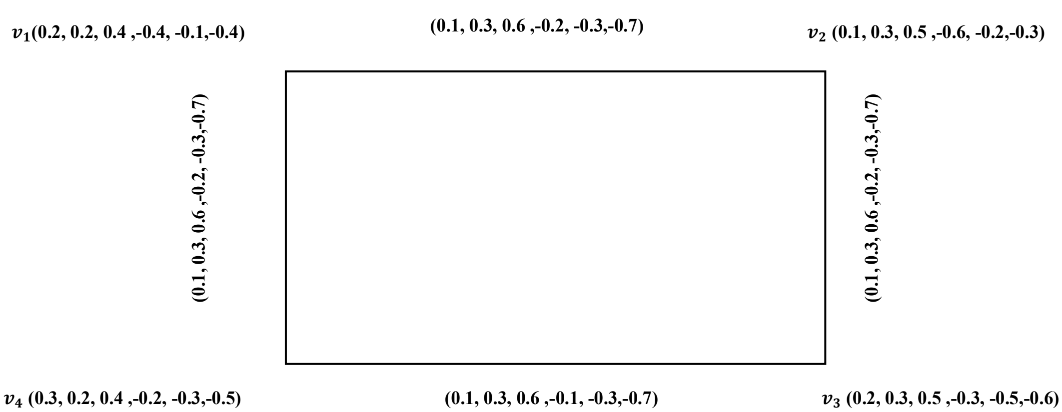

Figure 9: *Constant bipolar single valued neutrosophic graph G.*

In this example, the degree of v_1, v_2, v_3, v_4 is (0.2, 0.6, 1.2, -0.4, -0.6, -1.4).

O(G)= (0.8, 1, 1.8, -1.5, -1.1, -1.8)

S(G) = (0.4, 1.2, 2.4, -0.7, -1.2, -2.8)

Remark 3.25. G is a (k_i, k_j, k_l, k_m, k_n, k_o)-constant BSVNG iff δ = Δ= k, where k = k_i+ k_j+ k_l +k_m+ k_n+ k_o.

Definition 3.26. A bipolar single valued neutrosophic graph G= (A, B) is called strong bipolar single valued neutrosophic graph if

$T_B^P(u,v)$ =min ($T_A^P(u)$, $T_A^P(v)$),

$I_B^P(u,v)$ =max ($I_A^P(u)$, $I_A^P(v)$),

$F_B^P(u,v)$ =max ($F_A^P(u)$, $F_A^P(v)$),

$T_B^N(u,v)$ =max ($T_A^N(u)$, $T_A^N(v)$),

$I_B^N(u,v)$ =min ($I_A^N(u)$, $I_A^N(v)$),

$F_B^N(u,v)$ = min ($F_A^N(u)$, $F_A^N(v)$) for all (u, v) ∈ E

Example 3.27. Consider a strong BSVN-graph G such that V = {v_1, v_2, v_3, v_4}and E = {(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)}

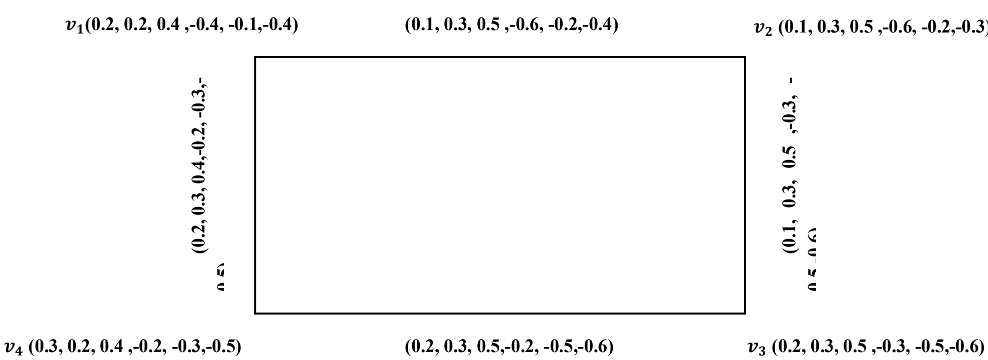

Figure 10: *Strong bipolar single valued neutrosophic graph G.*

Definition 3.28. A bipolar single valued neutrosophic graph G= (A, B) is called complete if

$T_B^P(u,v)$ =min ($T_A^P(u)$, $T_A^P(v)$),

$I_B^P(u,v)$ =max ($I_A^P(u)$, $I_A^P(v)$),

$F_B^P(u,v)$ =max ($F_A^P(u)$, $F_A^P(v)$),

$T_B^N(u,v)$ =max ($T_A^N(u)$, $T_A^N(v)$),

$I_B^N(u,v)$ =min ($I_A^N(u)$, $I_A^N(v)$),

$F_B^N(u,v)$ = min ($F_A^N(u)$, $F_A^N(v)$) for all u, v ∈ V.

