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Minimum Dominating Color Energy of a Graph

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Abstract: In this paper, we introduce the concept of minimum dominating color energy of a graph, $E_c^D(G)$ and compute the minimum dominating color energy $E_c^D(G)$ of few families of graphs. Further, we establish the bounds for minimum dominating color energy.

Key Words: Minimum dominating set, Smarandachely dominating, minimum dominating color eigenvalues, minimum dominating color energy of a graph.

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§1. Introduction

Let G be a graph with n vertices $v_1, v_2, \cdots, v_n$ and m edges. Let $A = (a_{ij})$ be the adjacency matrix of the graph. The eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of A, assumed in non increasing order, are the eigenvalues of the graph G. As A is real symmetric, the eigenvalues of G are real with sum equal to zero. The energy $E(G)$ of G is defined to be the sum of the absolute values of the eigenvalues of G.

$$E(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{1}$$

The concept of graph energy originates from chemistry to estimate the total π-electron energy of a molecule. In chemistry the conjugated hydrocarbons can be represented by a graph called molecular graph. Here every carbon atom is represented by a vertex and every carbon-carbon bond by an edge and hydrogen atoms are ignored. The eigenvalues of the molecular graph represent the energy level of the electron in the molecule. An interesting quantity in Hückel theory is the sum of the energies of all the electrons in a molecule, the so called π-electron energy of a molecule.

Prof.Chandrashekara Adiga et al.[5] have defined color energy $E_c(G)$ of a graph G. Rajesh Khanna et al.[2] have defined the minimum dominating energy. Motivated by these two papers, we introduced the concept of minimum dominating color energy $E_c^D(G)$ of a graph G and computed minimum dominating chromatic energies of star graph, complete graph, crown graph, and cocktail party graphs. Upper and lower bounds for $E_c^D(G)$ are also established.

This paper is organized as follows. In section 3, we define minimum dominating color energy of a graph. In section 4,minimum dominating color spectrum and minimum dominating

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color energies are derived for some families of graphs. In section 5 Some properties of minimum dominating color energy of a graph are discussed. In section 6 bounds for minimum dominating color energy of a graph are obtained. section 7 consist some open problems.

§2. Minimum Dominating Energy of a Graph

Let G be a simple graph of order n with vertex set $V = v_1, v_2, v_3, \cdots, v_n$ and edge set E. A subset $D \subseteq V$ is a dominating set if D is a dominating set and every vertex of $V - D$ is adjacent to at least one vertex in D, and generally, for $\forall O \subset V$ with $\langle O \rangle_G$ isomorphic to a special graph, for instance a tree, a Smarandachely dominating set D_S on O of G is such a subset $D_S \subseteq V - O$ that every vertex of $V - D_S - O$ is adjacent to at least one vertex in D_S. Obviously, if $O = \emptyset$, D_S is nothing else but the usual dominating set of graph. Any dominating set with minimum cardinality is called a minimum dominating set. Let D be a minimum dominating set of a graph G. The minimum dominating matrix of G is the $n \times n$ matrix defined by $A_D(G) = (a_{ij})$ ([2]) where

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent,} \\ 1 & \text{if } i = j \text{ and } v_i \in D, \\ 0 & \text{otherwise} \end{cases}$$

The characteristic polynomial of $A_D(G)$ is denoted by $f_n(G, \lambda) = det(\lambda I - A_D(G))$. The minimum dominating eigenvalues of the graph G are the eigenvalues of $A_D(G)$.

Since $A_D(G)$ is real and symmetric, its eigenvalues are real numbers and are labelled in non-increasing order $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$ The minimum dominating energy of G is defined as

$$E_c^D(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{2}$$

§3. Coloring and Color Energy

A coloring of graph G is a coloring of its vertices such that no two adjacent vertices receive the same color. The minimum number of colors needed for coloring of a graph G is called chromatic number and is denoted by $\chi(G)$ ([19]).

