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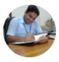

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Acceptance sampling plans for two-stage process for multiple manufacturing lines under neutrosophic statistics

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Abstract. In this paper, a new acceptance-sampling plan has been introduced for the two-stage process for multiple lines under the neutrosophic statistics. The parameters of the proposed sampling plan have been determined by satisfying the given risks using the optimization solution under the neutrosophic statistical interval method (NSIM) Using the specific producer's and consumer's risks, the parameters of the proposed plan have also been determined under neutrosophic operating function (NOF). The comparison based on the sample size of the proposed and the existing plans has been given at different plan parameters. The tables are provided, and an industrial example is illustrated for the practical use of the proposed sampling plan. The comparison reveals that the proposed plan is more efficient, flexible, and adequate to be used under uncertainty.

Keywords: Neutrosophic statistic, EWMA, sampling plan, producer's and consumer's risks, multistage neutrosophic, average sample number

1. Introduction

Acceptance sampling plans are most commonly espoused in the manufacturing industries to provide safeguards to the producer and consumer, to increase the quality of the product and to insert the pressure on the producer [1]. Two types of sampling plans are adopted, i.e., attribute plans based upon the go/no go bifurcations of the products and the variable plans based upon the measurements of the interested quality characteristics such as height, weight, length, etc. for the protection of the wishes of the two parties. The variable sampling plans provide more details of the products as compared to the attribute sampling plans which are easier to apply and understand. For more detail, the readers may refer to [2].

The performance of the proposed plan is judged through constructing the operating characteristic (OC) curve which quantifies risks of parties, the vendor and the buyer. The OC curve portrays the discriminating power of the sampling plan with the probability of accepting the individual lot against the fraction nonconforming. The vendor (producer) generally concentrate on the specific quality level of the product generally called the acceptable quality level (AQL) with at least $(1-\alpha)$ probability of accepting the lot. The AQL also characterize the maximum level of the nonconforming item in a lot supposed to be accepted on average. Likewise, the buyer (consumer) generally concentrates on the low-

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est point on the OC curve called the Lot tolerance percent defective (LTPD), which is the poorest quality level, with the lowest probability β of accepting the defective lot. For more details, readers are referred to [3–12].

When the acceptance or the rejection of the submitted lot depends upon the single sample then it is called a simple or single sampling scheme. There are many situations in which the decision of the submitted lot cannot be decided at first sample and the second sample help us to decide whether to accept or reject the lot as this decision depends upon the two-stage sample selection [13]. If the lot is too good or too bad, then at the first stage the lot is accepted or rejected. When we are not in a position of deciding its fate at the first stage then we switch to the second stage where we take another sample of the same size and then we decided the fate of the lot based on combining the information from both the samples [14]. In this modern era, the demand for the high-quality product has increased, and customer and producer want delivery of his order at a time. To meet the given order, the product is manufactured using multiple manufacturing lines. In this situation, the sampling plan designed for multiple independent lines is applied for the quick inspection of the product. According to [15] "Due to sufficient demands of consumers, it may not be possible to fulfill the demand in time using the process based on one manufacturing line. Therefore, the product is manufactured in the industry using multiple manufacturing lines". For more details, the reader may refer to [16, 17].

The technique of exponentially weighted moving average (EWMA) was developed by [18] for the quick monitoring of the process in the literature of control chart. This technique is applied extensively in the acceptance sampling plans. In EWMA statistic more weights (between 0 and 1) are assigned to the current observations and the most historic observations are assigned with least weights [7, 19–21].

In practice, sometimes, the experimenters are not certain about the proportion of defective in the lot. In this situation, the inspection of a lot is done using the fuzzy-based acceptance sampling plans. Therefore, the acceptance sampling plan using fuzzy logic is applied for the inspection of a lot of the product when some parameters are uncertain. Due to the wide applications of such sampling plans, several authors designed fuzzy-based sampling plans, including for example, [22, 23] designed the single fuzzy attribute acceptance plans; [24] worked on a fuzzy plan to select a process; [25] proposed sampling plan having fuzzy parameters; [26] studied OC curves using a fuzzy approach; [27] developed a fuzzy plan for the Poisson distribution; [28] proposed the fuzzy double sampling plan for the inspection of a lot; [29] proposed the fuzzy plan for the gamma distribution. More details on the fuzzy-based sampling plan can be seen in [30–35]. More information on the application of the fuzzy approach can be seen in [36–46].

Smarandache [47] stated that the neutrosophic logic which consists of truth, falsehood, and indeterminacy measures is the extension of the traditional fuzzy logic. Neutrosophic statistics is basically the extension of the classical statistics introduced by [48] when the data are imprecise, vague, ambiguous or incomplete. The neutrosophic statistics has been widely used in a variety of fields. The application of the neutrosophic statistics in the rock measuring issues can be seen in [49, 50]. For example, the application in the area of the control charts can be seen in [51, 52]. Recently, [53] introduced the neutrosophic statistics in the area of acceptance sampling plan; [54] designed a sudden death test under the neutrosophic statistics; [55] designed the sampling plan for the exponential distribution using the neutrosophic statistics; [56] proposed the sampling plan for the inspection at a single stage of the product coming from multiple independent lines.

By exploring the literature and the best of our knowledge, there is no work on the sampling plan using two-stage for multiple independent manufacturing lines under the neutrosophic statistics. There is a gap to develop a two-stage plan for the inspection of the product coming from for multiple independent manufacturing lines in uncertainty. In this paper, a new plan using the two-stage sampling for multiple lines under the neutrosophic statistics has been explored, which according to the best knowledge of the authors has not been studied yet. We expect that the proposed plan will be more efficient than the existing plan in terms of sample size under uncertainty environment. We expect that the proposed two-stage plan under neutrosophic statistics will be quite flexible, informative, and adequate to be used under uncertainty environment.

2. Design of the traditional neutrosophic plan

In this section, the design of the acceptance-sampling plan for the traditional neutrosophic statistic is explained. As mentioned by [57] "manufacturing process with multiple manufacturing lines often consists of multiple parallels independent manufacturing lines, with each manufacturing line having a machine or a group of machines performing necessary identical job operations. As the manufacturing lines have various process averages and standard deviations, the values of capability indices will be different for each manufacturing line. The combined output of all manufacturing lines leads to inaccurate yield measures of the process". Suppose the neutrosophic random variable (NRV), say $X_{Ni} \in [X_L, X_U]$ recoded from line j at time i have a neutrosophic sample number $n_{N1j} \in \{n_{L1j}, n_{U1j}\}$, where $j = 1, 2, 3, \ldots, k$. Also suppose that the lower and upper means are $\bar{x}_L = \sum_{i=1}^{n} x_i^L / n_L$, $\bar{x}_U = \sum_{i=1}^{n} x_i^U / n_U$, the lower and upper standard deviations, neutrosophic process capability index $\hat{S}_{pkjN}$ for k independent manufacturing lines and normal distribution of $\hat{S}_{pkjN}$ from [56] are as follows

$$s_L = \sqrt{\sum_{i=1}^{n} (x_i^L - \bar{x}_L)^2 / n_L},$$
$$s_U = \sqrt{\sum_{i=1}^{n} (x_i^U - \bar{x}_U)^2 / n_U}.$$