Example 3.29. Consider a complete BSVN-graph G such that V = $\{v_1, v_2, v_3, v_4\}$and E = $\{(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1), (v_1, v_3), (v_2, v_4)\}$

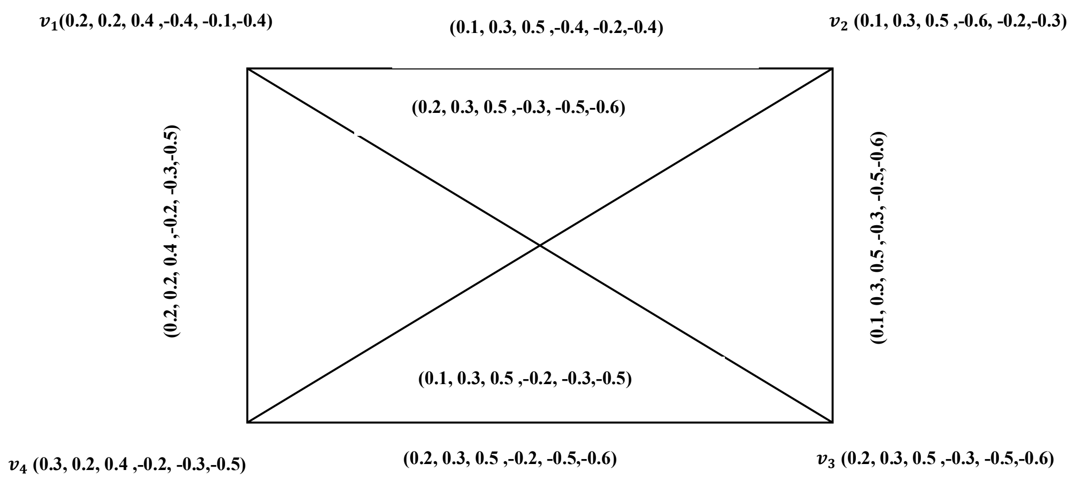

Figure 11: *Complete bipolar single valued neutrosophic graph G.*

$d(v_1)$= (0.5, 0.8, 1.4, -0 .9, -1, -1.5)
$d(\boldsymbol{v_2})$= (0.4, 0.9, 1.5, -1 .2, -1, -1.6)
$d(\boldsymbol{v_3})$= (0.4, 0.9, 1.5, -0 .7, -1.3, -1.7)
$d(\boldsymbol{v_4})$= (0.5, 0.8, 1.4, -0 .6, -1.1, -1.6)

Definition 3.30. The complement of a bipolar single valued neutrosophic graph G = (A, B) of a graph G^*= (V, E) is a bipolar single valued neutrosophic graph $\bar{G}$ = ($\bar{A}$, $\bar{B}$) of $\overline{G^*}$ = (V,V ×V), where $\bar{A}$ = A = (T_A^P, I_A^P, F_A^P, T_A^N , I_A^N, F_A^N) and $\bar{B}$ = ($\overline{T_B^P}$, $\overline{I_B^P}$, $\overline{F_B^P}$, $\overline{T_B^N}$, $\overline{I_B^N}$, $\overline{F_B^N}$)
is defined by
$\bar{T}_B^P$(u, v) = $\min\left(T_A^P(u), T_A^P(v)\right)$ - $T_B^P(u,v)$ for all u, v ∈ V, uv ∈ $\tilde{V}^2$
$\bar{I}_B^P$(u, v) = $\max\left(I_A^P(u), I_A^P(v)\right)$ - $I_B^P(u,v)$ for all u, v ∈ V, uv ∈ $\tilde{V}^2$
$\bar{F}_B^P$(u, v) = $\max\left(F_A^P(u), F_A^P(v)\right)$ - $F_B^P(u,v)$ for all u, v ∈ V, uv ∈ $\tilde{V}^2$
$\bar{T}_B^N$(u, v) = $\max\left(T_A^N(u), T_A^N(v)\right)$ - $T_B^N(u,v)$ for all u, v ∈ V, uv ∈ $\tilde{V}^2$
$\bar{I}_B^N$(u, v) = $\min\left(I_A^N(u), I_A^N(v)\right)$ - $I_B^N(u,v)$ for all u, v ∈ V, uv ∈ $\tilde{V}^2$
$\bar{F}_B^N$(u, v) = $\min\left(F_A^N(u), F_A^N(v)\right)$ - $F_B^N(u,v)$ for all u, v ∈ V, uv ∈ $\tilde{V}^2$

Proposition 3.31: The complement of complete BSVN-graph is a BSVN-graph with no edge. Or if G is a complete then in $\bar{G}$ the edge is empty.

