Reliability and Importance Discounting of Neutrosophic Masses

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!ÂÓÔÒÁÃÔ8 In this paper, we introduce for the first time the discounting of a neutrosophic mass in terms of reliability and respectively the importance of the source.

We show that reliability and importance discounts commute when dealing with classical masses.

ρ8)ÎÔÒÏÄÕÃÔÉÏÎ8 Let $\Phi = \{\Phi_1, \Phi_2, \dots, \Phi_n\}$ be the frame of discernment, where $n \geq 2$, and the set of **ÆÏÃÁÌ eleme ÎÔÓ**:

$$F = \{A_1, A_2, \dots, A_m\}, \text{ for } m \geq 1, F \subset G^{\Phi}. \ (1)$$

Let $G^{\Phi} = (\Phi, \cup, \cap, \mathcal{C})$ be the **ÆÕÓÉÏn ÓÐÁÃÅ**.

A **ÎÅÕÔÒÏÓÏÐÈÉÃ ÍÁÓÓ** is defined as follows:

$$m_n: G \to [0,1]^3$$

for any $x \in G$, $m_n(x) = (t(x), i(x), f(x))$, (2)

where $t(x) =$ believe that x will occur (truth);

$i(x) =$ indeterminacy about occurence;

and $f(x) =$ believe that x will not occur (falsity).

Simply, we say in neutrosophic logic:

$t(x)$ = believe in x;

$i(x)$ = believe in neut(x)
[the neutral of x, i.e. neither x nor anti(x)];

and $f(x)$ = believe in anti(x) [the opposite of x].

Of course, $t(x),\ i(x),\ f(x) \in [0,1]$, and

$$\sum_{x \in G}[t(x) + i(x) + f(x)] = 1, \quad (3)$$

while

$$m_n(\phi) = (0,0,0). \quad (4)$$

It is possible that according to some parameters (or data) a source is able to predict the believe in a hypothesis x to occur, while according to other parameters (or other data) the same source may be able to find the believe in x not occuring, and upon a third category of parameters (or data) the source may find some indeterminacy (ambiguity) about hypothesis occurence.

An element $x \in G$ is called **ÆÏÃÁÌ** if

$$n_m(x) \neq (0,0,0), \quad (5)$$

i.e. $t(x) > 0$ or $i(x) > 0$ or $f(x) > 0$.

Any **ÃÌÁÓÓÉÃÁl mÁÓÓ**:

$$m : G^{\phi} \to [0,1] \quad (6)$$

can be simply written as a neutrosophic mass as:

$$m(A) = (m(A), 0, 0). \quad (7)$$

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Let $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ be the reliability coefficient of the source, $\alpha \in [0,1]^3$.

Then, for any $x \in G^\theta \setminus \{\theta, I_t\}$,

where $\theta =$ the empty set

and $I_t =$ total ignorance,

$$m_n(x)_\alpha = \big(\alpha_1 t(x), \alpha_2 i(x), \alpha_3 f(x)\big), \ (8)$$

and

$$m_n(I_t)_\alpha = \Bigg(t(I_t) + (1-\alpha_1) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} t(x), \ i(I_t) + (1-\alpha_2) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} i(x), f(I_t) + (1-\alpha_3) \sum_{x \in G^\theta \setminus \{\phi, I_t\}} f(x) \Bigg) \quad (9),$$

and, of course,

$$m_n(\phi)_\alpha = (0, 0, 0).$$

The missing mass of each element x, for $x \neq \phi, x \neq I_t$, is transferred to the mass of the total ignorance in the following way:

$t(x) - \alpha_1 t(x) = (1 - \alpha_1) \cdot t(x)$ is transferred to $t(I_t)$, (10)

$i(x) - \alpha_2 i(x) = (1 - \alpha_2) \cdot i(x)$ is transferred to $i(I_t)$, (11)

and $f(x) - \alpha_3 f(x) = (1 - \alpha_3) \cdot f(x)$ is transferred to $f(I_t)$. (12)

