BROUMI, S[1,*], SMARANDACHE, F[2], TALEA, M[3], BAKALI, A[4]

[1,3] Laboratory of Information Processing, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco.
2 Department of Mathematics, University of New Mexico,705 Gurley Avenue, Gallup, NM 87301, USA.
4 Ecole Royale Navale, Boulevard Sour Jdid, B.P 16303 Casablanca, Morocco.
Corresponding author's E-Mail: broumisaid78@gmail.com

Operations on Interval Valued Neutrosophic Graphs

Abstract

Combining the single valued neutrosophic set with graph theory, a new graph model emerges, called single valued neutrosophic graph. This model allows attaching the truth-membership (t), indeterminacy–membership (i) and falsity- membership degrees (f) both to vertices and edges. Combining the interval valued neutrosophic set with graph theory, a new graph model emerges, called interval valued neutrosophic graph. This model generalizes the fuzzy graph, intuitionistic fuzzy graph and single valued neutrosophic graph. In this paper, the authors define operations of Cartesian product, composition, union and join on interval valued neutrosophic graphs, and investigate some of their properties, with proofs and examples.

Keywords

Neutrosophy, neutrosophic set, fuzzy set, fuzzy graph, neutrosophic graph, interval valued neutrosophic set, single valued neutrosophic graph, interval valued neutrosophic graph.

1. Introduction

The neutrosophy was pioneered by F. Smarandache (1995, 1998). It is a branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. The neutrosophic set proposed by Smarandache is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world, being a generalization of fuzzy set (Zadeh 1965; Zimmermann 1985), intuitionistic fuzzy set (Atanassov 1986; Atanassov 1999),interval valued fuzzy set (Turksen 1986) and interval valued intuitionistic fuzzy sets (Atanassov and Gargov 1989).The neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. If the range is restrained within the real standard unit interval [0, 1], the neutrosophic set easily applies to engineering problems. For this purpose, Wang et al. (2010) introduced the concept of single valued neutrosophic set (SVNS) as a subclass of the neutrosophic set. The same author introduced the notion of interval valued neutrosophic sets (Wang et al. 2005b, 2010) as subclass of neutrosophic sets in which the value of truth-membership, indeterminacy-membership and falsity-membership degrees are intervals of numbers instead of real numbers. The single valued neutrosophic set and the interval valued neutrosophic set have been applied in a wide variety of fields, including computer science, engineering, mathematics, medicine and economics (Ansari 2013a, 2013b, 2013c;Aggarwal 2010;Broumi 2014;Deli 2015;Hai-Long 2015;Liu and Shi 2015;Şahin 2015; Wang et al. 2005b;Ye 2014a, 2014b,2014c).

Graph theory has now become a major branch of applied mathematics and it is generally regarded as a branch of combinatorics. Graph is a widely used tool for solving combinatorial problems in different areas, such as geometry, algebra, number theory, topology, optimization and computer science. To be noted that, when there is uncertainty regarding either the set of vertices or edges, or both, the model becomes a fuzzy graph. Many works on fuzzy graphs, intuitionistic fuzzy graphs and interval valued intuitionistic fuzzy graphs (Antonios K et al. 2014; Bhattacharya 1987; Mishra and Pal 2013; Nagoor Gani and Shajitha Begum 2010; Nagoor Gani and Latha 2012; Nagoor Gani and Basheer Ahamed 2003;Parvathi and Karunambigai 2006; Shannon and Atanassov 1994) have been carried out and all of them have considered the vertex sets and edge sets as fuzzy and /or intuitionistic fuzzy sets. But, when the relations between nodes (or vertices) are indeterminate, the fuzzy graphs and intuitionistic fuzzy graphs fail to work. For this purpose, Smarandache (2015a, 2015b, 2015c) defined four main categories of neutrosophic graphs. Two are based on literal indeterminacy (I): I-edge neutrosophic graph and I-vertex neutrosophic graph. The two categories were deeply studied and gained popularity among the researchers (Garg et al. 2015,Vasantha Kandasamy2004, 2013, 2015) due to their applications via real world problems. The other neutrosophic graph categories are based on (t, i, f) components and are called:(t, i, f)-edge neutrosophic graph and (t, i, f)-vertex neutrosophic graph. These two categories are not developed at all.

Further on, Broumi et al. (2016b) introduced a new neutrosophic graph model, called single valued neutrosophic graph (SVNG), and investigated some of its properties as well. This model allows attaching the membership (t), indeterminacy (i) and non-membership degrees (f) both to vertices and edges. The single valued neutrosophic graph is a generalization of fuzzy graph and intuitionistic fuzzy graph. Broumi et al. (2016a) also introduced neighborhood degree of a vertex and closed neighborhood degree of a vertex in single valued neutrosophic graph, as a generalization of neighborhood degree of a vertex and closed neighborhood degree of a vertex in fuzzy graph and intuitionistic fuzzy graph. Moreover, Broumi et al. (2016c) introduced the concept of interval valued neutrosophic graph, as a generalization of single valued neutrosophic graph, and discussed some properties, with proofs and examples. In addition, Broumi et al.(2016c) introduced the concept of bipolar single valued neutrosophic graph, as a generalization of fuzzy graphs, intuitionistic fuzzy graph, N-graph, bipolar fuzzy graph and single valued neutrosophic graph, and studied some related properties.

In this paper, researchers' objective is to define some operations on interval valued neutrosophic graphs, and to investigate some properties.

2. Preliminaries

In this section, the authors mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, interval valued neutrosophic sets, fuzzy graphs, intuitionistic fuzzy graphs, interval valued intuitionistic fuzzy graphs, single valued neutrosophic graphs and interval valued neutrosophic graphs, relevant to the present work. The readers are referred for further details to (Broumi et al. 2016b;Mishra and Pal 2013;Nagoor Gani and Basheer Ahamed 2003;Parvathi and Karunambigai 2006;Smarandache 2006;Wang et al. 2010;Wang et al. 2005a).

Definition 1 (Smarandache 2006)

Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set A (NS A) is an object having the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈X}, where the functions T, I, F: X→]$^-0,1^+$[define respectively a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element x ∈X to the set A with the condition:

$$^-0 \le T_A(x)+ I_A(x)+ F_A(x) \le 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of]$^-0,1^+$[.

Since it is difficult to apply NSs to practical problems, Wang et al. 2010 introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

Definition 2 (Wang et al. 2010)

Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x)$∈ [0, 1]. A SVNS A can be written as

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}$$
(2)

Definition 3 (Wang et al. 2005a)

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS A) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, one has that

$$T_A(x) = [T_{AL}(x), T_{AU}(x)],$$

$$I_A(x) = [I_{AL}(x), I_{AU}(x)],$$

$F_A(x) = [F_{AL}(x), F_{AU}(x)] \subseteq [0, 1]$, and

$$0 \le T_A(x)+ I_A(x)+ F_A(x) \le 3. \quad (3)$$

Definition 4 (Wang et al. 2005a)

An IVNS A is contained in the IVNS B, A ⊆ B, if and only if

$$T_{AL}(x) \le T_{BL}(x),\ T_{AU}(x) \le T_{BU}(x),$$
$$I_{AL}(x) \ge I_{BL}(x),\ I_{AU}(x) \ge I_{BU}(x),$$
$$F_{AL}(x) \ge F_{BL}(x),\ F_{AU}(x) \ge F_{BU}(x),\quad \text{for any x in X.} \tag{4}$$

Definition 5 (Wang et al. 2005a)

The union of two interval valued neutrosophic sets A and B is an interval neutrosophic set C, written as C = A ∪ B, whose truth-membership, indeterminacy-membership, and false membership are related to those A and B by

$$T_{CL}(x) = \max\,(T_{AL}(x),\ T_{BL}(x))$$

$$T_{CU}(x) = \max\,(T_{AU}(x),\ T_{BU}(x))$$

$$I_{CL}(x) = \min\,(I_{AL}(x),\ I_{BL}(x))$$

$$I_{CU}(x) = \min\,(I_{AU}(x),\ I_{BU}(x))$$

$$F_{CL}(x) = \min\,(F_{AL}(x),\ F_{BL}(x))$$

$$F_{CU}(x) = \min\,(F_{AU}(x),\ F_{BU}(x)),\ \text{for all x in X.} \tag{5}$$

Definition 6 (Wang et al 2005a)

Let X and Y be two non-empty crisp sets. An interval valued neutrosophic relation R(X, Y) is a subset of product space X × Y, and is characterized by the truth membership function T_R(x, y), the indeterminacy membership function I_R(x, y), and the falsity membership function F_R(x, y), where x ∈ X and y ∈ Y and T_R(x, y),I_R(x, y),F_R(x, y) ⊆ [0, 1].

Definition 7 (Nagoor Gani and Basheer Ahamed 2003)

A fuzzy graph is a pair of functions G = (σ, μ), where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ, i.e.σ: V → [0,1] and μ: VxV→[0,1], such that μ(uv) ≤ σ(u) ∧σ(v), for all u, v ∈ V where uv denotes the edge between u and v and σ(u) ∧σ(v) denotes the minimum of σ(u) and σ(v). σ is called the fuzzy vertex set of G andμ is called the fuzzy edge set of G.

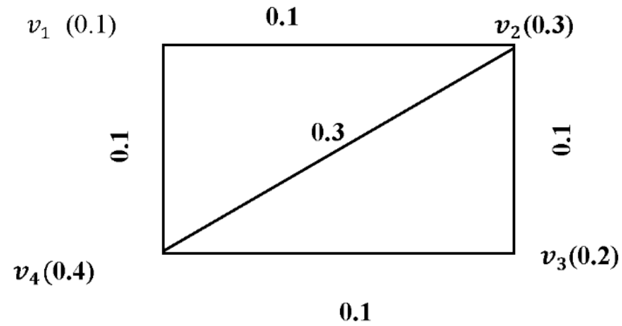

Figure 1: Fuzzy Graph

Definition 8 (Nagoor Gani and Basheer Ahamed 2003)

The fuzzy subgraph H=(τ,ρ) is called a fuzzy subgraph of G =(σ, μ) if τ(u) ≤ σ(u) for all u ∈ V and ρ(u, v) ≤ μ(u, v) for all u, v ∈ V.

