A Congruence with Smarandache's Function

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Smarandache's function is defined thus:
$S(n) =$ is the smallest integer such that $S(n)!$ is divisible by n. [1]
In this article we are going to look at the value that has $S(2^k - 1) \pmod k$ for all k integer, $2 \leq k \leq 97$.

One can observe in the following table that gives the continuation $S(2^k - 1) \equiv 1 \pmod k$ in the majority of cases, there are only 4 exeptions for $2 \leq k \leq 97$.

k	$S(2^k - 1)$	$S(2^k - 1) \, (mod\ k)$
2	3	1
3	7	1
4	5	1
5	31	1
6	7	1
7	127	1
8	17	1
9	73	1
10	31	1
11	89	1
12	13	1
13	8191	1
14	127	1
15	151	1
16	257	1
17	131071	1
18	73	1
19	524287	1

k	$S(2^k-1)$	$S(2^k-1) \pmod k$	k	$S(2^k-1)$	$S(2^k-1) \pmod k$
20	41	1	59	3203431780337	1
21	337	1	60	1321	1
22	683	1	61	2305843009213693951	1
23	178481	1	62	2147483647	1
24	241	1	63	649657	1
25	1801	1	64	6700417	1
26	8191	1	65	145295143558111	1
27	262657	1	66	599479	1
28	127	**15**	67	761838257287	1
29	2089	1	68	131071	**35**
30	331	1	69	10052678938039	1
31	2147483647	1	70	122921	1
32	65537	1	71	212885833	1
33	599479	1	72	38737	1
34	131071	1	73	9361973132609	1
35	122921	1	74	616318177	1
36	109	1	75	10567201	1
37	616318177	1	76	525313	1
38	524287	1	77	581283643249112959	1
39	121369	1	78	22366891	1
40	61681	1	79	1113491139767	1
41	164511353	1	80	4278255361	1
42	5419	1	81	97685839	1
43	2099863	1	82	8831418697	1
44	2113	1	83	57912614113275649087721	1
45	23311	1	84	14449	1
46	2796203	1	85	9520972806333758431	1
47	13264529	1	86	2932031007403	1
48	673	1	87	9857737155463	1
49	4432676798593	1	88	2931542417	1
50	4051	1	89	618970019642690137449562111	1
51	131071	1	90	18837001	1
52	8191	**27**	91	23140471537	1
53	20394401	1	92	2796203	**47**
54	262657	1	93	658812288653553079	1
55	201961	1	94	165768537521	1
56	15790321	1	95	30327152671	1
57	1212847	1	96	22253377	1
58	3033169	1	97	13842607235828485645766393	1

One can see from the table that there are only 4 exeptions for $2 \leq k \leq 97$.

We can see in detail the 4 exeptions in a table:

$k = 28 = 2^2 \cdot 7$	$S(2^{28} - 1) \equiv 15 \ (mod\ 28)$
$k = 52 = 2^2 \cdot 13$	$S(2^{52} - 1) \equiv 27 \ (mod\ 52)$
$k = 68 = 2^2 \cdot 17$	$S(2^{68} - 1) \equiv 35 \ (mod\ 68)$
$k = 92 = 2^2 \cdot 23$	$S(2^{92} - 1) \equiv 47 \ (mod\ 92)$

One can observe in these 4 cases that $k = 2^2 \cdot p$ with p prime and moreover $S(2^k - 1) \equiv \frac{k}{2} + 1 \ (mod\ k)$

Unsolved Question:

One can obtain a general formula that gives us, in function of k the value $S(2^k - 1) \ (mod\ k)$ for all pisitive integer values of k ?.

References:

[1] SMARANDACHE NOTIONS JOURNAL Vol. 9, No. 1-2,(1998) pp 21-26.

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