Scientia Magna
Vol. 14 (2019), No. 1, 1-8

A survey on Smarandache notions in number theory VI: Smarandache Ceil function

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Abstract In this paper we give a survey on recent results on Smarandache Ceil function.

Keywords Smarandache notion, Smarandache Ceil function, sequence, mean value.
2010 Mathematics Subject Classification 11A07, 11B50, 11L20, 11N25.

§1. Definition and simple properties

For any fixed positive integer k and any positive integer n, the famous Smarandache ceil function $S_k(n)$ is defined as follows:

$$S_k(n) = \min\left\{m \in \mathbb{N} :\ n \mid m^k\right\}. \tag{1.1}$$

Many people had studied elementary properties of $S_k(n)$, and obtained some interesting results.

Z. Xu [18]. *Define $\Omega(n) = \Omega(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_r^{\alpha_r}) = \alpha_1 + \alpha_2 + \cdots + \alpha_r$. Let k be a given positive integer. Then for any real number $x \geq 3$, we have the asymptotic formula*

$$\sum_{n \leq x} \Omega\left(S_k(n)\right) = x \ln\ln x + Ax + O\left(\frac{x}{\ln x}\right),$$

where $A = \gamma + \sum\limits_{p}\left(\ln\left(1 - \frac{1}{p}\right) + \frac{1}{p}\right)$, γ is the Euler constant and $\sum\limits_{p}$ denotes the sum over all the primes.

J. Li [8]. *Define $\Omega(n) = \Omega(p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_r^{\alpha_r}) = \alpha_1 + \alpha_2 + \cdots + \alpha_r$. Let k be a given positive integer. Then for any integer $n \geq 3$, we have the asymptotic formula*

$$\Omega\left(S_k(n!)\right) = \frac{n}{k}(\ln\ln n + C) + O\left(\frac{n}{\ln n}\right),$$

where C is a computable constant.

Y. Wang [15]. *Let k be a fixed positive integer, then for any integer $n \geq 3$, we have the asymptotic formula*

$$\ln(S_k(n!)) = \frac{n \ln n}{k} + O\left(n\right).$$

§2. Mean values of the Smarandache Ceil function

L. Ding [1]. *Let $x \geq 2$, for any fixed positive integer k, we have the asymptotic formula*

$$\sum_{n \leq x} S_k(n) = \frac{x^2 \zeta(2k-1)}{2} \prod_p \left[1 - \frac{1}{p(p+1)}\left(1 + \frac{1}{p^{2k-3}}\right)\right] + O\left(x^{\frac{3}{2}+\varepsilon}\right),$$

where $\zeta(s)$ is the Riemann zeta function, $\prod_p$ denotes the product over all prime p, and ε is any fixed positive number.

C. Wu [16]. 1) *For any fixed positive integer $k \geq 2$ and any positive integer n, let $a_k(n)$ denote the k-th power complements of n. Then we have*

$$(S_k(n))^k = a_k(n) \cdot n.$$

2) *Let k be a fixed positive integer. For any real number $x \geq 1$, we have the asymptotic formula*

$$\sum_{n \leq x} S_k(n) = \frac{\zeta(2k-1)}{2} x^2 \prod_p \left(1 - \frac{1}{p^2+p} - \frac{1}{p^{2k-1}+p^{2k-2}}\right) + O\left(x^{\frac{3}{2}+\varepsilon}\right),$$

where $\zeta(s)$ is the Riemann zeta function, $\varepsilon > 0$ is any fixed positive number.

X. Wang [13]. *For any real number $x \geq 2$, we have the asymptotic formula*

$$\sum_{n \leq x} \frac{1}{S_2(n)} = \frac{3 \ln^2 x}{2\pi^2} + A_1 \ln x + A_2 + O\left(x^{-\frac{1}{4}+\varepsilon}\right),$$

where A_1 and A_2 are two computable constants, ε is any fixed positive integer.

Y. Wang [14]. 1) *For any real number $\alpha > 1$ and integer $k \geq 2$, we have the identity*

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{S_k^{\alpha}(n)} = \frac{2^\alpha - k - 1}{2^\alpha + k - 1} \prod_p \left(1 + \frac{k}{p^\alpha - 1}\right),$$

where $\prod_p$ denotes the product over all prime p.

