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3-Difference Cordial Labeling of Corona Related Graphs

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Abstract: Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \cdots, k\}$ be a map where k is an integer $2 \le k \le p$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate 3-difference cordial labeling behavior of $DT_n \odot K_1$ $DT_n \odot 2K_1$,$DT_n \odot K_2$ and some more graphs.

Key Words: Difference cordial labeling, Smarandachely k-difference cordial labeling, path, complete graph, triangular snake, corona.

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§1. Introduction

All Graphs in this paper are finite ,undirect and simple. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. Ponraj et al. [3], has been introduced the concept of k-difference cordial labeling of graphs and studied the 3-difference cordial labeling behavior of of some graphs. In [4,5,6,7] they investigate the 3-difference cordial labeling behavior of path, cycle, complete graph, complete bipartite graph, star, bistar, comb, double comb, quadrilateral snake, $C_4^{(t)}$, $S(K_{1,n})$, $S(B_{n,n})$ and carona of some graphs with double alternate triangular snake double alternate quadrilateral snake . In this paper we examine the 3-difference cordial labeling behavior of $DT_n \odot K_1$ $DT_n \odot 2K_1$,$DT_n \odot K_2$ etc. Terms are not defined here follows from Harary [2].

§2. k-Difference Cordial Labeling

Definition 2.1 *Let G be a (p, q) graph and let $f : V(G) \to \{1, 2, \cdots, k\}$ be a map. For*

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each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph.

On the other hand, if $|v_f(i) - v_f(j)| \geq 1$ or $|e_f(0) - e_f(1)| \geq 1$, such a labeling is called a Smarandachely k-difference cordial labeling of G.

A double triangular snake DT_n consists of two triangular snakes that have a common path. That is a double triangular snake is obtained from a path $u_1u_2\cdots u_n$ by joining u_i and u_{i+1} to two new vertices v_i $(1 \leq i \leq n-1)$ and w_i $(1 \leq i \leq n-1)$.

First we investigate the 3-difference cordial labeling behavior of $DT_n \odot K_1$.

Theorem 2.1 *$DT_n \odot K_1$ is 3-difference cordial.*

Proof Let $V(DT_n \odot K_1) = V(DT_n)\bigcup\{x_i : 1 \leq i \leq n\}\bigcup\{v_i', w_i' : 1 \leq i \leq n-1\}$ and $E(DT_n \odot K_1) = E(DT_n)\bigcup\{u_ix_i : 1 \leq i \leq n\}\bigcup\{v_iv_i', w_iw_i' : 1 \leq i \leq n-1\}$.

Case 1. n is even.

First we consider the path vertices u_i. Assign the label 1 to all the path vertices u_i $(1 \leq i \leq n)$. Then assign the label 2 to the path vertices $v_1, v_3, v_5, \cdots$ and assign the label 1 to the path vertices $v_2, v_4, v_6, \cdots$. Now we consider the vertices w_i. Assign the label 2 to all the vertices w_i $(1 \leq i \leq n-1)$. Next we move to the vertices v_i' and w_i'. Assign the label 2 to the vertices v_{2i+1}' for all the values of $i = 0, 1, 2, 3, \cdots$ and assign the label 1 to the vertices v_{2i} for $i = 1, 2, 3, \cdots$. Next we assign the label 1 to the vertex w_1' and assign the label 3 to the vertices $w_2', w_3', w_4', \cdots$ Finally assign the label 3 to all the vertices of x_i $(1 \leq i \leq n)$. The vertex condition and the edge conditions are $v_f(1) = v_f(2) = \frac{6n-3}{3}$ and $v_f(3) = \frac{6n-2}{3}$ and $e_f(0) = 4n-4$ and $e_f(1) = 4n-3$.

Case 2. n is odd.

Assign the label to the path vertices u_i $(1 \leq i \leq n)$, v_i $(1 \leq i \leq n-1)$, w_i $(1 \leq i \leq n-1)$, v_i' $(1 \leq i \leq n-1)$, x_i $(1 \leq i \leq n)$ as in case 1. Then assign the label 3 to all the vertices w_i' $(1 \leq i \leq n-1)$. Since $e_f(0) = 4n-3$, $e_f(1) = 4n-4$ and $v_f(1) = v_f(3) = 2n-1$ and $v_f(2) = 2n-2$, $DT_n \odot K_1$ is 3-difference cordial. □

Next investigation about $DT_n \odot 2K_1$.

