# On neutrosophic topology



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## Abstract

**Purpose** – Recently, Smarandache generalized the Atanassov's intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). Also, this author defined the notion of neutrosophic topology on the non-standard interval. One can expect some relation between the intuitionistic fuzzy topology (IFT) on an IFS and the neutrosophic topology. This paper aims to show that this is false.

**Design/methodology/approach** – The possible relation between the IFT and the neutrosophic topology is studied.

**Findings** – Relations on neutrosophic topology and IFT are found.

**Research limitations/implications** – Clearly, this paper is confined to IFSs and NSs.

**Practical implications** – The main applications are in the mathematical field.

**Originality/value** – The paper shows original results on fuzzy sets and topology.

**Keywords** Cybernetics, Set theory, Topology

**Paper type** Research paper

## 1. Introduction

In various recent papers, Smarandache (2002, 2003, 2005) generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). In Smarandache (2005) some distinctions between NSs and IFSs are underlined.

The notion of IFS defined by Atanassov (1983) has been applied by Çoker (1997) for study intuitionistic fuzzy topological spaces (IFTSs). This concept has been developed by many authors (Bayhan and Çoker, 2003; Çoker, 1996, 1997; Çoker and Eş, 1995; Eş and Çoker, 1996; Gürçay *et al.*, 1997; Hanafy, 2003; Hur *et al.*, 2004; Lee and Lee, 2000; Lupiáñez, 2004a, b, 2006a, b, 2007; Turanh and Çoker, 2000).

Smarandache (2002) also defined the notion of neutrosophic topology on the non-standard interval.

One can expect some relation between the inuitionistic fuzzy topology on an IFS and the neutrosophic topology. We show in this paper that this is false. Indeed, the complement of an IFS $A$ is not the complement of $A$ in the neutrosophic operation, the union and the intersection of IFSs do not coincide with the corresponding operations for NSs, and finally an intuitionistic fuzzy topology (IFT) is not necessarily a neutrosophic topology.

## 2. Basic definitions

First, we present some basic definitions:

*Definition 1.* Let $X$ be a non-empty set. An IFS $A$, is an object having the form $A = \{ < x, \mu_A, \gamma_A > / x \in X\}$ where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set $A$, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$ (Atanassov, 1983).

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*Definition 2.* Let $X$ be a non-empty set, and the IFSs:

$$A = \{ < x, \mu_A, \gamma_A > | x \in X\}$$

$$B = \{ < x, \mu_B, \gamma_B > | x \in X\}$$

Let (Atanassov, 1988):

$$\bar{A} = \{ < x, \gamma_A, \mu_A > | x \in X\}$$

$$A \cap B = \{ < x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B > | x \in X\}$$

$$A \cup B = \{ < x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B > | x \in X\}.$$

*Definition 3.* Let $X$ be a non-empty set. Let $0_\sim = \{ < x, 0, 1 > | x \in X\}$ and $1_\sim = \{ < x, 1, 0 > | x \in X\}$ (Çoker, 1997).

*Definition 4.* An IFT on a non-empty set $X$ is a family $\tau$ of IFSs in $X$ satisfying:

- $0_\sim, 1_\sim \in \tau$;
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$; and
- $\cup G_j \in \tau$ for any family $\{G_j | j \in J\} \subset \tau$.

In this case, the pair $(X,\tau)$ is called an IFTS and any IFS in $\tau$ is called an intuitionistic fuzzy open set in $X$ (Çoker, 1997).

*Definition 5.* Let $T, I, F$ be real standard or non-standard subsets of the non-standard unit interval $]^-0, 1^+[$, with:

$$\sup T = t_{\sup}, \inf T = t_{\inf}$$

$$\sup I = i_{\sup}, \inf I = i_{\inf}$$

$$\sup F = f_{\sup}, \inf F = f_{\inf} \text{ and } n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup} \quad n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf},$$

$T, I, F$ are called neutrosophic components. Let $U$ be a universe of discourse, and $M$ a set included in $U$. An element $x$ from $U$ is noted with respect to the set $M$ as $x(T, I, F)$ and belongs to $M$ in the following way: it is $t$% true in the set, $i$% indeterminate (unknown if it is) in the set, and $f$% false, where $t$ varies in $T$, $i$ varies in $I$, $f$ varies in $F$. The set $M$ is called a NS (Smarandache, 2005).

*Remark.* All IFS is a NS.