2) *For any positive integer n, the dual function of $S_k(n)$ is defined as $\overline{S_k}(n) = \max\left\{m \in \mathbb{N} : \ m^k \mid n\right\}$. For any real number $\alpha > 1$ and integer $k \geq 2$, we have the identities*

$$\sum_{n=1}^{\infty} \frac{\overline{S_k}(n)}{n^\alpha} = \frac{\zeta(\alpha)\zeta(k\alpha - 1)}{\zeta(k\alpha)},$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\overline{S_k}(n)}{n^\alpha} = \frac{\zeta(\alpha)\zeta(k\alpha - 1)}{\zeta(k\alpha)} \left[\frac{(2^\alpha - 1)(2^{k\alpha-1} - 1)}{2^{\alpha-2}(2^{k\alpha} - 1)} - 1\right],$$

where $\zeta(s)$ is the Riemann zeta function.

D. Ren [12]. *Let $d(n)$ denote the Dirichlet divisor function, and let k be a given positive integer with $k \geq 2$. Then for any real number $x \geq 1$, we have the asymptotic formula*

$$\sum_{n \leq x} d\left(S_k(n)\right) = \frac{6\zeta(k) x \ln x}{\pi^2} \prod_p \left(1 - \frac{1}{p^k + p^{k-1}}\right) + Cx + O\left(x^{\frac{1}{2}+\varepsilon}\right),$$

where $\zeta(s)$ is the Riemann zeta function, C is a computable constant, and ε is any fixed positive number.

X. He and J. Guo [7]. 1) *Let $\alpha > 0$, $\sigma_\alpha(n) = \sum_{d|n} d^\alpha$. Then for any real number $x \geq 2$, and any fixed positive integer $k \geq 2$, we have the asymptotic formula*

$$\sum_{n\leq x} \sigma_\alpha\left(S_k(n)\right) = \frac{6x^{\alpha+1}\zeta(\alpha+1)\zeta(k(\alpha+1)-\alpha)}{(\alpha+1)\pi^2} R(\alpha+1) + O\left(x^{\alpha+\frac{1}{2}}+\varepsilon\right),$$

where $\zeta(s)$ is the Riemann zeta function, ε is any fixed positive number, and

$$R(\alpha+1) = \prod_p \left(1 - \frac{1}{p^{k(\alpha+1)-\alpha} - p^{(k-1)(\alpha+1)}}\right).$$

2) *Let $d(n)$ denote the Dirichlet divisor function. Then for any real number $x \geq 1$, and any fixed positive integer $k \geq 2$, we have the asymptotic formula*

$$\sum_{n\leq x} d\left(S_k(n)\right) = \frac{6\zeta(k)x\ln x}{\pi^2} \prod_p \left(1 - \frac{1}{p^k + p^{k-1}}\right) + Cx + O\left(x^{\frac{1}{2}+\varepsilon}\right),$$

where $\zeta(s)$ is the Riemann zeta function, C is a computable constant, and ε is any fixed positive number.

L. Zhang, M. Lv and W. Zhai [20]. *Let $d_3(n)$ denote the Piltz divisor function of dimensional 3, then for any real number $x \geq 2$, we have*

$$\sum_{n\leq x} d_3\left(S_k(n)\right) = xP_{2,k}(\log x) + O\left(x^{\frac{1}{2}} e^{-c\delta(x)}\right),$$

where $P_{2,k}(\log x)$ is a polynlmial of degree 2 in $\log x$, $\delta(x) = \log^{\frac{3}{5}} x (\log\log x)^{-\frac{1}{5}}$, $c > 0$ is an absulute constant.

Y. Zhang, H. Liu and P. Zhao [21]. *Let $d(n)$ denote the Dirichlet divisor function, $S_k(n)$ denote the Smarandache ceil function, then for any real number $\frac{1}{4} < \theta < \frac{1}{3}$, $x^{\theta+2\varepsilon} \leq y \leq x$, we have*

$$\sum_{x<n\leq x+y} d(S_k(n)) = H(x+y) - H(x) + O\left(yx^{-\frac{\varepsilon}{2}} + x^{\theta+\varepsilon}\right),$$

where $H(x) = t_1 x\log x + t_2 x$, ε denotes a fixed but sufficiently small positive constant.

