A New Understanding of Particles by $\overrightarrow{G}$-Flow Interpretation of Differential Equation

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Applying mathematics to the understanding of particles classically with an assumption that if the variables t and x_1, x_2, x_3 hold with a system of dynamical equations (1.4), then they are a point (t, x_1, x_2, x_3) in $\mathbb{R}^4$. However, if we put off this assumption, how can we interpret the solution space of equations? And are these resultants important for understanding the world? Recently, the author extended Banach and Hilbert spaces on a topological graph to introduce $\overrightarrow{G}$-flows and showed that all such flows on a topological graph $\overrightarrow{G}$ also form a Banach or Hilbert space, which enables one to find the multiverse solution of these equations on $\overrightarrow{G}$. Applying this result, this paper discusses the $\overrightarrow{G}$-flow solutions on Schrödinger equation, Klein-Gordon equation and Dirac equation, i.e., the field equations of particles, bosons or fermions, answers previous questions by "yes", and establishes the many world interpretation of quantum mechanics of H. Everett by purely mathematics in logic, i.e., mathematical combinatorics.

1 Introduction

Matter consists of bosons with integer spin n and fermions with half-integer spin $n/2$, $n \equiv 1 \,(\mathrm{mod}\, 2)$. The elementary particles consist of leptons and hadrons, i.e. mesons, baryons and their antiparticles, which are composed of quarks [16]. Thus, a hadron has an internal structure, which implies that all hadrons are not elementary but leptons are, viewed as point particles in elementary physics. Furthermore, there is also unmatter which is neither matter nor antimatter, but something in between [19-21]. For example, an atom of unmatter is formed either by electrons, protons, and antineutrons, or by antielectrons, antiprotons, and neutrons.

Usually, a particle is characterized by solutions of differential equation established on its wave function $\psi(t, x)$. In non-relativistic quantum mechanics, the wave function $\psi(t, x)$ of a particle of mass m obeys the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + U, \tag{1.1}$$

where, $\hbar = 6.582 \times 10^{-22}$MeVs is the Planck constant, U is the potential energy of the particle in applied field and

$$\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \text{ and } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Consequently, a free boson $\psi(t, x)$ hold with the Klein-Gordon equation

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\psi(x, t) + \left(\frac{mc}{\hbar}\right)^2 \psi(x, t) = 0 \tag{1.2}$$

and a free fermion $\psi(t, x)$ satisfies the Dirac equation

$$\left(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar}\right)\psi(t, x) = 0 \tag{1.3}$$

in relativistic forms, where,

$$\gamma^\mu = \left(\gamma^0, \gamma^1, \gamma^2, \gamma^3\right),$$
$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right),$$

c is the speed of light and

$$\gamma^0 = \begin{pmatrix} I_{2\times 2} & 0 \\ 0 & -I_{2\times 2} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

with the usual Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

It is well known that the behavior of a particle is on superposition, i.e., in two or more possible states of being. But *how to interpret this phenomenon in accordance with (1.1)–(1.3)* ? The many worlds interpretation on wave function of (1.1) by H. Everett [2] in 1957 answered the question in machinery, i.e., viewed different worlds in different quantum mechanics and the superposition of a particle be liked those separate arms of a branching universe ([15], also see [1]). In fact, H. Everett's interpretation claimed that the state space of particle is a multiverse, or parallel universe ([23, 24]), an application of philosophical law that *the integral always consists of its parts*, or formally, the following.

Definition 1.1([6],[18]) *Let $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$ be m mathematical or physical systems, different two by two. A Smarandache multisystem $\widetilde{\Sigma}$ is a union $\bigcup\limits_{i=1}^{m} \Sigma_i$ with rules $\widetilde{\mathcal{R}} = \bigcup\limits_{i=1}^{m} \mathcal{R}_i$ on $\widetilde{\Sigma}$, denoted by $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$.*

Furthermore, *things are inherently related, not isolated* in the world. Thus, every particle in nature is a union of elementary particles underlying a graph embedded in space, where, a graph G is said to be embeddable into a topological space $\mathscr{E}$ if there is a $1-1$ continuous mapping $f : G \to \mathscr{E}$ with $f(p) \neq f(q)$ if $p \neq q$ for $\forall p, q \in G$, i.e., edges only intersect at end vertices in $\mathscr{E}$. For example, a *planar graph* such as those shown in Fig. 1.

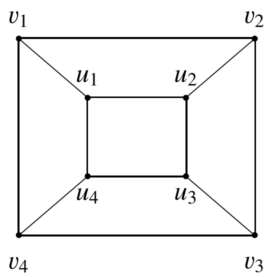

Fig. 1

Definition 1.2([6]) *For any integer $m \geq 1$, let $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ be a Smarandache multisystem consisting of mathematical systems $(\Sigma_1; \mathcal{R}_1), (\Sigma_2; \mathcal{R}_2), \cdots, (\Sigma_m; \mathcal{R}_m)$. An inherited topological structures $G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]$ on $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ is defined by*

$$V\left(G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]\right) = \{v_{\Sigma_1}, v_{\Sigma_2}, \cdots, v_{\Sigma_m}\},$$

$$E\left(G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]\right) = \{(v_{\Sigma_i}, v_{\Sigma_j}) | \Sigma_i \bigcap \Sigma_j \neq \emptyset, \ 1 \leq i \neq j \leq m\}$$

with a labeling $L : v_{\Sigma_i} \to L(v_{\Sigma_i}) = \Sigma_i$ and $L : (v_{\Sigma_i}, v_{\Sigma_j}) \to L(v_{\Sigma_i}, v_{\Sigma_j}) = \Sigma_i \bigcap \Sigma_j$, where $\Sigma_i \bigcap \Sigma_j$ denotes the intersection of spaces, or action between systems Σ_i with Σ_j for integers $1 \leq i \neq j \leq m$.

For example, let $\widetilde{\Sigma} = \bigcup_{i=1}^{4} \Sigma_i$ with $\Sigma_1 = \{a, b, c\}$, $\Sigma_2 = \{a, b\}$, $\Sigma_3 = \{b, c, d\}$, $\Sigma_4 = \{c, d\}$ and $\mathcal{R}_i = \emptyset$. Calculation shows that $\Sigma_1 \bigcap \Sigma_2 = \{a, b\}$, $\Sigma_1 \bigcap \Sigma_3 = \{b, c\}$, $\Sigma_1 \bigcap \Sigma_4 = \{c\}$, $\Sigma_2 \bigcap \Sigma_3 = \{b\}$, $\Sigma_2 \bigcap \Sigma_4 = \emptyset$, $\Sigma_3 \bigcap \Sigma_4 = \{c, d\}$. Such a graph $G^L\left[\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right]$ is shown in Fig. 2.

