A Multi-Spaçe Model for Chinese Bids Evaluaţion with Analyzing

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Abstract. A tendering is a negotiating process for a contract through by a tenderer issuing an invitation, bidders submitting bidding documents and the tenderer accepting a bidding by sending out a notification of award. As a useful way of purchasing, there are many norms and rulers for it in the purchasing guides of the World Bank, the Asian Development Bank, $\cdots$, also in contract conditions of various consultant associations. In China, there is a law and regulation system for tendering and bidding. However, few works on the mathematical model of a tendering and its evaluation can be found in publication. The main purpose of this paper is to construct a Smarandache multi-space model for a tendering, establish an evaluation system for bidding based on those ideas in the references [7] and [8] and analyze its solution by applying the decision approach for multiple objectives and value engineering. Open problems for pseudo-multi-spaces are also presented in the final section.

Key Words: tendering, bidding, evaluation, Smarandache multi-space, condition of successful bidding, decision of multiple objectives, decision of simply objective, pseudo-multiple evaluation, pseudo-multi-space.

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§1. Introduction

The tendering is an efficient way for purchasing in the market economy. According to the *Contract Law of the People's Republic of China* (Adopted at the second meeting of the Standing Committee of the 9th National People's Congress on March 15,1999), it is just a civil business through by a tenderer issuing a tendering announcement or an invitation, bidders submitting bidding documents compiled on the tendering document and the tenderer accepting a bidding after evaluation by sending out a notification of award. The process of this business forms a negotiating process of a contract. In China, there is an interval time for the acceptation of a bidding and becoming effective of the contract, i.e., the bidding is accepted as the tenderer send out the notification of award, but the contract become effective only as the tenderer and the successful bidder both sign the contract.

In the *Tendering and Bidding Law of the People's Republic of China* (Adopted at the 11th meeting of the Standing Committee of the 9th National People's Congress on August 30,1999), the programming and liability or obligation of the tenderer, the bidders, the bid evaluation committee and the government administration are stipulated in detail step by step. According to this law, the tenderer is on the side of raising and formulating rulers for a tender project and the bidders are on the side of response each ruler of the tender. Although the bid evaluation committee is organized by the tenderer, its action is independent on the tenderer. In tendering and bidding law and regulations of China, it is said that any unit or person can not disturbs works of the bid evaluation committee illegally. The action of them should consistent with the tendering and bidding law of China and they should place themselves under the supervision of the government administration.

The role of each partner can be represented by a tetrahedron such as those shown in Fig.1.

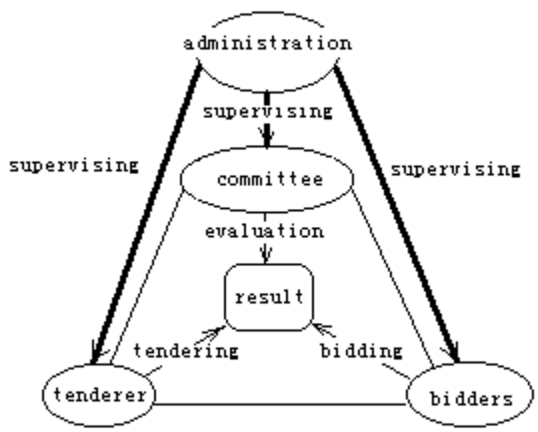

Fig.1

The 41th item in the *Tendering and Bidding Law of the People's Republic of China* provides conditions for a successful bidder:

(1) optimally responsive all of the comprehensive criterions in the tendering document;

(2) substantially responsive criterions in the tender document with the lowest evaluated bidding price unless it is lower than this bidder's cost.

The conditions (1) and (2) are often called the *comprehensive evaluation method* and the *lowest evaluated price method*. In the same time, these conditions also imply that the tendering system in China is a multiple objective system, not only evaluating in the price, but also in the equipments, experiences, achievements, staff and the programme, etc.. However, nearly all the encountered evaluation methods in China do not apply the scientific decision of multiple objectives. In where, the *comprehensive evaluation method* is simply replaced by the *100 marks* and the *lowest evaluated price method* by the *lowest bidding price method*. Regardless of whether different objectives being comparable, there also exist problems for the ability of bidders and specialists in the bid evaluation committee creating a false impression for the successful bidding price or the successful bidder. The tendering and bidding is badly in need of establishing a scientific evaluation system in accordance with these laws and regulations in China. Based on the reference [7] for Smarandache multi-spaces and the mathematical model for the tendering in [8], the main purpose of this paper is to establish a multi-space model for the tendering and a scientific evaluation system for bids by applying the approach in the multiple objectives and value engineering, which enables us to find a scientific approach for tendering and its management in practice. Some cases are also presented in this paper.

The terminology and notations are standard in this paper. For terminology and notation not defined in this paper can be seen in [7] for multi-spaces, in $[1]-[3]$ and [6] for programming, decision and graphs and in [8] for the tendering and bidding laws and regulations in China.

§2. A multi-space model for tendering

Under an idea of anti-thought or paradox for mathematics :*combining different fields into a unifying field*, Smarandache introduced the conception of multi-spaces in 1969([9]-[12]), including algebraic multi-spaces and multi-metric spaces. The contains the well-known Smarandache geometries($[5]-[6]$), which can be used to *General Relativity* and *Cosmological Physics*([7]). As an application to *Social Sciences*, multi-spaces can be also used to establish a mathematical model for tendering.

These algebraic multi-spaces are defined in the following definition.

Definition 2.1 *An algebraic multi-space $\sum$ with multiple m is a union of m sets $A_1, A_2, \cdots, A_m$*

$$\sum = \bigcup_{i=1}^{m} A_i,$$

where $1 \leq m < +\infty$ and there is an operation or ruler $\circ_i$ on each set A_i such that $(A_i \circ_i)$ is an algebraic system for any integer $i, 1 \leq i \leq m$.