Q. Feng and R. Wang [4]. *For any positive integer n, we define*

$$a_k(n) = \left[n^{\frac{1}{k}}\right], \qquad n = 0, 1, 2, 3, \cdots.$$

Let $\zeta(s)$ be the Riemann zeta function, X be any positive number, and

$$g(s) = \prod_p (1 + p^{1-s} - p^{1-ks} - p^{-s}).$$

1) *For any real number $x \geq 1$, $k \geq 3$, we have*

$$\sum_{n\leq x} S_k\left(a_k(n)\right) = \frac{1}{k}\zeta(k-1)g(1)x + O\left(x^{1-\frac{1}{2k}+X}\right).$$

2) *For any real number* $x \geq 1$, $k \leq 2$, *we have*

$$\sum_{n\leq x} S_k\left(a_k(n)\right) = \frac{k}{k^2-k+2}\zeta\left(\frac{2}{k}\right) g\left(\frac{2}{k}\right) x^{\frac{k^2-k+2}{k^2}} + O\left(x^{\frac{k^2-k+2}{k^2}+X}\right).$$

Q. Feng, J. Guo and R. Wang [5]. *For any positive integer* n *and any natural number* m, *we define*

$$a_m(n) = \max\left\{i^m : \ i^m \leq n, \ i \in \mathbb{N}\right\}.$$

1) *For any real number* $x \geq 1$, $n, m, k, t \in \mathbb{N}$, $m, t \geq 2$, $k = tm + 1$, *we have*

$$\begin{aligned}\sum_{n\leq x} S_k\left(a_m(n)\right) &= \frac{m}{m+1}x^{1+\frac{1}{m}}\zeta(2t-1)\zeta((2t-1)m+2)\\ &\times\prod_p\left[1-\frac{1}{p(p+1)}\left(1+\frac{1}{p^{2t-3}}+\frac{1}{p^{(2t-1)m-1}}\left(1-\frac{1}{p^{2t}}\right)\right)\right]+O\left(x^{1+\frac{1}{2m}+\varepsilon}\right),\end{aligned}$$

where $\zeta(s)$ *is the Riemann zeta function,* ε *is any positive real number.*

2) *For any real number* $x \geq 1$, $n, m, k, t \in \mathbb{N}$, $m = 2$, $t \geq 2$, $k = 2t + 1$, *we have*

$$\sum_{n\leq x} S_k\left(a_m(n)\right) = \frac{2}{3}x^{\frac{3}{2}}\zeta(4t)\prod_p\left[1-\frac{1}{p(p+1)}\left(1+\frac{1}{p^{2t-1}}+\frac{1}{p^{2(t-1)}}\left(1-\frac{1}{p^{2t}}\right)\right)\right]+O\left(x^{\frac{5}{4}+\varepsilon}\right),$$

where $\zeta(s)$ *is the Riemann zeta function,* ε *is any positive real number.*

3) *For any real number* $x \geq 1$, $n, m, k, t \in \mathbb{N}$, $m, t \geq 2$, $k = tm$, *we have*

$$\sum_{n\leq x} S_k\left(a_m(n)\right) = \frac{m}{m+1}x^{1+\frac{1}{m}}\zeta(2t-1)\prod_p\left(1-\frac{p^{2t}+p^3}{p^{2t+2}+p^{2t+1}}\right)+O\left(x^{1+\frac{1}{2m}+\varepsilon}\right),$$

where $\zeta(s)$ *is the Riemann zeta function,* ε *is any positive real number.*

4) *For any real number* $x \geq 1$, $n, m, k, t \in \mathbb{N}$, $m, t \geq 2$, $m = kt$, *we have*

$$\sum_{n\leq x} S_k\left(a_m(n)\right) = \frac{m}{m+1}x^{1+\frac{t}{m}}+O\left(x^{1+\frac{t}{2m}+\varepsilon}\right),$$

where ε *is any positive real number.*

J. Xu [17]. *For any fixed positive integer* k *and any integer* n, *we define*

$$\begin{aligned}c_k(n) &= \min\left\{m^k : \ m^k \geq n, \ m \in \mathbb{N}^+\right\},\\ d_k(n) &= \max\left\{m^k : \ m^k \leq n, \ m \in \mathbb{N}^+\right\}.\end{aligned}$$

For any real number $x > 2$, *we have the asymptotic formula*

$$\sum_{n\leq x} S_k\left(c_k(n)\right) = \frac{x^2}{2}+O\left(x^{\frac{2k-1}{k}}\right), \qquad \sum_{n\leq x} S_k\left(d_k(n)\right) = \frac{x^2}{2}+O\left(x^{\frac{2k-1}{k}}\right).$$