Consider the vertex colored graph. Then entries of the matrix $A_c(G)$ are as follows ([5]):

If $c(v_i)$ is the color of v_i, then

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent with } c(v_i) \neq c(v_j), \\ -1 & \text{if } v_i \text{ and } v_j \text{ are non-adjacent with } c(v_i) = c(v_j), \\ 0 & \text{otherwise.} \end{cases}$$

The characteristic polynomial of $A_c(G)$ is denoted by $f_n(G, \rho) = det(\rho I - A_c(G))$. The color eigenvalues of the graph G are the eigenvalues of $A_c(G)$.

Since $A_c(G)$ is real and symmetric, its eigen values are real numbers and are labelled in non-increasing order $\rho_1 \geq \rho_2 \geq \rho_3 \geq \cdots \geq \rho_n$ The color energy of G is defined as

$$E_c(G) = \sum_{i=1}^{n} |\rho_i|. \tag{3}$$

§4. The Minimum Dominating Color Energy of a Graph

Let G be a simple graph of order n with vertex set $V = v_1, v_2, v_3, \cdots, v_n$ and edge set E. Let D be the minimum dominating set of a graph G. The minimum dominating color matrix of G is the $n \times n$ matrix defined by $A_c^D(G) = (a_{ij})$ where

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent with } c(v_i) \neq c(v_j) \text{ or if } i = j \text{ and } v_i \in D, \\ -1 & \text{if } v_i \text{ and } v_j \text{ are non adjacent with } c(v_i) = c(v_j), \\ 0 & \text{otherwise} \end{cases}$$

The characteristic polynomial of $A_c^D(G)$ is denoted by $f_n(G,\lambda) = det(\lambda I - A_c^D(G))$. The minimum dominating color eigenvalues of the graph G are the eigenvalues of $A_c^D(G)$.

Since $A_c^D(G)$ is real and symmetric, its eigenvalues are real numbers and are labelled in non-increasing order $\lambda_1 \geq \lambda_2 \geq \lambda_3 .... \geq \lambda_n$ The minimum dominating color energy of G is defined as

$$E_c^D(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{4}$$

If the color used is minimum then the energy is called minimum dominating chromatic energy and it is denoted by $E_\chi^D(G)$. Note that the trace of $A_c^D(G) = |D|$.

§5. Minimum Dominating Color Energy of Some Standard Graphs

Theorem 5.1 *If K_n is the complete graph with n vertices has $E_\chi^D(G)(K_n) = (n-2) + \sqrt{n^2 - 2n + 5}$.*

Proof Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$. The minimum dominating set $= D = \{v_1\}$.

$$A_c^D(K_n) = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{pmatrix}_{n \times n}.$$

Its characteristic polynomial is

$$[\lambda+1]^{n-2}[\lambda^2-(n-1)\lambda-1].$$

The minimum dominating color eigenvalues are

$$spec_D(K_n)=\begin{pmatrix} -1 & \frac{n-1+\sqrt{(n^2-2n+5)}}{2} & \frac{n-1-\sqrt{(n^2-2n+5)}}{2} \\ n-2 & 1 & 1 \end{pmatrix}.$$

The minimum dominating color energy for complete graph is

$$\begin{aligned} E_\chi^D(K_n) &= |-1|(n-2)+|\frac{(n-1)+\sqrt{(n^2-2n+5)}}{2}| \\ &\quad +|\frac{(n-1)-\sqrt{(n^2-2n+5)}}{2}| \\ &= (n-2)+\sqrt{(n^2-2n+3)}, \end{aligned}$$

i.e.,

$$E_\chi^D(G)(K_n)=(n-2)+\sqrt{(n^2-2n+5)}.$$ □

Definition 5.2 *The crown graph S_n^0 for an integer $n\geq 3$ is the graph with vertex set $\{u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_n\}$ and edge set $\{u_iv_i: 1\leq i,j\leq n, i\neq j\}$. S_n^0 is therefore equivalent to the complete bipartite graph $K_{n,n}$ with horizontal edges removed.*