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Let $\beta \in [0,1]$ be the importance coefficient of the source. This discounting can be done in several ways.

a. For any $x \in G^{\theta} \setminus \{\phi\}$,

$$m_n(x)_{\beta_1} = \big(\beta \cdot t(x), i(x), f(x) + (1-\beta) \cdot t(x)\big), (13)$$

which means that $t(x)$, the believe in x, is diminished to $\beta \cdot t(x)$, and the missing mass, $t(x) - \beta \cdot t(x) = (1-\beta) \cdot t(x)$, is transferred to the believe in $anti(x)$.

b. Another way:

For any $x \in G^{\theta} \setminus \{\phi\}$,

$$m_n(x)_{\beta_2} = \big(\beta \cdot t(x), i(x) + (1-\beta) \cdot t(x), f(x)\big), (14)$$

which means that $t(x)$, the believe in x, is similarly diminished to $\beta \cdot t(x)$, and the missing mass $(1-\beta) \cdot t(x)$ is now transferred to the believe in $neut(x)$.

c. The third way is the most general, putting together the first and second ways.

For any $x \in G^{\theta} \setminus \{\phi\}$,

$$m_n(x)_{\beta_3} = \big(\beta \cdot t(x), i(x) + (1-\beta) \cdot t(x) \cdot \gamma, f(x) + (1-\beta) \cdot t(x) \cdot (1-\gamma)\big), (15)$$

where $\gamma \in [0,1]$ is a parameter that splits the missing mass $(1-\beta) \cdot t(x)$ a part to $i(x)$ and the other part to $f(x)$.

For $\gamma = 0$, one gets the first way of distribution, and when $\gamma = 1$, one gets the second way of distribution.

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a. *Reliability first, Importance second.*

For any $x \in G^\theta \setminus \{\phi, I_t\}$, one has after reliability α discounting, where

$$\alpha = (\alpha_1, \alpha_2, \alpha_3):$$

$$m_n(x)_\alpha = \big(\alpha_1 \cdot t(x), \alpha_2 \cdot t(x), \alpha_3 \cdot f(x)\big), \quad (16)$$

and
$$m_n(I_t)_\alpha = \left(t(I_t) + (1-\alpha_1) \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} t(x)\,, i(I_t) + (1-\alpha_2) \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} i(x)\,, f(I_t) + (1-\alpha_3) \cdot \sum_{x \in G^\theta \setminus \{\phi, I_t\}} f(x) \right) \stackrel{\text{def}}{=} \left(T_{I_t}, I_{I_t}, F_{I_t}\right). \quad (17)$$

Now we do the importance β discounting method, the third importance discounting way which is the most general:

$$m_n(x)_{\alpha\beta_3} = \big(\beta\alpha_1 t(x), \alpha_2 i(x) + (1-\beta)\alpha_1 t(x)\gamma, \alpha_3 f(x) + (1-\beta)\alpha_1 t(x)(1-\gamma)\big) \quad (18)$$

and

$$m_n(I_t)_{\alpha\beta_3} = \Big(\beta \cdot T_{I_t}, I_{I_t} + (1-\beta)T_{I_t} \cdot \gamma, F_{I_t} + (1-\beta)T_{I_t}(1-\gamma)\Big). \quad (19)$$

b. *Importance first, Reliability second.*

For any $x \in G^\theta \setminus \{\phi, I_t\}$, one has after importance β discounting (third way):

$$m_n(x)_{\beta_3} = \big(\beta \cdot t(x), i(x) + (1-\beta)t(x)\gamma, f(x) + (1-\beta)t(x)(1-\gamma)\big) \quad (20)$$

and

$$m_n(I_t)_{\beta_3} = \Big(\beta \cdot t\big(I_{I_t}\big), i\big(I_{I_t}\big) + (1-\beta)t(I_t)\gamma,\ f(I_t) + (1-\beta)t(I_t)(1-\gamma)\Big). \quad (21)$$