L. Qi and Y. Zhao [11]. *Let* $k \geq 2$, $m \geq 1$ *be two positive integers. For any real number* $x \geq 1$, *we have the asymptotic formula*

$$\sum_{n\leq x} \varphi^m\left(S_k(n)\right) = \frac{6\zeta(m+1)\zeta(k(m+1)-m)R(m+1)x^{m+1}}{\pi^2(m+1)}+O\left(x^{m+\frac{1}{2}+\varepsilon}\right),$$

where $\zeta(s)$ is the Riemann zeta function, $\varphi(n)$ is the Euler function, ε is any positive real number, and

$$R(m+1)=\prod_p\left[1-\frac{1}{1+p}\left(\frac{1}{p}+\frac{1}{p^{k(m+1)-m}}+\frac{1}{p^{m-1}}-\frac{1}{p^{(k-1)(m+1)-m}}-\left(1-\frac{1}{p^{k(m+1)}}\right)\cdot\frac{1}{p}\left(1-\frac{1}{p}\right)^m\right)\right].$$

E. Lv [10]. *Define*

$$U(1)=1,\qquad U(n)=\prod_{p|n}p.$$

Let $k\geq 2$ be a fixed positive integer. For any real number $x\geq 1$, we have the asymptotic formula

$$\begin{aligned}\sum_{n\leq x}(S_k(n)-U(n))^2 \quad &= \quad \frac{2\zeta(3)\zeta(3k-2)x^3}{\pi^2}\prod_p\left(1-\frac{1+p^{5-3k}}{p^2+p^3}\right)+\frac{2\zeta(3)x^3}{\pi^2}\prod_p\left(1-\frac{1}{p^2+p^3}\right)\\ &\quad -\frac{4\zeta(3)\zeta(3k-1)x^3}{\pi^2}\prod_p\left(1+\frac{p-p^2-p^4-p^{3k}}{p^{3k+3}+p^{3k+2}}\right)+O\left(x^{\frac{5}{2}+\varepsilon}\right),\end{aligned}$$

where $\zeta(s)$ is the Riemann zeta function, $\varepsilon>0$ is any positive real number.

Y. Xue and L. Gao [19]. *Define*

$$U(1)=1,\qquad U(n)=\prod_{p|n}p.$$

Let $k\geq 2$ be a fixed positive integer. For any real number $x\geq 1$, we have the asymptotic formula

$$\begin{aligned}\sum_{n\leq x}(S_k(n)+U(n))^3 \quad &= \quad \frac{3\zeta(4)\zeta(4k-3)x^4}{2\pi^2}\prod_p\left(1-\frac{1+p^{7-4k}}{p^3+p^4}\right)\\ &\quad +\frac{9\zeta(4)\zeta(4k-2)x^4}{2\pi^2}\prod_p\left(1-\frac{1+p^{3-4k}+p^{6-4k}-p^{2-4k}}{p^3+p^4}\right)\\ &\quad +\frac{9\zeta(4)\zeta(4k-1)x^4}{2\pi^2}\prod_p\left(1-\frac{1+p^{5-4k}-p^{1-4k}+p^{3-4k}}{p^3+p^4}\right)\\ &\quad +\frac{3\zeta(4)x^4}{2\pi^2}\prod_p\left(1-\frac{1}{p^3+p^4}\right)+O\left(x^{\frac{7}{2}+\varepsilon}\right),\end{aligned}$$

where $\zeta(s)$ is the Riemann zeta function, ε is any positive real number.

§3. The dual function of the Smarandache Ceil function

For any positive integer n and any fixed positive integer k, the dual function of $S_k(n)$ is defined as follows:

$$\overline{S_k}(n)=\max\left\{m\in\mathbb{N}:\ m^k\mid n\right\}.$$

J. Guo and Y. He [6]. 1) *Let $\alpha>0$, $\sigma_\alpha(n)=\sum_{d|n}d^\alpha$. Then for any real number $x\geq 1$ and any fixed positive integer $k\geq 2$, we have the asymptotic formula*

$$\sum_{n\leq x}\sigma_\alpha\left(\overline{S_k}(n)\right)=\begin{cases}\frac{k\zeta\left(\frac{\alpha+1}{k}\right)}{\alpha+1}x^{\frac{\alpha+1}{k}}+O\left(x^{\frac{\alpha+1}{2k}+\varepsilon}\right), & \text{if}\ \ \alpha>k-1,\\ \zeta(k-\alpha)x+O\left(x^{\frac{1}{2}+\varepsilon}\right), & \text{if}\ \ \alpha\leq k-1,\end{cases}$$

where $\zeta(s)$ is the Riemann zeta function, and ε is any fixed positive number.