*Definition 6.* Let $S_1$ and $S_2$ be two (uni-dimensional) real standard or non-standard subsets, then we define (Smarandache, 2003):

$$S_1 \oplus S_2 = \{x | x = s_1 + s_2, \quad \text{where } s_1 \in S_1 \text{ and } s_2 \in S_2\},$$

$$S_1 \ominus S_2 = \{x | x = s_1 - s_2, \quad \text{where } s_1 \in S_1 \text{ and } s_2 \in S_2\},$$

$$S_1 \odot S_2 = \{x | x = s_1 \cdot s_2, \quad \text{where } s_1 \in S_1 \text{ and } s_2 \in S_2\}.$$

*Definition 7.* One defines, with respect to the sets $A$ and $B$ over the universe $U$:

- Complement: if $x(T_1, I_1, F_1) \in A$ then $x(\{1^+\} \ominus T_1, \{1^+\} \ominus I_1, \{1^+\} \ominus F_1) \in C(A)$.
- Intersection: if $x(T_1, I_1, F_1) \in A$, $x(T_2, I_2, F_2) \in B$ then $x(T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2) \in A \cap B$.

- Union: if $x(T_1,I_1,F_1)\in A, x(T_2,I_2,F_2)\in B$ then $x(T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2) \in A \cup B$ (Smarandache, 2005).

## 3. Results

*Proposition 1.* Let $A$ be an IFS in $X$, and $j(A)$ be the corresponding NS. We have that the complement of $j(A)$ is not necessarily $j(\bar{A})$.

*Proof.* If $A=<x,\mu_A,\gamma_A>$ is $x(\mu_A(x),1-\mu_A(x)-\nu_A(x),\nu_A(x))\in j(A)$. Then:

$$\text{for } 0_\sim =< x,0,1> \text{ is } x(0,0,1)\in j(0_\sim)$$

$$\text{for } 1_\sim =< x,1,0> \text{ is } x(1,0,0)\in j(1_\sim)$$

$$\text{and for } \bar{A} =< x,\gamma_A,\mu_A> \text{ is } x(\gamma_A(x),1-\mu_A(x)-\nu_A(x),\mu_A(x))\in j(\bar{A}).$$

Thus, $1_\sim=\overline{0_\sim}$ and $j(1_\sim)\neq C(j(0_\sim))$ because $x(1,0,0,)\in j(1_\sim)$ but $x(\{1^+\},\{1^+\},\{0^+\})\in C(j(0_\sim))$. □

*Proposition 2.* Let $A$ and $B$ be two IFSs in $X$, and $j(A)$ and $j(B)$ be the corresponding NSs. We have that $j(A)\cup j(B)$ is not necessarily $j(A\cup B)$ and $j(A)\cap j(B)$ is not necessarily $j(A\cap B)$.

*Proof.* Let $A=<x,1/2,1/3>$ and $B=<x,1/2,1/2>$ (i.e. $\mu_A$, $\nu_A$, $\mu_B$, $\nu_B$ are constant maps).

Then, $A\cup B=<x,\mu_A\vee\mu_B,\gamma_A\wedge\gamma_B>=<x,1/2,1/3>$ and $x(1/2,1/6,1/3)\in j(A\cup B)$. On the other hand, $x(1/2,1/6,1/3)\in j(A)$, $x(1/2,0,1/2)\in j(B)$, $x(1,1/6,5/6)\in j(A)\oplus j(B)$, $x(1/4,0,1/6)\in j(A)\odot j(B)$ and $x(3/4,1/6,2/3)\in j(A)\cup j(B)$. Thus, $j(A\cup B)\neq j(A)\cup j(B)$.

Analogously, $A\cap B=<x,\mu_A\wedge\mu_B,\gamma_A\vee\gamma_B>=<x,1/2,1/2>$ and $x(1/2,0,1/2)\in j(A\cap B)$, but $x(1/4,0,1/6)\in j(A)\cap j(B)$. Thus, $j(A\cap B)\neq j(A)\cap j(B)$. □

*Definition 8.* Let us construct a neutrosophic topology on $\mathrm{NT}=]^-0,\ 1^+[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology gorms a neutrosophic topological space (Smarandache, 2002).

*Proposition 3.* Let $(X,\tau)$ be an IFTS. Then, the family $\{j(U)|U\in\tau\}$ is not necessarily a neutrosophic topology.

*Proof.* Let $\tau=\{1_\sim,0_\sim,A\}$ where $A=<x,1/2,1/2>$ then $x(1,0,0)\in j(1_\sim)$, $x\in(0,0,1)\in j(0_\sim)$ and $x(1/2,0,1/2)\in j(A)$. Thus, $\{j(1_\sim),j(0_\sim),j(A)\}$ is not a neutrosophic topology, because this family is not closed by finite intersections, indeed, $x(1/2,0,0)\in j(1_\sim)\cap j(A)$, and this NS is not in the family. □

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