Definition 9 (Parvathi and Karunambigai 2006)

An Intuitionistic fuzzy graph is of the form G=<V,E>,where V={v_1,v_2,….,v_n},such that μ_1:V→ [0,1] and γ_1:V→ [0,1] denote the degree of membership and non-membership of the elementv$_i$ ∈ V, respectively, and

$$0\leq \mu_1(v_i)+\gamma_1(v_i))\leq 1,\text{forevery } v_i \in V,(i=1, 2,\ldots\ldots n), \qquad (6)$$

E ⊆ VxVwhere μ_2:VxV→[0,1]and γ_2:VxV→ [0,1] are such that $\mu_2(v_i,v_j)\leq \min[\mu_1(v_i),\mu_1(v_j)]$ and $\gamma_2(v_i,v_j)\geq \max[\gamma_1(v_i),\gamma_1(v_j)]$, and

$$0\leq\mu_2(v_i,v_j)+\gamma_2(v_i,v_j)\leq 1 \text{ for every } (v_i,v_j) \in E,(i,j =1,2,\ldots\ldots n) \qquad (7)$$

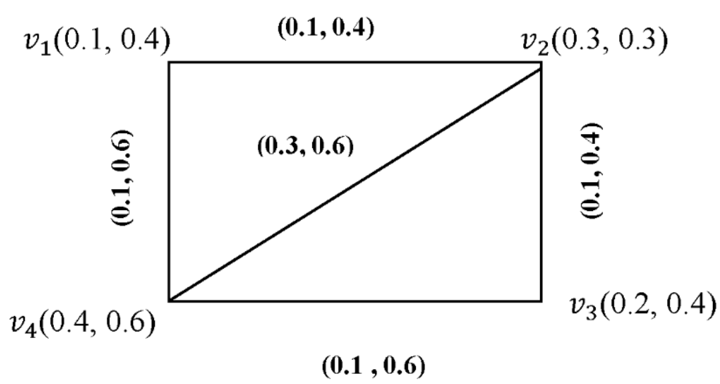

Figure 2: Intuitionistic Fuzzy Graph

Definition 10 (Mishra and Pal 2013)

An interval valued intuitionistic fuzzy graph (IVIFG) G= (A, B) satisfies the following conditions:

1.V= $\{v_1, v_2, \ldots, v_n\}$ such that M_{AL}:V→[0, 1],M_{AU}:V→[0, 1] and N_{AL}:V→[0,1], N_{AU}:V→[0, 1] denote the degree of membership and non-membership of the element $y \in$ V, respectively, and

$$0 \leq M_A(x) + N_A(x) \leq 1 \text{ for every } x \in V \tag{8}$$

2.The functions M_{BL}:V x V →[0, 1],M_{BU}:V x V →[0, 1] and N_{BL}:V x V →[0,1], N_{BU}:V x V →[0, 1] are denoted by

$$M_{BL}(xy) \leq \min [M_{AL}(x), M_{AL}(y)],\ M_{BU}(xy) \leq \min [M_{AU}(x), M_{AU}(y)]$$

$$N_{BL}(xy) \geq \max [N_{BL}(x), N_{BL}(y)],\ N_{BU}(xy) \geq \max[N_{BU}(x), N_{BU}(y)]$$

such that $0 \leq M_B(xy) + N_B(xy) \leq 1$, for every $xy \in$ E (9)

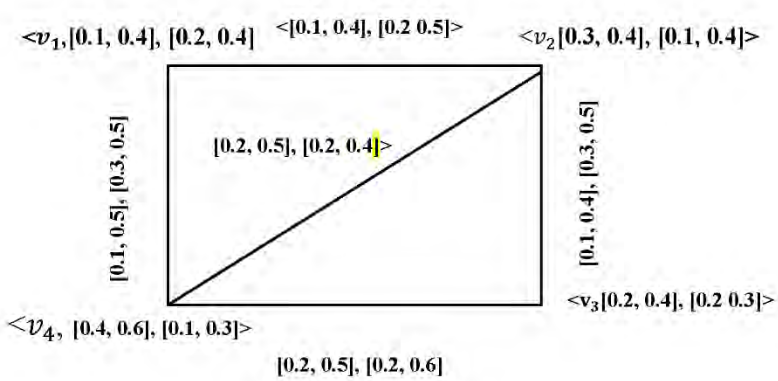

Figure 3: Interval valued intuitionistic graph

Definition 11 (Broumi et al. 2016b)

A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair G= (A, B), where:

1.The functions T_A:V→[0, 1], I_A:V→[0, 1] and F_A:V→[0, 1] denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element $v_i \in$ V, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3, \text{ for all } v_i \in V \text{ (i=1, 2, …,n)} \tag{10}$$

2. The functions T_B: E ⊆ V x V →[0, 1],I_B: E ⊆ V x V →[0, 1] and F_B: E ⊆ V x V →[0, 1] are defined by

$T_B(\{v_i, v_j\}) \leq \min [T_A(v_i), T_A(v_j)]$,

$I_B(\{v_i, v_j\}) \geq \max [I_A(v_i), I_A(v_j)]$ and

$F_B(\{v_i, v_j\}) \geq \max [F_A(v_i), F_A(v_j)]$ (11)

and denote the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i,v_j) \in$ E respectively, where

$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3$,

for all $\{v_i, v_j\} \in$ E (i, j = 1, 2,…, n). (12)

"A"is called the single valued neutrosophic vertex set of V, "B" - the single valued neutrosophic edge set of E, respectively. B is a symmetric single valued neutrosophic relation on A. The notation $(v_i, v_j)$is used for an element of E. Thus, G = (A, B) is a single valued neutrosophic graph of G^*= (V, E), if :

$T_B(v_i, v_j) \leq \min [T_A(v_i), T_A(v_j)]$,

$I_B(v_i, v_j) \geq \max [I_A(v_i), I_A(v_j)]$ and

$F_B(v_i, v_j) \geq \max [F_A(v_i), F_A(v_j)]$, for all $(v_i, v_j) \in$ E (13)

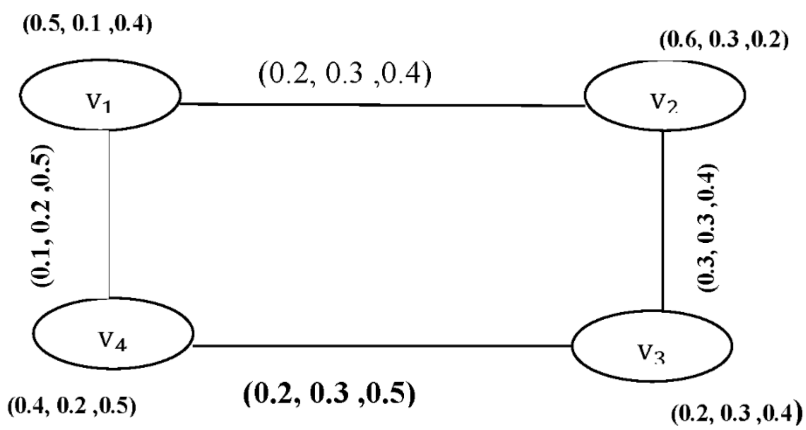

Figure 4: Single valued neutrosophic graph

Definition 12 (Broumi et al. 2016b)

Let G = (A, B) be a single valued neutrosophic graph. Then the degree of a vertex v is defined by d(v)= $(d_T(v), d_I(v), d_F(v))$, where

$d_T(v) = \sum_{u \neq v} T_B(u, v)$, $d_I(v) = \sum_{u \neq v} I_B(u, v)$ and $d_F(v) = \sum_{u \neq v} F_B(u, v)$ (14)

Definition 13 (Broumi et al. 2016b)

A single valued neutrosophic graph G= (A, B) and G^* is called strong neutrosophic graph

$$T_B(v_i, v_j) = \min\,[T_A(v_i), T_A(v_j)]$$

$$I_B(v_i, v_j) = \max\,[I_A(v_i), I_A(v_j)]$$

$$F_B(v_i, v_j) = \max\,[F_A(v_i), F_A(v_j)] \text{ for all } (v_i, v_j) \in \text{E}. \qquad (15)$$

Definition 14 (Broumi et al. 2016b)

The complement of a strong single valued neutrosophic graph G on G^* is strong single valued neutrosophic graph $\bar{G}$ on G^* where

1.$\bar{V}$ =V

2.$\overline{T_A}(v_i)= T_A(v_i)$,$\overline{I_A}(v_i)= I_A(v_i)$,$\overline{F_A}(v_i)= F_A(v_i)$, $v_j \in$ V.

3.$\overline{T_B}(v_i, v_j)= \min\left[T_A(v_i), T_A(v_j)\right]-T_B(v_i, v_j)$

$\overline{I_B}(v_i, v_j)= \max\left[I_A(v_i), I_A(v_j)\right]-I_B(v_i, v_j)$ and

$\overline{F_B}(v_i, v_j)= \max\left[F_A(v_i), F_A(v_j)\right]-F_B(v_i, v_j)$, for all $(v_i, v_j) \in$ E. (16)

Definition 15 (Broumi et al. 2016b)

A single valued neutrosophic graph G = (A, B) is called complete, if:

$T_B(v_i, v_j)= \min(T_A(v_i), T_A(v_j))$,

$I_B(v_i, v_j)= \max(I_A(v_i), I_A(v_j))$

and $F_B(v_i, v_j)= \max(F_A(v_i), F_A(v_j))$, for every $v_i, v_j \in$ V. (17)

Example 1

Consider a graph G^*= (V, E) such that V = {a, b, c, d} , E= {ab ,ac ,bc , cd}. Then, G= (A, B) is a single valued neutrosophic complete graph of G^*.

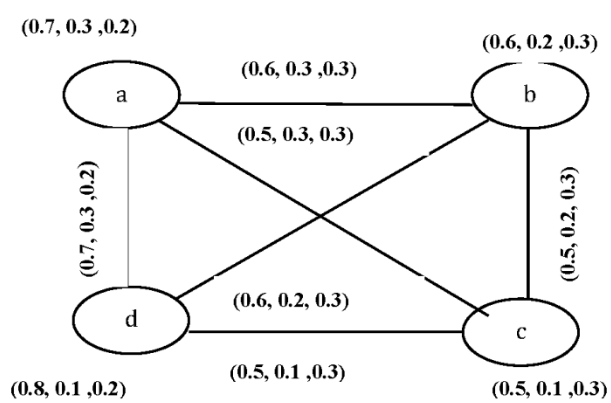

Figure 5: Complete single valued neutrosophic graph

3. Operations on Interval-Valued Neutrosophic Graphs

Throughout this section, G^* = (V, E) denotes a crisp graph, and G - an interval valued neutrosophic graph.

Definition 16

By an interval-valued neutrosophic graph of a graph G^*=(V,E)one means a pair G=(A,B), where A=< $[T_{AL}, T_{AU}]$, $[I_{AL}, I_{AU}]$, $[F_{AL}, F_{AU}]$>is an interval-valued neutrosophic set on V and B=< $[T_{BL}$, $T_{BU}]$, $[I_{BL}, I_{BU}]$, $[F_{BL}, F_{BU}]$ > is an interval-valued neutrosophic relation on E satisfying the following condition:

1.V= $\{v_1, v_2, \ldots, v_n\}$ such that T_{AL}:V→[0, 1],T_{AU}:V→[0, 1], I_{AL}:V→[0,1],I_{AU}:V→[0, 1] and F_{AL}:V→[0,1], F_{AU}:V→[0, 1] denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element $y \in$ V, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3,$$

for every$v_i \in$ V. (18)

2. The functions T_{BL}:V x V →[0, 1],T_{BU}:V x V →[0, 1],I_{BL}:V x V →[0, 1],I_{BU}:V x V →[0, 1] and F_{BL}:V x V →[0,1], F_{BU}:V x V →[0, 1], such that

$$T_{BL}(v_i, v_j) \leq \min\,[T_{AL}(v_i), T_{AL}(v_j)]$$

$$T_{BU}(v_i, v_j) \leq \min\,[T_{AU}(v_i), T_{AU}(v_j)]$$

$$I_{BL}(v_i, v_j) \geq \max\,[I_{BL}(v_i), I_{BL}(v_j)]$$

$$I_{BU}(v_i, v_j) \geq \max\,[I_{BU}(v_i), I_{BU}(v_j)]$$

and

$$F_{BL}(v_i, v_j) \geq \max\,[F_{BL}(v_i), F_{BL}(v_j)]$$

$$F_{BU}(v_i, v_j) \geq \max\,[F_{BU}(v_i), F_{BU}(v_j)] \quad (19)$$

denote the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $(v_i, v_j) \in$ E respectively, where

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3,$$

for all $(v_i, v_j) \in$ E. (20)

Example 2

Figure 5 is an example for IVNG, G = (A,B) defined on a graph G^*= (V, E)

such that V = {x, y, z}, E = {xy, yz, zx}, A is an interval valued neutrosophic set of V

A={ < x, [0.5, 0.7], [0.2, 0.3], [0.1, 0.3]>, <y, [0.6, 0.7],[0.2, 0.4], [0.1, 0.3]>, <z, [0.4, 0.6],[0.1, 0.3], [0.2, 0.4],>}, and B an interval valued neutrosophic set of E⊆V x V

B={ <xy, [0.3, 0.6], [0.2, 0.4], [0.2, 0.4]>, <yz, [0.3, 0.5],[0.2, 0.5], [0.2, 0.4]>, <xz, [0.3, 0.5],[0.3, 0.5], [0.2, 0.4]>}.

