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Palindromic Numbers and Iterations of the Pseudo-Smarandache Function

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Abstract

For $n \geq 1$, the Pseudo-Smarandache function Z(n) is the smallest integer m such that n evenly divides $1 + 2 + 3 + \ldots + m$. In this paper, some iterations of this function on palindromes that yield palindromes are demonstrated.

This paper was originally published in **Proceedings of the First International Conference on Smarandache Type Notions in Number Theory**, American Research Press, 1997.
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In his delightful book[1], Kenichiro Kashara introduced the Pseudo-Smarandache function.

Definition: For any $n \geq 1$, the value of the Pseudo-Smarandache function Z(n) is the smallest integer m such that n evenly divides $1 + 2 + 3 + \ldots + m$. It is well known that the sum is equivalent to

$$\frac{m(m+1)}{2}.$$

Having been defined only recently, many of the properties of this function remain to be discovered. In this short paper, we will tentatively explore the connections between Z(n) and a subset of the integers known as the palindromic numbers.

Definition: A number is said to be a palindrome if it reads the same forward and backwards. Examples of palindromes are 121, 34566543 and 1111111111.

There are some palindromic numbers n such that Z(n) is also palindromic. For example,

Z(909) = 404 and Z(2222) = 1111.

In this paper, we will not consider the trivial cases of the single digit numbers.

A simple computer program was used to search for values of n satisfying the above criteria. The range of the search was, $10 \leq n \leq 10000$. Of the 189 palindromic values of n within that range, 37, or slightly over 19%, satisfied the criteria.

Furthermore, it is sometimes possible to repeat the function and get another palindrome.

Z(909) = 404, Z(404) = 303.

Once again, a computer program was run looking for values of n within the range $1 \leq n \leq 10000$. Of the 37 values found in the previous test, 9 or slightly over 24%, exhibited the property of repeated palindromes.

Using the program to test for values of n such that n, Z(n), Z(Z(n)) and Z(Z(Z(n))) are all palindromic, we discovered that of the 9 found in the previous test, 2 or roughly 22% satisfy the new criteria.

Definition: Let $Z^k(n) = Z(Z(Z(\ldots(n)\ldots)$ where the Z function is executed k times. For notational purposes, let $Z^0(n) = n$.

Modifying the program to search for solutions for a value of n so that n and all iterations $Z^k(n)$ are palindromic for k = 1, 2, 3 and 4, we found that there were no solutions in the range $1 \leq n \leq 10000$. Given the percentages already encountered, this should not be a surprise. In fact, by expanding the search up through 100000, one solution was found.

Z(86868) = 17271, Z(17271) = 2222, Z(2222) = 1111, Z(1111) = 505.

Since Z(505) = 100, this is the largest such sequence in this region.

Computer searches for larger such sequences can be more efficiently carried out by using only palindromic numbers for n.

Unsolved Question: What is the largest value of m so that for some $Z^k(n)$ is a palindrome for all k = 0, 1, 2, . . . , m?

Unsolved Question: Do the percentages discussed previously accurately represent the general case?

Of course, an affirmative answer to the second question would mean that there is no largest value of m.

Conjecture: There is no largest value of m such that for some n, $Z^k(n)$ is a palindrome for all k = 0, 1, 2, 3, . . . , m.

There are solid arguments in support of the truth of this conjecture. Palindromes tend to be divisible by palindromic numbers, so if we take a palindromic, many of the numbers that divide it would be palindromic. Furthermore, that palindrome is often the product of two numbers, one of which is a palindrome. Numbers like the repunits, 11 . . .11 and those with a small number of different digits, like 1001 and 505 appeared quite regularly in the computer search.

Reference

1. K. Kashihara, **Comments and Topics on Smarandache Notions and Problems**, Erhus University Press, Vail, AZ, 1996.