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Base Solution
(The Smarandache Function)

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Definition of the Smarandache function S(n)

S(n) = the smallest positive integer such that S(n)! is divisible by n.

Problem A: Ashbacher's problem

For what triplets n, n-1, n-2 does the Smarandache function satisfy the Fibonacci reccurrence: $S(n)=S(n-1)+S(n-2)$. Solutions have been found for n=11, 121, 4902, 26245, 32112, 64010, 368140 and 415664. Is there a pattern that would lead to the proof that there is an infinite family of solutions?

The next three triplets n, n-1, n-2 for which the Smarandache funtion S(n) satisfies the relation $S(n)=S(n-1)+S(n-2)$ occur for n=2091206, n=2519648 and n=4573053. Apart from the triplet obtained from n=26245 the triplets have in common that one member is 2 times a prime and the other two members are primes.

This leads to a search for triplets restricted to integers which meet the following requirements:

$$n = xp^a \text{ with } a \leq p+1 \text{ and } S(x) < ap \tag{1}$$

$$n-1 = yq^b \text{ with } b \leq q+1 \text{ and } S(y) < bq \tag{2}$$

$$n-2 = zr^c \text{ with } c \leq r+1 \text{ and } S(z) < cr \tag{3}$$

p,q and r are primes. With then have $S(n)=ap$, $S(n-1)=bq$ and $S(n-2)=cr$. From this and by subtracting (2) from (1) and (3) from (2) we get

$$ap = bq + cr \tag{4}$$

$$xp^a - yq^b = 1 \tag{5}$$

$$yq^b - zr^c = 1 \tag{6}$$

Each solution to (4) generates infinitely many solutions to (5) which can be written in the form:

$$x = x_0 + q^b t, \qquad y = y_0 - p^a t \tag{5'}$$

where t is an integer and (x_0,y_0) is the principal solution, which can be otained using Euclid's algorithm.

Solutions to (5') are substituted in (6') in order to obtain integer solutions for z.

$$z = (yq^b - 1)/r^c \tag{6'}$$

Implementation:
Solutions were generated for (a,b,c)=(2,1,1), (a,b,c)=(1,2,1) and (a,b,c)=(1,1,2) with the parameter t restricted to the interval $-9 \leq t \leq 10$. The output is presented on page 5. Since the correctness of these calculations are easily verfied from factorisations of S(n), S(n-1), and S(n-2) some of these are given in an annex. This study strongly indicates that the set of solutions is infinite.

Problem B: Radu's problem

Show that, except for a finite set of numbers, there exists at least one prime number between S(n) and S(n+1).

The immediate question is <u>what would be this finite set?</u> I order to examine this the following more stringent problem (which replaces "between" with the requirement that S(n) and S(n+1) must also be composite) will be considered.

Find the set of consecutive integers n and n+1 for which two consecutive primes p_k and p_{k+1} exists so that $p_k < Min(S(n),S(n+1))$ and $p_{k+1} > Max(S(n),S(n+1))$.

Consider

$$n+1 = xp_r^s$$

$$n = yp_{r+1}^s$$

where p_r and p_{r+1} are consecutive primes. Subtract

$$xp_r^s - yp_{r+1}^s = 1 \tag{1}$$

The greatest common divisor $(p_r^s, p_{r+1}^s) = 1$ divides the right hand side of (1) which is the condition for this diophantine equation to have infinitely many integer solutions. We are interested in positive integer solutions (x,y) such that the following conditions are met.

I. $S(n+1) = sp_r$, i.e $S(x) < sp_r$

II. $S(n) = sp_{r+1}$, i.e $S(y) < sp_{r+1}$

in addition we require that the interval

III. $sp_r^s < q < sp_{r+1}^s$ is prime free, i.e. q is not a prime.

Euclid's algorithm has been used to obtain principal solutions (x_0, y_0) to (1). The general set of solutions to (1) are then given by

$$x = x_0 + p_{r+1}^s t, \qquad y = y_0 - p_r^s t$$

with t an integer.

Implementation:
The above algorithms have been implemented for various values of the parameters $d = p_{r+1} - p_r$, s and t. A very large set of solutions was obtained. There is no indication that the set would be finite. A pair of primes may produce several solutions. Within the limits set by the design of the program the largest prime difference for which a solution was found is $d=42$ and the largest exponent which produced solutions is 4. Some numerically large examples illustrating the above facts are given on page 6.

Problem C: Stuparu's problem

Consider numbers written in Smarandache Prime Base 1,2,3,5,7,11,.... given the example that 101 in Smarandache base means $1 \cdot 3 + 0 \cdot 2 + 1 \cdot 1 = 4_{10}$.

As this leads to several ways to translate a base 10 number into a Base Smarandache number it seems that further precisions are needed. Example

$$111_{\text{Smarandache}} = 1\cdot 3+1\cdot 2+1\cdot 1 = 6_{10}$$

$$1001_{\text{Smarandache}} = 1\cdot 5+0\cdot 3+0\cdot 2+1\cdot 1 = 6_{10}$$

Equipment and programs

Computer programs for this study were written in UBASIC ver. 8.77. Extensive use was made of NXTPRM(x) and PRMDIV(n) which are very convenient although they also set an upper limit for the search routines designed in the main program. Programs were run on a dtk 486/33 computer. Further numerical outputs and program codes are available on request.