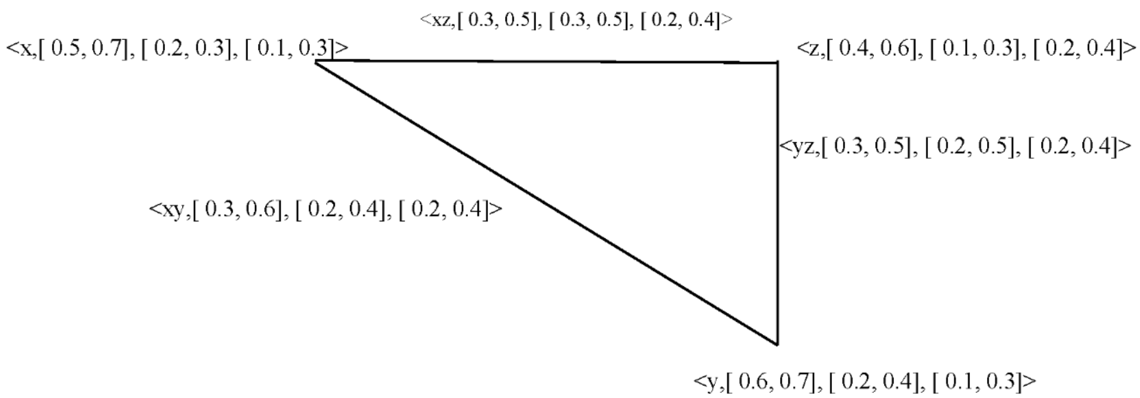

Figure 6: Interval valued neutrosophic graph

By routine computations, it is easy to see that G=(A,B) is an interval valued neutrosophic graph of G^*.

Here, the new concept of Cartesian product is given.

Definition 17

Let $G^* = G_1^* \times G_2^*$=(V, E) be the Cartesian product of two graphs where V = $V_1 \times V_2$ and E= $\{(x, x_2)\,(x, y_2)\,/x \in V_1,\, x_2y_2 \in E_2\} \cup \{(x_1, z)\,(y_1, z)\,/z \in V_2,\, x_1y_1 \in E_1\}$; then, the Cartesian product G = $G_1 \times G_2$ = ($A_1 \times A_2$, $B_1 \times B_2$) is an interval valued neutrosophic graph defined by

1) $(T_{A_1L} \times T_{A_2L})\,(x_1,x_2) = \min\,(T_{A_1L}(x_1), T_{A_2L}(x_2))$
$(T_{A_1U} \times T_{A_2U})\,(x_1,x_2) = \min\,(T_{A_1U}(x_1), T_{A_2U}(x_2))$
$(I_{A_1L} \times I_{A_2L})\,(x_1,x_2) = \max\,(I_{A_1L}(x_1), I_{A_2L}(x_2))$
$(I_{A_1U} \times I_{A_2U})\,(x_1,x_2) = \max\,(I_{A_1U}(x_1), I_{A_2U}(x_2))$
$(F_{A_1L} \times F_{A_2L})\,(x_1,x_2) = \max\,(F_{A_1L}(x_1), F_{A_2L}(x_2))$
$(F_{A_1U} \times F_{A_2U})\,(x_1,x_2) = \max\,(F_{A_1U}(x_1), F_{A_2U}(x_2))$

for all (x_1,x_2) $\in V$.

(21)

2) $(T_{B_1L} \times T_{B_2L})\,((x,x_2)(x,y_2)) = \min\,(T_{A_1L}(x), T_{B_2L}(x_2y_2))$
$(T_{B_1U} \times T_{B_2U})\,((x,x_2)(x,y_2)) = \min\,(T_{A_1U}(x), T_{B_2U}(x_2y_2))$
$(I_{B_1L} \times I_{B_2L})\,((x,x_2)(x,y_2)) = \max\,(I_{A_1L}(x), I_{B_2L}(x_2y_2))$
$(I_{B_1U} \times I_{B_2U})\,((x,x_2)(x,y_2)) = \max\,(I_{A_1U}(x), I_{B_2U}(x_2y_2))$
$(F_{B_1L} \times F_{B_2L})\,((x,x_2)\,(x,y_2)) = \max\,(F_{A_1L}(x), F_{B_2L}(x_2y_2))$

$(F_{B_1U} \times F_{B_2U})$ $((x,x_2)(x,y_2))$ = max$(F_{A_1U}(x)$, $F_{B_2U}(x_2y_2))$,

$\forall$ x $\in V_1$,$\forall x_2y_2 \in E_2$.
(22)

3) $(T_{B_1L} \times T_{B_2L})$ $((x_1,z)$ $(y_1,z))$ = min $(T_{B_1L}(x_1y_1)$, $T_{A_2L}(z))$
$(T_{B_1U} \times T_{B_2U})$ $((x_1,z)$ $(y_1,z))$ = min $(T_{B_1U}(x_1y_1)$, $T_{A_2U}(z))$
$(I_{B_1L} \times I_{B_2L})$ $((x_1$ $,z)$ $(y_1,z))$ = max $(I_{B_1L}(x_1y_1)$, $I_{A_2L}(z))$
$(I_{B_1U} \times I_{B_2U})$ $((x_1$ $,z)$ $(y_1,z))$ = max $(I_{B_1U}(x_1y_1)$, $I_{A_2U}(z))$
$(F_{B_1L} \times F_{B_2L})$ $((x_1$ $,z)$ $(y_1,z))$ = max $(F_{B_1L}(x_1y_1)$, $F_{A_2L}(z))$
$(F_{B_1U} \times F_{B_2U})$ $((x_1$ $,z)$ $(y_1,z))$ = max $(F_{B_1U}(x_1y_1)$, $F_{A_2U}(z))$

$\forall$ z $\in V_2$, $\forall x_1y_1 \in E_1$. (23)

Example 3

Let G_1^*= $(A_1$, $B_1)$ and G_2^*= $(A_2$, $B_2)$ be two graphs whereV_1 ={a, b}, V_2 ={c, d},E_1 ={a, b} and E_2 ={c, d}.Consider two interval valued neutrosophic graphs:

A_1={ <a, [0.5, 0.7], [0.2, 0.3], [0.1, 0.3]>, <b, [0.6, 0.7],[0.2, 0.4], [0.1, 0.3]>},

B_1={ <ab, [0.3, 0.6], [0.2, 0.4], [0.2, 0.4]>};

A_2={ <c, [0.4, 0.6], [0.2, 0.3], [0.1, 0.3]>, <d, [0.4, 0.7],[0.2, 0.4], [0.1, 0.3}>},

B_2={ <cd, [0.3, 0.5], [0.4, 0.5], [0.3, 0.5]>}.

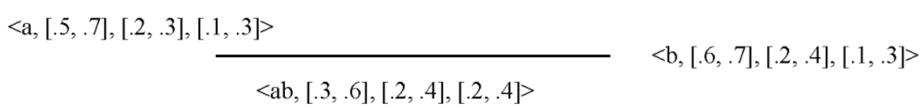

Figure 7: Interval valued neutrosophic graph G1

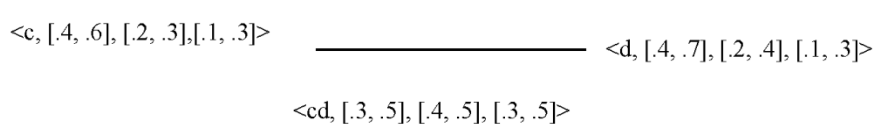

Figure 8: Interval valued neutrosophic graph G2

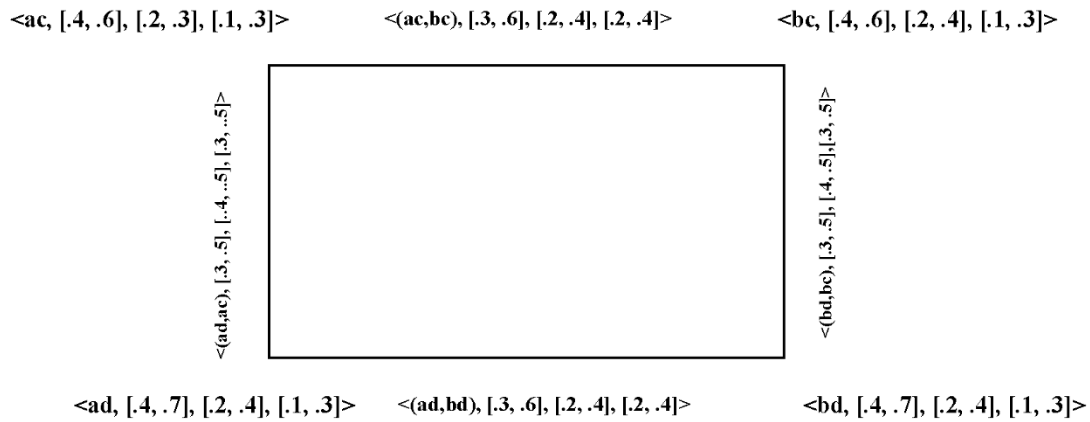

Figure 9: Cartesian product of interval valued neutrosophic graph

By routine computations, It is easy to see that $G_1 \times G_2$ is an interval-valued neutrosophic graph of $G_1^* \times G_2^*$.

Proposition 1

The Cartesian product $G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ of two interval valued neutrosophic graphs of the graphs G_1^* and G_2^* is an interval valued neutrosophic graph of $G_1^* \times G_2^*$.

Proof. Verifying only conditions for $B_1 \times B_2$, because conditions for $A_1 \times A_2$ areobvious.

Let $E = \{(x,x_2)(x,y_2) / x \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)(y_1, z) / z \in V_2, x_1y_1 \in E_1\}$

Considering $(x,x_2)(x,y_2) \in E$, one has:

$(T_{B_1L} \times T_{B_2L})((x,x_2)(x,y_2)) = \min(T_{A_1L}(x), T_{B_2L}(x_2y_2)) \le \min(T_{A_1L}(x), \min(T_{A_2L}(x_2), T_{A_2L}(y_2))) = \min(\min(T_{A_1L}(x), T_{A_2L}(x_2)), \min(T_{A_1L}(x), T_{A_2L}(y_2)))$

$= \min((T_{A_1L} \times T_{A_2L})(x,x_2), (T_{A_1L} \times T_{A_2L})(x,y_2)),$ (24)

$(T_{B_1U} \times T_{B_2U})((x,x_2)(x,y_2)) = \min(T_{A_1U}(x), T_{B_2U}(x_2y_2)) \le \min(T_{A_1U}(x), \min(T_{A_2U}(x_2), T_{A_2U}(y_2))) = \min(\min(T_{A_1U}(x), T_{A_2U}(x_2)), \min T_{A_1U}(x), T_{A_2U}(y_2))) = \min((T_{A_1U} \times T_{A_2U})(x,x_2), (T_{A_1U} \times T_{A_2U})(x,y_2)),$ (25)

$(I_{B_1L} \times I_{B_2L})((x,x_2)(x,y_2)) = \max(I_{A_1L}(x), I_{B_2L}(x_2y_2)) \ge \max(I_{A_1L}(x), \max(I_{A_2L}(x_2), I_{A_2L}(y_2))) = \max(\max(I_{A_1L}(x), I_{A_2L}(x_2)), \max(I_{A_1L}(x), I_{A_2L}(y_2))) = \max((I_{A_1L} \times I_{A_2L})(x,x_2), (I_{A_1L} \times I_{A_2L})(x,y_2)),$ (26)