Now we do the reliability $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ discounting, and one gets:

$$m_n(x)_{\beta_3\alpha} = \big(\alpha_1 \cdot \beta \cdot t(x), \alpha_2 \cdot i(x) + \alpha_2(1-\beta)t(x)\gamma, \alpha_3 \cdot f(x) + \alpha_3 \cdot (1-\beta)t(x)(1-\gamma)\big) \quad (22)$$

and

$$m_n(I_t)_{\beta_3\alpha} = \big(\alpha_1 \cdot \beta \cdot t(I_t), \alpha_2 \cdot i(I_t) + \alpha_2(1-\beta)t(I_t)\gamma, \alpha_3 \cdot f(I_t) + \alpha_3(1-\beta)t(I_t)(1-\gamma)\big). \quad (23)$$

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We see that (a) and (b) are in general different, so reliability of sources does not commute with the importance of sources.

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Let's consider a classical mass $m: G^\theta \to [0, 1]$ (24)

and the focal set $F \subset G^\theta$, $F = \{A_1, A_2, \dots, A_m\}, m \geq 1$, (25)

and of course $m(A_i) > 0$, for $1 \leq i \leq m$.

Suppose $m(A_i) = a_i \in (0,1]$. (26)

a. Reliability first, Importance second.

Let $\alpha \in [0, 1]$ be the reliability coefficient of $m\ (\cdot)$.

For $x \in G^\theta \setminus \{\phi, I_t\}$, one has $m(x)_\alpha = \alpha \cdot m(x)$, (27)

and $m(I_t) = \alpha \cdot m(I_t) + 1 - \alpha$. (28)

Let $\beta \in [0, 1]$ be the importance coefficient of $m\ (\cdot)$.

Then, for $x \in G^\theta \setminus \{\phi, I_t\}$,

$$m(x)_{\alpha\beta} = \big(\beta\alpha m(x), \alpha m(x) - \beta\alpha m(x)\big) = \alpha \cdot m(x) \cdot (\beta, 1-\beta), \quad (29)$$

considering only two components: believe that x occurs and, respectively, believe that x does not occur.

Further on,

$$m(I_t)_{\alpha\beta} = (\beta\alpha m(I_t) + \beta - \beta\alpha, \alpha m(I_t) + 1 - \alpha - \beta\alpha m(I_t) - \beta + \beta\alpha) = [\alpha m(I_t) + 1 - \alpha] \cdot (\beta, 1-\beta). \quad (30)$$

b. Importance first, Reliability second.

For $x \in G^{\theta} \setminus \{\phi, I_t\}$, one has

$$m(x)_{\beta} = \big(\beta \cdot m(x), m(x) - \beta \cdot m(x)\big) = m(x) \cdot (\beta, 1-\beta), \quad (31)$$

and $m(I_t)_{\beta} = \big(\beta m(I_t), m(I_t) - \beta m(I_t)\big) = m(I_t) \cdot (\beta, 1-\beta)$. (32)

Then, for the reliability discounting scaler α one has:

$$m(x)_{\beta\alpha} = \alpha m(x)(\beta, 1-\beta) = \big(\alpha m(x)\beta, \alpha m(x) - \alpha\beta m(m)\big) \quad (33)$$

and $m(I_t)_{\beta\alpha} = \alpha \cdot m(I_t)(\beta, 1-\beta) + (1-\alpha)(\beta, 1-\beta) = [\alpha m(I_t) + 1 - \alpha] \cdot (\beta, 1-\beta) = (\alpha m(I_t)\beta, \alpha m(I_t) - \alpha m(I_t)\beta) + (\beta - \alpha\beta, 1 - \alpha - \beta + \alpha\beta) = (\alpha\beta m(I_t) + \beta - \alpha\beta, \alpha m(I_t) - \alpha\beta m(I_t) + 1 - \alpha - \beta - \alpha\beta)$. (34)

Hence (a) and (b) are equal in this case.