Proof

Let G= (V, E) be a complete BSVN-graph. $T_B^P(u,v)$ =min ($T_A^P(u)$, $T_A^P(v)$),
So $T_B^P(u,v)$ =min ($T_A^P(u)$, $T_A^P(v)$), $T_B^N(u,v)$ =max ($T_A^N(u)$, $T_A^N(v)$),

$I_B^P(u,v)$ =max $(T_A^P(u), T_A^P(v))$, $I_B^N(u,v)$ =min $(I_A^N(u), I_A^N(v))$,

$F_B^P(u,v)$ =max $(T_A^P(u), T_A^P(v))$, $F_B^N(u,v)$ =min $(F_A^N(u), F_A^N(v)$ for all $u, v \in$ V

Hence in $\bar{G}$,
$\bar{T}_B^P$= $\min\left(T_A^P(u), T_A^P(v)\right)$ - $T_B^P(u,v)$ for all $u, v \in$ V
= $\min\left(T_A^P(u), T_A^P(v)\right)$ - $\min\left(T_A^P(u), T_A^P(v)\right)$ for all $u, v \in$ V
= 0 for all $u, v \in$ V
and
$\bar{I}_B^P$= $\max\left(I_A^P(u), I_A^P(v)\right)$ - $I_B^P(u,v)$ for all $u, v \in$ V
= $\max\left(I_A^P(u), I_A^P(v)\right)$ - $\max\left(I_A^P(u), I_A^P(v)\right)$ for all $u, v \in$ V
= 0 for all $u, v \in$ V
Also
$\bar{F}_B^P$= $\max\left(F_A^P(u), F_A^P(v)\right)$ - $F_B^P(u,v)$ for all $u, v \in$ V
= $\max\left(F_A^P(u), F_A^P(v)\right)$ - $\max\left(F_A^P(u), F_A^P(v)\right)$ for all $u, v \in$ V
= 0 for all $u, v \in$ V
Similarly
$\bar{T}_B^N$= $\max\left(T_A^N(u), T_A^N(v)\right)$ - $T_B^N(u,v)$ for all $u, v \in$ V
= $\max\left(T_A^N(u), T_A^N(v)\right)$ - $\max\left(T_A^N(u), T_A^N(v)\right)$ for all $u, v \in$ V
= 0 for all $u, v \in$ V
and
$\bar{I}_B^P$= $\min\left(I_A^N(u), I_A^N(v)\right)$ - $I_B^N(u,v)$ for all $u, v \in$ V
= $\min\left(I_A^N(u), I_A^N(v)\right)$ - $\min\left(I_A^N(u), I_A^N(v)\right)$ for all $u, v \in$ V
= 0 for all $u, v \in$ V
Also
$\bar{F}_B^N$= $\min\left(F_A^N(u), F_A^N(v)\right)$ - $F_B^N(u,v)$ for all $u, v \in$ V
= $\min\left(F_A^N(u), F_A^N(v)\right)$ - $\min\left(F_A^N(u), F_A^N(v)\right)$ for all $u, v \in$ V
= 0 for all $u, v \in$ V
$(\bar{T}_B^P, \bar{I}_B^P, \bar{F}_B^P, \bar{T}_B^N, \bar{I}_B^N, \bar{F}_B^N)$
Thus $(\bar{T}_B^P, \bar{I}_B^P, \bar{F}_B^P, \bar{T}_B^N, \bar{I}_B^N, \bar{F}_B^N)$= (0, 0, 0, 0, 0)
Hence the edge set of $\bar{G}$ is empty if G is a complete BSVNG.

Definition 3.32: A regular BSVN-graph is a BSVN-graph where each vertex has the same number of open neighbors degree. d_N(v)= $(d_{NT}^P(v), d_{NI}^P(v), d_{NF}^P(v), d_{NT}^N(v), d_I^N(v), d_{NF}^N(v))$.