Theorem 2.2 *$DT_n \odot 2K_1$ is 3-difference cordial.*

Proof Let $V(DT_n \odot 2K_1) = V(DT_n)\bigcup\{x_i, y_i : 1 \leq i \leq n\}\bigcup\{v_i', v_i'', w_i', w_i'' : 1 \leq i \leq n-1\}$ and $E(DT_n \odot 2K_1) = E(DT_n)\bigcup\{u_ix_i, u_iy_i : 1 \leq i \leq n\}\bigcup\{v_iv_i', v_iv_i'', w_iw_i', w_iw_i'' : 1 \leq i \leq n-1\}$.

Case 1. n is even.

Consider the path vertices u_i. Assign the label 1 to the path vertex u_1. Now we assign the labels 1,1,2,2 to the vertices u_2, u_3, u_4, u_5 respectively. Then we assign the labels 1,1,2,2 to the next four vertices u_6, u_7, u_8, u_9 respectively. Proceeding like this we assign the label to the next four vertices and so on. If all the vertices are labeled then we stop the process. Otherwise there are some non labeled vertices are exist. If the number of non labeled vertices are less than or equal to 3 then assign the labels 1,1,2 to the non labeled vertices. If it is two then assign the label 1,1 to the non labeled vertices. If only one non labeled vertex is exist then assign the label 1 only. Next we consider the label v_i. Assign the label 2 to the vertex v_1. Then we assign the label 2 to the vertices $v_2, v_4, v_6, \cdots$ and assign the label 3 to the vertices $v_3, v_5, v_7, \cdots$. Next we move to the vertices x_i and y_i. Assign the label 2 to the vertices x_1 and x_2 and we assign the label 3 to the vertices y_1 and y_2. Now we assign the label 1 to the vertices x_{4i+1} and x_{4i} for all the values of $i = 1, 2, 3, \cdots$. Then we assign the label 1 to the vertices x_{4i+3} for $i = 0, 1, 2, 3, \cdots$. Next we assign the label 2 to the vertices x_{4i+2} for all the values of $i = 1, 2, 3, \cdots$. Now we assign the label 3 to the vertices y_{4i+3} for i=0,1,2,3,... For all the values of $i = 1, 2, 3, \cdots$. assign the label 3 to the vertices y_{4i+1} and y_{4i+2}. Then we assign the label 2 to the vertices y_{4i} for $i = 1, 2, 3, \cdots$. Now we consider the vertices v'_i and v''_i. For all the values of i=1,2,3... assign the label 1 to the vertices v'_{4i+1}, v'_{4i+2}. Assign the label 1 to the vertices v'_{4i} for $i = 1, 2, 3, \cdots$. Then we assign the label 2 to the vertices v'_{4i+3} for all the values of $i = 0, 1, 2, 3, \cdots$. Consider the vertices v''_i. Assign the label 3 to the path vertex v''_{4i+1}, v''_{4i+2} and v''_{4i+3} for all the values of $i = 0, 1, 2, 3, \cdots$. Next we assign the label 2 to the vertices v''_{4i} for $i = 1, 2, 3, \cdots$. Now we assign the label 3 to the vertices $w_i (1 \le i \le n-1)$. Next we move to the vertices w'_i and w''_i. Assign the label 1 to all the vertices of $w'_i (1 \le i \le n-1)$ and we assign the label 2 to all the vertices of $w''_i (1 \le i \le n-1)$. Since $v_f(1) = v_f(2) = v_f(3) = 3n-2$ and $e_f(0) = \frac{11n-10}{2}$ and $e_f(1) = \frac{11n-8}{2}$, this labeling is 3-difference cordial labeling.

Case 2. n is odd.