$(I_{B_1U} \times I_{B_2U})((x,x_2)(x,y_2)) = \max(I_{A_1U}(x), I_{B_2U}(x_2y_2)) \ge \max(I_{A_1U}(x), \max(I_{A_2U}(x_2), I_{A_2U}(y_2))) = \max(\max(I_{A_1U}(x), I_{A_2U}(x_2)), \max(I_{A_1U}(x), I_{A_2U}(y_2))) = \max((I_{A_1U} \times I_{A_2U})(x,x_2), (I_{A_1U} \times I_{A_2U})(x,y_2)),$ (27)

$(F_{B_1L}\times F_{B_2L})$ $((x,x_2)\ (x,y_2))$ = max $(F_{A_1L}(x), F_{B_2L}(x_2y_2)) \geq$ max $(F_{A_1L}(x)$, max$(F_{A_2L}(x_2), F_{A_2L}(y_2)))$ = max(max $(F_{A_1L}(x), F_{A_2L}(x_2))$, max $(F_{A_1L}(x), F_{A_2L}(y_2)))$ = max $((F_{A_1L}\times F_{A_2L})\ (x,x_2), (F_{A_1L}\times F_{A_2L})\ (x,y_2))$, (28)

$(F_{B_1U}\times F_{B_2U})$ $((x,x_2)\ (x,y_2))$ = max $(F_{A_1U}(x), F_{B_2U}(x_2y_2)) \geq$ max $(F_{A_1U}(x)$, max$(F_{A_2U}(x_2), F_{A_2U}(y_2)))$ = max(max $(F_{A_1U}(x), F_{A_2U}(x_2))$, max $(F_{A_1U}(x), F_{A_2U}(y_2)))$= max $((F_{A_1U}\times F_{A_2U})\ (x,x_2), (F_{A_1U}\times F_{A_2U})\ (x,y_2))$. (29)

Similarly, for$(x_1, z)\ (y_1, z) \in E$, one has:

$(T_{B_1L}\times T_{B_2L})$ $((x_1, z)\ (y_1, z))$ = min $(T_{B_1L}(x_1y_1), T_{A_2L}(z)) \leq$ min $(min(T_{A_1L}(x_1), T_{A_1L}(y_1))), T_{A_2L}(z)))$ = min(min $(T_{A_1L}(x), T_{A_2L}(z))$, min $(T_{A_1L}(y_1), T_{A_2L}(z)))$= min $((T_{A_1L}\times T_{A_2L})\ (x_1,z), (T_{A_1L}\times T_{A_2L})\ (y_1,z))$, (30)

$(T_{B_1U}\times T_{B_2U})$ $((x_1, z)\ (y_1, z))$ = min $(T_{B_1U}(x_1y_1), T_{A_2U}(z)) \leq$ min $(min(T_{A_1U}(x_1), T_{A_1U}(y_1))), T_{A_2U}(z)))$= min(min $(T_{A_1U}(x), T_{A_2U}(z))$, min $(T_{A_1U}(y_1), T_{A_2U}(z)))$ = min $((T_{A_1U}\times T_{A_2U})\ (x_1\ ,z), (T_{A_1U}\times T_{A_2U})\ (y_1\ ,z))$, (31)

$(I_{B_1L}\times I_{B_2L})$ $((x_1, z)\ (y_1, z))$ = max $(I_{B_1L}(x_1y_1), I_{A_2L}(z)) \geq$ max$(max(I_{A_1L}(x_1), I_{A_1L}(y_1))), I_{A_2L}(z)))$ = max(max $(I_{A_1L}(x), I_{A_2L}(z))$, max $(I_{A_1L}(y_1), I_{A_2L}(z)))$ = max $((I_{A_1L}\times I_{A_2L})\ (x_1,z), (I_{A_1L}\times I_{A_2L})\ (y_1,z))$, (32)

$(I_{B_1U}\times I_{B_2U})$ $((x_1, z)\ (y_1, z))$ = max $(I_{B_1U}(x_1y_1), I_{A_2U}(z)) \geq$ max $(max(I_{A_1U}(x_1), I_{A_1U}(y_1))), I_{A_2U}(z)))$ = max(max $(I_{A_1U}(x), I_{A_2U}(z))$, max $(I_{A_1U}(y_1), I_{A_2U}(z)))$ = max $((I_{A_1U}\times I_{A_2U})\ (x_1,z), (I_{A_1U}\times I_{A_2U})\ (y_1,z))$, (33)

$(F_{B_1L}\times F_{B_2L})$ $((x_1, z)\ (y_1, z))$ = max$(F_{B_1L}(x_1y_1), F_{A_2L}(z)) \geq$ max $(max(F_{A_1L}(x_1), F_{A_1L}(y_1))), F_{A_2L}(z)))$ = max(max $(F_{A_1L}(x), F_{A_2L}(z))$, max $(F_{A_1L}(y_1), F_{A_2L}(z)))$ = max $((F_{A_1L}\times F_{A_2L})\ (x_1\ ,z), (F_{A_1L}\times F_{A_2L})\ (y_1\ ,z))$, (34)

$(F_{B_1U}\times F_{B_2U})$ $((x_1, z)\ (y_1, z))$ = max $(F_{A_1U}(x_1y_1), F_{B_2U}(z)) \geq$ max (max$(F_{A_1U}(x_1), F_{A_1U}(y_1))), F_{A_2U}(z)))$ = max(max $(F_{A_1U}(x), F_{A_2U}(z))$, max $(F_{A_1U}(y_1), F_{A_2U}(z)))$ = max $((F_{A_1U}\times F_{A_2U})\ (x_1,z), (F_{A_1U}\times F_{A_2U})\ (y_1,z))$. (35)

This completes the proof.

Definition 18

Let $G^* = G_1^*\times G_2^*$=$(V_1\times V_2$, E) be the composition of two graphs where E= $\{(x, x_2)\ (x, y_2)\ /x \in V_1,\ x_2y_2 \in E_2\} \cup \{(x_1,\ z)\ (y_1, z)\ /z \in V_2,\ x_1y_1 \in E_1\} \cup \{(x_1, x_2)\ (y_1, y_2)\ |x_1y_1 \in E_1,\ x_2 \neq y_2\}$,then the composition of interval valued neutrosophic graphs $G_1[\ G_2]$ = $(A_1 \circ A_2, B_1 \circ B_2)$ is an interval valued neutrosophic graphs defined by:

1. $(T_{A_1L} \circ T_{A_2L})\ (x_1,x_2)$ = min $(T_{A_1L}(x_1), T_{A_2L}(x_2))$ (36)
   $(T_{A_1U} \circ T_{A_2U})\ (x_1,x_2)$ = min $(T_{A_1U}(x_1), T_{A_2U}(x_2))$

$(I_{A_1L} \circ I_{A_2L})$ (x_1,x_2) = max $(I_{A_1L}(x_1), I_{A_2L}(x_2))$
$(I_{A_1U} \circ I_{A_2U})$ (x_1, x_2) = max $(I_{A_1U}(x_1), I_{A_2U}(x_2))$
$(F_{A_1L} \circ F_{A_2L})$ (x_1, x_2) = max $(F_{A_1L}(x_1), F_{A_2L}(x_2))$
$(F_{A_1U} \circ F_{A_2U})$ (x_1, x_2) = max $(F_{A_1U}(x_1), F_{A_2U}(x_2))$ $\forall\, x_1 \in V_1, x_2 \in V_2$;

2. $(T_{B_1L} \circ T_{B_2L})$ $((x,x_2)(x,y_2))$ = min $(T_{A_1L}(x), T_{B_2L}(x_2y_2))$ (37)
$(T_{B_1U} \circ T_{B_2U})$ $((x, x_2)(x, y_2))$ = min $(T_{A_1U}(x), T_{B_2U}(x_2y_2))$
$(I_{B_1L} \circ I_{B_2L})$ $((x, x_2)(x, y_2))$ = max $(I_{A_1L}(x), I_{B_2L}(x_2y_2))$
$(I_{B_1U} \circ I_{B_2U})$ $((x, x_2)(x, y_2))$ = max $(I_{A_1U}(x), I_{B_2U}(x_2y_2))$
$(F_{B_1L} \circ F_{B_2L})$ $((x,x_2)(x,y_2))$ = max $(F_{A_1L}(x), F_{B_2L}(x_2y_2))$
$(F_{B_1U} \circ F_{B_2U})$ $((x,x_2)(x,y_2))$ = max $(F_{A_1U}(x), F_{B_2U}(x_2y_2))$ $\forall\, x \in V_1, \forall x_2y_2 \in E_2$;

3. $(T_{B_1L} \circ T_{B_2L})$ $((x_1,z)(y_1,z))$ = min $(T_{B_1L}(x_1y_1), T_{A_2L}(z))$
(38)
$(T_{B_1U} \circ T_{B_2U})$ $((x_1,z)(y_1,z))$ = min $(T_{B_1U}(x_1y_1), T_{A_2U}(z))$
$(I_{B_1L} \circ I_{B_2L})$ $((x_1,z)(y_1,z))$ = max $(I_{B_1L}(x_1y_1), I_{A_2L}(z))$
$(I_{B_1U} \circ I_{B_2U})$ $((x_1,z)(y_1,z))$ = max $(I_{B_1U}(x_1y_1), I_{A_2U}(z))$
$(F_{B_1L} \circ F_{B_2L})$ $((x_1,z)(y_1,z))$ = max $(F_{B_1L}(x_1y_1), F_{A_2L}(z))$
$(F_{B_1U} \circ F_{B_2U})$ $((x_1,z)(y_1,z))$= max $(F_{B_1U}(x_1y_1), F_{A_2U}(z))$ $\forall$ z $\in V_2, \forall x_1y_1 \in E_1$;

4. $(T_{B_1L} \circ T_{B_2L})$ $((x_1,x_2)(y_1,y_2))$ = min $(T_{A_2L}(x_2), T_{A_2L}(y_2), T_{B_1L}(x_1y_1))$
(39)
$(T_{B_1U} \circ T_{B_2U})$ $((x_1,x_2)(y_1,y_2))$ = min $(T_{A_2U}(x_2), T_{A_2U}(y_2), T_{B_1U}(x_1y_1))$

$(I_{B_1L} \circ I_{B_2L})$ $((x_1,x_2)(y_1,y_2))$ = max $(I_{A_2L}(x_2), I_{A_2L}(y_2), I_{B_1L}(x_1y_1))$

$(I_{B_1U} \circ I_{B_2U})$ $((x_1,x_2)(y_1,y_2))$ = max $(I_{A_2U}(x_2), I_{A_2U}(y_2), I_{B_1U}(x_1y_1))$

$(F_{B_1L} \circ F_{B_2L})$ $((x_1,x_2)(y_1,y_2))$ = max $(F_{A_2L}(x_2), F_{A_2L}(y_2), F_{B_1L}(x_1y_1))$

$(F_{B_1U} \circ F_{B_2U})((x_1, x_2)(y_1, y_2))$ = max $(F_{A_2U}(x_2), F_{A_2U}(y_2), F_{B_1U}(x_1y_1))$,
$\forall\, (x_1,x_2)(y_1,y_2) \in E^0$-E,where E^0= E $\cup$ $\{(x_1,x_2)(y_1,y_2) \mid x_1y_1 \in E_1, x_2 \neq y_2\}$.