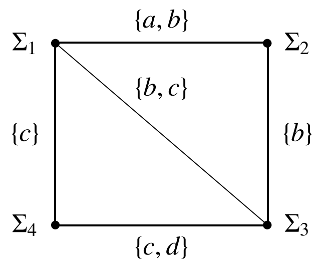

Fig. 2

Generally, a particle should be characterized by $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$ in theory. However, we can only verify it by some of systems $(\Sigma_1; \mathcal{R}_1)$, $(\Sigma_2; \mathcal{R}_2)$, $\cdots$, $(\Sigma_m; \mathcal{R}_m)$ for the limitation of human beings because he is also a system in $\left(\widetilde{\Sigma}; \widetilde{\mathcal{R}}\right)$. Clearly, the underlying graph in H. Everett's interpretation on wave function is in fact a binary tree and there are many such traces in the developing of physics. For example, a baryon is predominantly formed from three quarks, and a meson is mainly composed of a quark and an antiquark in the models of Sakata, or Gell-Mann and Ne'eman on hadrons ([14]), such as those shown in Fig. 3, where, $q_i \in \{\mathbf{u}, \mathbf{d}, \mathbf{c}, \mathbf{s}, \mathbf{t}, \mathbf{b}\}$ denotes a quark for $i = 1, 2, 3$ and $\overline{q}_2 \in \left\{\overline{\mathbf{u}}, \overline{\mathbf{d}}, \overline{\mathbf{c}}, \overline{\mathbf{s}}, \overline{\mathbf{t}}, \overline{\mathbf{b}}\right\}$, an antiquark. Thus, the underlying graphs $\overrightarrow{G}$ of a meson, a baryon are respectively $\overrightarrow{K}_2$ and $\overrightarrow{K}_3$ with actions. In fact, a free quark was not found in experiments until today. So it is only a machinery model on hadrons. Even so, it characterizes well the known behavior of particles.

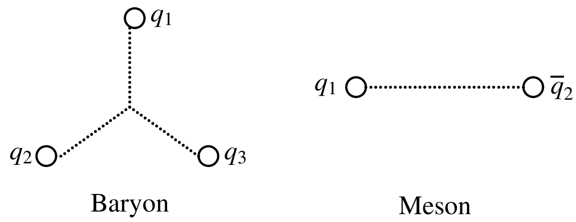

Fig. 3

It should be noted that the geometry on Definition 1.1–1.2 can be also used to characterize particles by combinatorial fields ([7]), and there is a priori assumption for discussion in physics, namely, *the dynamical equation of a subparticle of a particle is the same of that particle*. For example, the dynamical equation of quark is nothing else but the Dirac equation (1.3), a characterizing on quark from the macroscopic to the microscopic, the quantum level in physics. However, (1.3) cannot provide such a solution on the behaviors of 3 quarks. We can only interpret it similar to that of H. Everett, i.e., there are 3 parallel equations (1.3) in discussion, a seemly rational interpretation in physics, but not perfect for mathematics. Why this happens is because the interpretation of solution of equation. Usually, we identify a particle to the solution of its equation, i.e., if the variables t and x_1, x_2, x_3 hold with a system of dynamical equations

$$\mathscr{F}_i\left(t, x_1, x_2, x_3, u_t, u_{x_1}, \cdots, u_{x_1 x_2}, \cdots\right) = 0,$$

$$\text{with} \quad 1 \leq i \leq m, \tag{1.4}$$

the particle in $\mathbb{R} \times \mathbb{R}^3$ is a point (t, x_1, x_2, x_3), and if more than one points (t, x_1, x_2, x_3) hold with (1.4), the particle is nothing else but consisting of all such points. However, the solutions of (1.1)–(1.3) are all definite on time t. *Can this interpretation be used for particles in all times*? Certainly not because a particle can be always decomposed into elementary particles, and it is a little ambiguous which is a point, the particle itself or its one of elementary particles sometimes.

This speculation naturally leads to a question on mathematics, i.e., *what is the right interpretation on the solution of differential equation accompanying with particles*? Recently, the author extended Banach spaces on topological graphs $\overrightarrow{G}$ with operator actions in [13], and shown all of these extensions are also Banach space, particularly, the Hilbert space with unique correspondence in elements on linear continuous functionals, which enables one to solve linear functional equations in such extended space, particularly, solve differential equations on a topological graph, i.e., find multiverse solutions for equations. This scheme also enables us to interpret the superposition of particles in accordance with mathematics in logic.

The main purpose of this paper is to present an interpretation on superposition of particles by $\overrightarrow{G}$-flow solutions of (1.1)–(1.3) in accordance with mathematics. Certainly, the geometry on non-solvable differential equations discussed in [9]–[12] brings us another general way for holding behaviors of particles in mathematics. For terminologies and notations not mentioned here, we follow references [16] for elementary particles, [6] for geometry and topology, and [17]–[18] for Smarandache multi-spaces, and all equations are assumed to be solvable in this paper.

2 Extended Banach $\vec{G}$-flow space

2.1 Conservation laws

A conservation law, such as those on energy, mass, momentum, angular momentum and electric charge states that a particular measurable property of an isolated physical system does not change as the system evolves over time, or simply, constant of being. Usually, a local conservation law is expressed mathematically as a continuity equation, which states that the amount of conserved quantity at a point or within a volume can only change by the amount of the quantity which flows in or out of the volume. According to Definitions 1.1 and 1.2, a matter in the nature is nothing else but a Smarandache system $\left(\Sigma;\widetilde{\mathcal{R}}\right)$, or a topological graph $G^L\left[\left(\Sigma;\widetilde{\mathcal{R}}\right)\right]$ embedded in $\mathbb{R}^3$, hold with conservation laws

$$\sum_k \mathbf{F}(\mathbf{v})_k^- = \sum_l \mathbf{F}(\mathbf{v})_l^+$$

on $\forall v \in V\left(G^L\left[\left(\Sigma;\widetilde{\mathcal{R}}\right)\right]\right)$, where, $\mathbf{F}(\mathbf{v})_k^-$, $k \geq 1$ and $\mathbf{F}(\mathbf{v})_l^+$, $l \geq 1$ denote respectively the input or output amounts on a particle or a volume v.

2.2 $\vec{G}$-flow spaces

Classical operation systems can be easily extended on a graph $\overrightarrow{G}$ constraint on conditions for characterizing the unanimous behaviors of groups in the nature, particularly, go along with the physics. For this objective, let $\overrightarrow{G}$ be an oriented graph with vertex set $V(G)$ and arc set $X(G)$ embedded in $\mathbb{R}^3$ and let $(\mathscr{A};\circ)$ be an operation system in classical mathematics, i.e., for $\forall a,b \in \mathscr{A}$, $a\circ b \in \mathscr{A}$. Denoted by $\overrightarrow{G}^L_{\mathscr{A}}$ all of those labeled graphs $\overrightarrow{G}^L$ with labeling $L: X\left(\overrightarrow{G}\right) \to \mathscr{A}$. Then, we can extend operation $\circ$ on elements in $\overrightarrow{G}^{\mathscr{A}}$ by a ruler following:

R: *For* $\forall \overrightarrow{G}^{L_1}, \overrightarrow{G}^{L_2} \in \overrightarrow{G}^L_{\mathscr{A}}$, *define* $\overrightarrow{G}^{L_1} \circ \overrightarrow{G}^{L_2} = \overrightarrow{G}^{L_1\circ L_2}$, *where* $L_1 \circ L_2 : e \to L_1(e) \circ L_2(e)$ *for* $\forall e \in X\left(\overrightarrow{G}\right)$.

For example, such an extension on graph $\overrightarrow{C}_4$ is shown in Fig. 4, where, $\mathbf{a}_3=\mathbf{a}_1\circ\mathbf{a}_2$, $\mathbf{b}_3=\mathbf{b}_1\circ\mathbf{b}_2$, $\mathbf{c}_3=\mathbf{c}_1\circ\mathbf{c}_2$, $\mathbf{d}_3=\mathbf{d}_1\circ\mathbf{d}_2$.