Notice that if $i \neq j, 1 \leq i, j \leq m$, there must not be $A_i \bigcap A_j = \emptyset$, which are just correspondent with the characteristics of a tendering. Thereby, we can construct a Smarandache multi-space model for a tendering as follows.

Assume there are m evaluation items $A_1, A_2, \cdots, A_m$ for a tendering $\widetilde{A}$ and there are n_i evaluation indexes $a_{i1}, a_{i2}, \cdots, a_{in_i}$ for each evaluation item $A_i, 1 \leq i \leq m$. By applying mathematics, this tendering can be represented by

$$\widetilde{A} = \bigcup_{i=1}^{m} A_i,$$

where, for any integer $i, 1 \leq i \leq m$,

$$(A_i, \circ_i) = \{a_{i1}, a_{i2}, \cdots, a_{in_i} | \circ_i\}$$

is an algebraic system. Notice that we do not define other relations of the tendering $\widetilde{A}$ and evaluation indexes a_{ij} with $A_i, 1 \leq i \leq m$ unless $A_i \subseteq \widetilde{A}$ and $a_{ij} \in A_i$ in this multi-space model.

Now assume there are $k, k \geq 3$ bidders $R_1, R_2, \cdots, R_k$ in the tendering $\widetilde{A}$ and the bidding of bidder $R_j, 1 \leq j \leq k$ is

$$R_j(\widetilde{A}) = R_j \left(\begin{array}{c} A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \right) = \left(\begin{array}{l} R_j(A_1) \\ R_j(A_2) \\ \cdots \\ R_j(A_m) \end{array} \right)$$

.

According to the successful bidding criterion in the *Tendering and Bidding Law of the People's Republic of China* and regulations, the bid evaluation committee needs to determine indexes $i_1, i_2, \cdots, i_k$, where $\{i_1, i_2, \cdots, i_k\} = \{1, 2, \cdots, k\}$ such that there is an ordered sequence

$$R_{i_1}(\widetilde{A}) \succ R_{i_2}(\widetilde{A}) \succ \cdots \succ R_{i_k}(\widetilde{A})$$

for these bidding $R_1(\widetilde{A}), R_2(\widetilde{A}), \cdots, R_k(\widetilde{A})$ of bidders $R_1, R_2, \cdots, R_k$. Here, these bidders R_{i_1}, R_{i_2} and R_{i_3} are *pre-successful bidders* in succession determined by the bid evaluation committee in the laws and regulations in China.

Definition 2.2 *An ordered sequence for elements in the symmetry group S_n on $\{1, 2, \cdots, m\}$ is said an alphabetical sequence if it is arranged by the following criterions:*

(i) $(1, 0 \cdots, 0) \succeq P$ for any permutation $P \in S_n$.

(ii) if integers $s_1, s_2, \cdots, s_h \in \{1, 2, \cdots, m\}, 1 \leq h < m$ and permutations $(s_1, s_2, \cdots, s_h, t, \cdots), (s_1, s_2, \cdots, s_h, l, \cdots) \in S_n$, then

$$(s_1, s_2, \cdots, s_h, t, \cdots) \succ (s_1, s_2, \cdots, s_h, l, \cdots)$$

if and only if $t < l$. Let $\{x_{\sigma_i}\}_1^n$ be a sequence, where $\sigma_1 \succ \sigma_2 \succ \cdots \succ \sigma_n$ and $\sigma_i \in S_n$ for $1 \leq i \leq n$, then the sequence $\{x_{\sigma_i}\}_1^n$ is said an alphabetical sequence.

Now if $x_\sigma \succ x_\tau$, x_σ is preferable than x_τ in order. If $x_\sigma \succeq x_\tau$, then x_σ is preferable or equal with x_τ in order. If $x_\sigma \succeq x_\tau$ and $x_\tau \succeq x_\sigma$, then x_σ is equal x_τ in order, denoted by $x_\sigma \approx x_\tau$.

We get the following result for an evaluation of a tendering.

Theorem 2.1 *Let $O_1, O_2, O_3 \cdots$ be ordered sets. If $R_j(\widetilde{A}) \in O_1 \times O_2 \times O_3 \times \cdots$ for any integer $j, 1 \leq j \leq k$, then there exists an arrangement $i_1, i_2, \cdots, i_k$ for indexes $1, 2, \cdots, k$ such that*

$$R_{i_1}(\widetilde{A}) \succeq R_{i_2}(\widetilde{A}) \succeq \cdots \succeq R_{i_k}(\widetilde{A}).$$

Proof By the assumption, for any integer $j, 1 \leq j \leq k$,

$$R_j(\widetilde{A}) \in O_1 \times O_2 \times O_3 \times \cdots.$$

Whence, $R_j(\widetilde{A})$ can be represented by

$$R_j(\widetilde{A}) = (x_{j1}, x_{j2}, x_{j3}, \cdots),$$

where $x_{jt} \in O_t, t \geq 1$. Define a set

$$S_t = \{x_{jt}; 1 \leq j \leq m\}.$$

Then the set $S_t \subseteq O_t$ is finite. Because the set O_t is an ordered set, so there exists an order for elements in S_t. Not loss of generality, assume the order is

$$x_{1t} \succeq x_{2t} \succeq \cdots \succeq x_{mt},$$

for elements in S_t. Then we can apply the alphabetical approach to $R_{i_1}(\widetilde{A}), R_{i_2}(\widetilde{A})$, $\cdots, R_{i_k}(\widetilde{A})$ and get indexes $i_1, i_2, \cdots, i_k$ such that

$$R_{i_1}(\widetilde{A}) \succeq R_{i_2}(\widetilde{A}) \succeq \cdots \succeq R_{i_k}(\widetilde{A}). \quad \natural$$

If we choose $O_i, i \geq 1$ to be an ordered function set in Theorem 2.1, particularly, let $O_1 = \{f\}, f : A_i \to R, 1 \leq i \leq m$ be a monotone function set and $O_t = \emptyset$ for $t \geq 2$, then we get the next result.