Example 4

Let G_1^*= (V_1, E_1) and G_2^*= (V_2, E_2) be two graphs such that V_1 ={a, b}, V_2 ={c, d},E_1 ={a, b} and E_2 ={c, d}. Consider two interval-valued neutrosophic graphs:

A_1={ < a, [0.5, 0.7], [0.2, 0.3], [0.1, 0.3]>, <b, [0.6, 0.7],[0.2, 0.4], [0.1, 0.3},

B_1={ < ab, [0.3, 0.6], [0.2, 0.4], [0.2, 0.4]>};

A_2={ < c, [0.4, 0.6], [0.2, 0.3], [0.1, 0.3]>, <d, [0.4, 0.7],[0.2, 0.4], [0.1, 0.3},

B_2={ < cd, [0.3, 0.5], [0.2, 0.5], [0.3, 0.5]>}.

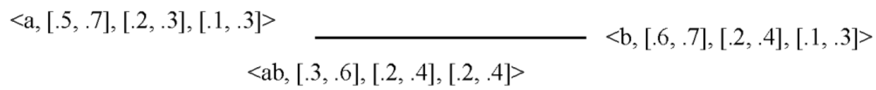

Figure 10: Interval valued neutrosophic graph G1

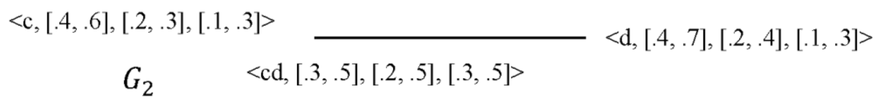

Figure 11: Interval valued neutrosophic graph G2

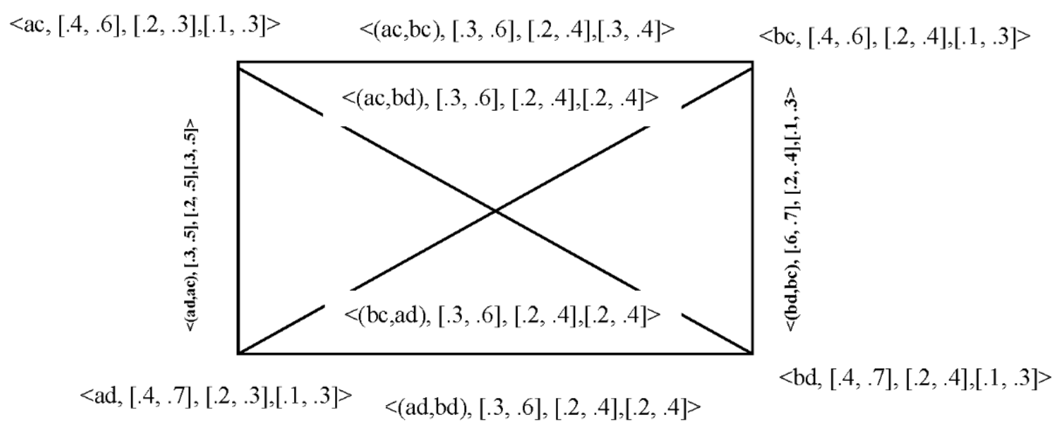

Figure 12: Composition of interval valued neutrosophic graph.

Proposition2

The composition$G_1[\,G_2]$ =($A_1 \circ A_2$, $B_1 \circ B_2$) of two interval valued neutrosophic graphs of the graphs $G_1^*$and G_2^* is an interval valued neutrosophic graph of $G_1^*[G_2^*]$.

Proof**.** Verifying only conditions for $\mathrm{B}_1 \circ \mathrm{B}_2$, because conditions for $\mathrm{A}_1 \circ \mathrm{A}_2$ are obvious. Let E= $\{(x,x_2)\ (x,y_2)\ /x_1 \in V_1, x_2y_2 \in E_2\} \cup \{(x_1, z)\ (y_1, z)\ /z \in V_2, x_1y_1 \in E_1\}$.Considering$(x,x_2)$ $(x,y_2) \in E$, one has:

$(T_{B_1L} \circ T_{B_2L})\ ((x, x_2)\ (x, y_2))$ = min $(T_{A_1L}(x), T_{B_2L}(x_2y_2)) \leq$ min $(T_{A_1L}(x)$, min$(T_{A_2L}(x_2), T_{A_2L}(y_2)))$= min(min $(T_{A_1L}(x), T_{A_2L}(x_2))$, min $(T_{A_1L}(x), T_{A_2L}(y_2)))$= min $((T_{A_1L} \circ T_{A_2L})\ (x,x_2), (T_{A_1L} \circ T_{A_2L})\ (x,y_2))$, (40)

$(T_{B_1U} \circ T_{B_2U})$ $((x, x_2)\ (x, y_2))$ = min $(T_{A_1U}(x),\ T_{B_2U}(x_2y_2)) \leq$ min $(T_{A_1U}(x)$, min$(T_{A_2U}(x_2)$,$T_{A_2U}(y_2)))$= min(min $(T_{A_1U}(x)$,$T_{A_2U}(x_2))$, min $(T_{A_1U}(x)$,$T_{A_2U}(y_2)))$= min $((T_{A_1U} \circ T_{A_2U})$ $(x$,$x_2)$,$(T_{A_1U} \circ T_{A_2U})$ $(x$,$y_2))$, (41)

$(I_{B_1L} \circ I_{B_2L})$ $((x, x_2)\ (x, y_2))$ = max $(I_{A_1L}(x),\ I_{B_2L}(x_2y_2)) \geq$ max $(I_{A_1L}(x)$, max$(I_{A_2L}(x_2), I_{A_2L}(y_2)))$ = max(max $(I_{A_1L}(x), I_{A_2L}(x_2))$, max $(I_{A_1L}(x), I_{A_2L}(y_2)))$ = max$((I_{A_1L} \circ I_{A_2L})$ $(x$,$x_2)$,$(I_{A_1L} \circ I_{A_2L})$ $(x$,$y_2))$, (42)

$(I_{B_1U} \circ I_{B_2U})$ $((x, x_2)\ (x, y_2))$ = max $(I_{A_1U}(x),\ I_{B_2U}(x_2y_2)) \geq$ max $(I_{A_1U}(x)$, max$(I_{A_2U}(x_2)$,$I_{A_2U}(y_2)))$ = max(max $(I_{A_1U}(x)$,$I_{A_2U}(x_2))$, max $(I_{A_1U}(x)$,$I_{A_2U}(y_2)))$ = max $((I_{A_1U} \circ I_{A_2U})$ $(x$,$x_2)$,$(I_{A_1U} \circ I_{A_2U})$ $(x$,$y_2))$, (43)

$(F_{B_1L} \circ F_{B_2L})$ $((x, x_2)\ (x, y_2))$ = max $(F_{A_1L}(x),\ F_{B_2L}(x_2y_2)) \geq$ max $(F_{A_1L}(x)$, max$(F_{A_2L}(x_2)$,$F_{A_2L}(y_2)))$ = max(max $(F_{A_1L}(x)$,$F_{A_2L}(x_2))$, max $(F_{A_1L}(x)$,$F_{A_2L}(y_2)))$ = max $((F_{A_1L} \circ F_{A_2L})$ $(x$,$x_2)$,$(F_{A_1L} \circ F_{A_2L})$ $(x$,$y_2))$, (44)

$(F_{B_1U} \circ F_{B_2U})$ $((x, x_2)\ (x, y_2))$ = max $(F_{A_1U}(x),\ F_{B_2U}(x_2y_2)) \geq$ max $(F_{A_1U}(x)$, max$(F_{A_2U}(x_2)$,$F_{A_2U}(y_2)))$ = max(max $(F_{A_1U}(x)$,$F_{A_2U}(x_2)$, max $(F_{A_1U}(x)$,$F_{A_2U}(y_2))$ = max$((F_{A_1U} \circ F_{A_2U})$ $(x$,$x_2)$,$(F_{A_1U} \circ F_{A_2U})$ $(x$,$y_2))$. (45)

In the case (x_1, z) $(y_1, z) \in E$, the proof is similar.

In the case $(x_1$,$x_2)$ $(y_1$,$y_2) \in E^0$-E.

$(T_{B_1L} \circ T_{B_2L})((x_1$,$x_2)$ $(y_1$,$y_2))$ = min $(T_{A_2L}(x_2)$, $T_{A_2L}(y_2)$, $T_{B_1L}(x_1y_1)) \leq$ min $(T_{A_2L}(x_2)$, $T_{A_2L}(y_2)$,min $(T_{A_1L}(x_1)$, $T_{A_1L}(y_1)))$ = min(min $(T_{A_1L}(x_1)$, $T_{A_2L}(x_2))$, min $(T_{A_1L}(y_1)$,$T_{A_2L}(y_2)))$ = min $((T_{A_1L} \circ T_{A_2L})$ $(x_1$,$x_2)$,$(T_{A_1L} \circ T_{A_2L})$ $(y_1$,$y_2))$, (46)

$(T_{B_1U} \circ T_{B_2U})$ $((x_1, x_2)\ (y_1, y_2))$ = min $(T_{A_2U}(x_2)$, $T_{A_2U}(y_2)$, $T_{B_1L}(x_1y_1)) \leq$ min $(T_{A_2U}(x_2)$, $T_{A_2U}(y_2)$,min $(T_{A_1U}(x_1)$, $T_{A_1U}(y_1)))$ = min(min $(T_{A_1U}(x_1)$,$T_{A_2U}(x_2))$, min $(T_{A_1U}(y_1)$,$T_{A_2U}(y_2)))$ = min $((T_{A_1U} \circ T_{A_2U})$ $(x_1$,$x_2)$,$(T_{A_1U} \circ T_{A_2U})$ $(y_1$,$y_2))$,(47)

$(I_{B_1L} \circ I_{B_2L})$ $((x_1$,$x_2)$ $(y_1$,$y_2))$ = max $(I_{A_2L}(x_2)$, $I_{A_2L}(y_2)$, $I_{B_1L}(x_1y_1)) \geq$ max $(I_{A_2L}(x_2)$, $I_{A_2L}(y_2)$,max $(I_{A_1L}(x_1)$, $I_{A_1L}(y_1)))$ = max(max $(I_{A_1L}(x_1)$, $I_{A_2L}(x_2))$, max $(I_{A_1L}(y_1)$,$I_{A_2L}(y_2)))$ = max $((I_{A_1L} \circ I_{A_2L})$ $(x_1$,$x_2)$,$(I_{A_1L} \circ I_{A_2L})$ $(y_1$,$y_2))$, (48)

$(I_{B_1U} \circ I_{B_2U})$ $((x_1$,$x_2)$ $(y_1$,$y_2))$ = max $(I_{A_2U}(x_2)$, $I_{A_2U}(y_2)$, $I_{B_1L}(x_1y_1)) \geq$ max $(I_{A_2U}(x_2)$, $I_{A_2U}(y_2)$,max $(I_{A_1U}(x_1)$, $I_{A_1U}(y_1)))$ = max(max $(I_{A_1U}(x)$, $I_{A_2U}(x_2))$, max $(I_{A_1U}(y_1)$,$I_{A_2U}(y_2)))$ = max $((I_{A_1U} \circ I_{A_2U})$ $(x_1$,$x_2)$,$(I_{A_1U} \circ I_{A_2U})$ $(y_1$,$y_2))$, (49)

$(F_{B_1L} \circ F_{B_2L})$ $((x_1, x_2)\ (y_1, y_2))$ = max $(F_{A_2L}(x_2)$, $F_{A_2L}(y_2)$, $F_{B_1L}(x_1y_1)) \geq$ max $(F_{A_2L}(x_2)$, $F_{A_2L}(y_2)$,max $(F_{A_1L}(x_1)$, $F_{A_1L}(y_1)))$ = max(max $(F_{A_1L}(x), F_{A_2L}(x_2))$, max $(F_{A_1L}(y_1)$,$F_{A_2L}(y_2)))$= max $((F_{A_1L} \circ F_{A_2L})$ $(x_1$,$x_2)$,$(F_{A_1L} \circ F_{A_2L})$ $(y_1$,$y_2))$, (50)

$(F_{B_1U} \circ F_{B_2U})$ $((x_1, x_2)$ $(y_1, y_2))$ = max$(F_{A_2U}(x_2)$, $F_{A_2U}(y_2)$, $F_{B_1L}(x_1y_1)) \geq$ max $(F_{A_2U}(x_2)$, $F_{A_2U}(y_2)$,max $(F_{A_1U}(x_1)$, $F_{A_1U}(y_1)))$ = max(max $(F_{A_1U}(x), F_{A_2U}(x_2))$, max $(F_{A_1U}(y_1), F_{A_2U}(y_2)))$ = max $((F_{A_1U} \circ F_{A_2U})$ $(x_1, x_2), (F_{A_1U} \circ F_{A_2U})$ $(y_1, y_2))$. (51)

This completes the proof.