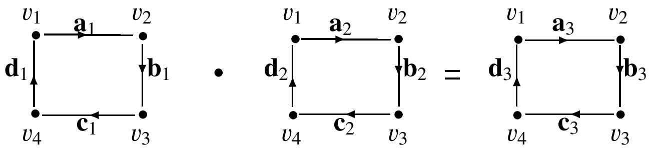

Fig. 4

Clearly, $\overrightarrow{G}^{L_1}\circ\overrightarrow{G}^{L_2} \in \overrightarrow{G}^L_{\mathscr{A}}$ by definition, i.e., $\overrightarrow{G}^L_{\mathscr{A}}$ is also an operation system under ruler $\mathbf{R}$, and it is commutative if $(\mathscr{A},\circ)$ is commutative,

Furthermore, if $(\mathscr{A},\circ)$ is an algebraic group, $\overrightarrow{G}^L_{\mathscr{A}}$ is also an algebraic group because

(1) $\left(\overrightarrow{G}^{L_1} \circ \overrightarrow{G}^{L_2}\right)\circ\overrightarrow{G}^{L_3} = \overrightarrow{G}^{L_1} \circ \left(\overrightarrow{G}^{L_2} \circ \overrightarrow{G}^{L_3}\right)$ for $\forall \overrightarrow{G}^{L_1}, \overrightarrow{G}^{L_2}, \overrightarrow{G}^{L_3} \in \overrightarrow{G}^{\mathscr{A}}$ because

$$(L_1(e)\circ L_2(e))\circ L_3(e) = L_1(e)\circ(L_2(e)\circ L_3(e))$$

for $e \in X\left(\overrightarrow{G}\right)$, i.e., $\overrightarrow{G}^{(L_1\circ L_2)\circ L_3} = \overrightarrow{G}^{L_1\circ(L_2\circ L_3)}$.

(2) there is an identify $\overrightarrow{G}^{L_{1_{\mathscr{A}}}}$ in $\overrightarrow{G}^L_{\mathscr{A}}$, where $L_{1_{\mathscr{A}}} : e \to 1_{\mathscr{A}}$ for $\forall e \in X\left(\overrightarrow{G}\right)$;

(3) there is an uniquely element $\overrightarrow{G}^{L^{-1}}$ for $\forall \overrightarrow{G}^L \in \overrightarrow{G}^L_{\mathscr{A}}$.

However, for characterizing the unanimous behaviors of groups in the nature, the most useful one is the extension of vector space $(\mathscr{V};+,\cdot)$ over field $\mathcal{F}$ by defining the operations $+$ and $\cdot$ on elements in $\overrightarrow{G}^{\mathscr{V}}$ such as those shown in Fig. 5 on graph $\overrightarrow{C}_4$, where $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, $\mathbf{d}$, $\mathbf{a}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$, $\mathbf{d}_i \in \mathscr{V}$ for $i = 1,2,3$, $\mathbf{x}_3=\mathbf{x}_1+\mathbf{x}_2$ for $\mathbf{x}=\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ or $\mathbf{d}$ and $\alpha \in \mathcal{F}$.

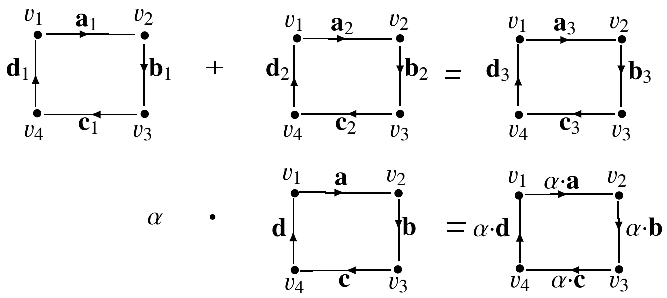

Fig. 5

A $\overrightarrow{G}$-*flow* on $\overrightarrow{G}$ is such an extension hold with $L(u,v) = -L(v,u)$ and conservation laws

$$\sum_{u\in N_G(v)} L(v,u) = \mathbf{0}$$

for $\forall v \in V\left(\overrightarrow{G}\right)$, where $\mathbf{0}$ is the zero-vector in $\mathscr{V}$. Thus, a $\overrightarrow{G}$-flow is a subfamily of $\overrightarrow{G}^L_{\mathscr{V}}$ limited by conservation laws. For example, if $\overrightarrow{G} = \overrightarrow{C}_4$, there must be $\mathbf{a}$=$\mathbf{b}$=$\mathbf{c}$=$\mathbf{d}$, $\mathbf{a}_i$=$\mathbf{b}_i$=$\mathbf{c}_i$=$\mathbf{d}_i$ for $i = 1,2,3$ in Fig. 5.

Clearly, all conservation $\overrightarrow{G}$-flows on $\overrightarrow{G}$ also form a vector space over $\mathcal{F}$ under operations + and $\cdot$ with zero vector $\mathbf{O} = \overrightarrow{G}^{L_\mathbf{0}}$, where $L_\mathbf{0}:\ e \to \mathbf{0}$ for $\forall e \in X\left(\overrightarrow{G}\right)$. Such an extended vector space on $\overrightarrow{G}$ is denoted by $\overrightarrow{G}^{\mathscr{V}}$.

Furthermore, if $(\mathscr{V};+,\cdot)$ is a Banach or Hilbert space with inner product $\langle\cdot,\cdot\rangle$, we can also introduce the *norm* and *inner product* on $\overrightarrow{G}^{\mathscr{V}}$ by

$$\left\|\overrightarrow{G}^L\right\| = \sum_{(u,v)\in X\left(\overrightarrow{G}\right)} \|L(u,v)\|$$

or

$$\left\langle\overrightarrow{G}^{L_1},\overrightarrow{G}^{L_2}\right\rangle = \sum_{(u,v)\in X\left(\overrightarrow{G}\right)} \langle L_1(u,v), L_2(u,v)\rangle$$

for $\forall\overrightarrow{G}^L,\overrightarrow{G}^{L_1},\overrightarrow{G}^{L_2} \in \overrightarrow{G}^{\mathscr{V}}$, where $\|L(u,v)\|$ is the norm of $L(u,v)$ in $\mathscr{V}$. Then it can be verified that

(1) $\left\|\overrightarrow{G}^L\right\| \ge 0$ and $\left\|\overrightarrow{G}^L\right\| = 0$ if and only if $\overrightarrow{G}^L = \mathbf{O}$;

(2) $\left\|\overrightarrow{G}^{\xi L}\right\| = \xi\left\|\overrightarrow{G}^L\right\|$ for any scalar ξ;

(3) $\left\|\overrightarrow{G}^{L_1}+\overrightarrow{G}^{L_2}\right\| \le \left\|\overrightarrow{G}^{L_1}\right\| + \left\|\overrightarrow{G}^{L_2}\right\|$;

(4) $\left\langle\overrightarrow{G}^L,\overrightarrow{G}^L\right\rangle = \sum\limits_{(u,v)\in X\left(\overrightarrow{G}\right)} \langle L(u^v), L(u^v)\rangle \ge 0$ and $\left\langle\overrightarrow{G}^L,\overrightarrow{G}^L\right\rangle = 0$ if and only if $\overrightarrow{G}^L = \mathbf{O}$;

(5) $\left\langle\overrightarrow{G}^{L_1},\overrightarrow{G}^{L_2}\right\rangle = \overline{\left\langle\overrightarrow{G}^{L_2},\overrightarrow{G}^{L_1}\right\rangle}$ for $\forall\overrightarrow{G}^{L_1},\overrightarrow{G}^{L_2}\in\overrightarrow{G}^{\mathscr{V}}$;

(6) For $\overrightarrow{G}^L,\overrightarrow{G}^{L_1},\overrightarrow{G}^{L_2}\in\overrightarrow{G}^{\mathscr{V}}$ and $\lambda,\mu\in\mathcal{F}$,

$$\left\langle\lambda\overrightarrow{G}^{L_1}+\mu\overrightarrow{G}^{L_2},\overrightarrow{G}^L\right\rangle = \lambda\left\langle\overrightarrow{G}^{L_1},\overrightarrow{G}^L\right\rangle + \mu\left\langle\overrightarrow{G}^{L_2},\overrightarrow{G}^L\right\rangle.$$

The following result is obtained by showing that Cauchy sequences in $\overrightarrow{G}^{\mathscr{V}}$ is converges hold with conservation laws.