Theorem 2.2 *Let $R_j : A_i \to R, 1 \leq i \leq m, 1 \leq j \leq k$ be monotone functions. Then there exists an arrangement $i_1, i_2, \cdots, i_k$ for indexes $1, 2, \cdots, k$ such that*

$$R_{i_1}(\widetilde{A}) \succeq R_{i_2}(\widetilde{A}) \succeq \cdots \succeq R_{i_k}(\widetilde{A}).$$

We also get the following consequence for evaluation numbers by Theorem 2.2.

Corollary 2.1 *If $R_j(A_i) \in [-\infty, +\infty] \times [-\infty, +\infty] \times [-\infty, +\infty] \times \cdots$ for any integers $i, j, 1 \leq i \leq m, 1 \leq j \leq k$, then there exists an arrangement $i_1, i_2, \cdots, i_k$ for indexes $1, 2, \cdots, k$ such that*

$$R_{i_1}(\widetilde{A}) \succeq R_{i_2}(\widetilde{A}) \succeq \cdots \succeq R_{i_k}(\widetilde{A}).$$

Notice that in the above ordered sequence, if we arrange $R_{i_s} \succ R_{i_l}$ or $R_{i_l} \succ R_{i_s}$ further in the case of $R_{i_s} \approx R_{i_l}, s \neq l$, then we can get an ordered sequence

$$R_{i_1}(\widetilde{A}) \succ R_{i_2}(\widetilde{A}) \succ \cdots \succ R_{i_k}(\widetilde{A}),$$

and the pre-successful bidders accordance with the laws and regulations in China.

§3. A mathematical analog for bids evaluation

For constructing an evaluation system of bids by the multi-space of tendering, the following two problems should be solved in the first.

Problem 1 *For any integers* $i, j, 1 \le i, j \le m$*, how to determine* $R_j(A_i)$ *on account of the responsiveness of a bidder* R_j *on indexes* $a_{i1}, a_{i2}, \cdots, a_{in_i}$*?*
Problem 2 *For any integer* $j, 1 \le j \le m$*, how to determine* $R_j(\widetilde{A})$ *on account of the vector* $(R_j(A_1), R_j(A_2), \cdots, R_j(A_m))^t$*?*

Different approaches for solving Problems 1 and 2 enable us to get different mathematical analogs for bids evaluation.

3.1. **An approach of multiple objectives decision**

This approach is originated at the assumption that $R_j(A_1), R_j(A_2), \cdots, R_j(A_m), 1 \le j \le m$ are independent and can not compare under a unified value unit. The objectives of tendering is multiple, not only in the price, but also in the equipments, experiences, achievements, staff and the programme, etc., which are also required by the 41th item in the *Tendering and Bidding Law of the People's Republic of China*.

According to Theorems $2.1 - 2.2$ and their inference, we can establish a programming for arranging the order of each evaluation item $A_i, 1 \le i \le m$ and getting an ordered sequence of bids $R_1(\widetilde{A}), R_2(\widetilde{A}), \cdots, R_k(\widetilde{A})$ of a tendering $\widetilde{A} = \bigcup\limits_{i=1}^{m} A_i$, as follows:

STEP 1 *determine the order of the evaluation items* $A_1, A_2, \cdots, A_m$*. For example, for* $m = 5$*,* $A_1 \succ A_2 \approx A_3 \succ A_4 \approx A_5$ *is an order of the evaluation items* A_1, A_2, A_3, A_4, A_5*.*

STEP 2 *for two bids* $R_{j_1}(A_i), R_{j_2}(A_i), j_1 \neq j_2, 1 \le i \le m$*, determine the condition for* $R_{j_1}(A_i) \approx A_{j_2}(A_2)$*. For example, let* A_1 *be the bidding price. Then* $R_{j_1}(A_1) \approx R_{j_2}(A_1)$ *providing* $|R_{j_1}(A) - R_{j_2}(A_1)| \le 100$*(10 thousand yuan).*

STEP 3 *for any integer* $i, 1 \le i \le m$*, determine the order of* $R_1(A_i), R_2(A_i), \cdots, R_k(A_i)$*. For example, arrange the order of bidding price from lower to higher and the bidding programming dependent on the evaluation committee.*

STEP 4 *alphabetically arrange* $R_1(\widetilde{A}), R_2(\widetilde{A}), \cdots, R_k(\widetilde{A})$*, which need an approach for arranging equal bids* $R_{j_1}(\widetilde{A}) \approx R_{j_2}(\widetilde{A})$ *in order. For example, arrange them by the ruler of lower price preferable and get an ordered sequence*

$$R_{i_1}(\widetilde{A}) \succ R_{i_2}(\widetilde{A}) \succ \cdots \succ R_{i_k}(\widetilde{A})$$

of these bids $R_1(\widetilde{A}), R_2(\widetilde{A}), \cdots, R_k(\widetilde{A})$*.*

Notice that we can also get an ordered sequence through by defining the weight functions

$$\omega(\widetilde{A}) = H(\omega(A_1), \omega(A_2), \cdots, \omega(A_m))$$

and

$$\omega(A_i) = F(\omega(a_{i1}), \omega(a_{i2}), \cdots, \omega(a_{in_i})).$$

For the weight function in detail, see the next section.