Then

$$\bar{x}_{jN} \in \left\{ \bar{x}_L = \sum_{i=1}^{n} x_i^L / n_L, \bar{x}_U = \sum_{i=1}^{n} x_i^U / n_U \right\}; \bar{x}_{jN} \in [\bar{x}_{jL}, \bar{x}_{jU}] \tag{1}$$

$$s_{jN} \in \left\{ s_L = \sqrt{\sum_{i=1}^{n} (x_i^L - \bar{x}_L)^2 / n_L}, s_U = \sqrt{\sum_{i=1}^{n} (x_i^U - \bar{x}_U)^2 / n_U} \right\}; s_{jN} \in [s_{jL}, s_{jU}] \tag{2}$$

$$\widehat{S}^{M}_{pkN} = \frac{1}{3}\Phi^{-1}\left\{ \left[\frac{1}{k} \sum_{j=1}^{k} (2\Phi(3\widehat{S}_{pkjN}) - 1) + 1 \right] \Big/ 2 \right\}; \quad \widehat{S}^{M}_{pkN} \in \left\{ \widehat{S}^{M}_{pkL}, \widehat{S}^{M}_{pkU} \right\} \tag{3}$$

where $\Phi^{-1}(x)$ denotes the inverse of standard normal distribution.

$$\widehat{S}_{pkjN} = \frac{1}{3}\Phi^{-1}\left\{ \frac{1}{2}\Phi\left(\frac{USL - \widehat{X}_{jN}}{S_{jN}} \right) + \frac{1}{2}\Phi\left(\frac{\widehat{X}_{jN} - LSL}{S_{jN}} \right) \right\}; \quad \widehat{S}^{M}_{pkjN} \in \left\{ \widehat{S}^{M}_{pkjL}, \widehat{S}^{M}_{pkjU} \right\} \tag{4}$$

or

$$\hat{S}^{M}_{pkN} \sim N_N \left(S^{M}_{pkN}, \frac{D_N^2 \phi^2(3D_N)}{2k^2 n_N \phi^2(3S^{M}_{pkN})} \right); \quad n_N \in \{n_L, n_U\}, \; \hat{S}^{M}_{pkN} \in \{\hat{S}^{M}_{pkL}, \; \hat{S}^{M}_{pkU}\} \tag{5}$$

where

$$D_N = (1/3)\Phi^{-1}\{[k(2\Phi(3S^{M}_{pkN}) - 1) - (k-2)]/2\} \tag{6}$$

and $N_N(x)$ represents the neutrosophic normal distribution.

Stage-1: Select NRV of size $n_{N1j} \in \{n_{L1j}, n_{U1j}\}$, $j = 1, 2, 3, \ldots, k$ and recoded from line j at time i. Determine $\hat{S}^{M}_{pkN}$. If $\hat{S}^{M}_{pkN1} \geq c_{N1}$, where $c_{N1} \in \{c_{L1}, c_{U1}\}$ accept a lot of product and implement Step-2, if $\hat{S}^{M}_{pkN} < c_{N1}$, reject a lot of the product.

Stage-2: Select NRV $n_{N2j} \in \{n_{L2j}, n_{U2j}\}$, $j = 1, 2, 3, \ldots, k$ and recoded from line j at time i. Accept the lot if $\hat{S}^{M}_{pkN2} \geq c_{N2}$, where $c_{N2} \in \{c_{L2}, c_{U2}\}$ is neutrosophic acceptance number.

The proposed sampling plan is the extensions of several acceptance-sampling plans. The proposed plan reduces to [56] plan when $\hat{S}^{M}_{pkN1} = \hat{S}^{M}_{pkN2} = \hat{S}^{M}_{pkN}$. The proposed sampling plan reduces to [15] plan if no uncertain observations/parameters are noted.

The neutrosophic operating characteristic (NOC) function of the traditional plan can be derived as follows:

$$L_N\left(\hat{S}^{M}_{pkN}\right) = \pi_{AN}\left(\hat{S}^{M}_{pkN1}\right) \times \pi_{AN}\left(\hat{S}^{M}_{pkN2}\right) \tag{7}$$

For-k stage, the NOC is given by

$$L_N(\hat{S}^{M}_{pkN}) = \prod_{w=1}^{k} \pi_{AN}(\hat{S}^{M}_{pkN_w}) \; w = 1, 2, 3, \ldots, k \tag{8}$$

By following, [15] and [56], we have

$$\pi_{AN}(\overset{\frown}{S}{}^{M}_{pkN1}) = 1 - \Phi \left((c - \overset{\frown}{S}{}^{M}_{pkN1} / \sqrt{\left[\frac{D_{N1}^{2} \phi^{2}(3D_{N1})}{2k^{2}(n_{N1})\phi^{2}(3S^{M}_{pkN1})} \right]} \right) \quad (9)$$

$$\pi_{AN}(\overset{\frown}{S}{}^{M}_{pkN2}) = 1 - \Phi \left((c - \overset{\frown}{S}{}^{M}_{pkN2} / \sqrt{\left[\frac{D_{N2}^{2} \phi^{2}(3D_{N2})}{2k^{2}(n_{N2})\phi^{2}(3S^{M}_{pkN2})} \right]} \right) \quad (10)$$

The average sample number (ASN) of the proposed plan is given by

$$n_{N1} + n_{N2}\pi_{AN}(\hat{S}^{M}_{pkN1}) + n_{N3}\pi_{AN}(\hat{S}^{M}_{pkN1}) \times \pi_{AN}(\hat{S}^{M}_{pkN2}) + \ldots ; n_{N1j} \in \left\{ n_{L1j}, n_{U1j} \right\}, \; n_{N2j} \in \left\{ n_{L2j}, n_{U2j} \right\} \quad (11)$$

The plan parameters of the proposed sampling will be determined using the following non-optimization solution under the neutrosophic statistical interval method.

Minimize

$$n_{N1} + n_{N2}\pi_{AN}(\hat{S}^{M}_{pkN1}) + n_{N3}\pi_{AN}(\hat{S}^{M}_{pkN1}) \times \pi_{AN}(\hat{S}^{M}_{pkN2}) + \ldots ; n_{N1j} \in \left\{ n_{L1j}, n_{U1j} \right\}, \; n_{N2j} \in \left\{ n_{L2j}, n_{U2j} \right\} \quad (12a)$$

Subject to

$$L_{N}(C_{AQL}) = \prod\nolimits_{w=1}^{k} \pi_{AN}(\hat{S}^{M}_{pkN_w}) \geq 1 - \alpha \quad (12b)$$

$$L_{N}(C_{LTPD}) = \prod\nolimits_{w=1}^{k} \pi_{AN}(\hat{S}^{M}_{pkN_w}) \leq \beta \quad (12c)$$

A neutrosophic acceptance sampling plan is considered as an efficient and effective if it fulfills the requirements of the producer's risk β and the consumer's risk α. Therefore, the NOC must pass through the points of the AQL and LTPD.