The following example shows that there is no relationship between regular BSVN-graph and a constant BSVN-graph

Example 3.33. Consider a graph G^* such that V= $\{v_1, v_2, v_3, v_4\}$, E = $\{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$. Let A be a single valued neutrosophic subset of V and le B a single valued neutrosophic subset of E denoted by

	v_1	v_2	v_3	v_4
T_A^P	0.2	0.2	0.2	0.2
I_A^P	0.2	0.2	0.2	0.2

	v_1v_2	v_2v_3	v_3v_4	v_4v_1
T_B^P	0.1	0.1	0.1	0.2
I_B^P	0.3	0.3	0.5	0.3

F_A^P	0.4	0.4	0.4	0.4
T_A^N	-0.4	-0.4	-0.4	-0.4
I_A^N	-0.1	-0.4	-0.1	-0.1
F_A^N	-0.4	-0.4	-0.4	-0.4

F_B^P	0.6	0.6	0.6	0.5
T_B^N	-0.2	-0.1	-0.1	-0.2
I_B^N	-0.3	-0.6	-0.6	-0.3
F_B^N	-0.5	-0.7	-0.7	-0.5

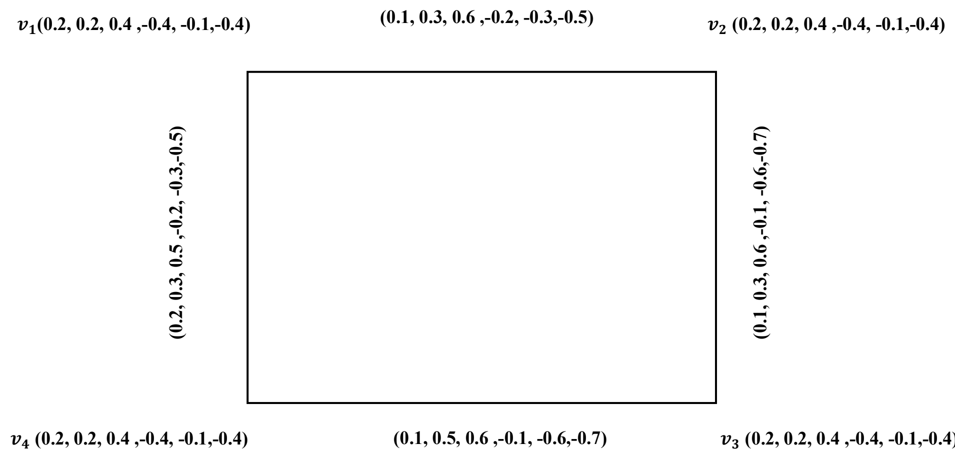

Figure 12: *Regular bipolar single valued neutrosophic graph G.*

By routing calculations show that G is regular BSVN-graph since each open neighbors degree is same, that is (0.4, 0.4, 0.8, -0.8, -0.2, -0.8). But it is not constant BSVN-graph since degree of each vertex is not same.

Definition 3.34: Let G = (V, E) be a bipolar single valued neutrosophic graph. Then the totally degree of a vertex v ∈ V is defined by

td(v)= $(td_T^P(v), td_I^P(v), td_F^P(v), td_T^N(v), td_I^N(v), td_F^N(v))$ where

$td_T^P(v)$ =$\sum_{u\neq v} T_B^P(u,v)$ +$T_A^P(v)$ denotes the totally positive T- degree of a vertex v,

$td_I^P(v)$=$\sum_{u\neq v} I_B^P(u,v)$ +$I_A^P(v)$ denotes the totally positive I- degree of a vertex v,

$td_F^P(v)$ =$\sum_{u\neq v} F_B^P(u,v)$ +$F_A^P(v)$ denotes the totally positive F- degree of a vertex v,

$td_T^N(v)$ =$\sum_{u\neq v} T_B^N(u,v)$ +$T_A^N(v)$ denotes the totally negative T- degree of a vertex v,

$td_I^N(v)$=$\sum_{u\neq v} I_B^N(u,v)$ +$I_A^N(v)$ denotes the totally negative I- degree of a vertex v,

$td_F^N(v)$ =$\sum_{u\neq v} F_B^N(u,v)$ +$F_A^N(v)$ denotes the totally negative F- degree of a vertex v

If each vertex of G has totally same degree **m = (**$m_1, m_2, m_3, m_4, m_5, m_6$**)**, then G is called a **m**-totally constant BSVN-Graph.