Theorem 3.1 *The ordered sequence of bids of a tendering $\widetilde{A}$ can be gotten by the above programming.*

Proof Assume there are k bidders in this tendering. Then we can alphabetically arrange these bids $R_{i_1}(\widetilde{A}), R_{i_2}(\widetilde{A}), \cdots, R_{i_k}(\widetilde{A})$ and get

$$R_{i_1}(\widetilde{A}) \succeq R_{i_2}(\widetilde{A}) \succeq \cdots \succeq R_{i_k}(\widetilde{A}).$$

Now applying the arranging approach in the case of $R_{j_1}(\widetilde{A}) \approx R_{j_2}(\widetilde{A})$, we finally obtain an ordered sequence

$$R_{i_1}(\widetilde{A}) \succ R_{i_2}(\widetilde{A}) \succ \cdots \succ R_{i_k}(\widetilde{A}). \quad \natural$$

Example 3.1 *There are* 3 *evaluation items in a building construction tendering $\widetilde{A}$ with A_1 =price, A_2=programming and A_3=similar achievements in nearly* 3 *years. The order of the evaluation items is $A_1 \succ A_3 \succ A_2$ and $R_{j_1}(A_i) \approx R_{j_2}(A_i), 1 \leq i \leq 3$ providing $|R_{j_1}(A_1) - R_{j_2}(A_1)| \leq 150$, $R_{j_1}(A_2)$ and $R_{j_2}(A_2)$ are in the same rank or the difference of architectural area between $R_{j_1}(A_3)$ and $R_{j_2}(A_3)$ is not more than* $40000m^2$. *For determining the order of bids for each evaluation item, it applies the rulers that from the lower to the higher for the price, from higher rank to a lower rank for the programming by the bid evaluation committee and from great to small amount for the similar achievements in nearly* 3 *years and arrange $R_{j_1}(\widetilde{A}), R_{j_2}(\widetilde{A})$, $1 \leq j_1, j_2 \leq k$ =bidders by the ruler of lower price first for two equal bids in order $R_{j_1}(\widetilde{A}) \approx R_{j_2}(\widetilde{A})$.*

There were 4 *bidders R_1, R_2, R_3, R_4 in this tendering. Their bidding prices are in table* 1.

bidder	R_1	R_2	R_3	R_4
A_1	3526	3166	3280	3486

table 1

Applying the arrangement ruler for A_1, the order for $R_2(A_1), R_3(A_1), R_4(A_1)$, $R_1(A_1)$ is

$$R_2(A_1) \approx R_3(A_1) \succ R_4(A_1) \approx R_1(A_1).$$

The evaluation order for A_2 by the bid evaluation committee is $R_3(A_2) \approx R_2(A_2) \succ R_1(A_2) \succ R_4(A_2)$. They also found the bidding results for A_3 are in table 2.

bidder	R_1	R_2	R_3	R_4
$A_3(m^2)$	250806	210208	290108	300105

table 2

Whence the order of $R_4(A_3), R_3(A_3), R_1(A_3), R_2(A_3)$ is

$$R_4(A_3) \approx R_3(A_3) \succ R_1(A_3) \approx R_2(A_3).$$

Therefore, the ordered sequence for these bids $R_1(\widetilde{A}), R_2(\widetilde{A}), R_3(\widetilde{A})$ *and* $R_4(\widetilde{A})$ *is*

$$R_3(\widetilde{A}) \succ R_2(\widetilde{A}) \succ R_4(\widetilde{A}) \succ R_1(\widetilde{A}).$$

Let the order of evaluation items be $A_1 \succ A_2 \succ \cdots \succ A_m$. Then we can also get the ordered sequence of a tendering by applying a graphic method. By the terminology in graph theory, to arrange these bids of a tendering is equivalent to find a directed path passing through all bidders $R_1, R_2, \cdots, R_k$ in a graph $G[\widetilde{A}]$ defined in the next definition. Generally, the graphic method is more convenience in the case of less bidders, for instance 7 bidders for a building construction tendering in China.

Definition 3.1 *Let* $R_1, R_2, \cdots, R_k$ *be all these* k *bidders in a tendering* $\widetilde{A} = \bigcup\limits_{i=1}^{m} A_i$. *Define a directed graph* $G[\widetilde{A}] = (V(G[\widetilde{A}]), E(G[\widetilde{A}]))$ *as follows.*

$V(G[\widetilde{A}]) = \{R_1, R_2, \cdots, R_k\} \times \{A_1, A_2, \cdots, A_m\}$,

$E(G[\widetilde{A}]) = E_1 \bigcup E_2 \bigcup E_3$.

Where E_1 *consists of all these directed edges* $(R_{j_1}(A_i), R_{j_2}(A_i))$, $1 \leq i \leq m, 1 \leq j_1, j_2 \leq k$ *and* $R_{j_1}(A_i) \succ R_{j_2}(A_i)$ *is an adjacent order. Notice that if* $R_s(A_i) \approx R_l(A_i) \succ R_j(A_i)$, *then there are* $R_s(A_i) \succ R_j(A_i)$ *and* $R_l(A_i) \succ R_j(A_i)$ *simultaneously.* E_2 *consists of edges* $R_{j_1}(A_i)R_{j_2}(A_i), 1 \leq i \leq m, 1 \leq j_1, j_2 \leq k$, *where* $R_{j_1}(A_i) \approx R_{j_2}(A_i)$ *and* $E_3 = \{R_j(A_i)R_j(A_{i+1}) | 1 \leq i \leq m-1, 1 \leq j \leq k\}$.

For example, the graph $G[\widetilde{A}]$ for Example 3.1 is shown in Fig.2.