The plan parameters of the proposed plan are determined through the grid search method using Equations (12a)to (12c). The plan parameters are selected by the grid search method. During, this process, it is noted that several combinations of the parameters are available that meet the given constraints. Among these combinations, we choose the plan parameters having the smaller average sample number (ASN). Tables 1 and 2 show the estimated design parameters for $k = 2$ and 3 at the risk levels of 0.05 and 0.10 of the producer and consumer, respectively, for various values of AQL and LTPD. The parametric values of n_1, n_2, c_1, c_2, and ASN are generated using the R-codes under different process settings. The similar tables can be generated for different parametric values under the above settings.

3. Design of neutrosophic EWMA plan

In this section, the design of the proposed sampling plan for the neutrosophic EWMA statistic is explained. The proposed plan under the neutrosophic statistics is stated as

Stage-1: Select NRV $n_{N1j} \in \{n_{L2j}, n_{U1j}\}$, $j = 1, 2, 3, \ldots, k$ and recoded from line j at time i. Compute the following statistics

$$\hat{S}^{M\ EWMA_i}_{pkN1} = \lambda \hat{S}^{M}_{pkN} + (1 - \lambda)\hat{S}^{MEWMA_{i-1}}_{pkN};$$

$$\hat{S}^{MEWMA_i}_{pkN1} \in \left[\hat{S}^{MEWMA_i}_{pkL1}, \hat{S}^{MEWMA_i}_{pkU1} \right].$$

Table 1
Design parameters of the traditional multistage neutrosophic plan when $k = 2$

C_{AQL}	C_{LTPD}	$\alpha = 0.05, \beta = 0.05$			$\alpha = 0.05, \beta = 0.10$		
		$n_1 = n_2$	$c_1 = c_2$	ASN	$n_1 = n_2$	$c_1 = c_2$	ASN
1.00	0.90	[284, 413]	[0.9247, 0.9378]	[562.4, 817.7]	[234, 314]	[0.9190, 0.9304]	[462.84, 620.9]
1.10	1.00	[382, 498]	[1.0256, 1.0391]	[757.2, 983.7]	[287, 339]	[1.0194, 1.0255]	[567.0, 669.9]
1.20	1.10	[438, 530]	[1.1255, 1.1256]	[866.7, 1053.1]	[354, 461]	[1.1180, 1.1249]	[700.1, 913.7]
1.30	1.20	[538, 795]	[1.2253, 1.2412]	[1065.2, 1570.4]	[470, 551]	[1.2188, 1.2225]	[931.3, 1093.4]
1.40	1.30	[694, 833]	[1.3296, 1.3303]	[1372.9, 1654.0]	[578, 681]	[1.3176, 1.3258]	[1147.0, 1350.0]
1.50	1.40	[813, 849]	[1.4249, 1.4326]	[1613.3, 1677.5]	[649, 729]	[1.4182, 1.4213]	[1286.3, 1446.1]
1.60	1.50	[880, 1136]	[1.5275, 1.5295]	[1740.6, 2257.2]	[676, 887]	[1.5188, 1.5210]	[1335.7, 1761.7]
1.70	1.60	[1096, 1209]	[1.6300, 1.6339]	[2168.8, 2391.3]	[822, 1102]	[1.6216, 1.6274]	[1623.9, 2184.8]
1.80	1.70	[1151, 1346]	[1.7278, 1.7296]	[2276.7, 2669.3]	[869, 1160]	[1.7187, 1.7207]	[1716.8, 2304.7]
1.90	1.80	[1424, 1564]	[1.8290, 1.8333]	[2821.1, 3096.2]	[990, 1230]	[1.8186, 1.8278]	[1956.8, 2429.6]
2.00	1.90	[1472, 1539]	[1.9289, 1.9294]	[2910.7, 3045.5]	[1115, 1314]	[1.9198, 1.9245]	[2202.55, 2598.8]

Table 2
Design parameters of the traditional multistage nutrosophic plan when $k = 3$

C_{AQL}	C_{LTPD}	$\alpha = 0.05, \beta = 0.05$			$\alpha = 0.05, \beta = 0.10$		
		$n_1 = n_2$	$c_1 = c_2$	ASN	$n_1 = n_2$	$c_1 = c_2$	ASN
1.00	0.90	[246, 352]	[0.9293, 0.9305]	[486.0, 700.4]	[193, 283]	[0.9191, 0.9282]	[389.7, 561.5]
1.10	1.00	[324, 462]	[1.0249, 1.0414]	[642.3, 912.5]	[257, 377]	[1.0186, 1.0233]	[508.6. 750.1]
1.20	1.10	[409, 609]	[1.1275, 1.1308]	[809.4, 1212.5]	[339, 460]	[1.1168, 1.1234]	[672.2, 914.7]
1.30	1.20	[529, 599]	[1.2268, 1.2357]	[1048.6, 1183.2]	[458, 548]	[1.2236, 1.2272]	[906.6, 1086.8]
1.40	1.30	[681, 799]	[1.3238, 1.3354]	[1353.9, 1582.9]	[501, 651]	[1.3199, 1.3308]	[991.7, 1287.5]
1.50	1.40	[795, 869]	[1.4302, 1.4356]	[1574.4, 1717.6]	[581, 724]	[1.4218, 1.4236]	[1147.9, 1436.6]
1.60	1.50	[873, 1051]	[1.5268, 1.5343]	[1730.3, 2081.6]	[619, 885]	[1.5189, 1.5222]	[1222.4, 1758.9]
1.70	1.60	[1008, 1343]	[1.6276, 1.6279]	[1996.9, 2674.6]	[774, 917]	[1.6202, 1.6266]	[1530.6, 1813.4]
1.80	1.70	[1110, 1323]	[1.7291, 1.7342]	[2194.6, 2617.5]	[861, 999]	[1.7191, 1.7237]	[1702.8, 1977.3]
1.90	1.80	[1215, 1443]	[1.8260, 1.8331]	[2405.5, 2854.8]	[963, 1206]	[1.8188, 1.8221]	[1904.3, 2393.0]
2.00	1.90	[1383, 1518]	[1.9290, 1.9325]	[2733.2, 2999.2]	[1051, 1188]	[1.9193, 1.9226]	[2076.0, 2349.1]

If $\hat{S}_{pkN1}^{MEWMA_i} \geq C_{N1}$, where $C_{N1} \in \{C_{L1}, C_{U1}\}$ accept a lot of product and implement Step-2, if $\hat{S}_{pkN}^{MEWMA_i} < C_{N1}$, reject a lot of the product.

Stage-2: Select NRV $n_{N2j} \in \{n_{L2j}, n_{U2j}\}$, $j = 1, 2, 3, \ldots, k$ and recoded from line j at time i. Accept the lot if $\hat{S}_{pkN_2}^{MEWMA_i} \geq C_{N2}$, where $C_{N2} \in \{C_{L2}, C_{U2}\}$ is neutrosophic acceptance number.