Example 3.35. Let us consider a bipolar single valued neutrosophic graph G= (A, B) of G^* = (V, E), such that V = {v_1, v_2, v_3, v_4}, E = {(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)}

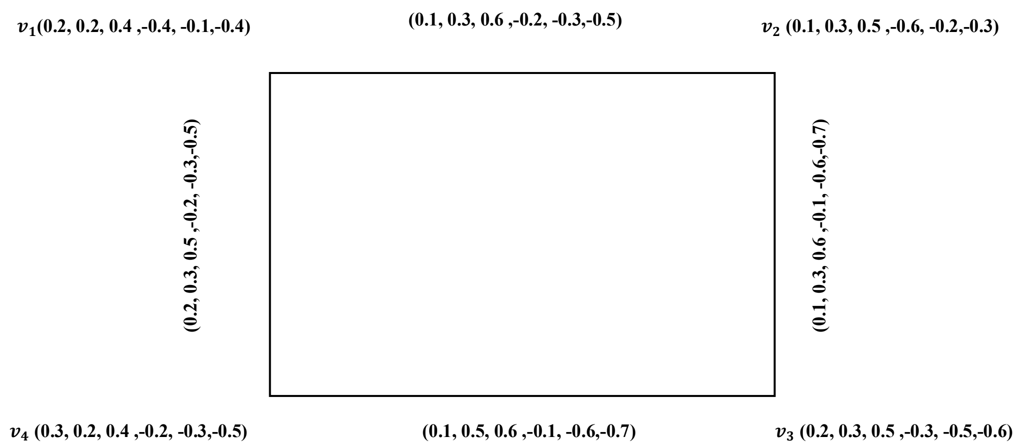

Figure 13: *Totally degree of a bipolar single valued neutrosophic graph G.*

In this example, the totally degree of v_1 is (0.5, 0.8, 1.4, -0.8, -0.7, -1.4). The totally degree of v_2 is (0.3, 0.9, 1.7, -0.9, -1.1, -1.5). The totally degree of v_3 is (0.4, 1.1, 1.7, -0.5, -1.7, -2). The totally degree of v_4 is (0.6, 1, 1.5, -0.5, -1.1, -1,7).

Definition 3.36: A totally regular BSVN-graph is a BSVN-graph where each vertex has the same number of closed neighbors degree, it is noted d[v]

Example 3.37. Let us consider a BSVN-graph G= (A, B) of G^* = (V, E), such that V = $\{v_1, v_2, v_3, v_4\}$ and E = {(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)}

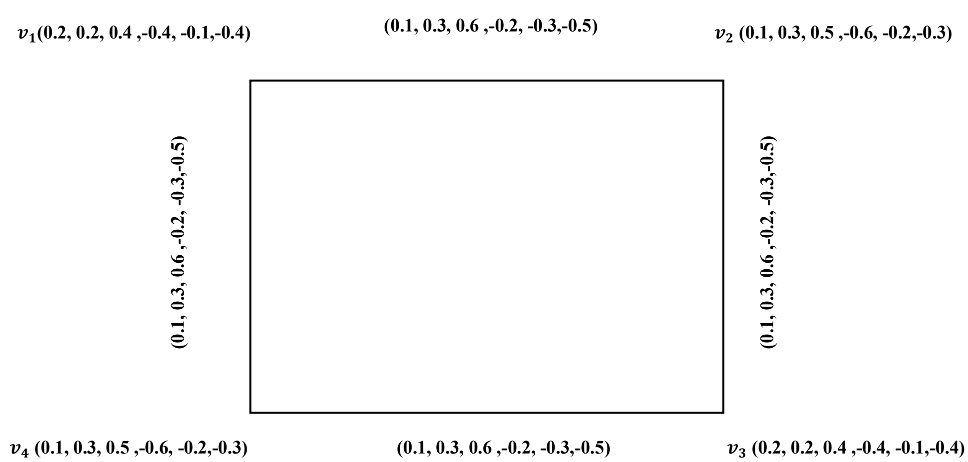

Figure 14: *Degree of a bipolar single valued neutrosophic graph G.*

By routing calculations, we show that G is regular BSVN-graph since the degree of v_1, v_2, v_3, and v_4 is (0.2, 0.6, 1.2, -0.4, -0.6, -1). It is neither totally regular BSVN-graph not constant BSVN-graph.

4. Conclusion

In this paper, we have introduced the concept of bipolar single valued neutrosophic graphs and described degree of a vertex, order, size of bipolar single valued neutrosophic graphs, also we have introduced the notion of complement of a bipolar single valued neutrosophic graph, strong bipolar single valued neutrosophic graph, complete bipolar single valued neutrosophic graph, regular bipolar single valued neutrosophic graph. Further, we are going to study some types of single valued neutrosophic graphs such irregular and totally irregular single valued neutrosophic graphs and bipolar single valued neutrosophic graphs.

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