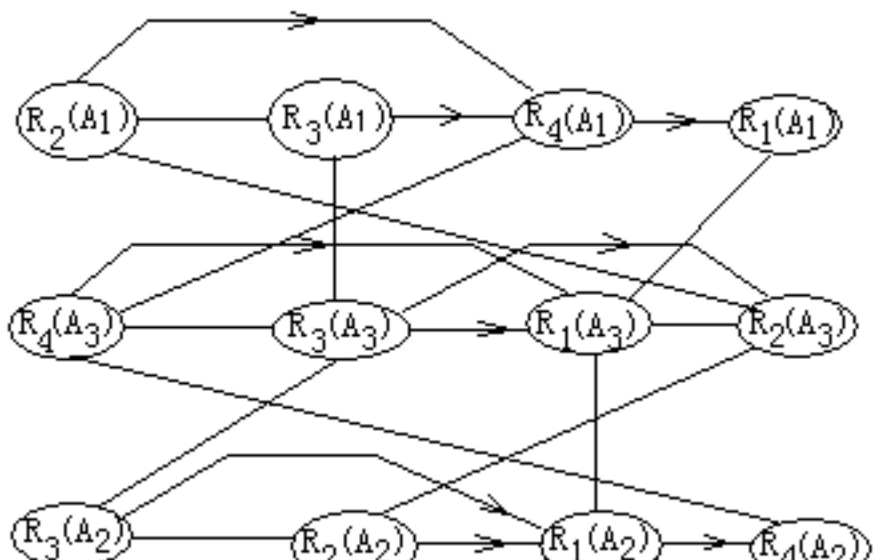

Fig.2

Now we need to find a directed path passing through R_1, R_2, R_3, R_4 with start vertex $R_2(A_1)$ or $R_3(A_1)$. By the ruler in an alphabetical order, we should travel starting from the vertex $R_3(A_1)$ passing through A_2, A_3 and then arriving at A_1. Whence, we find a direct path correspondent with the ordered sequence

$$R_3(\widetilde{A}) \succ R_2(\widetilde{A}) \succ R_4(\widetilde{A}) \succ R_1(\widetilde{A}).$$

3.2. **An approach of simply objective decision**

This approach is established under the following considerations for Problems 1 and 2.

Consideration 1 *In these evaluation items $A_1, A_2, \cdots, A_m$ of a tendering $\widetilde{A}$, seek the optimum of one evaluation item. For example, seek the lowest bidding price in a construction tendering for a simply building or seek the optimum of design scheme in a design project tendering, etc..*
Consideration 2 *The value of these evaluation items $A_1, A_2, \cdots, A_m$ is comparable which enables us to measure each of them by a unify unit and to construct various weighted functions on them. For example, the 100 marks and the lowest evaluated price method widely used in China are used under this consideration.*

3.2.1. The optimum of one objective

Assume the optimal objective being A_1 in a tendering $\widetilde{A} = \bigcup\limits_{i=1}^{m} A_i$. We need to determine the acceptable basic criterions for all other items $A_2, A_3, \cdots, A_k$, then arrange $R_1(A_1), R_2(A_1), \cdots, R_l(A_1)$ among these acceptable bids $R_1, R_2, \cdots, R_l$ for items $A_2, A_3, \cdots, A_k$ in $R_i, 1 \leq i \leq k$. For example, evaluating these items $A_2, A_3, \cdots, A_k$ by qualification or by weighted function on $A_2, A_3, \cdots, A_k$ up to these criterions, then arrange these acceptable bids $R_1, R_2, \cdots, R_l$ under their response to A_1 and the order of $R_i(\widetilde{A})$, $R_i(\widetilde{A})$ if $R_i(A_1) \approx R_j(A_1)$. According to Theorem 3.1, we get the following result.

Theorem 3.2 *The approach of one optimal objective can get an ordered sequence of bids for a tendering $\widetilde{A}$.*

Example 3.2 *The optimum of design scheme is the objective in a design project tendering $\widetilde{A}$ which is divided into* 5 *ranks A, B, C, D, E and other evaluation items such as human resources, design period and bidding price by a qualifiable approach if the bidding price is in the interval of the service fee norm of China. The final order of bids is determined by the order of design schemes with qualifiable human resources, design period and bidding price and applying the ruler of lower price first for two equal design scheme in order.*

There were 8 *bidders in this tendering. Their bidding prices are in table* 3.

bidder	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
bidding price	251	304	268	265	272	283	278	296

table 3

After evaluation for these human resources, design period and bidding price, 4 bidders are qualifiable unless the bidder R_5 in human resources. The evaluation result for bidding design schemes is in table 4.

rank	*A*	*B*	*C*	*D*	*E*
design scheme	R_3R_6	R_1	R_2R_8	R_7	R_4

table 4

Therefore, the ordered sequence for bids is

$$R_3(A) \widetilde{\succ R_6}(\widetilde{A}) \succ R_1(\widetilde{A}) \succ R_8(\widetilde{A}) \succ R_2(\widetilde{A}) \succ R_7(\widetilde{A}) \succ R_4(\widetilde{A}).$$

Example 3.3 *The optimum objective in a tendering $\widetilde{A}$ for a construction of a dwelling house is the bidding price A_1. All other evaluation items, such as qualifications, management persons and equipments is evaluated by a qualifiable approach.*

There were 7 bidders $R_i, 1 \leq i \leq 7$ in this tendering. The evaluation of price is by a weighted function approach, i.e., determine the standard price S first, then calculate the mark N of each bidder by the following formulae

$$S = \frac{(\sum_{i=1}^{7} A_i - \max\{R_i(A_1)|1 \leq i \leq 7\} - \min\{R_i(A_1)|1 \leq i \leq 7\}}{5},$$

$$N_i = 100 - t \times |\frac{R_i(A_1) - S}{S}| \times 100, \quad 1 \leq i \leq 7,$$

where, if $R_i(A_1) - S > 0$ then $t = 6$ and if $R_i(A_1) - S < 0$ then $t = 3$.