The NOC for this case is given by

$$L_N\left(\hat{S}_{pkN}^{M}\right) = \pi_{AN}\left(\hat{S}_{pkN1}^{M}\right) \times \pi_{AN}\left(\hat{S}_{pkN2}^{M}\right);$$

$$L_N\left(\hat{S}_{pkN}^{M}\right) \in \left[L_N\left(\hat{S}_{pkL}^{M}\right), L_N\left(\hat{S}_{pkU}^{M}\right)\right] \tag{13}$$

where

$$\pi_{AN}(\hat{S}_{pkN1}^{M}) = 1 - \Phi \left(\frac{c - S_{pkN1}^{M}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D_{N1}^2\varphi^2(3D_{N1})}{2k^2(nN_1)\varphi^2\left(3S_{pkN1}^{M}\right)}\right]}}\right),$$

$$\pi_{AN}\left(\hat{S}_{pkN1}^{M}\right) \in \left[\pi_{AL}\left(\hat{S}_{pkL1}^{M}\right), \pi_{AU}\left(\hat{S}_{pkU1}^{M}\right)\right] \tag{14}$$

$$\pi_{AN}(\hat{S}_{pkN_2}^{M}) = 1 - \Phi \left(\frac{c - S_{pkN_2}^{M}}{\sqrt{(\lambda/(2-\lambda))\left[\frac{D_{N_2}^2\varphi^2(3D_{N2})}{2k^2(nN_2)\varphi^2\left(3S_{pkN_2}^{M}\right)}\right]}}\right),$$

$$\pi_{AN}\left(\hat{S}_{pkN2}^{M}\right) \in \left[\pi_{AL}\left(\hat{S}_{pkL2}^{M}\right), \pi_{AU}\left(\hat{S}_{pkU2}^{M}\right)\right] \tag{15}$$

The NOC function of the proposed plan is derived with probabilities of the risks attached with the producer and the consumer as $\pi_{AN}(\hat{S}_{pkN_1}^{MEWMA_i})$ and $\pi_{AN}(\hat{S}_{pkN_2}^{MEWMA_i})$, respectively. Two points on the NOC function are given as $\left(\pi_{AN}(\hat{S}_{pkN_1}^{M}) \geq 1-\alpha\right)$ and $\left(\pi_{AN}(\hat{S}_{pkN_2}^{M}), \beta\right)$ which satisfy the producer's risk and the consumer's risk using the following neutrosophic non-linear optimization solution.

Minimize

$$\begin{aligned} &n_{N1} + n_{N2}\pi_{AN}(\hat{S}_{pkN_1}^{M}) + n_{N3}\pi_{AN}(\hat{S}_{pkN_1}^{M}) \\ &\quad \times \pi_{AN}(\hat{S}_{pkN_2}^{M}) + \ldots, n_{N1j} \in \{n_{L1j}, n_{U1j}\}, \\ &\quad \in \{n_{L2j}, n_{U2j}\} \end{aligned} \tag{16a}$$

Subject to

$$L_N(C_{AQL}) = \prod\nolimits_{w=1}^{k} \pi_{AN}(\hat{S}_{pkN_W}^{M}) \geq 1-\alpha \tag{16b}$$

$$L_N(C_{LTPD}) = \prod\nolimits_{w=1}^{k} \pi_{AN}(\hat{S}_{pkN_W}^{M}) \leq \beta \tag{16c}$$

The design parameters of the proposed Neutrosophic plan are estimated using the search grid method and Equations (13) through (17) mentioned above. Tables 3 and 4 show the estimated design parameters for $k = 2, \lambda = 0.10$, and for $k = 3, \lambda = 0.10$ for the risk levels of 0.05 and 0.10 of the producer and consumer, respectively, for various values of AQL and LTPD. The parametric values of n_1, n_2, c_1, c_2, and ASN are generated using the R-codes under different process settings. The similar tables can be generated for different parametric values under the above settings.

Table 3
Design Parameters of the multistage neutrosophic EWMA plan when $k = 2, \lambda = 0.10$

C_{AQL}	C_{LTPD}	$\alpha = 0.05, \beta = 0.05$			$\alpha = 0.05, \beta = 0.10$		
		$n_1 = n_2$	$c_1 = c_2$	ASN	$n_1 = n_2$	$c_1 = c_2$	ASN
1.00	0.90	[14,23]	[0.9256, 0.9394]	[27.6, 45.5]	[13, 22]	[0.9231, 0.9232]	[25.7, 43.9]
1.10	1.00	[22, 29]	[1.0306, 1.0394]	[43.6, 57.4]	[16, 23]	[1.0193, 1.0324]	[31.7, 45.5]
1.20	1.10	[28, 41]	[1.1300, 1.1350]	[55.5, 81.6]	[20, 34]	[1.1187, 1.1402]	[39.6, 67.1]
1.30	1.20	[32, 55]	[1.2252, 1.2273]	[63.5, 109.8]	[23, 32]	[1.2189, 1.2285]	[45.5, 63.4]
1.40	1.30	[34, 48]	[1.3288, 1.3323]	[67.2, 95.4]	[30, 35]	[1.3194, 1.3262]	[59.4, 69.3]
1.50	1.40	[40, 63]	[1.4260, 1.4327]	[79.2, 125.4]	[32, 40]	[1.4197, 1.4253]	[63.3, 79.2]
1.60	1.50	[54, 78]	[1.5245, 1.5276]	[107.4, 155.6]	[38, 45]	[1.5208, 1.5282]	[75.1, 88.9]
1.70	1.60	[56, 72]	[1.6261, 1.6363]	[111.0, 142.6]	[46, 68]	[1.6188, 1.6351]	[91.2, 134.6]
1.80	1.70	[64, 84]	[1.7273, 1.7313]	[126.8, 167.0]	[58, 83]	[1.7235, 1.7388]	[114.9, 164.1]
1.90	1.80	[66, 98]	[1.8282, 1.8346]	[130.4, 194.6]	[61, 91]	[1.8180, 1.8182]	[121.1, 181.6]
2.00	1.90	[83, 97]	[1.9294, 1.9357]	[164.3, 191.9]	[64, 95]	[1.9223, 1.9263]	[126.5, 188.9]

Table 4
Design parameters of the multistage neutrosophic EWMA plan when $k = 3, \lambda = 0.10$

C_{AQL}	C_{LTPD}	$\alpha = 0.05, \beta = 0.05$			$\alpha = 0.05, \beta = 0.10$		
		$n_1 = n_2$	$c_1 = c_2$	ASN	$n_1 = n_2$	$c_1 = c_2$	ASN
1.00	0.90	[12, 22]	[0.9255, 0.9333]	[23.7, 43.8]	[11, 16]	[0.9179, 0.9276]	[21.8, 31.8]
1.10	1.00	[19, 23]	[1.0233, 1.0278]	[37.8, 45.8]	[16, 21]	[1.0166, 1.0339]	[31.8, 41.6]
1.20	1.10	[21, 33]	[1.1263, 1.1316]	[41.6, 65.7]	[17, 24]	[1.1197, 1.1279]	[33.6, 47.6]
1.30	1.20	[28, 34]	[1.2274, 1.2370]	[55.5, 67.2]	[21, 34]	[1.2196, 1.230]	[41.5, 67.6]
1.40	1.30	[32, 45]	[1.3267, 1.340]	[63.4, 89.0]	[28, 39]	[1.3241, 1.3243]	[55.4, 77.6]
1.50	1.40	[39, 49]	[1.4286, 1.4330]	[77.2, 97.2]	[31, 42]	[1.4202, 1.4212]	[61.3, 83.6]
1.60	1.50	[43, 61]	[1.5264, 1.5366]	[85.1, 120.9]	[37, 44]	[1.5178, 1.5273]	[73.4, 87.1]
1.70	1.60	[50, 71]	[1.6276, 1.6381]	[98.9, 140.6]	[41, 53]	[1.6221, 1.6225]	[81.0, 105.3]
1.80	1.70	[57, 81]	[1.7292, 1.7352]	[112.6, 160.7]	[46, 59]	[1.7208, 1.7275]	[90.9, 116.8]
1.90	1.80	[62, 80]	[1.8263, 1.8264]	[122.6, 159.1]	[54, 65]	[1.8212, 1.8271]	[106.8, 128.6]
2.00	1.90	[80, 97]	[1.9256, 1.9304]	[158.8, 192.8]	[62, 74]	[1.9192, 1.930]	[122.8, 146.2]