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1. Classical mass.

The following classical is given on $\theta = \{A, B\}$:

	A	B	AUB
m	0.4	0.5	0.1

(35)

Let $\alpha = 0.8$ be the reliability coefficient and $\beta = 0.7$ be the importance coefficient.

a. Reliability first, Importance second.

	A	B	AUB
m_{α}	0.32	0.40	0.28
$m_{\alpha\beta}$	(0.224, 0.096)	(0.280, 0.120)	(0.196, 0.084)

(36)

We have computed in the following way:

$$m_\alpha(A) = 0.8m(A) = 0.8(0.4) = 0.32, (37)$$

$$m_\alpha(B) = 0.8m(B) = 0.8(0.5) = 0.40, (38)$$

$$m_\alpha(AUB) = 0.8(\text{AUB}) + 1 - 0.8 = 0.8(0.1) + 0.2 = 0.28, (39)$$

and $m_{\alpha\beta}(B) = \big(0.7m_\alpha(A), m_\alpha(A) - 0.7m_\alpha(A)\big) = \big(0.7(0.32), 0.32 - 0.7(0.32)\big) = (0.224, 0.096), (40)$

$$m_{\alpha\beta}(B) = \big(0.7m_\alpha(B), m_\alpha(B) - 0.7m_\alpha(B)\big) = \big(0.7(0.40), 0.40 - 0.7(0.40)\big) = (0.280, 0.120), (41)$$

$$m_{\alpha\beta}(AUB) = \big(0.7m_\alpha(AUB), m_\alpha(AUB) - 0.7m_\alpha(AUB)\big) = \big(0.7(0.28), 0.28 - 0.7(0.28)\big) = (0.196, 0.084). (42)$$

b. Importance first, Reliability second.

	A	B	AUB
m	0.4	0.5	0.1
m_β	(0.28, 0.12)	(0.35, 0.15)	(0.07, 0.03)
$m_{\beta\alpha}$	(0.224, 0.096	(0.280, 0.120)	(0.196, 0.084)

(43)

We computed in the following way:

$$m_\beta(A) = \big(\beta m(A), (1-\beta)m(A)\big) = \big(0.7(0.4), (1-0.7)(0.4)\big) = (0.280, 0.120), (44)$$

$$m_\beta(B) = \big(\beta m(B), (1-\beta)m(B)\big) = \big(0.7(0.5), (1-0.7)(0.5)\big) = (0.35, 0.15), (45)$$

$$m_\beta(AUB) = \big(\beta m(AUB), (1-\beta)m(AUB)\big) = \big(0.7(0.1), (1-0.1)(0.1)\big) = (0.07, 0.03), (46)$$

and $m_{\beta\alpha}(A) = \alpha m_\beta(A) = 0.8(0.28, 0.12) = \big(0.8(0.28), 0.8(0.12)\big) = (0.224, 0.096), (47)$

$$m_{\beta\alpha}(B) = \alpha m_\beta(B) = 0.8(0.35, 0.15) = \big(0.8(0.35), 0.8(0.15)\big) = (0.280, 0.120), (48)$$

$$m_{\beta\alpha}(AUB) = \alpha m(AUB)(\beta, 1-\beta) + (1-\alpha)(\beta, 1-\beta) = 0.8(0.1)(0.7, 1 - 0.7) + (1-0.8)(0.7, 1-0.7) = 0.08(0.7, 0.3) + 0.2(0.7, 0.3) = (0.056, 0.024) + (0.140, 0.060) = (0.056 + 0.140, 0.024 + 0.060) = (0.196, 0.084). \quad (49)$$

Therefore reliability discount commutes with importance discount of sources when one has classical masses.

The result is interpreted this way: believe in A is 0.224 and believe in $nonA$ is 0.096, believe in B is 0.280 and believe in $nonB$ is 0.120, and believe in total ignorance AUB is 0.196, and believe in non-ignorance is 0.084.