Theorem 5.3 *If S_n^0 is a crown graph of order $2n$ then $E_\chi^D(S_n^0)=(2n-3)+\sqrt{(4n^2+4n-7)}$.*

Proof Let S_n^0 be a crown graph of order $2n$ with vertex set $\{u_1,u_2,\cdots,u_n,v_1,v_2,\cdots,v_n\}$ and minimum dominating set $=D=\{u_1,v_1\}$. Since $\chi(S_n^0)=2$, we have

$$A_\chi(S_n^0)=\begin{pmatrix} 1 & -1 & \cdots & -1 & -1 & 0 & 1 & \cdots & 1 & 1 \\ -1 & 0 & \cdots & -1 & -1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & \cdots & 0 & -1 & 1 & 1 & \cdots & 0 & 1 \\ -1 & -1 & \cdots & -1 & 0 & 1 & 1 & \cdots & 1 & 0 \\ 0 & 1 & \cdots & 1 & 1 & 1 & -1 & \cdots & -1 & -1 \\ 1 & 0 & \cdots & 1 & 1 & -1 & 0 & \cdots & -1 & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 0 & 1 & -1 & -1 & \cdots\ 0 & -1 & \\ 1 & 1 & \cdots & 1 & 0 & -1 & -1 & \cdots & -1 & 0 \end{pmatrix}_{2n\times 2n}.$$

Its characteristic polynomial is

$$\lambda^{n-1}[\lambda-1][\lambda-2]^{n-2}[\lambda^2+(2n-5)\lambda-(6n-8)]$$

and its minimum dominating color eigenvalues are

$$spec_{\chi}^{D}(S_n^0) = \begin{pmatrix} 0 & 1 & 2 & \frac{-(2n-5)+\sqrt{(4n^2+4n-7)}}{2} & \frac{-(2n-5)-\sqrt{(4n^2+4n-7)}}{2} \\ n-1 & 1 & n-2 & 1 & 1 \end{pmatrix}.$$

The minimum dominating color energy of S_n^0 is

$$\begin{aligned} E_{\chi}^{D}(S_n^0) &= |0|(n-1)+|1|(n-1)+|2|+|\frac{-(2n-5)+\sqrt{(4n^2+4n-7)}}{2}| \\ &\quad +|\frac{-(2n-5)-\sqrt{(4n^2+4n-7)}}{2}| \\ &= (2n-3)+\sqrt{4n^2+4n-7}, \end{aligned}$$

i.e.,

$$E_{\chi}^{D}(S_n^0) = (2n-3)+\sqrt{4n^2+4n-7}. \qquad \square$$

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Theorem 5.4 *If $K_{1,n-1}$ is a star graph of order n, then*

(i) $E_{\chi}(K_{1,n-1}) = \sqrt{5}$ *for* $n = 2$;

(ii) $E_{\chi}(K_{1,n-1}) = (n-2)+\sqrt{(n^2-2n+3)} for n \geq 3.$.

Proof Let $K_{1,n-1}$ be a colored graph on n vertices. Minimum dominating set is $D = \{v_0\}$. Then we have

$$A_{\chi}(K_{1,n-1}) \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & -1 & \cdots & -1 & -1 \\ 1 & -1 & 0 & \cdots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & -1 & -1 & \cdots & 0 & -1 \\ 1 & -1 & -1 & \cdots & -1 & 0 \end{pmatrix}_{n\times n}.$$

Case 1. The characteristic equation for $n = 2$ is $\lambda^2 - \lambda - 1 = 0$ and the minimum dominating color eigenvalues for $n = 2$ are $= \dfrac{1 \pm \sqrt{5}}{2}$. Whence, $E_{\chi}^{D}(K_{1,n-1}) = \sqrt{5}$.