Table 5
Comparison of the sample size for proposed and existing neutrosophic plans when $k = 3$

C_{AQL}	C_{LTPD}	$\alpha = 0.05, \beta = 0.05$				$\alpha = 0.05, \beta = 0.10$			
		Traditional plan		EWMA plan for $\lambda = 0.10$		Traditional plan		EWMA plan for $\lambda = 0.10$	
		Proposed $n_1 = n_2$	Existing n	Proposed $n_1 = n_2$	Existing n	Proposed $n_1 = n_2$	Existing n	Proposed $n_1 = n_2$	Existing n
1.00	0.90	[246, 352]	[775, 905]	[12, 22]	[28, 61]	[193, 283]	[717, 799]	[11, 16]	[41, 86]
1.10	1.00	[324, 462]	[692, 875]	[19, 23]	[32, 89]	[257, 377]	[829, 969]	[16, 21]	[42, 78]
1.20	1.10	[409, 609]	[792, 961]	[21, 33]	[36, 93]	[339, 460]	[878, 986]	[17, 24]	[38, 81]
1.30	1.20	[529, 599]	[813, 960]	[28, 34]	[57, 76]	[458, 548]	[837, 972]	[21, 34]	[33, 70]
1.40	1.30	[681, 799]	[1248, 1424]	[32, 45]	[49, 81]	[501, 651]	[1160, 1433]	[28, 39]	[38, 61]
1.50	1.40	[795, 869]	[1198, 1364]	[39, 49]	[59, 97]	[581, 724]	[1203, 1387]	[31, 42]	[44, 77]
1.60	1.50	[873, 1051]	[1781, 1943]	[43, 61]	[112, 148]	[619, 885]	[1186, 1481]	[37, 44]	[48, 73]
1.70	1.60	[1008, 1343]	[1488, 1600]	[50, 71]	[105, 143]	[774, 917]	[1451, 1735]	[41, 53]	[65, 97]
1.80	1.70	[1110, 1323]	[1891, 1995]	[57, 81]	[81, 112]	[861, 999]	[1827, 2017]	[46, 59]	[93, 136]
1.90	1.80	[1215, 1443]	[2144, 2319]	[62,80]	[142, 161]	[963, 1206]	[1884, 2064]	[54, 65]	[97, 132]
2.00	1.90	[1383, 1518]	[2439, 2572]	[80, 97]	[127, 173]	[1051, 1188]	[2178, 2376]	[62, 74]	[106, 129]

4. Comparison of the proposed sampling plan

As mentioned earlier, [56] designed the sampling plan for multiple manufacturing lines using the single sampling plan under the neutrosophic statistics. In the area of acceptance sampling plans, a plan having the smaller sample size is said to be more efficient than the other sampling plan. In this Section, the comparative study of the proposed sampling plan under neutrosophic statistics and the plan proposed by [56] in terms of neutrosophic sample size is discussed. Table 5 presents the comparison of the proposed and the existing neutrosophic plan by [56] for $k = 3$. For rational comparison, we used the same plan parameters of $k = 3, \alpha = 0.05, \beta = 0.05$ for different settings of the AQL and LTPD with $\lambda = 0.10$.

Table 6
Stage-1 data from Integrated Circuit [56]

Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
[8.0584,8.6021]	[9.8074,9.8074]	[9.0717,9.0717]	[8.0032,8.0032]	[9.7851,9.7851]	**[8.2868,9.9745]**
[8.1291,8.1291]	[9.9382,9.9382]	[8.4836,8.4826]	[8.123,8.123]	[9.7055,9.7055]	[9.3139,9.3139]
[8.2945,8.2945]	[9.7405,9.7405]	[8.8189,8.8189]	[8.3639,8.3639]	[9.5367,9.5367]	[8.9274,8.9274]
[8.4041,8.4041]	[9.7714,9.7714]	[8.7452,8.7452]	[7.8184,7.8184]	[9.7462,9.7462]	[8.8426,8.8426]
[7.7078,9.0267]	[9.9143,9.9143]	**[9.1409,9.7321]**	[7.9084,7.9084]	[9.8264,9.8264]	**[8.6537,9.5664]**
[7.9494,7.9494]	[9.7012,9.7012]	[9.1566,9.1566]	[8.4156,8.4156]	[9.8275,9.8275]	[8.9899,8.9899]
[8.1295,8.1295]	[9.7931,9.7931]	[9.0607,9.0607]	[7.9281,7.9281]	[9.8429,9.8429]	[9.2326,9.2326]
[8.0902,8.0902]	[9.6253,9.6253]	**[8.7977,9.6640]**	[8.0968,8.0968]	[9.8771,9.8771]	[8.9034,8.9034]
[8.5301,8.5301]	[9.7565,9.7565]	[9.1246,9.1246]	[7.9588,7.9588]	[9.8513,9.8513]	[9.0191,9.0191]
[8.224,8.7721]	[9.7282,9.7282]	[8.9472,8.9472]	[7.9915,7.9915]	[9.8122,9.8122]	[9.7331,9.7331]
[8.2187,8.2187]	[9.7240,9.7240]	**[8.8287,9.3749]**	**[7.9198,9.0781]**	[9.7033,9.7033]	[8.9759,8.9759]
[8.5358,8.5358]	[9.5096,9.5096]	**[9.3396,9.6910]**	[8.2088,8.2088]	[9.7222,9.7222]	[9.1025,9.1025]
[7.9681,8.9214]	[9.5861,9.5861]	[9.3521,9.3521]	[8.3243,8.3243]	[9.6909,9.6909]	[9.2906,9.2906]
[8.0459,9.2937]	[9.8542,9.8542]	[8.9805,8.9805]	[7.9551,7.9551]	[9.6893,9.6893]	**[8.5702,10.2741]**
[8.4624,8.4624]	[9.5363,9.5363]	**[8.4378,8.8956]**	[8.1552,8.1552]	[9.550,9.550]	[8.7624,8.7624]
[8.0555,8.0555]	[9.3871,9.3871]	**[8.4064,9.2068]**	[8.2015,8.2015]	[9.6551,9.6551]	[8.7146,8.7146]
[7.80,8.642]	[10.0586,10.0586]	[9.0542,9.0542]	**[8.1281,8.8649]**	[9.6923,9.6923]	[9.4084,9.4048]
[7.8861,7.8861]	[10.0815,10.0815]	[8.7694,8.7694]	[7.9844,7.9844]	[9.7844,9.7844]	**[8.4058,10.1128]**
[7.8745,7.8745]	[9.9046,9.9046]	[9.2664,9.2664]	[8.2926,8.2926]	[9.7143,9.7143]	[8.8485,8.8485]
[8.4662,8.4662]	[9.7467,9.7467]	**[8.5586,10.0325]**	[8.1087,8.1087]	[9.6478,9.6478]	[9.0328,9.0328]
[8.0458,8.0458]	[9.6709,6709]	[9.1405,9.1405]	[8.3324,8.3324]	[9.6576,9.6576]	[9.6685,9.6685]
[8.0045,8.7302]	[9.7344,7344]	[8.9628,8.9628]	[8.4791,8.4791]	[9.5404,9.5404]	[9.3008,9.3008]
[8.0504,8.0504]	[9.6369,9.6369]	[9.1336,9.1336]	[8.2146,8.2146]	[9.7357,9.7357]	**[9.0707,10.2547]**
[8.009,8.009]	[9.6128,9.6128]	[9.4906,9.4906]	**[8.1495,8.9654]**	[9.8667,9.8667]	[9.4463,9.4463]