Theorem 2.1([13]) *For any topological graph $\overrightarrow{G}$, $\overrightarrow{G}^{\mathscr{V}}$ is a Banach space, and furthermore, if $\mathscr{V}$ is a Hilbert space, $\overrightarrow{G}^{\mathscr{V}}$ is a Hilbert space also.*

According to Theorem 2.1, the operators action on Banach or Hilbert space $(\mathscr{V};+,\cdot)$ can be extended on $\overrightarrow{G}^{\mathscr{V}}$, for example, the linear operator following.

Definition 2.2 *An operator $\mathbf{T}:\overrightarrow{G}^{\mathscr{V}}\to\overrightarrow{G}^{\mathscr{V}}$ is linear if*

$$\mathbf{T}\left(\lambda\overrightarrow{G}^{L_1}+\mu\overrightarrow{G}^{L_2}\right) = \lambda\mathbf{T}\left(\overrightarrow{G}^{L_1}\right)+\mu\mathbf{T}\left(\overrightarrow{G}^{L_2}\right)$$

for $\forall\overrightarrow{G}^{L_1},\overrightarrow{G}^{L_2}\in\overrightarrow{G}^{\mathscr{V}}$ and $\lambda,\mu\in\mathscr{F}$, and is continuous at a $\overrightarrow{G}$-flow $\overrightarrow{G}^{L_0}$ if there always exist a number $\delta(\varepsilon)$ for $\forall\epsilon > 0$ such that

$$\left\|\mathbf{T}\left(\overrightarrow{G}^L\right)-\mathbf{T}\left(\overrightarrow{G}^{L_0}\right)\right\| < \varepsilon$$

if

$$\left\|\overrightarrow{G}^L-\overrightarrow{G}^{L_0}\right\| < \delta(\varepsilon).$$

The following interesting result generalizes the result of Fréchet and Riesz on linear continuous functionals, which opens us mind for applying $\overrightarrow{G}$-flows to hold on the nature.

Theorem 2.3([13]) *Let $\mathbf{T}:\overrightarrow{G}^{\mathscr{V}}\to\mathbb{C}$ be a linear continuous functional. Then there is a unique $\overrightarrow{G}^{\widehat{L}}\in\overrightarrow{G}^{\mathscr{V}}$ such that*

$$\mathbf{T}\left(\overrightarrow{G}^L\right) = \left\langle\overrightarrow{G}^L,\overrightarrow{G}^{\widehat{L}}\right\rangle$$

for $\forall\overrightarrow{G}^L\in\overrightarrow{G}^{\mathscr{V}}$.

Particularly, if all flows $L(u,v)$ on arcs (u,v) of $\overrightarrow{G}$ are state function, we extend the differential operator on $\overrightarrow{G}$-flows. In fact, a *differential operator* $\dfrac{\partial}{\partial t}$ or $\dfrac{\partial}{\partial x_i}$: $\overrightarrow{G}^{\mathscr{V}}\to\overrightarrow{G}^{\mathscr{V}}$ is defined by

$$\frac{\partial}{\partial t}:\ \overrightarrow{G}^L\to\overrightarrow{G}^{\frac{\partial L}{\partial t}},\quad \frac{\partial}{\partial x_i}:\ \overrightarrow{G}^L\to\overrightarrow{G}^{\frac{\partial L}{\partial x_i}}$$

for integers $1\le i\le 3$. Then, for $\forall\mu,\lambda\in\mathcal{F}$,

$$\begin{aligned}&\frac{\partial}{\partial t}\left(\lambda\overrightarrow{G}^{L_1}+\mu\overrightarrow{G}^{L_2}\right)\\ &= \frac{\partial}{\partial t}\left(\overrightarrow{G}^{\lambda L_1+\mu L_2}\right) = \overrightarrow{G}^{\frac{\partial}{\partial t}(\lambda L_1+\mu L_2)}\\ &= \overrightarrow{G}^{\frac{\partial}{\partial t}(\lambda L_1)+\frac{\partial}{\partial t}(\mu L_2)} = \overrightarrow{G}^{\frac{\partial}{\partial t}(\lambda L_1)}+\overrightarrow{G}^{\frac{\partial}{\partial t}(\mu L_2)}\\ &= \frac{\partial}{\partial t}\overrightarrow{G}^{(\lambda L_1)}+\frac{\partial}{\partial t}\overrightarrow{G}^{(\mu L_2)}\\ &= \lambda\frac{\partial}{\partial t}\overrightarrow{G}^{L_1}+\mu\frac{\partial}{\partial t}\overrightarrow{G}^{L_2},\end{aligned}$$

i.e.,

$$\frac{\partial}{\partial t}\left(\lambda\overrightarrow{G}^{L_1}+\mu\overrightarrow{G}^{L_2}\right) = \lambda\frac{\partial}{\partial t}\overrightarrow{G}^{L_1}+\mu\frac{\partial}{\partial t}\overrightarrow{G}^{L_2}.$$

Similarly, we know also that

$$\frac{\partial}{\partial x_i}\left(\lambda\overrightarrow{G}^{L_1}+\mu\overrightarrow{G}^{L_2}\right) = \lambda\frac{\partial}{\partial x_i}\overrightarrow{G}^{L_1}+\mu\frac{\partial}{\partial x_i}\overrightarrow{G}^{L_2}$$

for integers $1 \leq i \leq 3$. Thus, operators $\dfrac{\partial}{\partial t}$ and $\dfrac{\partial}{\partial x_i}$, $1 \leq i \leq 3$ are all linear on $\overrightarrow{G}^{\mathscr{V}}$.

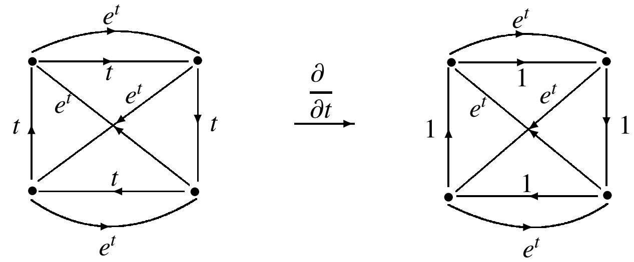

Fig. 6

Similarly, we introduce *integral operator* $\int : \overrightarrow{G}^{\mathscr{V}} \to \overrightarrow{G}^{\mathscr{V}}$ by

$$\int : \overrightarrow{G}^{L} \to \overrightarrow{G}^{\int L dt}, \quad \overrightarrow{G}^{L} \to \overrightarrow{G}^{\int L dx_i}$$

for integers $1 \leq i \leq 3$ and know that

$$\int \left(\mu \overrightarrow{G}^{L_1} + \lambda \overrightarrow{G}^{L_2}\right) = \mu \int \left(\overrightarrow{G}^{L_1}\right) + \lambda \int \left(\overrightarrow{G}^{L_2}\right)$$

for $\forall \mu, \lambda \in \mathscr{F}$,

$$\int \circ \left(\frac{\partial}{\partial t}\right) \text{ and } \int \circ \left(\frac{\partial}{\partial x_i}\right) : \overrightarrow{G}^{L} \to \overrightarrow{G}^{L} + \overrightarrow{G}^{L_c},$$

where L_c is such a labeling that $L_c(u, v)$ is constant for $\forall (u, v) \in X\left(\overrightarrow{G}\right)$.