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Let's consider the third way of redistribution of masses related to importance coefficient of sources. $\beta = 0.7$, but $\gamma = 0.4$, which means that 40% of β is redistributed to $i(x)$ and 60% of β is redistributed to $f(x)$ for each $x \in G^{\theta} \setminus \{\phi\}$; and $\alpha = 0.8$.

a. Reliability first, Importance second.

	A	B	AUB
m	0.4	0.5	0.1
m_{α}	0.32	0.40	0.28
$m_{\alpha\beta}$	(0.2240, 0.0384, 0.0576)	(0.2800, 0.0480, 0.0720)	(0.1960, 0.0336, 0.0504).

(50)

We computed m_α in the same way.

But:

$$m_{\alpha\beta}(A) = \big(\beta \cdot m_\alpha(A), i_\alpha(A) + (1-\beta)m_\alpha(A) \cdot \gamma, f_\alpha(A) + (1-\beta)m_\alpha(A)(1-\gamma)\big) = \big(0.7(0.32), 0 + (1-0.7)(0.32)(0.4), 0 + (1-0.7)(0.32)(1-0.4)\big) = (0.2240, 0.0384, 0.0576). \quad (51)$$

Similarly for $m_{\alpha\beta}(B)$ and $m_{\alpha\beta}(AUB)$.

b. *Importance first, Reliability second.*

	A	B	AUB
m	0.4	0.5	0.1
m_β	(0.280, 0.048, 0.072)	(0.350, 0.060, 0.090)	(0.070, 0.012, 0.018)
$m_\beta\alpha$	(0.2240, 0.0384, 0.0576)	(0.2800, 0.0480, 0.0720)	(0.1960, 0.0336, 0.0504).

(52)

We computed $m_\beta(\cdot)$ in the following way:

$$m_\beta(A) = \big(\beta \cdot t(A), i(A) + (1-\beta)t(A) \cdot \gamma, f(A) + (1-\beta)t(A)(1-\gamma)\big) = \big(0.7(0.4), 0 + (1-0.7)(0.4)(0.4), 0 + (1-0.7)0.4(1-0.4)\big) = (0.280, 0.048, 0.072). \quad (53)$$

Similarly for $m_\beta(B)$ and $m_\beta(AUB)$.

To compute $m_{\beta\alpha}(\cdot)$, we take $\alpha_1 = \alpha_2 = \alpha_3 = 0.8$, (54)

in formulas (8) and (9).

$$m_{\beta\alpha}(A) = \alpha \cdot m_\beta(A) = 0.8(0.280, 0.048, 0.072)$$
$$= \big(0.8(0.280), 0.8(0.048), 0.8(0.072)\big)$$
$$= (0.2240, 0.0384, 0.0576). (55)$$

Similarly $m_{\beta\alpha}(B) = 0.8(0.350, 0.060, 0.090) = (0.2800, 0.0480, 0.0720)$. (56)

For $m_{\beta\alpha}(AUB)$ we use formula (9):

$$m_{\beta\alpha}(AUB) = \big(t_\beta(AUB) + (1-\alpha)\big[t_\beta(A) + t_\beta(B)\big],\ i_\beta(AUB) + (1-\alpha)\big[i_\beta(A) + i_\beta(B)\big], f_\beta(AUB) + (1-\alpha)\big[f_\beta(A) + f_\beta(B)\big]\big)$$
$$= (0.070 + (1-0.8)[0.280 + 0.350], 0.012 + (1-0.8)[0.048 + 0.060], 0.018 + (1-0.8)[0.072 + 0.090])$$
$$= (0.1960, 0.0336, 0.0504).$$

Again, the reliability discount and importance discount commute.

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In this paper we have defined a new way of discounting a classical and neutrosophic mass with respect to its importance. We have also defined the discounting of a neutrosophic source with respect to its reliability.

In general, the reliability discount and importance discount do not commute. But if one uses classical masses, they commute (as in Examples 1 and 2).

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