About Smarandache-Multiplicative Functions

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The main objective of this note is to introduce the notion of the S-multiplicative function and to give some simple properties concerning it. The name of S-multiplicative is short for Smarandache-multiplicative and reflects the main equation of the Smarandache function.

Definition 1. A function $f: N^* \rightarrow N^*$ is called S-multiplicative if :

(1) $(a,b) = 1 \Rightarrow f(a * b) = \max \{ f(a), f(b) \}$

The following functions are obviously S-multiplicative:

1. The constant function $f: N^* \rightarrow N^*$, $f(n) = 1$.
2. The Erdos function $f: N^* \rightarrow N^*$, $f(n) = \max \{ p \mid p \text{ is prime and } n:p \}$. [1].
3. The Smarandache function $S: N^* \rightarrow N$, $S(n) = \max \{ p \mid p! : n \}$. [3].

Certainly, many properties of multiplicative functions[2] can be translated for S-multiplicative functions. The main important property of this function is presented in the following.

Definition 2. If $f: N^* \rightarrow N$ is a function, then $\bar{f}: N^* \rightarrow N$ is defined by

$$\bar{f}(n) = \min \{ f(d) \mid n:d \}.$$

Theorem 1. If f is S-multiplicative function, then $\bar{f}$ is S-multiplicative.

Proof. This proof is made using the following simple remark:

(2). $(d|a*b \wedge (a,b)=1) \Rightarrow ((\exists d_1 \mid a)(\exists d_2 | b)(d_1, d_2) = 1 \wedge d = (d_1 * d_2)$

If d_1 and d_2 satisfy (2), then $f(d_1 * d_2) = \max\{f(d_1), f(d_2)\}$.

Let a,b be two natural numbers, such that $(a,b) = 1$. Therefore, we have

(3) $\bar{f}(a * b) = \min\limits_{d|a*b} f(d) = \min\limits_{d_1|a, d_2|a} f(d_1 * d_2) = \min\limits_{d_1|a} \min\limits_{d_2|a} \max \{ f(d_1), f(d_2) \}$.

Applying the distributing property of the max and min functions, equation (3) is transformed as follows:

$\bar{f}(a*b) = \max \{ \min\limits_{d_1|a} f(d_1), \min\limits_{d_2|a} f(d_2) \} = \max \{ \bar{f}(a), \bar{f}(b) \}$. Therefore,

the function $\bar{f}$ is S-multiplicative.

We believe that many other properties can be deduced for S-multiplicative functions. Therefore, it will be in our attention to further investigate these functions.

References

1. Erdos, P.: (1974) **Problems and Result in Combinatorial Number Theory**, Bordaux.

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3. F. Smarandache:(1980) 'A Function in Number Theory', Analele Univ. Timisoara, XVIII.