After evaluation, all bidders are qualifiable in qualifications, management persons and equipments. Their bidding prices are in table 5.

bidder	R_1	R_2	R_3	R_4	R_5	R_6	R_7
A_1	3518	3448	3682	3652	3490	3731	3436

table 5

According to these formulae, we get that $S = 3558$ and the mark of each bidder as those shown in table 6.

bidder	R_1	R_2	R_3	R_4	R_5	R_6	R_7
mark	96.70	91.27	79.12	84.16	94.27	73.84	89.68

table 6

Therefore, the ordered sequence of bids is

$$R_1(\widetilde{A}) \succ R_5(\widetilde{A}) \succ R_2(\widetilde{A}) \succ R_7(\widetilde{A}) \succ R_4(\widetilde{A}) \succ R_3(\widetilde{A}) \succ R_6(\widetilde{A}).$$

3.2.2. The pseudo-optimum of multiple objectives

This approach assumes that there is a unifying unit between these evaluation items $A_1, A_2, \cdots, A_m$ in an interval $[a, b]$. Whence it can be transformed into case 3.2.1 and sought the optimum of one objective. Not loss of generality, we assume the unifying unit is ϖ and

$$\varpi(A_i) = f_i(\varpi), \quad 1 \leq i \leq m,$$

where f_i denotes the functional relation of the metric $\varpi(A_i)$ with unit ϖ. Now the objective of tendering turns to a programming of one objective

$$\max_{\varpi} F(f_1(\varpi), f_2(\varpi), \cdots, f_m(\varpi)) \text{ or } \min_{\varpi} F(f_1(\varpi), f_2(\varpi), \cdots, f_m(\varpi)),$$

where F denotes the functional relation of the tendering $\widetilde{A}$ with these evaluation items $A_1, A_2, \cdots, A_m$, which can be a weighted function, such as a linear function

$$F(f_1(\varpi), f_2(\varpi), \cdots, f_m(\varpi)) = \sum_{i=1}^{m} f_i(\varpi)$$

or an ordered sequence. According to Theorem 3.2, we know the following result.

Theorem 3.3 *If the function F of a tendering $\widetilde{A}$ only has one maximum value in $[a, b]$, then there exists an ordered sequence for these bids $R_i(\widetilde{A}), 1 \leq i \leq k$ after determined how to arrange $R_i(\widetilde{A})$ and $R_j(\widetilde{A})$ when $F(R_i(\widetilde{A})) = F(R_j(\widetilde{A})), i \neq j$.*

The 100 marks and the lowest evaluated price method widely used in China both are applications of this approach. In the 100 marks, the weight function is a linear function

$$F(f_1(\varpi), f_2(\varpi), \cdots, f_m(\varpi)) = \sum_{i=1}^{m} f_i(\varpi)$$

with $0 \leq F(f_1(\varpi), f_2(\varpi), \cdots, f_m(\varpi)) \leq 100, f_i \geq 0, 1 \leq i \leq m$. In the lowest evaluated price method, each difference of an evaluation item $A_i, 2 \leq i \leq m$ is changed to the bidding price $\varpi(A_1)$, i.e.,

$$f_i = (R(A_i) - S(A_i))\varpi(A_1), \ 1 \leq i \leq m,$$

where $S(A_i)$ is the standard line for A_i, $\varpi(A_i)$ is one unit difference of A_i in terms of A_1. The weighted function of the lowest evaluated price method is

$$F(\varpi(A_1), f_2(\varpi(A_1)), \cdots, f_m(\varpi(A_1))) = (1 + \sum_{j=2}^{m}(R(A_i) - S(A_i)))\varpi(A_1).$$

For example, we can fix one unit difference of a technological parameter 15, i.e., $\varpi(A_1) = 15$ ten thousand dollars in terms of the bidding price.

§4. **Weighted functions and their construction**

We discuss weighted functions on the evaluation items or indexes in this section. First, we give a formal definition for weighted functions.

Definition 4.1 *For a tendering $\widetilde{A} = \bigcup_{i=1}^{m} A_i$, where $A_i = \{a_{i1}, a_{i2}, \cdots, a_{in}\}, 1 \le i \le m$ with k bidders $R_1, R_2, \cdots, R_k$, if there is a continuous function $\omega : \widetilde{A} \to [a, b] \subset (-\infty, +\infty)$ or $\omega : A_i \to [a, b] \subset (-\infty, +\infty), 1 \le i \le m$ such that for any integers $l, s, 1 \le l, s \le k$, $R_l(\omega(\widetilde{A})) > R_s(\omega(\widetilde{A}))$ or $R_l(\omega(\widetilde{A})) = R_s(\omega(\widetilde{A}))$ as $R_l(\widetilde{A}) \succ R_s(\widetilde{A})$ or $R_l(\widetilde{A}) \approx R_s(\widetilde{A})$ and $R_l(\omega(A_i) > R_s(\omega(A_i))$ or $R_l(\omega(A_i)) = R_s(\omega(A_i))$ as $R_l(A_i) \succ R_s(A_i)$ or $R_l(A_i) \approx R_s(A_i), 1 \le i \le m$, then ω is called a weighted function for the tendering $\widetilde{A}$ or the evaluation items $A_i, 1 \le i \le m$.*

According to the decision theory of multiple objectives([3]), the weighted function $\omega(A_i)$ must exists for any integer $i, 1 \le i \le m$. but generally, the weight function $\omega(\widetilde{A})$ does not exist if the values of these evaluation items $A_1, A_2, \cdots, A_m$ can not compare. There are two choice for the weighted function $\omega(A_i)$.