2) *Let $d(n)$ denote the Dirichlet divisor function. Then for any real number $x \geq 1$ and any fixed positive integer $k \geq 2$, we have*

$$\sum_{n \leq x} d\left(\overline{S_k}(n)\right) = \zeta(k)x + O\left(x^{\frac{1}{2}+\varepsilon}\right),$$

where $\zeta(s)$ is the Riemann zeta function, and ε is any fixed positive number.

Y. Lu [9]. *Let $d(n)$ denote the Dirichlet divisor function, and let $k \geq 2$ be a fixed integer. Then for any real number $x > 1$, we have the asymptotic formula*

$$\sum_{n \leq x} d\left(\overline{S_1}(n)\right) = x \ln x + (2\gamma - 1)x + O\left(x^{\frac{1}{3}}\right),$$

$$\sum_{n \leq x} d\left(\overline{S_k}(n)\right) = \zeta(k)x + \zeta\left(\frac{1}{k}\right)x^{\frac{1}{k}} + O\left(x^{\frac{1}{k+1}}\right),$$

where γ is the Euler constant, and $\zeta(s)$ is the Riemann zeta function.

L. Ding [2]. 1) *Let $x \geq 2$, for any fixed positive integer $k > 2$, we have the asymptotic formula*

$$\sum_{n \leq x} \overline{S_k}(n) = \frac{\zeta(k-1)}{\zeta(k)}x + O\left(x^{\frac{1}{2}+\varepsilon}\right),$$

where $\zeta(s)$ is the Riemann zeta function, and ε is any fixed positive number.

2) *For $k = 2$, we have the asymptotic formula*

$$\sum_{n \leq x} \overline{S_2}(n) = x\left(\frac{3}{\pi^2}\ln x + C\right) + O\left(x^{\frac{3}{4}+\varepsilon}\right),$$

where C is a computable constant, and ε is any fixed positive number.

Q. Feng and J. Guo [3]. *For any positive integer n and any fixed positive integer $k \geq 2$, we define*

$$c_k(n) = \min\left\{m \in \mathbb{N} :\ nm = t^k,\ t \in \mathbb{N}\right\}.$$

1) *For any real number $x \geq 1$, k, $n \in \mathbb{N}$, $k \geq 2$, we have*

$$\begin{aligned}\sum_{n \leq x} S_k(n)c_k(n) &= \frac{6}{(k+1)\pi^2}x^{k+1}\zeta(k+2)\zeta(k^2+k-1) \\ &\quad \times \prod_p \left(1 - \frac{1}{p^{k-1}(p+1)}\left(\frac{1}{p^2} + \frac{1}{p^{k^2-1}}\right)\right) + O\left(x^{k+\frac{1}{2}+\varepsilon}\right),\end{aligned}$$

where $\zeta(s)$ is the Riemann zeta function, and ε is any fixed positive number.

2) *For any real number $x \geq 1$, k, $n \in \mathbb{N}$, $k \geq 2$, we have*

$$\sum_{n \leq x} S_k(c_k(n)) = \frac{3}{\pi^2}x^2 \prod_p \left(1 + \frac{R(2)}{(p+1)(p^2-2)}\right) + O\left(x^{\frac{3}{2}+\varepsilon}\right),$$

where ε is any fixed positive number, and

$$R(2) = 1 - \frac{1}{p^{2(k-2)}} + \left(p^2\left(1 - \frac{1}{p^{2(k-1)}}\right) + p^3 - p\right)\frac{1}{p^{2k-1}}.$$

3) *For any real number $x \geq 1$, k, $n \in \mathbb{N}$, $k \geq 2$, we have*

$$\begin{aligned}\sum_{n\leq x}\overline{S_k}(n)c_k(n) &= \frac{6}{k\pi^2}x^k\zeta(k+1)\zeta(k^2-1)\\ &\times\prod_p\left(1-\frac{1}{p^k(p+1)}\left(1+\frac{1}{p^{k^2-k-1}}-\frac{1}{p^{k^2-1}}\right)\right)+O\left(x^{k+\frac{1}{2}+\varepsilon}\right),\end{aligned}$$

where $\zeta(s)$ is the Riemann zeta function, and ε is any fixed positive number.

4) *For any real number $x \geq 1$, k, $n \in \mathbb{N}$, $k \geq 2$, we have*

$$\sum_{n\leq x}\overline{S_k}(c_k(n)) = x + O\left(x^{\frac{1}{2}+\varepsilon}\right),$$

where ε is any fixed positive number

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