3 Particle equations in $\vec{G}$-flow space

We are easily find particle equations with nonrelativistic or relativistic mechanics in $\overrightarrow{G}^{\mathscr{V}}$. Notice that

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi, \qquad -i\hbar \nabla \psi = \overrightarrow{p}^2 \psi$$

and

$$E = \frac{1}{2m} \overrightarrow{p}^2 + U,$$

in classical mechanics, where ψ is the state function, $E, \overrightarrow{p}, U$ are respectively the energy, the momentum, the potential energy and m the mass of the particle. Whence,

$$\begin{aligned}
\mathbf{O} &= \overrightarrow{G}^{(E - \frac{1}{2m}\overrightarrow{p}^2 - U)\psi} \\
&= \overrightarrow{G}^{E\psi} - \overrightarrow{G}^{\frac{1}{2m}\overrightarrow{p}^2\psi} - \overrightarrow{G}^{U\psi} \\
&= \overrightarrow{G}^{i\hbar \frac{\partial \psi}{\partial t}} - \overrightarrow{G}^{-\frac{\hbar}{2m}\nabla^2 \psi} - \overrightarrow{G}^{U\psi} \\
&= i\hbar \frac{\partial \overrightarrow{G}^{L_\psi}}{\partial t} + \frac{\hbar}{2m} \nabla^2 \overrightarrow{G}^{L_\psi} - \overrightarrow{G}^{L_U} \overrightarrow{G}^{L_\psi},
\end{aligned}$$

where $L_\psi : e \to$ state function and $L_U : e \to$ potential energy on $e \in X\left(\overrightarrow{G}\right)$. According to the conservation law of energy, there must be $\overrightarrow{G}^{U} \in \overrightarrow{G}^{\mathscr{V}}$. We get the Schrödinger equation in $\overrightarrow{G}^{\mathscr{V}}$ following.

$$-i\hbar \frac{\partial \overrightarrow{G}^{L_\psi}}{\partial t} = \frac{\hbar}{2m} \nabla^2 \overrightarrow{G}^{L_\psi} - \widehat{U} \overrightarrow{G}^{L_\psi}, \tag{3.1}$$

where $\widehat{U} = \overrightarrow{G}^{L_U} \in \overrightarrow{G}^{\mathscr{V}}$. Similarly, by the relativistic energy-momentum relation

$$E^2 = c^2 \overrightarrow{p}^2 + m^2 c^4$$

for bosons and

$$E = c\alpha_k \overrightarrow{p}_k + \alpha_0 mc^2$$

for fermions, we get the Klein-Gordon equation and Dirac equation

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \overrightarrow{G}^{L_\psi} + \left(\frac{cm}{\hbar}\right) \overrightarrow{G}^{L_\psi} = \mathbf{O} \tag{3.2}$$

and

$$\left(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar}\right) \overrightarrow{G}^{L_\psi} = \mathbf{O}, \tag{3.3}$$

of particles in $\overrightarrow{G}^{\mathscr{V}}$ respectively. Particularly, let $\overrightarrow{G}$ be such a topological graph with one vertex but only with one arc. Then, (3.1)–(3.3) are nothing else but (1.1)–(1.3) respectively. However, (3.1)–(3.3) conclude that we can find $\overrightarrow{G}$-flow solutions on (1.1)–(1.3), which enables us to interpret mathematically the superposition of particles by multiverse.

4 $\vec{G}$-flows on particle equations

Formally, we can establish equations in $\overrightarrow{G}^{\mathscr{V}}$ by equations in Banach space $\mathscr{V}$ such as (3.1)–(3.3). However, the important thing is not just on such establishing but finding $\overrightarrow{G}$-flows on equations in $\mathscr{V}$ and then interpret the superposition of particles by $\overrightarrow{G}$-flows.

4.1 $\vec{G}$-flow solutions on equation

Theorem 2.3 concludes that there are $\overrightarrow{G}$-flow solutions for a linear equations in $\overrightarrow{G}^{\mathscr{V}}$ for Hilbert space $\mathscr{V}$ over field $\mathscr{F}$, including algebraic equations, linear differential or integral equations without considering the topological structure. For example, let $ax = b$. We are easily getting its $\overrightarrow{G}$-flow solution $x = \overrightarrow{G}^{a^{-1}L}$ if we view an element $b \in \mathscr{V}$ as $b = \overrightarrow{G}^{L}$, where $L(u, v) = b$ for $\forall (u, v) \in X\left(\overrightarrow{G}\right)$ and $0 \neq a \in \mathscr{F}$, such as those shown in Fig. 7 for $\overrightarrow{G} = \overrightarrow{C}_4$ and $a = 3$, $b = 5$.

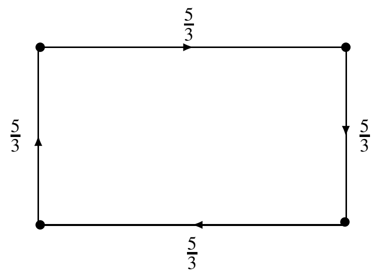

Fig. 7

Generally, we know the following result:

Theorem 4.1([13]) *A linear system of equations*

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

with $a_{ij}, b_j \in \mathscr{F}$ for integers $1 \le i \le n, 1 \le j \le m$ holding with

$$\mathrm{rank}\left[a_{ij}\right]_{m\times n} = \mathrm{rank}\left[a_{ij}\right]^{+}_{m\times(n+1)}$$

has $\overrightarrow{G}$-flow solutions on infinitely many topological graphs $\overrightarrow{G}$, where

$$\left[a_{ij}\right]^{+}_{m\times(n+1)} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & L_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & L_2 \\ \dots & \dots & .. & \dots & .. \\ a_{m1} & a_{m2} & \cdots & a_{mn} & L_m \end{bmatrix}.$$

We can also get $\overrightarrow{G}$-flow solutions for linear partial differential equations ([14]). For example, the Cauchy problems on differential equations

$$\frac{\partial X}{\partial t} = c^2 \sum_{i=1}^{n} \frac{\partial^2 X}{\partial x_i^2}$$

with initial value $X|_{t=t_0} = \overrightarrow{G}^{L'} \in \overrightarrow{G}^{\mathscr{V}}$ is also solvable in $\overrightarrow{G}^{\mathscr{V}}$ if $L'(u,v)$ is continuous and bou- nded in $\mathbb{R}^n$ for $\forall(u,v) \in X\left(\overrightarrow{G}\right)$ and $\forall \overrightarrow{G}^{L'} \in \overrightarrow{G}^{\mathscr{V}}$. In fact, $X = \overrightarrow{G}^{L_F}$ with $L_F : (u,v) \to F(u,v)$ for $\forall(u,c) \in X\left(\overrightarrow{G}\right)$, where

$$\begin{aligned} F(u,v) &= \frac{1}{(4\pi t)^{\frac{n}{2}}}\left(\int_{-\infty}^{+\infty} e^{-\frac{(x_1-y_1)^2+\cdots+(x_n-y_n)^2}{4t}}\right. \\ &\times \left. L'(u,v)(y_1,\cdots,y_n)dy_1\cdots dy_n\right) \end{aligned}$$

is such a solution.