Table 7
Stage-2 data from Integrated Circuit.

Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
[8.0708, 8.0708]	[9.5133, 9.5133]	[8.6584, 8.6584]	[8.3185, 8.3185]	[9.9076, 9.9076]	**[9.4311, 9.1388]**
[8.3509, 8.3509]	[9.6289, 9.6289]	[9.3193, 9.3193]	[7.8323, 7.8323]	[9.6366, 9.6366]	[8.6268, 8.6268]
[7.8602, 7.8602]	[9.7673, 9.7673]	[8.8598, 8.8598]	[8.0665, 8.0665]	[9.7275, 9.7275]	[8.5388, 8.5388]
[8.2283, 8.2283]	[9.6516, 9.6516]	[8.9783, 8.9783]	**[7.9771, 8.1034]**	[9.4666, 9.4666]	[9.1463, 9.1463]
[8.2108, 8.2108]	[10.0002, 10.0002]	**[8.9256, 9.6205]**	[7.9914, 7.9914]	**[9.7932, 9.6365]**	[9.2710, 9.2710]
[8.3085, 8.3103]	[9.6452, 9.6452]	[8.8284, 8.8284]	[8.2473, 8.2473]	[9.6184, 9.6184]	**[8.5749, 9.2096]**
[8.1447, 8.1447]	[9.9697, 9.9697]	[8.6173, 8.6173]	[8.3019, 8.3019]	[9.9736, 9.9736]	[9.2501, 9.2501]
[8.2874, 8.2874]	[9.7209, 9.7209]	[9.0968, 9.0968]	[8.1039, 8.1039]	[9.6291, 9.6291]	[9.430, 9.430]
[8.2931, 8.2931]	[9.6153, 9.6153]	[8.8931, 8.8931]	[8.2821, 8.2821]	[9.5490, 9.5490]	**[8.5334, 8.7648]**
[7.9639, 8.4273]	[9.6081, 9.6081]	[8.5153, 8.5153]	[8.4999, 8.4999]	[9.5421, 9.5421]	[8.5644, 8.5644]
[7.9715, 8.3501]	**[9.6636, 9.8742]**	[9.1225, 9.1225]	**[7.9666, 8.2191]**	[9.8959, 9.8959]	[8.6912, 8.6912]
[8.0278, 8.0278]	[9.8082, 9.8082]	[9.2602, 9.2602]	**[8.2123, 8.0858]**	**[9.6344, 9.7619]**	[8.7276, 8.7276]
[8.1233, 8.1233]	[9.6968, 9.6968]	**[8.8786, 9.3549]**	[7.8696, 7.8696]	[10.0278, 10.0278]	[9.1666, 9.1666]
[7.8822, 7.8822]	[9.7702, 9.7702]	[9.4992, 9.4992]	[8.1297, 8.1297]	[9.8847, 9.8847]	[8.6763, 8.6763]
[8.1990, 8.1990]	[9.7158, 9.7158]	[9.4527, 9.4527]	[7.5497, 7.5497]	[9.9141, 9.9141]	[9.5516, 9.5516]
[7.8548, 8.4414]	[9.6738, 9.6738]	[9.5686, 9.5686]	[7.9525, 7.9525]	[9.9832, 9.9832]	**[9.7439, 9.2329]**
[8.3947, 8.3947]	[9.4693, 9.4693]	**[9.0160, 9.0290]**	**[8.2020, 7.8922]**	[9.5640, 9.5640]	**[8.8642, 9.6527]**
[8.3639, 8.3639]	[9.7897, 9.7897]	[8.7788, 8.7788]	**[7.9584, 8.5099]**	[9.7901, 9.7901]	[8.5171, 8.5171]
[8.1526, 8.1526]	**[9.5057, 9.6895]**	[9.4034, 9.4034]	[8.0077, 8.0077]	**[9.6312, 9.6734]**	[8.6802, 8.6802]
[8.1437, 8.1437]	[9.8515, 9.8515]	[8.8452, 8.8452]	[7.8801, 7.8801]	[9.7792, 9.7792]	[8.6721, 8.6721]
[8.2153, 8.2153]	[9.8294, 9.8294]	[8.5760, 8.5760]	[7.8115, 7.8115]	[9.8181, 9.8181]	[8.6131, 8.6131]
[8.0206, 8.0206]	[9.6377, 9.6377]	[9.0379, 9.0379]	[8.4469, 8.4469]	[9.7221, 9.7221]	**[8.9924, 9.1311]**
[7.9505, 8.0401]	[9.5844, 9.5844]	[8.9846, 8.9846]	**[7.9840, 8.0550]**	[9.5426, 9.5426]	[8.8524, 8.8524]
[8.0408, 8.0408]	[9.8111, 9.8111]	[9.1359, 9.1359]	[8.0597, 8.0597]	[9.5720, 9.5720]	[9.1835, 9.1835]

It can be seen that the neutrosophic sample size for the lot sentencing is smaller for the proposed scheme as compared with the existing scheme by [56]. It can be seen from Table 5 that when $k = 3$, $\alpha = 0.05$, $\beta = 0.05$, $C_{AQL} = 1.00$, $C_{LTPD} = 0.90$ the values of $n_N \in \{n_L, n_U\}$ for the proposed chart are 246 and 352 while the existing plan shows 775 and 905 for the same process settings. The same results can

Table 8
Neutrosophic statistics of IC

Lines	$\bar{X}_{jN}$	S_{jN}	$\widehat{S}_{pkjN}$
		Stage-1	
1	[8.1250, 8.3103]	[0.2027, 0.3620]	[1.0947, 0.8316]
2	[9.7350, 9.7350]	[0.1351, 0.1351]	[1.9267, 1.9267]
3	[8.9910, 9.2469]	[0.3286, 0.4211]	[1.5210, 1.0597]
		Stage-2	
1	[8.0981, 8.1416]	[0.1930, 0.2052]	[1.0996, 1.1084]
2	[9.7194, 9.7279]	[0.1492, 0.1466]	[1.8091, 1.7975]
3	[8.9698, 9.0151]	[0.3338, 0.3333]	[1.4919, 1.4986]

be seen for other process settings given in Table 5. So it can be claimed without any doubt that the proposed plan of the neutrosophic EWMA statistics utilizes a much smaller sample size for deciding about the lot acceptance or rejection.