First we consider the path vertices u_i. Assign the label 1,1,2,2 to the first four path vertices u_1, u_2, u_3, u_4 respectively. Then we assign the labels 1,1,2,2 to the next four vertices u_5, u_6, u_7, u_8 respectively. Continuing like this assign the label to the next four vertices and so on. If all the vertices are labeled then we stop the process. Otherwise there are some on labeled vertices are exist. If the number of non labeled vertices are less than or equal to 3 then assign the labels 1,1,2 to the non labeled vertices. If it is 2 assign the labels 1,1 to the non labeled vertices. If only one non labeled vertex exist then assign the label 1 to that vertex. Consider the vertices v_i. Assign the label 2 to the vertices $v_1, v_3, v_5, \cdots$ and we assign the label 3 to the vertices $v_2, v_4, v_6, \cdots$. Next we move to the vertices w_i. Assign the label to the vertices w_i $(1 \le i \le n-)$ as in case 1. Now we consider the vertices x_i and y_i. Assign the label 2 to the vertices x_{4i+1} for all the values of $i = 0, 1, 2, 3, \cdots$. For all the values of i=0,1,2,3,... assign the label 1 to the vertices x_{4i+2} and x_{4i+3}. Then we assign the label 1 to the vertices x_{4i} for all the values of $i = 1, 2, 3, \cdots$. Next we assign the label 3 to the vertices y_{4i+1} and y_{4i+2} for all the values of $i = 0, 1, 2, 3, \cdots$ and we assign the label 3 to the vertices y_{4i} for $i = 1, 2, 3, \cdots$. Then we assign the label 2 to the vertices y_{4i+3} for all values $i = 0, 1, 2, 3, \cdots$. Next we move to the vertices v'_i and v''_i. For all the values of $i = 0, 1, 2, 3, \cdots$ assign the label 1 to the vertices v'_{4i+1} and v'_{4i+3}. Now we assign the label 1 to the vertices v'_{4i} for $i = 1, 2, 3, \cdots$. Next we assign the label 2 to the vertices v'_{4i+2} for $i = 01, 2, 3, \cdots$. Consider the vertices v'_i. Assign the label

3 to the vertices v''_{4i+1} and v''_{4i+2} for all the values of $i = 0, 1, 2, 3, \cdots$ and we assign the label 1 to the vertices v_{4i} for $i = 1, 2, 3, \cdots$. For the values of $i = 0, 1, 2, 3, \cdots$ assign the label 2 to the vertices v_{4i+3}. Finally we consider the vertices w'_i and w''_i. Assign the label to the vertices w'_i $(1 \leq i \leq n-1)$ and w''_i $(1 \leq i \leq n-1)$ as in case 1. The vertex and edge condition are $v_f(1) = v_f(2) = v_f(3) = 3n-2$ and $e_f(0) = e_f(1) = \frac{11n-9}{2}$. □

We now investigate the graph $DT_n \odot K_2$.

Theorem 2.3 *$DT_n \odot K_2$ is 3-difference cordial.*

Proof Let $V(DT_n \odot K_2) = V(DT_n) \bigcup \{x_i, y_i : 1 \leq i \leq n\} \bigcup \{v'_i, v''_i, w'_i, w''_i : 1 \leq i \leq n-1\}$ and $E(DT_n \odot K_2) = E(DT_n) \bigcup \{u_ix_i, u_iy_i, x_iy_i : 1 \leq i \leq n\} \bigcup \{v_iv'_i, v_iv''_i, v'_iv''_i, w_iw'_i, w_iw''_i, w'_iw''_i : 1 \leq i \leq n-1\}$.

Case 1. n is even.

Consider the path vertices u_i. Assign the label 1 to the path vertices $u_1, u_2, u_3, \cdots$. Then we assign the labels 2 to the vertices $v_1, v_2, v_3, \cdots$. Next we assign the labels 3 to the vertices w_1, w_2, w_3, w_4. Now we consider the vertices v'_i and v''_i. Assign the label 2 to the vertex v'_1. Then we assign the label 1 to the vertices $v'_2, v'_3, v'_4, v'_6, \cdots$. Now we assign the label 3 to the vertices $v''_1, v''_2, v''_3, v''_4, \cdots$. Next we move to the vertices w'_i and w''_i. Assign the label 1 to the vertex w'_1. Then we assign the label 1 to the vertices $w'_2, w'_4, w'_6, \cdots$ and assign the label 2 to the vertices $w'_3, w'_5, w'_7, \cdots$. Assign the label 2 to the vertices $w''_1, w''_2, w''_3, w''_4, \cdots$. Finally we move to the vertices x_i and y_i. Assign the label 1 to the vertices $x_1, x_3, x_5, \cdots$ and we assign the label 2 to the vertices $x_2, x_4, x_6, \cdots$ then we assign the label 3 to the vertices $y_1, y_2, y_3, \cdots$. Clearly in this case the vertex and edge condition is given in $v_f(1) = v_f(2) = v_f(3) = 3n-2$ and $e_f(0) = 7n-5$ and $e_f(1) = 7n-6$.

