Some Problems Concerning The Smarandache Deconstructive Sequence*

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The Smarandache Deconstructive Sequence (SDS(n)) of integers is constructed by sequentially repeating the digits 1-9 in the following way:

1, 23, 456, 7891, 23456, 789123, 4567891, 23456789, 123456789, 1234567891, ...

and first appeared in the collection by Smarandache[1]. In a later collection by Kashihara[2], the question was asked:

How many primes are there in this sequence?

In this article, we will briefly explore that question and raise a few others concerning this sequence.

The values of the first thirty elements of this sequence appear in Table 1. From the list, it seems clear that the trailing digits repeat the pattern,

1, 3, 6, 1, 6, 3, 1, 9, 9, 1, 3, 6, 1, 6, 3, 1, 9, 9, 1, . . .

and it is simple to prove that this is indeed the case. Given the rules used in the construction of this sequence, the remaining columns also have similar patterns.

It is also simple to prove that every third element in the sequence is evenly divisible by 3. More specifically, 3 | SDS(n) if and only if 3 | n.

The list contains the eight primes

23, 4567891, 23456789, 1234567891, 23456789123456789, 23456789123456789123,
4567891234567891234567891, 1234567891234567891234567891.

If we do not consider the first element in the list, the percentage of primes is $\frac{8}{29} = 0.276$. Given this, admittedly slim, numeric evidence and the regular nature of the digits, the author is confident enough to offer the following conjecture.

Conjecture 1: The Smarandache Deconstructive Sequence contains an infinite number of primes.

Two out of every nine numbers end in 6. In examining the factorizations of these numbers, we see that 456 is divisible by 2^3, 23456 by 2^5, and all others by 2^7. This prompts the question:

Table 1.

1	1
2	23
3	456
4	7891
5	23456
6	789123
7	4567891
8	23456789
9	123456789
10	1234567891
11	23456789123
12	456789123456
13	7891234567891
14	23456789123456
15	789123456789123
16	4567891234567891
17	23456789123456789
18	123456789123456789
19	1234567891234567891
20	23456789123456789123
21	456789123456789123456
22	7891234567891234567891
23	23456789123456789123456
24	789123456789123456789123
25	4567891234567891234567891
26	23456789123456789123456789
27	123456789123456789123456789
28	1234567891234567891234567891
29	23456789123456789123456789123
30	456789123456789123456789123456

Question 1: Does every even element of the Smarandache Deconstructive Sequence contain at least three instances of the prime 2 as a factor?

Even more specifically,

Question 2: If we form a sequence from the elements of SDS(n) that end in a 6, do the powers of 2 that divide them form a monotonically increasing sequence?

The following is prompted by examining the divisors of the elements of the sequence.

Question 3: Let k be the largest integer such that $3^k \mid n$ and j the largest integer such that $3^j \mid SDS(n)$. Is it true that k is always equal to j?

And we close with the question

Question 4: Are there any other patterns of divisibility in this sequence?

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References

1. F. Smarandache, *Only Problems, Not Solutions*, Xiquan Publishing House, Phoenix, Arizona, 1993.
2. K. Kashihara, *Comments and Topics on Smarandache Notions and Problems*, Erhus University Press, Vail, Arizona, 1996.