Choice 1 *the monotone functions in the interval $[a, b]$, such as the linear functions.*

Choice 2 *The continuous functions only with one maximum value in the interval $[a, b]$, such as $\omega(A_i) = -2x^2 + 6x + 12$ or*

$$\omega(A_i) = \begin{cases} x, & \text{if} \quad 0 \le x \le 2, \\ -x + 4, & \text{if} \quad x \ge 4. \end{cases}$$

As examples of concrete weighted functions ω, we discuss the tendering of civil engineering constructions.

4.1. **The weighted function for the bidding price**

Let A_1 be the bidding price. We often encounter the following weighted function $\omega(A_1)$ in practice.

$$\omega(R_i(A_1)) = -\varsigma \times \frac{R_i(A_1) - S}{S} + \zeta$$

where,

$$S = \frac{R_1(A_1) + R_2(A_1) + \cdots + R_k(A_1)}{k}$$

or

$$S = \begin{cases} \frac{R_1(A_1)+R_2(A_1)+\cdots+R_k(A_1)-M-N}{k-2}, & k \geq 5, \\ \frac{R_1(A_1)+R_2(A_1)+\cdots+R_k(A_1)}{k}, & 3 \leq k \leq 4 \end{cases}$$

or

$$S = T \times A\% + \frac{R_1(A_1) + R_2(A_1) + \cdots + R_k(A_1)}{k} \times (1 - A\%).$$

Where T,$A\%$,k, M and N are the pre-price of the tender, the percentage of T in S, the number of bidders and the maximum and minimum bidding price, respectively, $R_i(A_1), i = 1, 2, \cdots, k$ denote the bidding prices and ς, ζ are both constants.

There is a postulate in these weighted functions, i.e., each bidding price is random and accord with the normal distribution. Then the best excepted value of this civil engineering is the arithmetic mean of these bidding prices. However, each bidding price is not random in fact. It reflects the bidder's expected value and subjectivity in a tendering. We can not apply any definite mathematics to fix its real value. Therefore, this formula for a weighted function can be only seen as a game, not a scientific decision.

By the view of scientific decision, we can apply weighted functions according to the expected value and its cost in the market, such as

(1) *the linear function*

$$\omega(R_i(A_1)) = -p \times \frac{R_i(A_1) - N}{M - N} + q$$

in the interval $[N, M]$*, where* M, N *are the maximum and minimum bidding prices* p *is the deduction constant and* q *is a constant such that* $R_i(\omega(A_1)) \geq 0, 1 \leq i \leq k$*. The objective of this approach is seek a lower bidding price.*

(2) *non-linear functions in the interval* $[N, M]$*, such as*

$$\omega(R_i(A_1)) = -p \times \frac{R_i(A_1) - \frac{T+\sum_{j=1}^{k} R_i(A_1)}{k+1}}{+} q,$$

$$\omega(R_i(A_1)) = -p \times \frac{R_i(A_1) - \sqrt[k+1]{R_1(A_1)R_2(A_1)\cdots R_k(A_1)T}}{\sqrt[k+1]{R_1(A_1)R_2(A_1)\cdots R_k(A_1)T}} + q$$

or

$$\omega(R_i(A_1)) = -p \times \frac{R_i(A_1) - \sqrt{\frac{R_1^2(A_1)+R_2^2(A_1)+\cdots+R_k^2(A_1)+T^2}{k+1}}}{\sqrt{\frac{R_1^2(A_1)+R_2^2(A_1)+\cdots+R_k^2(A_1)+T^2}{k+1}}} + q$$

etc.. If we wish to analog a curve for these bidding prices and choose a point on this curve as $\omega(R_i(A_1))$, we can apply the value of a polynomial of degree $k+1$

$$f(x) = a_{k+1}x^{k+1} + a_k x^k + \cdots + a_1 x + a_0$$

by the undetermined coefficient method. Arrange the bidding prices and pre-price of the tender from lower to higher. Not loss of generality, let it be $R_{j_1}(A_1) \succ R_(j_2)(A_1) \succ \cdots \succ T \succ \cdots \succ R_{j_k}(A_1)$. Choose $k+2$ constants $c_1 > c_2 > \cdots > c_{k+1} > 0$, for instance $k+1 > k > \cdots > 1 > 0$. Solving the equation system

$$\begin{aligned}
&R_{j_1}(A_1) = a_{k+1}c_1^{k+1} + a_k c_1^k + \cdots + a_1 c_1 + a_0 \\
&R_{j_2}(A_1) = a_{k+1}c_2^{k+1} + a_k c_2^k + \cdots + a_1 c_2 + a_0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&R_{j_{k-1}}(A_1) = a_{k+1}c_k^{k+1} + a_k c_k^k + \cdots + a_1 c_k + a_0 \\
&R_{j_k}(A_1) = a_0
\end{aligned}$$

we get a polynomial $f(x)$ of degree $k+1$. The bidding price has an acceptable difference in practice. Whence, we also need to provide a bound for the difference which does not affect the ordered sequence of bids.