Case 2. The characteristic equation for $n \geq 3$ is $(\lambda-1)^{n-2}(\lambda^2+(n-3)\lambda-(2n-3)) = 0$ and The minimum dominating color eigenvalues for $n \geq 3$ are

$$\begin{pmatrix} 1 & \frac{(n-3)+\sqrt{(n^2+2n-3)}}{2} & \frac{(n-3)-\sqrt{(n^2+2n-3)}}{2} \\ n-2 & 1 & 1 \end{pmatrix}.$$

Its minimum dominating color energy is

$$\begin{aligned} E_\chi^D(K_{1,n-1}) &= |1|(n-2) + |\frac{n-3+\sqrt{(n^2+2n-3)}}{2}| \\ &\quad +|\frac{n-3-\sqrt{(n^2+2n-3)}}{2}| \\ &= (n-2)+\sqrt{(n^2-2n+3)}. \end{aligned}$$

Therefore,

$$E_\chi^D(K_{1,n-1}) = (n-2)+\sqrt{(n^2-2n+3)}. \qquad \square$$

Definition 5.5 *The cocktail party graph, denoted by $K_{n\times 2}$, is graph having vertex set $V = \bigcup_{i=1}^{n}\{u_i, v_i\}$ and edge set $E = \{u_iu_j, v_iv_j, u_iv_j, v_iu_j : 1 \le i < j \le n\}$. This graph is also called as complete n-partite graph.*

Theorem 5.6 *If $K_{n\times 2}$ is a cocktail party graph of order $2n$, then $E_\chi^D(K_{n\times 2}) = (4n-5) + \sqrt{(4n^2-4n+9)}$.*

Proof Let $K_{n\times 2}$ be a cocktail party graph of order $2n$ with $V(K_{n\times 2}) = \{v_1, v_2, \cdots, v_n, u_1, u_2, \cdots, u_n\}$. The minimum dominating set $= D = \{u_1, v_1\}$. Then,

$$A_\chi^D(K_{n\times 2}) = \begin{pmatrix} 1 & -1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & \cdots & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & -1 & \cdots & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 0 & \cdots & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & \cdots & 0 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & \cdots & -1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & \cdots & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 & \cdots & 1 & 1 & -1 & 0 \end{pmatrix}_{2n\times 2n}.$$

Its characteristic equation is

$$[\lambda+3]^{n-2}[\lambda-1]^{n-1}[\lambda-2][\lambda^2-(2n-5)\lambda-4(n-1)]$$

with the minimum dominating color eigenvalues

$$Espec_\chi^D(K_n \times 2) = \begin{pmatrix} -3 & 1 & 2 & \frac{2n-5+\sqrt{(4n^2-4n+9)}}{2} & \frac{2n-5-\sqrt{(4n^2-4n+9)}}{2} \\ n-2 & n-1 & 1 & 1 & 1 \end{pmatrix}.$$

and the minimum dominating color energy,

$$\begin{aligned} E_\chi^D(K_n \times 2) &= |-3|(n-2)+1(n-1)+|2|+|\frac{2n-5+\sqrt{(4n^2-4n+9)}}{2}| \\ &\quad +|\frac{2n-5-\sqrt{(4n^2-4n+9)}}{2}| \\ &= (4n-5)+\sqrt{(4n^2-4n+9)}. \end{aligned}$$

This completes the proof. □

Definition 5.7 *The friendship graph, denoted by $F_3^{(n)}$, is the graph obtained by taking n copies of the cycle graph C_3 with a vertex in common.*

Theorem 5.8 *If $F_3^{(n)}$ is a friendship graph, then $E_\chi^D(F_3^{(n)}) = (3n-2)+\sqrt{(n^2+6n+1)}$.*