Case 2. n is odd.

Assign the label to the vertices u_i $(1 \leq i \leq n)$, v_i $(1 \leq i \leq n-1)$ and w_i $(1 \leq i \leq n-1)$ as in case 1. Consider the vertices v'_i and v''_i. Assign the label 1 to the vertices $v'_1, v'_2, v'_3, v'_4, \cdots$. Then assign the label to the vertices v''_i $(1 \leq i \leq n-1)$ as in case 1. Now we move to the vertices w'_i and w''_i. Assign the label 1 to the vertices $w'_1, w'_3, w'_5, \cdots$ and we assign the label 3 to the vertices $w'_2, w'_4, w'_6, \cdots$. Next we assign the label to the vertices w''_i $(1 \leq i \leq n-1)$ as in case 1. Now we consider the vertices x_i and y_i. Assign the label 2 to the vertices $x_1, x_3, x_5, \cdots$ and we assign the label 1 to the vertices $x_2, x_4, x_6, \cdots$. Then we assign the label to the vertices y_i $(1 \leq i \leq n)$ as in case 1. Since $v_f(1) = v_f(2) = v_f(3) = 3n-2$ and $e_f(0) = 7n-6$ and $e_f(1) = 7n-5$, this labeling is 3-difference cordial labeling. □

A double quadrilateral snake DQ_n consists of two quadrilateral snakes that have a common path. Let $V(DQ_n) = \{u_i : 1 \leq i \leq n\} \bigcup \{v_i, w_i, xi, y_i : 1 \leq i \leq n-1\}$ and $E(DQ_n) = \{u_iu_{i+1}, v_iw_i, x_iy_i, w_iu_{i+1}, y_iu_{i+1} : 1 \leq i \leq n-1\}$.

Now we investigate the graphs $DQ_n \odot K_1$,$DQ_n \odot 2K_1$ and $DQ_n \odot K_2$.

Theorem 2.4 *$DQ_n \odot K_1$ is 3-difference cordial.*

Proof Let $V(DQ_n \odot K_1) = V(DQ_n) \bigcup \{u'_i : 1 \leq i \leq n\} \bigcup \{v'_i, w'_i, x'_i, y'_i : 1 \leq i \leq n-1\}$ and $E(DQ_n \odot K_1) = E(DQ_n) \bigcup \{u_i u'_i : 1 \leq i \leq n\} \bigcup \{u_i v'_i, w_i w'_i, x_i x'_i, y_i, y'_i : 1 \leq i \leq n-1\}$. Assign the label 1 to the path vertex u_1. Next we assign the labels 1,1,2 to the vertices u_2, u_3, u_4 respectively. Then we assign the labels 1,1,2 to the next three path vertices u_5, u_6, u_7 respectively. Proceeding like this we assign the label to the next three vertices and so on. If all the vertices are labeled then we stop the process. Otherwise there are some non labeled vertices are exist. If the number of non labeled vertices are less than or equal to 2 then assign the labels 1,1 to the non labeled vertices. If only one non labeled vertex exist then assign the label 1 only. Now we consider the vertices v_i and w_i. Assign the label 2 to the vertices v_{3i+1} and v_{3i+2} for all the values of i=0,1,2,3... For all the vales of $i = 1, 2, 3, \cdots$ assign the label 1 to the vertices v_{3i}. Then we assign the label 3 to the vertices w_i $(1 \leq i \leq n)$. Next we move to the vertices x_i and y_i. Assign the labels 2,3 to the vertices x_1 and y_1 respectively. Then we assign the label 2 to the vertices $x_2, x_5, x_8, \cdots$. Now we assign the label 1 to the vertices $x_3, x_6, x_9, \cdots$ and the vertices $x_4, x_7, x_{10}, \cdots$. Assign the label 3 to the vertices $y_1, y_2, y_3, \cdots$. We consider the vertices u'_i. Assign the labels 2,3 to the vertices u'_1 and u'_2 respectively. Now we assign the label 1 to the vertices $u'_3, u'_6, u'_9, \cdots$ and we assign the label 3 to the vertices $u'_4, u'_7, u'_{10}, \cdots$. Then we assign the label 2 to the vertices $u'_5, u'_8, u'_{11}, \cdots$. Next we move to the vertices v'_i and w'_i. Assign the the label 3 to the vertex w'_1. Now assign the label 1 to all the vertices of v'_i $(1 \leq i \leq n-1)$ and we assign the label 2 to the vertices $w'_2, w'_3, w'_4, \cdots$. We consider the vertices x'_i and y'_i. Assign the label 2,1 to the vertices x'_1 and y'_1 respectively. Also we assign the label 2 to the vertices $x'_2, x'_5, x'_8, \cdots$ and the vertices $x'_3, x'_6, x'_9, \cdots$. Then we assign the label 1 to the vertices $x'_4, x'_7, x'_{10}, \cdots$. Next we assign the label 3 to the vertices $y'_2, y'_3, y'_4, ...$ The vertex condition is $e_f(0) = 6n-5$ and $e_f(1) = 6n-6$. Also the edge condition is given in Table 1 following. $\square$