Generally, if $\overrightarrow{G}$ can be decomposed into circuits $\overrightarrow{C}$, the next result concludes that we can always find $\overrightarrow{G}$-flow solutions on equations, no matter what the equation looks like, linear or non-linear ([13]).

Theorem 4.2 *If the topological graph $\overrightarrow{G}$ is strong-connected with circuit decomposition*

$$\overrightarrow{G} = \bigcup_{i=1}^{l} \overrightarrow{C}_i$$

such that $L(u,v) = L_i(\boldsymbol{x})$ for $\forall(u,v) \in X\left(\overrightarrow{C}_i\right)$, $1 \le i \le l$ and the Cauchy problem

$$\begin{cases} \mathscr{F}_i(\boldsymbol{x}, u, u_{x_1}, \cdots, u_{x_n}, u_{x_1x_2}, \cdots) = 0 \\ u|_{\boldsymbol{x}_0} = L_i(\boldsymbol{x}) \end{cases}$$

is solvable in a Hilbert space $\mathscr{V}$ on domain $\Delta \subset \mathbb{R}^n$ for integers $1 \le i \le l$, then the Cauchy problem

$$\begin{cases} \mathscr{F}_i(\boldsymbol{x}, X, X_{x_1}, \cdots, X_{x_n}, X_{x_1x_2}, \cdots) = 0 \\ X|_{\boldsymbol{x}_0} = \overrightarrow{G}^{L} \end{cases}$$

such that $L(u,v) = L_i(\boldsymbol{x})$ for $\forall(u,v) \in X\left(\overrightarrow{C}_i\right)$ is solvable for $X \in \overrightarrow{G}^{\mathscr{V}}$.

In fact, such a solution is constructed by $X = \overrightarrow{G}^{L_{u(\mathbf{x})}}$ with $L_{u(\mathbf{x})}(u,v) = u(\mathbf{x})$ for $(u,v) \in X\left(\overrightarrow{G}\right)$ by applying the input and the output at vertex v all being $u(\mathbf{x})$ on $\overrightarrow{C}$, which implies that all flows at vertex $v \in V\left(\overrightarrow{G}\right)$ is conserved.

4.2 $\vec{G}$-flows on particle equation

The existence of $\overrightarrow{G}$-flow solutions on particle equations (1.1)–(1.3) is clearly concluded by Theorem 4.2, also implied by (3.1)–(3.3) for any $\overrightarrow{G}$. However, the superposition of a particle P shows that there are $N \ge 2$ states of being associated with a particle P. Considering this fact, a convenient $\overrightarrow{G}$-flow model for elementary particle fermions, the lepton or quark P is by a bouquet $\overrightarrow{B}_N^{L_\psi}$, and an antiparticle $\overline{P}$ of P presented by $\overrightarrow{B}_N^{L_{\psi^{-1}}}$ with all inverse states on its loops, such as those shown in Fig. 8.

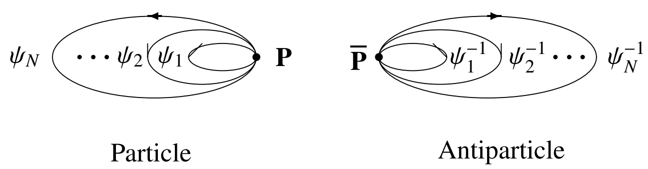

Fig. 8

An elementary unparticle is an intermediate form between an elementary particle and its antiparticle, which can be presented by $\overrightarrow{B}_N^{L_\psi^C}$, where $L_\psi^C : e \to L_{\psi^{-1}}(e)$ if $e \in C$ but $L_\psi^C : e \to L_\psi(e)$ if $e \in X\left(\overrightarrow{B}_N\right) \setminus C$ for a subset $C \subset X\left(\overrightarrow{B}_N\right)$, such as those shown in Fig. 9,

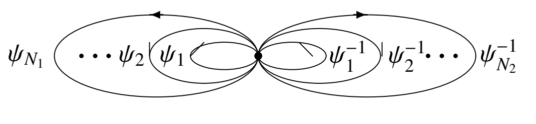

Fig. 9 Unparticle

where $N_1, N_2 \geq 1$ are integers. Thus, an elementary particle with its antiparticles maybe annihilate or appears in pair at a time, which consists in an elementary unparticle by combinations of these state functions with their inverses.

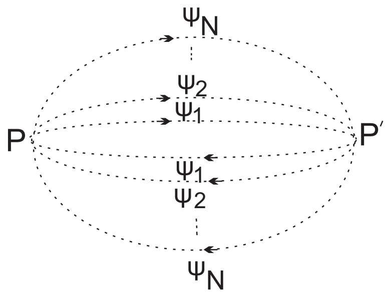

Fig. 10 $\overrightarrow{D^{\perp}}^{L_\psi}_{0,2N,0}$

For those of mediate interaction particle quanta, i.e., boson, which reflects interaction between particles. Thus, they are conveniently presented by dipole $\overrightarrow{D^{\perp}}^{L_\psi}_{0,2N,0}$ but with dotted lines, such as those in Fig. 10, in which the vertex P, P' denotes particles, and arcs with state functions $\psi_1, \psi_2, \cdots, \psi_N$ are the N states of P. Notice that $\overrightarrow{B}^{L_\psi}_N$ and $\overrightarrow{D^{\perp}}^{L_\psi}_{0,2N,0}$ both are a union of N circuits.

According to Theorem 4.2, we consequently get the following conclusion.

Theorem 4.3 *For an integer $N \geq 1$, there are indeed $\overrightarrow{D^{\perp}}^{L_\psi}_{0,2N,0}$ -flow solution on Klein-Gordon equation (1.2), and $\overrightarrow{B}^{L_\psi}_N$ -flow solution on Dirac equation (1.3).*

Generally, this model enables us to know that the $\overrightarrow{G}$-flow constituents of a particle also.

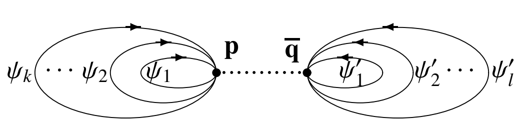

Fig. 11 Meson

Thus, if a particle $\widetilde{P}$ is consisted of l elementary particles $P_1, P_1, \cdots, P_l$ underlying a graph $\overrightarrow{G}\left[\widetilde{P}\right]$, its $\overrightarrow{G}$-flow is obtained by replace each vertex v by $\overrightarrow{B}^{L_{\psi_v}}_{N_v}$ and each arc e by $\overrightarrow{D^{\perp}}^{L_{\psi_e}}_{0,2N_e,0}$ in $\overrightarrow{G}\left[\widetilde{P}\right]$, denoted by $\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right]$. For example, the model of Sakata, or Gell-Mann and Ne'eman on hadrons claims that the meson and the baryon are respectively the dipole $\overrightarrow{D^{\perp}}^{L_{\psi_e}}_{k,2N,l}$-flow shown in Fig. 11 and the triplet $\overrightarrow{G}$-flow $\overrightarrow{C^{\perp}}^{L_\psi}_{k,l,s}$ shown in Fig. 12,