5. Applications

In this section, we present the application of the proposed sampling plan in gold bumping factor situated in Taiwan and the pre-amplifier of micro-electro-mechanical systems (MEMS) dataset.

5.1. Example 1

The gold bumping data is taken from [58]. The final gold product is manufactured using several multiple manufacturing lines. According to [58] "For a new mass production product, FHD1080H (FHD, 1920 × 1080 RGB), a quality practitioner of (integrated circuit) IC design house employs a sampling plan for the gold bump product acceptance determination. Because the manufacturer's factory involves three manufacturing lines, the inspection data are collected from the lines separately. In current factory practices, five designate die sites are inspected on a wafer at the location of the top, center, bottom, left, and right. In addition, four bump sites including the left side of the top, the right side of the top, the left side of down, and the right side of down are inspected on one die site". As pointed out by [56] that the gold bumping data is measurement and may consist of some unclear, fuzzy, and indeterminate observations. Therefore, this type of data cannot be used for the inspection of the product under the classical statistics. In addition, the experimenter is interested to use a two-stage plan to ensure the

Table 9
Stage-1 data of the absolute reference voltage

Line 1	Line 2	Line 3	Line 4
[1.5101, 1.5101]	**[1.5057, 1.5050]**	[1.4929, 1.4929]	[1.5092, 1.5092]
[1.5077, 1.5077]	[1.4937, 1.4937]	[1.4933, 1.4933]	[1.5065, 1.5065]
[1.5104, 1.5092]	[1.4969, 1.4969]	**[1.4939, 1.4930]**	[1.5097, 1.5097]
[1.5098, 1.5098]	[1.4938, 1.4938]	**[1.4879, 1.4880]**	[1.5090, 1.5090]
[1.5052, 1.5052]	[1.4987, 1.4987]	[1.4958, 1.4958]	[1.5035, 1.5035]
[1.5074, 1.5074]	[1.4929, 1.4929]	[1.4935, 1.4935]	[1.5062, 1.5062]
[1.5050, 1.5050]	**[1.4924, 1.4863]**	[1.4992, 1.4992]	**[1.5033, 1.5030]**
[1.5108, 1.5067]	[1.4925, 1.4925]	[1.4932, 1.4932]	[1.5101, 1.5101]
[1.5094, 1.5094]	**[1.4956, 1.4942]**	[1.4912, 1.4912]	[1.5084, 1.5084]
[1.5091,1.5078]	[1.4929, 1.4929]	**[1.4952, 1.4978]**	[1.5081, 1.5081]
[1.4988, 1.5007]	[1.4943, 1.4943]	[1.4967, 1.4967]	[1.4961, 1.4961]
[1.5079, 1.5079]	[1.4977, 1.4977]	**[1.4943, 1.4960]**	[1.5067, 1.5067]
[1.5077, 1.5077]	**[1.4948, 1.4920]**	[1.4919, 1.4919]	**[1.5064, 1.5060]**
[1.5023, 1.5023]	**[1.4999, 1.4997]**	[1.4936, 1.4936]	[1.5002, 1.5002]
[1.5069, 1.5069]	[1.4979, 1.4979]	[1.4957, 1.4957]	[1.5055, 1.5055]
[1.5038, 1.5031]	[1.4991, 1.4991]	[1.4988, 1.4988]	**[1.5020, 1.5013]**
[1.5065, 1.4992]	[1.5010, 1.5010]	**[1.4904, 1.4964]**	[1.5050, 1.5050]
[1.5068, 1.5068]	[1.4912, 1.4912]	[1.4922, 1.4922]	[1.5054, 1.5054]
[1.5064, 1.5064]	**[1.4945, 1.4909]**	[1.4924, 1.4924]	[1.5050, 1.5050]
[1.5003, 1.5003]	[1.4976, 1.4976]	[1.4979, 1.4979]	[1.4978, 1.4978]
[1.5065, 1.5065]	[1.4993, 1.4993]	[1.4954, 1.4954]	[1.5051, 1.5051]
[1.5019, 1.5019]	[1.4965, 1.4965]	[1.4976, 1.4976]	[1.4997, 1.4997]
[1.5068, 1.5070]	**[1.5019, 1.5021]**	[1.4962, 1.4962]	[1.5054, 1.5054]
[1.5028, 1.5028]	[1.4913, 1.4913]	**[1.4920, 1.4958]**	[1.5007, 1.5007]
[1.5043, 1.5043]	**[1.4967, 1.4892]**	[1.4944, 1.4944]	**[1.5025, 1.5011]**
[1.5044, 1.5044]	[1.4967, 1.4967]	[1.4919, 1.4919]	**[1.5026, 1.5020]**
[1.5057, 1.5057]	[1.4919, 1.4919]	[1.4910, 1.4910]	[1.5041, 1.5041]
[1.5025, 1.4956]	[1.4936, 1.4936]	**[1.4950, 1.4945]**	[1.5005, 1.5005]
[1.5079, 1.5079]	[1.5041, 1.5041]	[1.4912, 1.4912]	[1.5067, 1.5067]
[1.5079, 1.5079]	[1.4964, 1.4964]	**[1.4953, 1.4985]**	[1.5067, 1.5067]