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod 3$	$\frac{10n-9}{3}$	$\frac{10n-6}{3}$	$\frac{10n-9}{3}$
$n \equiv 1 \pmod 3$	$\frac{10n-10}{3}$	$\frac{10n-7}{3}$	$\frac{10n-7}{3}$
$n \equiv 2 \pmod 3$	$\frac{10n-8}{3}$	$\frac{10n-8}{3}$	$\frac{10n-8}{3}$

Table 1

Theorem 2.5 *$DQ_n \odot 2K_1$ is 3-difference cordial.*

Proof Let $V(DQ_n \odot 2K_1) = V(DQ_n) \bigcup \{u'_i, u''_i : 1 \leq i \leq n\} \bigcup \{v'_i, v''_i, w'_i, w''_i, x'_i, x''_i, y'_i, y''_i : 1 \leq i \leq n-1\}$ and $E(DQ_n \odot 2K_1) = E(DQ_n) \bigcup \{u_i u'_i, u_i u''_i : 1 \leq i \leq n\} \bigcup \{v_i v'_i, v_i v''_i, w_i w'_i, w_i w''_i, x_i x'_i, x_i x''_i, y_i y''_i, y_i y''_i : 1 \leq i \leq n-1\}$. First we consider the path vertices u_i. Assign the label 1 to the path vertices $u_1, u_3, u_5, \cdots$ and we assign the label 2 to the path vertices $u_2, u_4, u_6, \cdots$. Clearly the last vertex u_n received the label 2 or 1 according as $n \equiv 0 \bmod 2$ or $n \equiv 1 \pmod 2$. Next we move to the vertices v_i and w_i. Assign the label 1 to all the vertices of v_i $(1 \leq i \leq n)$ and we assign the label 3 to the vertices $w_1, w_2, w_3, ...$ Then we assign the label to the vertices x_i $(1 \leq i \leq n-1)$ is same as assign the label to the vertices v_i $(1 \leq i \leq n-1)$ and we assign the label to the vertices y_i $(1 \leq i \leq n-1)$ is same as assign the label to the vertices w_i $(1 \leq i \leq n-1)$. Next we move to the vertices u'_i and v''_i. Assign the label 2 to the vertices

$v'_1, v'_2, v'_3, \cdots$ then we assign the label 3 to the vertex v''_1. Assign the label 3 to the vertices v''_{2i} for all the values of $i = 1, 2, 3, \cdots$ and we assign the label 2 to the vertices v''_{2i+1} for $i = 1, 2, 3, \cdots$. Next we consider the vertices w'_i and w''_i. Assign the label 1 to the vertices $w'_1, w'_2, w'_3, \cdots$ and we assign the label 3 to the vertices $w''_1, w''_2, w''_3, \cdots$. Next we move to the vertices x'_i and x''_i. Assign the label 1 to all the vertices of x'_i $(1 \leq i \leq n-1)$ and we assign the label 2 to all the vertices of x''_i $(1 \leq i \leq n-1)$. Now we assign the label 2 to the vertices $y'_1, y'_2, y'_3, \cdots$ and we assign the label 3 to the vertices $y''_1, y''_2, y''_3, \cdots$. Finally we move to the vertices u'_i and u''_i. Assign the label 2 to the vertices $u'_1, u'_3, u'_5, \cdots$ and we assign the label 1 to the vertices $u'_2, u'_4, u'_6, \cdots$. Next we assign the label 2 to the vertices $u''_1, u''_3, u''_5, \cdots$ and we assign the label 3 to the vertices $u''_2, u''_4, u''_6, \cdots$. The vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{15n-12}{3}$. Also the edge condition is given in Table 2. □