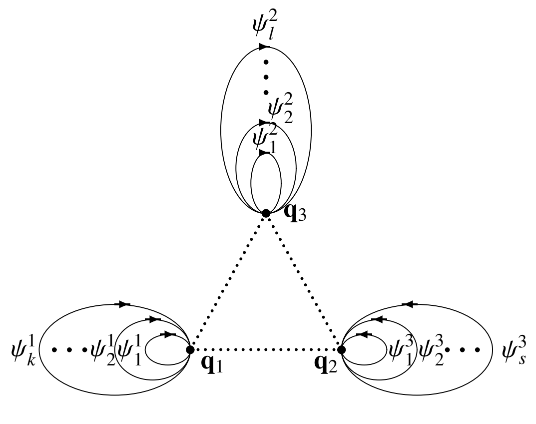

Fig. 12 Baryon

Theorem 4.4 *If $\widetilde{P}$ is a particle consisted of elementary particles $P_1, P_1, \cdots, P_l$ for an integer $l \geq 1$, then $\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right]$ is a $\overrightarrow{G}$-flow solution on the Schrödinger equation (1.1) whenever λ_G is finite or infinite.*

Proof If λ_G is finite, the conclusion follows Theorem 4.2 immediately. We only consider the case of $\lambda_G \to \infty$. In fact, if $\lambda_G \to \infty$, calculation shows that

$$\begin{aligned} & i\hbar \lim_{\lambda_G\to\infty}\left(\frac{\partial}{\partial t}\left(\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right]\right)\right) \\ &= \lim_{\lambda_G\to\infty}\left(i\hbar\frac{\partial}{\partial t}\left(\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right]\right)\right) \\ &= \lim_{\lambda_G\to\infty}\left(-\frac{\hbar^2}{2m}\nabla^2\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right] + \overrightarrow{G}^{L_U}\right) \\ &= -\frac{\hbar^2}{2m}\nabla^2 \lim_{\lambda_G\to\infty}\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right] + \overrightarrow{G}^{L_U}, \end{aligned}$$

i.e.,

$$\begin{aligned} & i\hbar \lim_{\lambda_G\to\infty}\left(\frac{\partial}{\partial t}\left(\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right]\right)\right) \\ &= -\frac{\hbar^2}{2m}\nabla^2 \lim_{\lambda_G\to\infty}\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right] + \overrightarrow{G}^{L_U}. \end{aligned}$$

In particular,

$$i\hbar \lim_{N\to\infty}\left(\frac{\partial \overrightarrow{B}^{L_\psi}_N}{\partial t}\right) = -\frac{\hbar^2}{2m}\nabla^2 \lim_{N\to\infty}\overrightarrow{B}^{L_\psi}_N + \overrightarrow{G}^{L_U},$$

$$i\hbar \lim_{N\to\infty}\frac{\partial}{\partial t}\left(\overrightarrow{D^{\perp}}^{L_\psi}_{0,2N,0}\right) = -\frac{\hbar^2}{2m}\nabla^2 \lim_{N\to\infty}\overrightarrow{D^{\perp}}^{L_\psi}_{0,2N,0} + \overrightarrow{G}^{L_U}$$

for bouquets and dipoles. □

5 $\vec{G}$-flow interpretation on particle superposition

The superposition of a particle P is depicted by a Hilbert space $\mathscr{V}$ over complex field $\mathbb{C}$ with orthogonal basis $|1\rangle, |2\rangle, \cdots, |n\rangle, \cdots$ in quantum mechanics. In fact, the linearity of Schrödinger equation concludes that all states of particle P are in such a space. However, an observer can grasp only one state, which promoted H. Everett devised a multiverse consisting of states in splitting process, i.e., the quantum effects spawn countless branches of the universe with different events occurring in each, not influence one another, such as those shown in Fig. 13, and the observer selects by randomness, where the multiverse is $\bigcup_{i\geq 1} \mathscr{V}_i$ with $\mathscr{V}_{kl} = \mathscr{V}$ for integers $k \geq 1,\ 1 \leq l \leq 2^k$ but in different positions.

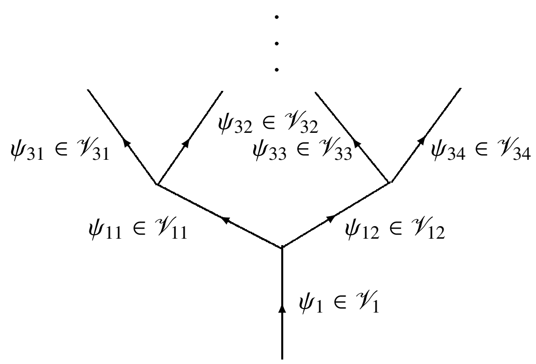

Fig. 13

Why it needs an interpretation on particle superposition in physics lies in that we characterize the behavior of particle by dynamic equation on state function and interpret it to be the solutions, and different quantum state holds with different solution of that equation. However, we can only get one solution by solving the equation with given initial datum once, and hold one state of the particle P, i.e., the solution correspondent only to one position but the particle is in superposition, which brought the H. Everett interpretation on superposition. It is only a biological mechanism by infinite parallel spaces $\mathscr{V}$ but loses of conservations on energy or matter in the nature, whose independently runs also overlook the existence of universal connection in things, a philosophical law.

Even so, it can not blot out the ideological contribution of H. Everett to sciences a shred because all of these mentions are produced by the interpretation on mathematical solutions with the reality of things, i.e., scanning on local, not the global. However, if we extend the Hilbert space $\mathscr{V}$ to $\overrightarrow{B}_N^{L_\psi}$, $\overrightarrow{D}^{\perp L_\psi}_{0,2N,0}$ or $\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right]$ in general, i.e., $\overrightarrow{G}$-flow space $\overrightarrow{G}^{\mathscr{V}}$, where $\overrightarrow{G}$ is the underling topological graph of P, the situation has been greatly changed because $\overrightarrow{G}^{\mathscr{V}}$ is itself a Hilbert space, and we can identify the $\overrightarrow{G}$-flow on $\overrightarrow{G}$ to particle P, i.e.,

$$P = \overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right] \tag{5.1}$$

for a globally understanding the behaviors of particle P whatever $\lambda_G \to \infty$ or not by Theorem 4.4. For example, let $P = \overrightarrow{B}_N^{L_\psi}$, i.e., a free particle such as those of electron e^-, muon μ^-, tauon τ^-, or their neutrinos ν_e, ν_μ, ν_τ. Then the superposition of P is displayed by state functions ψ on N loops in $\overrightarrow{B}_N$ hold on its each loop with

$$\text{input } \psi_i = \text{ ouput } \psi_i \text{ at vertex } P$$

for integers $1 \leq i \leq N$. Consequently,

$$\text{input } \sum_{i\in I} \psi_i = \text{ ouput } \sum_{i\in I} \psi_i \text{ at vertex } P$$

for $\forall I \subset \{1, 2, \cdots, N\}$, the conservation law on vertex P. Furthermore, such a $\overrightarrow{B}_N^{L_\psi}$ is not only a disguise on P in form but also a really mathematical element in Hilbert space $\overrightarrow{B}^{\mathscr{V}}$, and can be also used to characterize the behavior of particles such as those of the decays or collisions of particles by graph operations. For example, the β-decay $n \to p+e^-+\mu_e^-$ is transferred to a decomposition formula