Definition 19

The union $G_1 \cup G_2 = (A_1 \cup A_2, B_1 \cup B_2)$ of two interval valued neutrosophic graphs of the graphs G_1^* and G_2^* is an interval-valued neutrosophic graph of $G_1^* \cup G_2^*$.

1) $(T_{A_1L} \cup T_{A_2L})(x) = T_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$,
$(T_{A_1L} \cup T_{A_2L})(x) = T_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$,
$(T_{A_1L} \cup T_{A_2L})(x) = \max(T_{A_1L}(x), T_{A_2L}(x))$ if $x \in V_1 \cap V_2$,
(52)
2) $(T_{A_1U} \cup T_{A_2U})(x) = T_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$,
$(T_{A_1U} \cup T_{A_2U})(x) = T_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$,
$(T_{A_1U} \cup T_{A_2U})(x) = \max(T_{A_1U}(x), T_{A_2U}(x))$ if $x \in V_1 \cap V_2$,
(53)
3) $(I_{A_1L} \cup I_{A_2L})(x) = I_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$,
$(I_{A_1L} \cup I_{A_2L})(x) = I_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$,
$(I_{A_1L} \cup I_{A_2L})(x) = \min(I_{A_1L}(x), I_{A_2L}(x))$ if $x \in V_1 \cap V_2$, (54)
4) $(I_{A_1U} \cup I_{A_2U})(x) = I_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$,
$(I_{A_1U} \cup I_{A_2U})(x) = I_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$,
$(I_{A_1U} \cup I_{A_2U})(x) = \min(I_{A_1U}(x), I_{A_2U}(x))$ if $x \in V_1 \cap V_2$,
(55)
5) $(F_{A_1L} \cup F_{A_2L})(x) = F_{A_1L}(x)$ if $x \in V_1$ and $x \notin V_2$,
$(N_{A_1L} \cup N_{A_2L})(x) = F_{A_2L}(x)$ if $x \notin V_1$ and $x \in V_2$,
$(N_{A_1L} \cup N_{A_2L})(x) = \min(F_{A_1L}(x), F_{A_2L}(x))$ if $x \in V_1 \cap V_2$,
(56)
6) $(F_{A_1U} \cup F_{A_2U})(x) = F_{A_1U}(x)$ if $x \in V_1$ and $x \notin V_2$,
$(F_{A_1U} \cup F_{A_2U})(x) = F_{A_2U}(x)$ if $x \notin V_1$ and $x \in V_2$,
$(F_{A_1U} \cup F_{A_2U})(x) = \min(F_{A_1U}(x), F_{A_2U}(x))$ if $x \in V_1 \cap V_2$,
(57)
7) $(T_{B_1L} \cup T_{B_2L})(xy) = T_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$,
$(T_{B_1L} \cup T_{B_2L})(xy) = T_{B_2L}(xy)$ if $xy \notin E_1$ and $xy \in E_2$,
$(T_{B_1L} \cup T_{B_2L})(xy) = \max(T_{B_1L}(xy), T_{B_2L}(xy))$ if $xy \in E_1 \cap E_2$, (58)
8) $(T_{B_{1U}} \cup T_{B_2U})(xy) = T_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$,
$(T_{B_1U} \cup T_{B_2U})(xy) = T_{B_2U}(xy)$ if $xy \notin E_1$ and $xy \in E_2$,
$(T_{B_1U} \cup T_{B_2U})(xy) = \max(T_{B_1U}(xy), T_{B_2U}(xy))$ if $xy \in E_1 \cap E_2$,
(59)
9) $(I_{B_1L} \cup I_{B_2L})(xy) = I_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$,
$(I_{B_1L} \cup M_{B_2L})(xy) = I_{B_2L}(xy)$ if $xy \notin E_1$ and $xy \in E_2$,

$(I_{B_1L} \cup I_{B_2L})\,(xy) = \min\,(I_{B_1L}(xy), I_{B_2L}(xy)\,)$ if $xy \in E_1 \cap E_2$, (60)

10) $(I_{B_1U} \cup I_{B_2U})\,(xy) = I_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$,

$(I_{B_1U} \cup I_{B_2U})\,(xy) = I_{B_2U}(xy)$ if $xy \notin E_1$ and $xy \in E_2$,

$(I_{B_1U} \cup I_{B_2U})\,(xy) = \min\,(I_{B_1U}(xy), I_{B_2U}(xy)\,)$ if $xy \in E_1 \cap E_2$, (61)

11) $(F_{B_1L} \cup F_{B_2L})\,(xy) = F_{B_1L}(xy)$ if $xy \in E_1$ and $xy \notin E_2$,

$(F_{B_1L} \cup F_{B_2L})\,(xy) = F_{B_2L}(xy)$ if $xy \notin E_1$ and $xy \in E_2$,

$(F_{B_1L} \cup F_{B_2L})\,(xy) = \min\,(F_{B_1L}(xy), F_{B_2L}(xy)\,)$ if $xy \in E_1 \cap E_2$, (62)

12) $(F_{B_1U} \cup F_{B_2U})\,(xy) = F_{B_1U}(xy)$ if $xy \in E_1$ and $xy \notin E_2$,

$(F_{B_1U} \cup F_{B_2U})\,(xy) = F_{B_2U}(xy)$ if $xy \notin E_1$ and $xy \in E_2$,

$(F_{B_1U} \cup F_{B_2U})\,(xy) = \min\,(F_{B_1U}(xy), F_{B_2U}(xy)\,)$ if $xy \in E_1 \cap E_2$. (63)

Proposition 3

Let G_1 and G_2 are two interval valued neutrosophic graphs, then $G_1 \cup G_2$ is an interval valued neutrosophic graph.

P**roof**. Verifying only conditions for $B_1 \circ B_2$, because conditions for $A_1 \circ A_2$ are obvious.

Let x y $\in E_1 \cap E_2$.

Then:

$(T_{B_1L} \cup T_{B_2L})(xy) = \max(T_{B_1L}(xy), T_{B_2L}(xy)) \leq \max(\min(T_{A_1L}(x), T_{A_1L}(y)), \min(T_{A_2L}(x), T_{A_2L}(y))) = \min(\max(T_{A_1L}(x), T_{A_2L}(x)), \max(T_{A_1L}(y), T_{A_2L}(y))) = \min((T_{A_1L} \cup T_{A_2L})\,(x), (T_{A_1L} \cup T_{A_2L})\,(y)))$; (64)

$(T_{B_1U} \cup T_{B_2U})(xy) = \max(T_{B_1U}(xy), T_{B_2U}(xy)) \leq \max(\min(T_{A_1U}(x), T_{A_1U}(y)), \min(T_{A_2U}(x), T_{A_2U}(y))) = \min(\max(T_{A_1U}(x), T_{A_2U}(x)), \max(T_{A_1U}(y), T_{A_2U}(y))) = \min((T_{A_1U} \cup T_{A_2U})\,(x), (T_{A_1U} \cup T_{A_2U})\,(y)))$; (65)

$(I_{B_1L} \cup I_{B_2L})(xy) = \min(I_{B_1L}(xy), I_{B_2L}(xy)) \geq \min(\max(I_{A_1L}(x), I_{A_1L}(y)), \max(I_{A_2L}(x), I_{A_2L}(y))) = \min(\min(I_{A_1L}(x), I_{A_2L}(x)), \min(I_{A_1L}(y), I_{A_2L}(y))) = \max((I_{A_1L} \cup I_{A_2L})\,(x), (I_{A_1L} \cup I_{A_2L})\,(y)))$; (66)

$(I_{B_1U} \cup I_{B_2U})(xy) = \min(I_{B_1U}(xy), I_{B_2U}(xy)) \geq \min(\max(I_{A_1U}(x), I_{A_1U}(y)), \max(I_{A_2U}(x), I_{A_2U}(y))) = \max(\min(I_{A_1U}(x), I_{A_2U}(x)), \min(I_{A_1U}(y), I_{A_2U}(y))) = \max((I_{A_1U} \cup I_{A_2U})\,(x), (I_{A_1U} \cup I_{A_2U})\,(y)))$; (67)

$(F_{B_1L} \cup F_{B_2L})(xy) = \min(F_{B_1L}(xy), F_{B_2L}(xy)) \geq \min(\max(F_{A_1L}(x), F_{A_1L}(y)), \max(F_{A_2L}(x), F_{A_2L}(y))) = \min(\min(F_{A_1L}(x), F_{A_2L}(x)), \min(F_{A_1L}(y), F_{A_2L}(y))) = \max((F_{A_1L} \cup F_{A_2L})\,(x), (F_{A_1L} \cup F_{A_2L})\,(y)))$; (68)

$(F_{B_1U} \cup F_{B_2U})(xy) = \min(F_{B_1U}(xy), F_{B_2U}(xy)) \geq \min(\max(F_{A_1U}(x), F_{A_1U}(y)), \max(F_{A_2U}(x), F_{A_2U}(y))) = \max(\min(F_{A_1U}(x), F_{A_2U}(x)), \min(F_{A_1U}(y), F_{A_2U}(y))) = \max((F_{A_1U} \cup F_{A_2U})\,(x), (F_{A_1U} \cup F_{A_2U})\,(y)))$. (69)

This completes the proof.

Example *5*

Let $G_1^*= (V_1, E_1)$ and $G_2^*= (V_2, E_2)$ be two graphs such that $V_1 =\{v_1, v_2, v_3, v_4, v_5\}. V_2 =\{v_1, v_2, v_3, v_4\}, E_1 =\{v_1v_2, v_1v_5, v_2v_3, v_5v_3, v_5v_4, v_4v_3\}$ and $E_2 =\{v_1v_2, v_2v_3, v_2v_4, v_3v_{34}, v_4v_1\}$. Consider two interval valued neutrosophic graphs $G_1= (A_1, B_1)$ and $G_2= (A_2, B_2)$.