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 2$	$\frac{17n-16}{2}$	$\frac{17n-14}{2}$
$n \equiv 1 \pmod 2$	$\frac{17n-15}{2}$	$\frac{17n-15}{2}$

Table 2

Theorem 2.6 *$DQ_n \odot K_2$ is 3-difference cordial.*

Proof Let $V(DQ_n \odot K_2) = V(DQ_n) \bigcup \{u'_i, u''_i : 1 \leq i \leq n\} \bigcup \{v'_i, v''_i, w'_i, w''_i, x'_i, x''_i, y'_i, y''_i : 1 \leq i \leq n-1\}$ and $E(DQ_n \odot K_2) = E(DQ_n) \bigcup \{u_i u'_i, u_i u''_i, u'_i u''_i : 1 \leq i \leq n\} \bigcup \{v_i v'_i, v_i v''_i, v'_i v''_i, w_i w'_i, w_i w''_i, w'_i w''_i, x_i x'_i, x_i x''_i, x'_i x''_i, y_i y''_i, y_i y''_i, y'_i y''_i : 1 \leq i \leq n-1\}$. First we consider the path vertices u_i. Assign the label 1 to the vertex u_1. Then we assign the label 1 to the vertices $u_2, u_4, u_6, \cdots$ and we assign the label 2 to the path vertices $u_1, u_3, u_5, \cdots$. Note that in this case the last vertex u_n received the label 1or 2 according as $n \equiv 0 \pmod 2$ or $n \equiv 1 \pmod 2$. Next we move to the vertices v_i and w_i. Assign the label 2 to the vertex v_1. Then we assign the label 3 to all the vertices of w_i $(1 \leq i \leq n-1)$. Assign the label 1 to the vertices $v_2, v_3, v_4, ...$ We consider the vertices x_i and y_i. Assign the label to the vertices x_i $(1 \leq i \leq n-1)$ is same as assign the label to the vertices v_i $(1 \leq i \leq n-1)$ and assign the label to the vertices y_i $(1 \leq i \leq n-1)$ is same as assign the label to the vertices w_i $(1 \leq i \leq n-1)$. Next we move to the vertices v'_i and v''_i. Assign the label 2 to the vertices $v'_1, v'_2, v'_3, \cdots$ and assign the label 3 to the vertices $v''_1, v''_2, v''_3, \cdots$. Consider the vertices x'_i and x''_i. Assign the label 1 to all the vertices of x'_i $(1 \leq i \leq n-1)$. Assign the label 2 to the vertex x''_1. Then we assign the label 3 to the vertices $x''_2, x''_3, x''_4, \cdots$. Now we assign the label 2 to all the vertices of w'_i $(1 \leq i \leq n-1)$ and assign the label 3 to all the vertices of w''_i $(1 \leq i \leq n-1)$. Now we move to the vertices y'_i and y''_i. Assign the label 1 to the vertices $y'_1, y'_2, y'_3, \cdots$ and we assign the label 2 to the vertices $y''_1, y''_2, y''_3, \cdots$. Next we move to the vertices u'_i and u''_i. Assign the label 1,3 to the vertices u'_1 and u''_1 respectively. Assign the label 1 to the vertices u'_{2i} for all the values of $i = 1, 2, 3, \cdots$ and assign the label 2 to the vertices u_{2i+1} for $i = 1, 2, 3, \cdots$ then we assign the label 2 to the vertices $u''_2, u''_3, u''_4, \cdots$. The vertex and edge conditions are

$$v_f(1) = v_f(2) = v_f(3) = \frac{15n-12}{3}$$

and

$$e_f(0) = 11n - 9, \quad e_f(1) = 11n - 10. \qquad \square$$

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