$$\overrightarrow{C}^{\perp L_{\psi n}}_{k,l,s} = \overrightarrow{C}^{\perp L_{\psi p}}_{k_1,l_1,s_1} \bigcup \overrightarrow{B}^{L_{\psi e}}_{N_1} \bigcup \overrightarrow{B}^{L_{\psi\mu}}_{N_2},$$

on graph, where, $\overrightarrow{C}^{\perp L_{\psi p}}_{k_1,l_1,s_1}$, $\overrightarrow{B}^{L_{\psi e}}_{N_1}$, $\overrightarrow{B}^{L_{\psi\mu}}_{N_2}$ are all subgraphs of $\overrightarrow{C}^{\perp L_{\psi n}}_{k,l,s}$. Similarly, the β- collision $\nu_e+p \to n+e^+$ is transferred to an equality

$$\overrightarrow{B}^{L_{\psi\nu e}}_{N_1} \bigcup \overrightarrow{C}^{\perp L_{\psi p}}_{k_1,l_1,s_1} = \overrightarrow{C}^{\perp L_{\psi n}}_{k_2,l_2,s_2} \bigcup \overrightarrow{B}^{L_{\psi e}}_{N_2}.$$

Even through the relation (5.1) is established on the linearity, it is in fact truly for the linear and non-liner cases because the underlying graph of $\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right]$-flow can be decomposed into bouquets and dipoles, hold with conditions of Theorem 4.2. Thus, even if the dynamical equation of a particle P is non-linear, we can also adopt the presentation (5.1) to characterize the superposition and hold on the global behavior of P. Whence, it is a presentation on superposition of particles, both on linear and non-linear.

6 Further discussions

Usually, a dynamic equation on a particle characterizes its behaviors. But *is its solution the same as the particle*? Certainly not! Classically, a dynamic equation is established on characters of particles, and different characters result in different equations. Thus the superposition of a particle should be characterized by at least 2 differential equations. However, for a particle P, all these equations are the same one by chance, i.e., one of the Schrödinger equation, Klein-Gordon equation or Dirac equation, which lead to the many world interpretation of H. Everett, i.e., put a same equation or Hilbert space on different place for different solutions in Fig. 12. As it is shown in Theorems 4.1 – 4.4, we can interpret the solution of (1.1)–(1.3) to be a $\overrightarrow{G}^{L_\psi}\left[\overrightarrow{B}_v, \overrightarrow{D}_e\right]$-flow, which properly characterizes the superposition behavior of particles by purely mathematics.

The $\overrightarrow{G}$-flow interpretation on differential equation opens a new way for understanding the behavior of nature, particularly on superposition of particles. Generally, the dynamic equations on different characters maybe different, which will brings about contradicts equations, i.e., non-solvable equations. For example, we characterize the behavior of meson or baryon by Dirac equation (1.3). However, we never know the dynamic equation on quark. Although we can say it obeying the Dirac equation but it is not a complete picture on quark. If we find its equation some day, they must be contradicts because it appear in different positions in space for a meson or a baryon at least. As a result, the $\overrightarrow{G}$-solutions on non-solvable differential equations discussed in [9]–[12] are valuable for understanding the reality of the nature with $\overrightarrow{G}$-flow solutions a special one on particles.

As it is well known for scientific community, any science possess the falsifiability but which depends on known scientific knowledge and technical means at that times. Accordingly, it is very difficult to claim a subject or topic with logical consistency is truth or false on the nature sometimes, for instance the multiverse or parallel universes because of the limitation of knowing things in the nature for human beings. In that case, a more appreciated approach is not denied or ignored but tolerant, extends classical sciences and developing those of well known technical means, and then get a better understanding on the nature because the pointless argument would not essentially promote the understanding of nature for human beings ([3,4,22]).

Submitted on April 8, 2015 / Accepted on April 15, 2015

References

1. Bousso R. and Susskind L. Multiverse interpretation of quantum mechanics. *Physical Review*, 2012, D85 (4), DOI:10.1103; *Phys. Rev.* D.85.045007.
2. Everett H. Relative state formulation of quantum mechanics. *Rev. Mod. Phys.*, 1957, v. 29, 454–462.
3. George E. Does the multiverse really exist? *Scientific American*, 2011, v. 305 (2), 38–43.
4. Kragh H. Contemporary history of cosmology and controversy over the multiverse. *Annals of Science*, 2009, v. 66 (4), 529.
5. Tian Ma. View Physics by Mathematics – Elementary Particles and Unified Field Theory (in Chinese). Science Press, Beijing, 2014.
6. Linfan Mao. Combinatorial Geometry with Applications to Field Theory. The Education Publisher, USA, 2011.
7. Linfan Mao. Combinatorial fields – an introduction. *International J. Math. Combin.*, 2009, v. 3, 1–22.
8. Linfan Mao. Relativity in combinatorial gravitational fields. *Progress in Physics*, 2010, v. 6 (3), 39–50.
9. Linfan Mao. Global stability of non-solvable ordinary differential equations with applications. *International J. Math. Combin.*, 2013, v. 1, 1–37.
10. Linfan Mao. Geometry on G^L-systems of homogenous polynomials. *International J. Contemp. Math. Sciences*, 2014, v. 9 (6), 287–308.
11. Linfan Mao. Non-solvable equation systems with graphs embedded in $\mathbf{R}^n$. in Proceedings of the First International Conference on Smarandache Multispace and Multistructure, The Education Publisher, July 2013.
12. Linfan Mao. Cauchy problem on non-solvable system of first order partial differential equations with applications. *Methods and Applications of Analysis*, (Accepted for publication).
13. Linfan Mao. Extended Banach $\overrightarrow{G}$-flow spaces on differential equations with applications. *Electronic J. Mathematical Analysis and Applications*, 2015, v. 3 (2), 59–91.
14. Nambu Y. Quarks: Frontiers in Elementary Particle Physics. World Scientific Publishing, 1985.
15. Byrne Peter The many worlds of Hugh Everett III. *Scientific American*, December 2007, 98–105.
16. Quang Ho-Kim and Pham Xuan Yem. Elementary Particles and Their Interactions. Springer-Verlag, Berlin-Heidelberg, 1998.
17. Smarandache F. *Paradoxist Geometry*, State Archives from Valcea, Rm. Valcea, Romania, 1969, and in Paradoxist Mathematics, Collected Papers (Vol. II), Kishinev University Press, Kishinev, 1997, 5–28.
18. Smarandache F. Multi-space and multi-structure, in Neutrosophy – Neutrosophic Logic, Set, Probability and Statistics, American Research Press, 1998.
19. Smarandache F. A new form of matter – unmatter, composed of particles and anti-particles. *Progress in Physics*, 2005, v. 1 (1), 9–11.
20. Smarandache F. Verifying unmatter by experiments, more types of unmatter. *Progress in Physics*, 2005, v. 1 (2), 113–116.
21. Smarandache F. and Rabounski D. Unmatter entities inside nuclei, predicted by the Brightsen nucleon cluster model. *Progress in Physics*, 2006, v. 2 (1), 14–18.
22. Steinhardt P. Theories of anything. http://www.edge.org/response-detail/25405, 2014.
23. Tegmark M. Parallel universes in Science and Ultimate Reality: From Quantum to Cosmos. in Barrow J. D., Davies P. C. W. and Harper C. L. (eds), Cambridge University Press, 2003.
24. Tegmark M. Our mathematical universe: My quest for the ultimate nature of reality. Knopf Doubleday Publishing Group, 2014.