4.2. **The weighted function for the programming**

Let A_2 be the evaluation item of programming with evaluation indexes $\{a_{21}, a_{22}, \cdots, a_{2n_2}\}$. It is difficult to evaluating a programming in quantify, which is not only for the tender, but also for the evaluation specialists. In general, any two indexes of A_2 are not comparable. Whence it is not scientific assigning numbers for each index since we can not explain why the mark of a programming is 96 but another is 88. This means that it should qualitatively evaluate a programming or a quantify after a qualitatively evaluation. Its weight function $\omega(R_i(A_2)), 1 \leq i \leq k$ can be chosen as a linear function

$$\omega(R_i(A_2)) = \omega(R_i(a_{21})) + \omega(R_i(a_{22})) + \cdots + \omega(R_i(a_{2n_2})).$$

For example, there are 4 evaluation indexes for the programming, and each with A, B, C, D ranks in a tendering. The corespondent mark for each rank is in table 7.

index	a_{21}	a_{22}	a_{23}	a_{24}
A	4	2	2	1
B	3	1.5	1.5	0.8
C	2	1	1	0.5
D	1	0.5	0.5	0.3

table 7

If the evaluation results for a bidding programming $R_i, 1 \le i \le 4$ are $\omega(R_i(a_{21})) = A$, $\omega(R_i(a_{22})) = B$, $\omega(R_i(a_{23})) = B$ and $\omega(R_i(a_{24})) = A$, then the mark of this programming is

$$\begin{aligned} R_i(\omega(A_2)) &= R_i(\omega(a_{21})) + R_i(\omega(a_{22})) + R_i(\omega(a_{23})) + R_i(\omega(a_{24})) \\ &= 4 + 3 + 1.5 + 1 = 9.5. \end{aligned}$$

By the approach in Section 3, we can alphabetically or graphicly arrange the order of these programming if we can determine the rank of each programming. Certainly, we need the order of these indexes for a programming first. The index order for programming is different for different constructions tendering.

§5. Further discussions

5.1 Let $\widetilde{A} = \bigcup\limits_{i=1}^{m} A_i$ be a Smarandache multi-space with an operation set $O(\widetilde{A}) = \{\circ_i; 1 \le i \le m\}$. If there is a mapping Θ on $\widetilde{A}$ such that $\Theta(\widetilde{A})$ is also a Smarandache multi-space, then $(\widetilde{A}, \Theta)$ is called a *pseudo-multi-space*. Today, nearly all geometries, such as the *Riemann* geometry, *Finsler* geometry and these pseudo-manifold geometries are particular cases of pseudo-multi-geometries.

For applying Smarandache multi-spaces to an evaluation system, choose $\Theta(\widetilde{A})$ being an order set. Then Theorem 3.1 only asserts that any subset of $\Theta(\widetilde{A})$ is an order set, which enables us to find the ordered sequence for all bids in a tendering. Particularly, if $\Theta(\widetilde{A})$ is continuous and $\Theta(\widetilde{A}) \subseteq [-\infty, +\infty]$, then Θ is a weighted function on $\widetilde{A}$ widely applied in the evaluation of bids in China. By a mathematical view, many problems on $(\widetilde{A}, \Theta)$ is valuable to research. Some open problems are presented in the following.

Problem 5.1 *Characterize these pseudo-multi-spaces* $(\widetilde{A}, \Theta)$*, particularly, for these cases of* $\Theta(\widetilde{A}) = \bigcup\limits_{i=1}^{n} [a_i, b_i]$, $\Theta(\widetilde{A}) = \bigcup\limits_{i=1}^{n} (G_i, \circ_i)$ *and* $\Theta(\widetilde{A}) = \bigcup\limits_{i=1}^{n} (R; +_i, \circ_i)$ *with* $(G_i, \circ_i)$ *and* $(R; +_i, \circ_i)$ *being a finite group or a ring for* $1 \le i \le n$.

Problem 5.2 *Let* $\Theta(\widetilde{A})$ *be a group, a ring or a filed. Can we find an ordered sequence for a finite subset of* $\widetilde{A}$*?*

Problem 5.3 *Let* $\Theta(\widetilde{A})$ *be* n *lines or* n *planes in an Euclid space* $\mathbf{R}^n$. *Characterize these pseudo-multi-spaces* $(\widetilde{A}, \Theta)$. *Can we find an arrangement for a finite subset of* $\widetilde{A}$*?*

5.2 The evaluation approach in this paper can be also applied to evaluate any multiple objectives, such as the evaluation of a scientific project, a personal management system, an investment of a project, $\cdots$, etc..

References

[1] G.Chartrand and L.Lesniak,*Graphs & Digraphs*, Wadsworth, Inc., California, 1986.

[2] T.Chen, *Decision Approaches in Multiple Objectives*(in Chinese), in *The Handbook of Modern Engineering Mathematics*, Vol.IV, Part 77,1357-1410. Central China Engineering College Press, 1987.

[3] P.C.Fishburn, *Utility Theory for Decision Making*, New York, Wiley, 1970.

[4] D.L.Lu, X.S.Zhang and Y.Y.Mi, An offer model for civil engineering construction, *Chinese OR Transaction*, Vol.5, No.4(2001)41-52.

[5] L.F.Mao, *On Automorphisms groups of Maps, Surfaces and Smarandache geometries*, *Sientia Magna*, Vol.1(2005), No.2, 55-73.

[6] L.F.Mao, *Automorphism Groups of Maps, Surfaces and Smarandache Geometries*, American Research Press, 2005.

[7] L.F.Mao, *Smarandache multi-space theory*, Hexis, Phoenix, AZ2006.

[8] L.F.Mao, *Chinese Construction Project Bidding Technique & Cases Analyzing–Smarandache Multi-Space Model of Bidding*,Xiquan Publishing House (Chinese Branch), America, 2006.

[9] F.Smarandache, Mixed noneuclidean geometries, *eprint arXiv: math/0010119*, 10/2000.

[10] F.Smarandache, *A Unifying Field in Logics. Neutrosopy: Neturosophic Probability, Set, and Logic*, American research Press, Rehoboth, 1999.

[11] F.Smarandache, Neutrosophy, a new Branch of Philosophy, *Multi-Valued Logic*, Vol.8, No.3(2002)(special issue on Neutrosophy and Neutrosophic Logic), 297-384.

[12] F.Smarandache, A Unifying Field in Logic: Neutrosophic Field, *Multi-Valued Logic*, Vol.8, No.3(2002)(special issue on Neutrosophy and Neutrosophic Logic), 385-438.