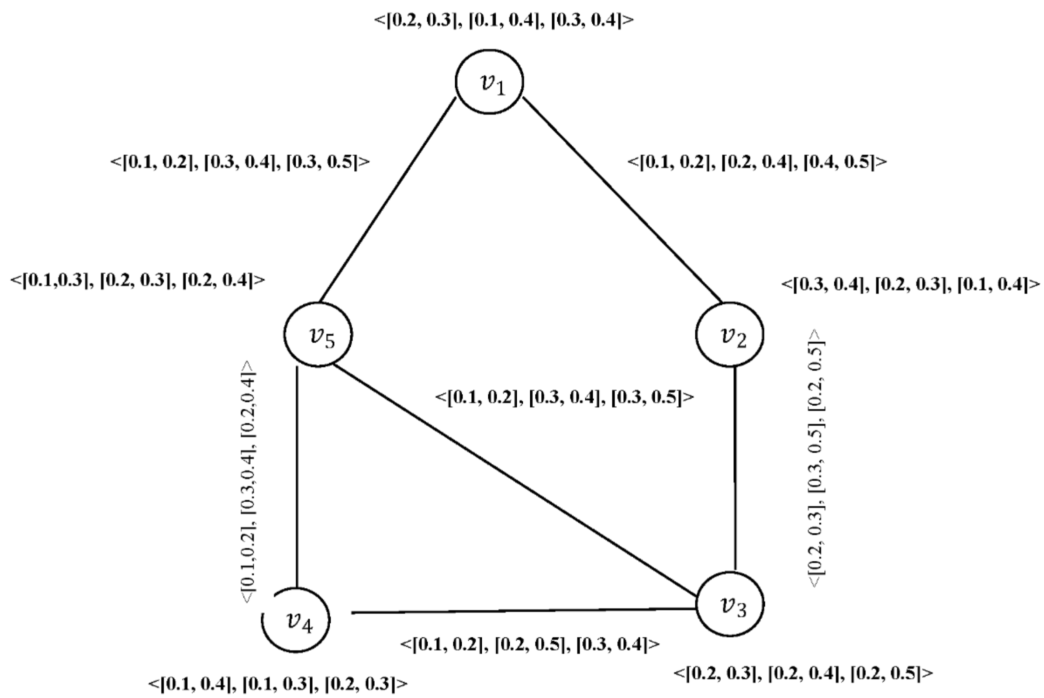

Figure 13: Interval valued neutrosophic graph G₁

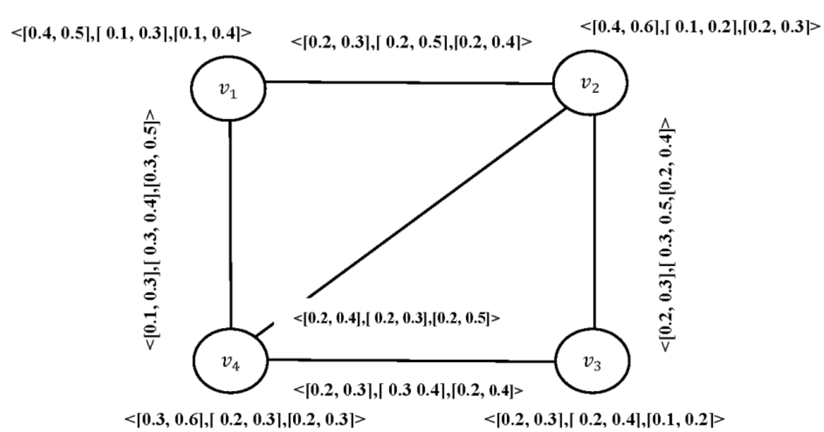

Figure 14: Interval valued neutrosophic graph G₁

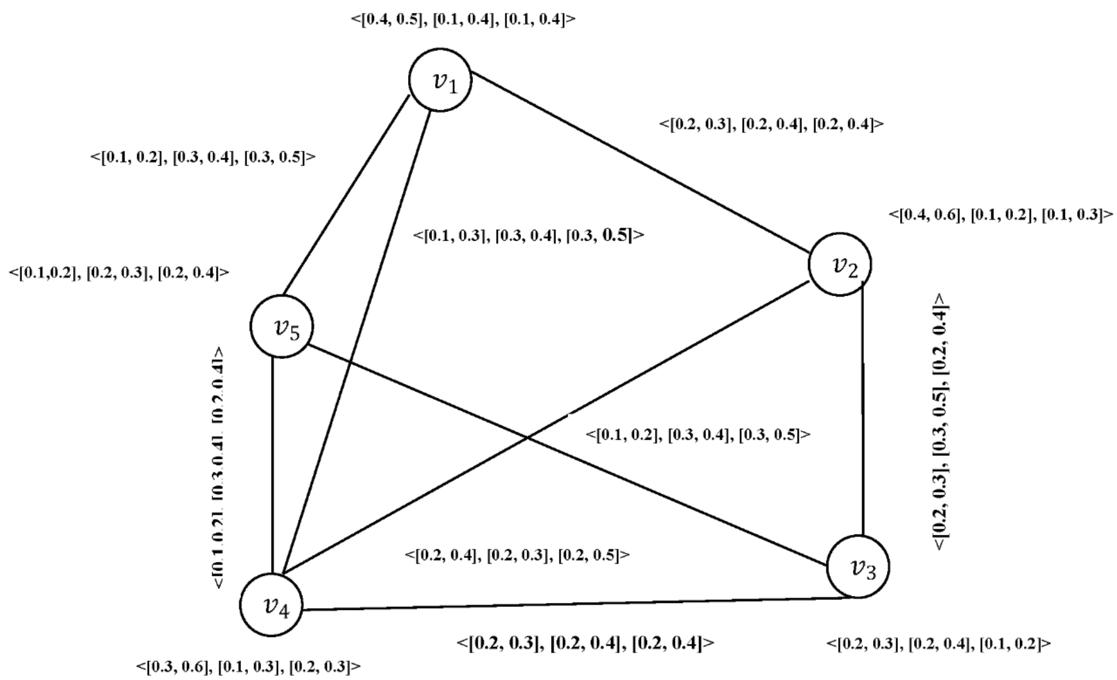

Figure 15: Interval valued neutrosophic graph $G_1 \cup G_2$

Definition 20

The join of $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$ interval valued neutrosophic graphs G_1 and G_2 of the graphs G_1^* and G_2^* is defined as follows:

1) $$(T_{A_1L} + T_{A_2L})(x) = \begin{cases} (T_{A_1L} \cup T_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ T_{A_1L}(x) & \text{if } x \in V_1 \\ T_{A_2L}(x) & \text{if } x \in V_2 \end{cases} \tag{70}$$

$$(T_{A_1U} + T_{A_2U})(x) = \begin{cases} (T_{A_1U} \cup T_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ T_{A_1U}(x) & \text{if } x \in V_1 \\ T_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

$$(I_{A_1L} + I_{A_2L})\,(x) = \begin{cases} (I_{A_1L} \cap I_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ I_{A_1L}(x) & \text{if } x \in V_1 \\ I_{A_2L}(x) & \text{if } x \in V_2 \end{cases}$$

$$(I_{A_1U} + I_{A_2U})\,(x) = \begin{cases} (I_{A_1U} \cap I_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ I_{A_1U}(x) & \text{if } x \in V_1 \\ I_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

$$(F_{A_1L} + F_{A_2L})\,(x) = \begin{cases} (F_{A_1L} \cap F_{A_2L})(x) & \text{if } x \in V_1 \cup V_2 \\ F_{A_1L}(x) & \text{if } x \in V_1 \\ F_{A_2L}(x) & \text{if } x \in V_2 \end{cases}$$

$$(F_{A_1U} + F_{A_2U})\,(x) = \begin{cases} (F_{A_1U} \cap F_{A_2U})(x) & \text{if } x \in V_1 \cup V_2 \\ F_{A_1U}(x) & \text{if } x \in V_1 \\ F_{A_2U}(x) & \text{if } x \in V_2 \end{cases}$$

2) $$(T_{B_1L} + T_{B_2L})\,(x\mathrm{y}) = \begin{cases} (T_{B_1L} \cup T_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ T_{B_1L}(xy) & \text{if } xy \in E_1 \\ T_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases} \tag{71}$$

$$(T_{B_1U} + T_{B_2U})\,(x\mathrm{y}) = \begin{cases} (T_{B_1U} \cup T_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ T_{B_1U}(xy) & \text{if } xy \in E_1 \\ T_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(I_{B_1L} + I_{B_2L})\,(x\mathrm{y}) = \begin{cases} (I_{B_1L} \cap I_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ I_{B_1L}(xy) & \text{if } xy \in E_1 \\ I_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(I_{B_1U} + I_{B_2U})\,(x\mathrm{y}) = \begin{cases} (I_{B_1U} \cap I_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ I_{B_1U}(xy) & \text{if } xy \in E_1 \\ I_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(F_{B_1L} + F_{B_2L})\,(x\mathrm{y}) = \begin{cases} (F_{B_1L} \cap F_{B_2L})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ F_{B_1L}(xy) & \text{if } xy \in E_1 \\ F_{B_2L}(xy) & \text{if } xy \in E_2 \end{cases}$$

$$(F_{B_1U} + F_{B_2U})\,(x\ \mathrm{y}) = \begin{cases} (F_{B_1U} \cap F_{B_2U})(\mathrm{xy}) & \text{if } xy \in E_1 \cup E_2 \\ F_{B_1U}(xy) & \text{if } xy \in E_1 \\ F_{B_2U}(xy) & \text{if } xy \in E_2 \end{cases}$$

3) $$\begin{aligned} &(T_{B_1L} + T_{B_2L})\,(x\ \mathrm{y}) = \min\,(T_{B_1L}(x), T_{B_2L}(x)) \\ &(T_{B_1U} + T_{B_2U})\,(xy) = \min\,(T_{B_1U}(x), T_{B_2U}(x)) \\ &(I_{B_1L} + I_{B_2L})\,(xy) = \max\,(I_{B_1L}(x), I_{B_2L}(x)) \\ &(I_{B_1U} + I_{B_2U})\,(x\ \mathrm{y}) = \max\,(I_{B_1U}(x), I_{B_2U}(x) \\ &(F_{B_1L} + F_{B_2L})\,(xy) = \max\,(F_{B_1L}(x), F_{B_2L}(x)) \\ &(F_{B_1U} + F_{B_2U})\,(x\ \mathrm{y}) = \max\,(F_{B_1U}(x), F_{B_2U}(x)) if xy\ \in\ E', \end{aligned} \tag{72}$$

where E' is the set of all edges joining the nodes of V_1 and V_2, assuming$V_1 \cap V_2$=$\emptyset$.

Example 6

Let G_1^*= (V_1, E_1) and G_2^*= (V_2, E_2) be two graphs such that V_1 ={u_1,u_2,u_3},V_2 ={v_1,v_2,v_3},E_1 ={u_1u_2, u_2u_3} and E_2 ={v_1v_2, v_2v_3}. Consider two interval valued neutrosophic graphs G_1= (A_1, B_1) and G_2= (A_2, B_2).