Table 10
Stage-2 data of the absolute reference voltage

Line 1	Line 2	Line 3	Line 4
[1.5043, 1.5043]	[1.4934, 1.4934]	[1.4940, 1.4940]	[1.5056, 1.5056]
[1.5092, 1.5079]	[1.4980, 1.4980]	[1.4942, 1.4942]	[1.5032, 1.5032]
[1.5170, 1.5170]	[1.4968, 1.4968]	**[1.4934, 1.4932]**	[1.5021, 1.5021]
[1.5081, 1.5081]	**[1.4946, 1.4960]**	[1.4976, 1.4976]	[1.5103, 1.5103]
[1.5071, 1.5071]	[1.4917, 1.4917]	**[1.4925, 1.4918]**	[1.5051, 1.5051]
[1.5042, 1.5042]	**[1.4974, 1.4980]**	[1.4961, 1.4961]	**[1.5069, 1.5065]**
[1.5075, 1.5075]	**[1.4938, 1.4942]**	[1.4934, 1.4934]	[1.4951, 1.4951]
[1.5100, 1.5100]	[1.4914, 1.4914]	[1.4932, 1.4932]	[1.5105, 1.5105]
[1.5107, 1.5088]	**[1.4944, 1.4964]**	[1.4948, 1.4948]	[1.5068, 1.5068]
[1.5068, 1.5068]	[1.5000, 1.5000]	[1.4935, 1.4935]	[1.5053, 1.5053]
[1.5056, 1.5056]	[1.4979, 1.4979]	[1.5003, 1.5003]	[1.5131, 1.5131]
[1.5084, 1.5084]	[1.4918, 1.4918]	[1.4910, 1.4910]	[1.5031, 1.5031]
[1.5018, 1.5002]	[1.5015, 1.5015]	**[1.4983, 1.4944]**	[1.5071, 1.5071]
[1.5058, 1.5058]	[1.4914, 1.4914]	[1.4965, 1.4965]	**[1.5046, 1.5032]**
[1.5034, 1.5034]	**[1.4988, 1.5006]**	[1.4973, 1.4973]	[1.5017, 1.5017]
[1.5085, 1.5085]	[1.4950, 1.4950]	**[1.4941, 1.4929]**	[1.5035, 1.5035]
[1.5108, 1.5108]	**[1.4907, 1.4957]**	[1.4948, 1.4948]	[1.5054, 1.5054]
[1.5067, 1.5067]	[1.4969, 1.4969]	[1.4969, 1.4969]	**[1.5033, 1.5018]**
[1.5047, 1.5023]	[1.4938, 1.4938]	[1.4926, 1.4926]	[1.5063, 1.5063]
[1.5053, 1.5036]	[1.5016, 1.5016]	**[1.4979, 1.4956]**	**[1.5070, 1.5010]**
[1.5088, 1.5088]	[1.4907, 1.4907]	[1.4938, 1.4938]	**[1.4984, 1.5003]**
[1.5043, 1.5043]	[1.5018, 1.5018]	[1.4883, 1.4883]	[1.5011, 1.5011]
[1.4999, 1.5001]	**[1.4924, 1.4929]**	[1.4946, 1.4946]	[1.5127, 1.5127]
[1.5083, 1.5083]	**[1.4953, 1.4951]**	[1.4984, 1.4984]	[1.5037, 1.5037]
[1.5103. 1.5103]	[1.4956, 1.4956]	[1.4913, 1.4913]	[1.5060, 1.5060]
[1.5037, 1.5031]	[1.4950, 1.4950]	[1.4909, 1.4909]	[1.5008, 1.5008]
[1.5026, 1.5026]	[1.4992, 1.4992]	**[1.4988, 1.4965]**	**[1.5030, 1.5014]**
[1.5020, 1.5020]	[1.4925, 1.4925]	[1.4941, 1.4941]	[1.5063, 1.5063]
[1.5063, 1.5060]	**[1.4925, 1.4945]**	[1.4898, 1.4898]	[1.5043, 1.5043]
[1.5100, 1.5100]	[1.4912, 1.4912]	**[1.4952, 1.4950]**	[1.5040, 1.5040]

high-quality of the product. Therefore, he decided to use the proposed sampling plan. The application of the proposed plan has been explained by using the integrated data of Stage-I given in Table 6 and integrated data of Stage-II given in Table 7. The mean and standard deviation of Stage-1 and Stage-II of Neutrosophic statistics of IC are given in Tables 8 and 9, respectively. The experimenters are interested to inspect the gold bombing product using the proposed sampling plan. For this experiment, let $k = 3, \lambda = 0.1$, $\alpha = \beta = 0.05$, $C_{AQL} = 1.50$, $C_{LTPD} = 1.40$. From Table 4, we note $n_N \in [39, 49]$ and $C_N \in [1.4286, 1.4330]$. Suppose the experimenters decide to select a random sample $n_1 = n_2 = 48$ from the production process. The statistics of this example based on two-stage process are shown in Table 8. According to the proposed plan at stage 1, accept a lot of the product as $\hat{S}^{MEWMA_i}_{pkN_1} \in [8.1746, 8.2819] > C_N \in [1.4286, 1.4330]$ and go to stage-2. We will finally accept the product as $\hat{S}^{MEWMA_i}_{pkN_2} \in [8.15597, 8.16097] > C_N \in [1.4286, 1.4330]$ at stage 2.

5.2. Example 2

MEMS is a system that converts pressure, sound, humidity, or other measurements into electrical signals. The main components of MEMS are amplifiers, mechanical sensors, and signal processing circuits. The pre-amplifier, designed to amplify and to filter the signals from MEMS sensors, is the pivotal component of MEMS as its performance affects the capability of MEMS. The absolute reference voltage (ARV) is the main quality characteristic of a pre-amplifier. The ARV must be resistant to environmental fluctuations such as humidity, temperature, or supply voltage. The variations in ARV needs careful monitoring and controlled to ensure the consistent performance of MEMS. For more details, see [59].

The ARV measurements of a certain G-sensor module are collected from four manufacturing lines and data are given in Tables 9, 10 which contain some unclear, fuzzy, and indeterminate observations as indicated by the bold entries. Due to the fuzzy nature of the ARV measurements, we use a two-stage

Table 11
Neutrosophic statistics of the absolute reference voltage

Lines	$\bar{X}_{jN}$	S_{jN}	$\widehat{S}_{pkjN}$
		Stage-1	
1	[1.5061, 1.5055]	[0.0030, 0.0034]	[1.0579, 1.0037]
2	[1.4964, 1.4956]	[0.0037, 0.0044]	[1.0939, 0.8840]
3	[1.4940, 1.4945]	[0.0026, 0.0027]	[1.2146, 1.2328]
4	[1.5046, 1.5045]	[0.0035, 0.0036]	[1.0594, 1.0422]
		Stage-2	
1	[1.5067, 1.5064]	[0.0035, 0.0036]	[0.8725, 0.8778]
2	[1.4952, 1.4957]	[0.0034, 0.0033]	[1.0684, 1.1450]
3	[1.4946, 1.4942]	[0.0028, 0.0026]	[1.2041, 1.2392]
4	[1.5049, 1.5046]	[0.0038, 0.0038]	[0.9612, 0.9858]

neutrosophic acceptance sampling plan to ensure the high-quality of the product as a replacement to traditional single sampling plans. The pre-amplifier considered in this study has a target ARV of 1.5V with LSL = 1.485V and USL = 1.515V. The two-stage neutrosophic test procedure is evaluated by using $k = 2, \lambda = 0.1, \alpha = \beta = 0.05$, $C_{AQL} = 1.20$, and $C_{LTPD} = 1.10$. Thus, the required sample size and neutrosophic acceptance number are $n_1 = n_2 = 30$ and $c_1 = c_2 = [1.130, 1.135]$, respectively. The neutrosophic statistics for ARV dataset are computed and provided in Table 11. As described in Section 3, we accept the lot at Stage-1 as $\hat{S}_{pkN_1}^{M\,EWMA_i} = [1.44701, 1.44863] > [1.130, 1.135]$ and move to Stage-2. At this stage, we finally accept the lot since $\hat{S}_{pkN_2}^{M\,EWMA_i} = [1.45903, 1.44843] > [1.130, 1.135]$.

6. Concluding remarks

In this paper, a new acceptance-sampling plan has been introduced for the two-stage process for multiple lines under the neutrosophic statistics. The proposed sampling plan is the generalization of several sampling plans. The proposed sampling plan is the extension of the plan developed using classical statistics. The proposed sampling plan can be applied when there is indeterminacy interval in the observations. The proposed sampling plan provides the plan parameters in indeterminacy interval range which are desirable in the indeterminate environment. The comparison indicates that the proposed plan is more efficient and adequate than the existing sampling to be applied for the inspection of a lot under the uncertainty. The proposed sampling plan using big data can be considered as future research.

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