ALGORITHM FOR LISTING OF SMARANDACHE FACTOR PARTITIONS

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ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION (**SFP**) , as follows:

Let $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r$ be a set of r natural numbers and $p_1, p_2, p_3, \ldots p_r$ be arbitrarily chosen distinct primes then $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ called the Smarandache Factor Partition of $(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ is defined as the number of ways in which the number

$$N = p_1^{\alpha 1}\ p_2^{\alpha 2}\ p_3^{\alpha 3} \ldots p_r^{\alpha r}$$ could be expressed as the product of its' divisors. For simplicity , we denote $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r) = F'(N)$,where

$$N = p_1^{\alpha_1}\ p_2^{\alpha_2}\ p_3^{\alpha_3} \ldots p_r^{\alpha_r} \ldots p_n^{\alpha_n}$$

and p_r is the r^{th} prime. $p_1 = 2$, $p_2 = 3$ etc.

In this note an algorithm to list out all the SFPs of a number without missing any is developed.

DISCUSSION:

DEFINITION: $F'_x(y)$ is defined as the number of those SFPs of y which involve terms not greater than x.

If F_1 be a factor partition of y :

$F_1 \longrightarrow x_1 \text{ X } x_2 \text{ X } x_3 \text{ X } \ldots x_r$, then F_1 is included in $F_x'(y)$ iff

$x_i \leq x$ for $1 \leq I \leq r$

clearly $F'_x(y) \leq F'(y)$, The equality holds good iff $x \geq y$.

Example: $F'_8(24) = 5$. Out of 7 only the last 5 are included in $F'_8(24)$.

(1) 24
(2) 12 X 2
(3) 8 X 3
(4) 6 X 4
(5) 6 X 2 X 2
(6) 4 X 3 X 2
(7) 3 X 2 X 2 X 2.

ALGORITHM: Let $d_1, d_2, d_3, \ldots d_r$ be the divisors of N in descending order. For listing the factor partitions following are the steps:

(A) (1) Start with $d_1 = N$.

(2) Write all the factor partitions involving d_2 and so on.

(B) While listing care should be taken that the terms from left to right should be written in descending order.

** At $d_k \geq N^{1/2} \geq d_{k+1}$, and onwards , step (B) will ensure that there is no repeatition.

Example: N = 36 , Divisors are 36 , 18, 12, 9 , 6, 4, 3 , 2 , 1.

36 ---→ 36
18 ---→ 18 X 2
12 ---→ 12 X 3
9 ---→ 9 X 4
9 X 2 X 2
6 ---→ 6 X 6
6 ---→ 6 X 3X 2
--------------------------------**$d_k = N^{1/2}$**
4 ---→ 4 X 3 X 3

3 --→3 X 3 X 2 X 2
2 --→NIL
1 --→NIL

FORMULA FOR F'(N)

$$F'(N) = \sum_{d_r/N} F'_{d_r}(N/d_r) \qquad \text{-------(8.1)}$$

Example:

$N = 216 = 2^3 3^3$

(1) 216 --→$F_{216}(1) = 1$
(2) 108 X 2 --→$F_{108}(2) = 1$
(3) 72 X 3 --→$F_{72}(3) = 1$
(4) 54 X 4 --→$F_{54}(4) = 2$
(5) 54 X 2 X 2
(6) 36 X 6 --→$F_{36}(6) = 2$
(7) 36 X 3 X 2
(8) 27 X 8 --→$F_{27}(8) = 3$
(9) 27 X 4 X 2
(10) 27 X 2 X 2 X 2
(11) 24 X 9 --→$F_{24}(9) = 2$
(12) 24 X 3 X 3
(13) 18 X 12 --→$F_{18}(12) = 4$
(14) 18 X 6 X 2
(15) 18 X 4 X 3
(16) 18 X 3 X 2 X 2
(17) 12 X 9 X 2 --→$F_{12}(18) = 3$
(18) 12 X 6 X 3
(19) 12 X 3 X 3 X 2
(20) 9 X 8 X 3 --→$F_9(24) = 5$
(21) 9 X 6 X 4
(22) 9 X 6 X 2 X 2
(23) 9 X 4 X 3 X 2
(24) 9 X 3 X 2 X 2
(25) 8 X 3 X 3 X 3 --→$F_8(27) = 1$
(26) 6 X 6 X 6 --→$F_6(36) = 4$
(27) 6 X 6 X 3 X 2
(28) 6 X 4 X 3 X 3
(29) 6 X 3 X 3 X 2 X 2
(30) 4 X 3 X 3 X 3 X 2 X 2 --→$F_4(54) = 1$
(31) 3 X 3 X 3 X 2 X 2 X 2 --→$F_3(72) = 1$
--→$F_2(108) = 0$
--→$F_1(216) = 0$

$$F'(216) = \sum_{d_r/N} F'_{d_r}(216/d_r) = 31$$

Remarks: This algorithm would be quite helpfull in developing a computer program for the listing of SFPs.

REFERENCES:

[1] "Amarnath Murthy" , 'Generalization Of Partition Function, Introducing 'Smarandache Factor Partition', SNJ, Vol. 11, No. 1-2-3, 2000.

[2] " The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texas at Austin, USA.