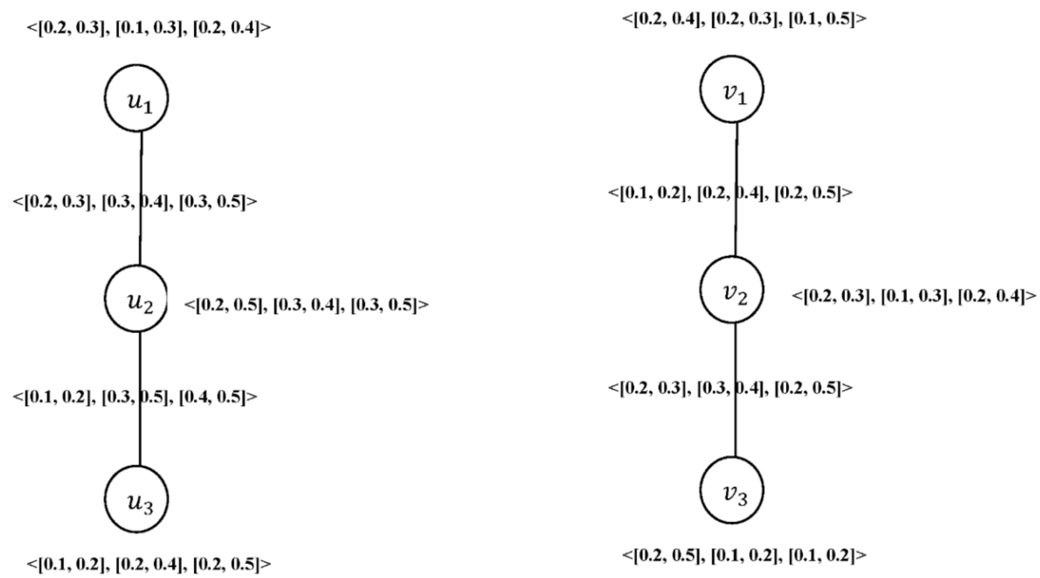

Figure 16: Interval valued neutrosophic graph ofG_1 and G_2

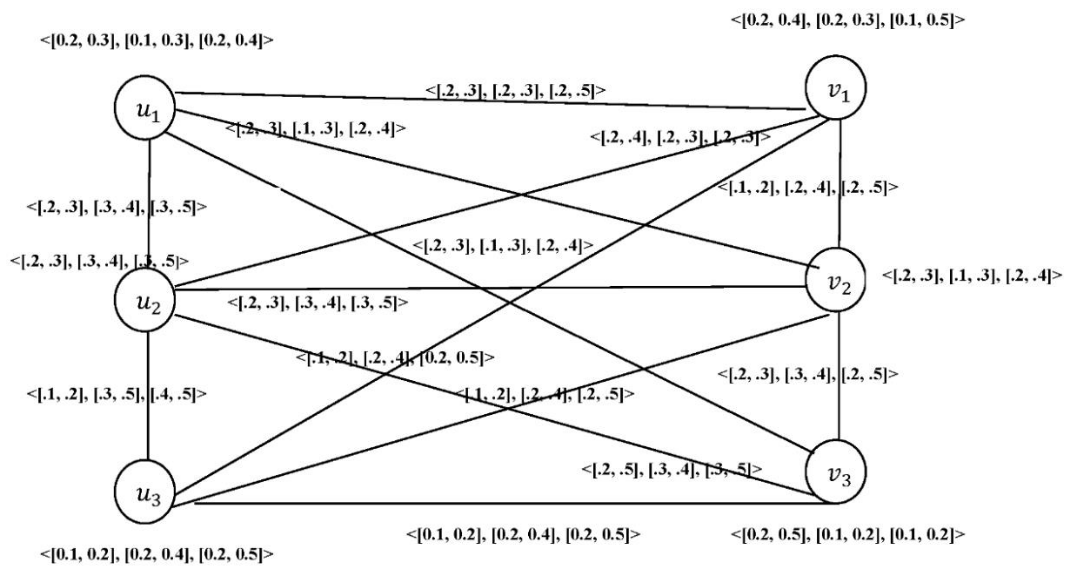

Figure 17: Interval valued neutrosophic graph of $G_1 + G_2$

5. Conclusion

The interval valued neutrosophic models give more precision, flexibility and compatibility to the system as compared to the classical, fuzzy, intuitionistic fuzzy and neutrosophic models. In this paper, the authors introduced some operations: Cartesian product, composition, union and join on interval valued neutrosophic graphs, and investigated some of their properties. In the future, the authors plan to study others operations, such as: tensor product and normal product of two interval valued neutrosophic graphs.

References

1. Ansari, A. Q., Biswas,R., &Aggarwal S.(2013a).Neutrosophic classifier: An extension of fuzzy classifier. Elsevier, Applied Soft Computing, 13, 563-573, http://dx.doi.org/10.1016/ j.asoc.2012.08.002.
2. Ansari, A. Q., Biswas,R., &Aggarwal, S.(2013b). (Presentation) Neutrosophication of Fuzzy Models, In: Proceedings of the IEEE Workshop On Computational Intelligence: Theories, Applications and Future Directions (hostedby IIT Kanpur), IEEE, 14th July 2013.
3. Ansari, A. Q., Biswas,R.,&Aggarwal, S. (2013c).Extension to fuzzy logic representation: Moving towards neutrosophic logic - A new laboratory rat. In: Proceedings of the IEEE Workshop On Fuzzy Systems (FUZZ), IEEE: 1 –8.DOI:10.1109/FUZZ-IEEE.2013.6622412.
4. Atanassov, K. (1999).Intuitionistic fuzzy sets: theory and applications, Physica, New York.
5. Atanassov, K. (1986). Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20: 87-96.
6. Atanassov, K., Gargov, G. (1989). Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol.31:343-349.
7. Antonios, K., Stefanos, S., Lazaros, I., &Elias, P. (2014).Fuzzy graphs: algebraic structure and syntactic recognition, Artificial Intelligence Review, vol 42, Issue 3:479-490.
8. Aggarwal, S., Biswas, R., &Ansari, A. Q.(2010). Neutrosophic modeling and control, Computer and Communication Technology (ICCCT), International Conference:718–723 DOI:10.1109/ICCCT.2010.5640435.
9. Broumi, S., Talea, M.,& Smarandache, F. (2016a) Single Valued Neutrosophic Graphs: Degree, Order and Size. IEEE World Congress on Computational Intelligence, 8 pages, accepted
10. Broumi, S., Talea, M., Bakali, A. and Smarandache, F. (2016b). Single Valued Neutrosophic Graphs, Journal of New Theory, N 10:86-101.
11. Broumi, S., Talea, M., Bakali, A and Smarandache, F. Interval Valued Neutrosophic Graphs. In press.
12. Broumi, S., Talea, M., Bakali, A., &Smarandache, F. (2016c). On Bipolar Single Valued Neutrosophic Graphs, Journal of New Theory, no. 11: 84-102.
13. Broumi, S., Smarandache, F., Talea, M. & Bakali, A. (2016). An Introduction to Bipolar Single Valued Neutrosophic Graph Theory. Applied Mechanics and Materials, vol.841:184-191, doi:10.4028/www.scienti_c.net/AMM.841.184
14. Broumi, S., Smarandache, F. (2014). New distance and similarity measures of interval neutrosophic sets.Information Fusion (FUSION), IEEE 17th International Conference:1 – 7.
15. Bhattacharya, P. (1987). Some remarks on fuzzy graphs, Pattern Recognition Letters, 6: 297-302.
16. Deli, I., Mumtaz, A. &Smarandache, F.(2015).Bipolar neutrosophic sets and their application based on multi-criteria decision making problems, Advanced Mechatronic Systems (ICAMechS), 2015 International Conference:249 – 254, DOI: 10.1109/ICAMechS.2015.7287068.
17. Garg, G., Bhutani, K., Kumar,M., & Aggarwal, S. (2015).Hybrid model for medical diagnosis using Neutrosophic Cognitive Maps with Genetic Algorithms, FUZZ-IEEE 2015 (IEEE International conference on fuzzy systems).
18. Hai-Long, Y., She, G., Yanhonge, & Xiuwu, L. (2015). On single valued neutrosophic relations, Journal of Intelligent & Fuzzy Systems:1-12.
19. Liu, P., &Shi, L. (2015).The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making, Neural Computing and Applications, 26 (2):457-471.
20. Mishra S. N. &Pal, A. (2013). Product of Interval Valued Intuitionistic fuzzy graph, Annals of Pure and Applied Mathematics, Vol. 5, No.1:37-46.
21. Nagoor, G.A., & Shajitha Begum, S. (2010). Degree, Order and Size in Intuitionistic Fuzzy Graphs, International Journal of Algorithms, Computing and Mathematics, (3)3:11-16.
22. Nagoor, G.A., & Latha, S.R. (2012). On Irregular Fuzzy Graphs. Applied Mathematical Sciences, Vol.6, no.11:517-523.

23. Nagoor, G. A., & Basheer Ahamed, M. (2003). Order and Size in Fuzzy Graphs, Bulletin of Pure and Applied Sciences, Vol 22E (No.1):145-148.
24. Parvathi, R., Karunambigai, M. G. (2006).Intuitionistic Fuzzy Graphs. Computational Intelligence, Theory and applications, International Conference in Germany,18 -20.
25. Shannon, A., & Atanassov, K. (1994). A first step of the intuitionistic fuzzy graph, Proceeding of the first Workshop on fuzzy based Expert Systems, Sofia, sept. 28-30: 59-61.
26. Şahin, R. (2015). Cross-entropy measure on interval neutrosophic sets and its applications in multicriteria decision making, Neural Computing and Applications:1-11.
27. Smarandache, F. (2015a).Symbolic Neutrosophic Theory, Brussels, Belgium: Europanova, 195p.
28. Smarandache, F. (2015b). Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies, Neutrosophic Sets and Systems, Vol. 9:58-63.
29. Smarandache, F. (2015c). Types of Neutrosophic Graphs and Neutrosophic Algebraic Structures together with their Applications in Technology. Seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania.
30. Smarandache, F. (2006). Neutrosophic set - a generalization of the intuitionistic fuzzy set.Granular Computing, 2006 IEEE International Conference:38–42. DOI: 10.1109/GRC.2006.1635754.
31. Smarandache, F. (2011). A geometric interpretation of the neutrosophic set - A generalization of the intuitionistic fuzzy set.Granular Computing (GrC), 2011 IEEE International Conference: 602 – 606.
32. Turksen, I. (1986). Interval valued fuzzy sets based on normal forms.Fuzzy Sets and Systems, vol. 20:191-210.
33. Vasantha Kandasamy, W. B., Smarandache, F.(2013).Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps, Kindle Edition.
34. Vasantha Kandasamy, W. B., Ilanthenral. K and Smarandache, F.(2015).Neutrosophic Graphs: A New Dimension to Graph Theory. Kindle Edition.
35. Vasantha Kandasamy, W.B., Smarandache,F. (2004). Analysis of social aspects of migrant laborers living with HIV/AIDS using Fuzzy Theory and Neutrosophic Cognitive Maps, Phoenix, AZ, USA: Xiquan.
36. Wang, H., Zhang, Y., & Sunderraman, R. (2005a).Truth-value based interval neutrosophic sets. Granular Computing, 2005 IEEE International Conference, Vol.1:274–277. DOI: 10.1109/GRC.2005.1547284.
37. Wang, H., Smarandache, F., Zhang,Y., &Sunderraman,R.(2010). Single valued Neutrosophic Sets, Multispace and Multistructure, 4:410-413.
38. Wang, H., Smarandache, F., Zhang,Y.Q and Sunderraman, R. (2005b).Interval Neutrosophic Sets and Logic: Theory and Applications in Computing. Phoenix, AZ, USA: Hexis.
39. Ye, J. (2014a). Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making, Journal of Intelligent & Fuzzy Systems (27):2231-2241.
40. Ye, J. (2014b). Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making, International Journal of Fuzzy Systems. Vol. 16, No. 2: 204-211.
41. Ye, J. (2014c). Similarity measures between interval neutrosophic sets and their applications in Multi-criteria decision-making. Journal of Intelligent and Fuzzy Systems, 26:165-172.
42. Zimmermann, H.J. (1985). Fuzzy Set Theory and its Applications. Boston, MA, USA: Kluwer-Nijhoff.
43. Zhang, H., Wang J, Chen, X. (2015a). An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets. Neural Computing and Applications, 1-13.
44. Zhang, H.Y., Ji, P., Wang, Q.J, Chen, H. X.(2015b). An Improved Weighted Correlation Coefficient Based on Integrated Weight for Interval Neutrosophic Sets and its Application in Multi-Criteria Decision-making Problems, International Journal of Computational Intelligence Systems, vol 8, Issue 6.
45. Zhang, H.Y., Wang, J.Q. & Chen, X.H. (2014). Interval neutrosophic sets and their application in multicriteria decision making problems. The Scientific World Journal,DOI:10.1155/2014/ 645953.
46. Zadeh, L. (1965).Fuzzy sets, Information and Control, 8:338-353.