Proof Let $F_3^{(n)}$ be a friendship graph with $V(F_3^{(n)}) = \{v_0, v_1, v_2, \ldots, v_n\}$. The minimum dominating set $= D = \{v_3\}$. Then,

$$A_\chi^D(F_3^{(n)}) == \begin{pmatrix} 0 & 1 & 1 & -1 & \cdots & -1 & 0 \\ 1 & 0 & 1 & 0 & \cdots & 0 & -1 \\ 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ -1 & 0 & 1 & 0 & \cdots & -1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 1 & -1 & \cdots & 0 & 1 \\ 0 & -1 & 1 & 0 & \cdots & 1 & 0 \end{pmatrix}_{(2n+1)\times(2n+1)}.$$

Its characteristic equation is

$$\lambda^{n-1}[\lambda+n][\lambda-2]^{n-1}[\lambda^2+(n-3)\lambda+(2-3n)]$$

with the minimum dominating color eigenvalues

$$Espec_\chi^D(F_3^{(n)}) = \begin{pmatrix} -n & 0 & 2 & \frac{-(n-3)+\sqrt{(n^2+6n+1)}}{2} & \frac{-(n-3)-\sqrt{(n^2+6n+1)}}{2} \\ 1 & n-1 & n-1 & 1 & 1 \end{pmatrix}.$$

and the minimum dominating color energy

$$\begin{aligned} E_\chi^D(F_3^{(n)}) &= |-n|+0+|2|(n-1)+|\frac{-(n-3)+\sqrt{(n^2+6n+1)}}{2}| \\ &\quad +|\frac{-(n-3)-\sqrt{(n^2+6n+1)}}{2}| \\ &= (3n-2)+\sqrt{(n^2+6n+1)}. \end{aligned}$$

This completes the proof. □

§6. Properties of Minimum Dominating Color Energy of a Graph

Theorem 6.1 *Let* $|\lambda I - A_C^D| = a_0\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + ... + a_n$ *be the characteristic polynomial of* A_c^D*. Then*

(*i*) $a_0 = 1$;
(*ii*) $a_1 = -|D|$;
(*iii*) $a_2 = (|D|_2) - (m + m_c)$.

where m *is the number of edges and* m_c *is the number of pairs of non-adjacent vertices receiving the same color in* G*.*

Proof (*i*) It follows from the definition, $P_c(G, \lambda) := det(\lambda I - A_c(G))$, that $a_0 = 1$.

(*ii*) The sum of determinants of all 1×1 principal submatrices of A_c^D is equal to the trace of A_c^D, which $\Rightarrow a_1 = (-1)^1$ trace of $[A_c^D(G)] = -|E|$.

(*iii*) The sum of determinants of all the 2×2 principal submatrices of $[A_c^D]$ is

$$\begin{aligned} a_2 &= (-1)^2 \sum_{1\le i<j\le n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} = \sum_{1\le i<j\le n} (a_{ii}a_{jj} - a_{ij}a_{ji}) \\ &= \sum_{1\le i<j\le n} a_{ii}a_{jj} - \sum_{1\le i<j\le n} a_{ji}a_{ij} \\ &= (D|2) - (m + \text{number of pairs of non} - \text{adjacent vertices} \\ &\qquad \text{receiving the same color in G}) \\ &= (D|2) - (m + m_c). \qquad \square \end{aligned}$$

Theorem 6.2 *If* $\lambda_1, \lambda_2, \cdots, \lambda_n$ *are eigenvalues of* $A_c^D(G)$*, then* $\sum_{i=1}^{n} \lambda_i = |D|$ *and* $\sum_{i=1}^{n} \lambda_i^2 = |D| + 2(m + m_c)$*, where* m_c *is the number of pairs of non-adjacent vertices receiving the same color in* G*.*

§7. Open Problems

Problem 1. *Determine the class of graphs whose minimum dominating color energy of a graph is equal to number of vertices.*

Problem 2. *Determine the class of graphs whose minimum dominating color energy of a graph equal to usual energy.*

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