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A FORMULA OF THE SMARANDACHE FUNCTION

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Abstract. In this paper we give a formula expressing the Smarandache function S(n) by means of n without using the factorization of n.

For any positive integer n, let S(n) denote the Smarandache function of n. Then we have

(1)
$$S(n) = \min\{a \mid a \in N, n \mid a!\},\$$

(See [1]). In this paper we give a formula of S(n) without using the factorization of n as follows:

Theorem. For any positive integer n, we have

(1)
$$S(n) = n+1 - \begin{bmatrix} n & -(n \sin(k! \pi / n))^2 \\ \sum_{k=1}^n n \end{bmatrix}$$

Proof. Let a = S(n). It is an obvious fact that $1 \le a \le n$. We see from (1) that

(2)
$$n \mid k!, k = a, a+1, ..., n.$$

It implies that

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(4)
$$n = n^{0} = 1, \quad k = a, a+1, ..., n.$$

On the other hand, since $n \not k!$ for k = 1, ..., a-1, we have sin $(k!\pi/n) \neq 0$ and

(5)
$$(n \sin \frac{k! \pi}{n})^2 \ge (n \sin \frac{\pi}{n})^2 > 1, \quad k = 1, ..., a - 1.$$

Hense, by (5), we get

(6)
$$0 < n$$
 $(k! \pi/n)^2 < 1/n, k = 1, ..., a-1.$

Therefore, by (4) and (6), we obtain

(7)
$$n+1-a < \sum_{k=1}^{n} n (n \sin(k! \pi/n))^2 < n+1-a+(a-1)/n < n+2-a.$$

Thus, by (7), we get (1) immediately. The theorem is proved.

Reference

1. F Smarandache, A function in the number theory, Smarandache function J. 1 (1990), No.1, 3 - 17.