Designing with Smarandache Sequences, Part 1: Concatenation Sequences

All kinds of things can be found among integer sequences, including the weird and nonsensical. Enter Smarandache sequences (S. sequences, for short), which are integer sequences due to Florentin Smarandache and his disciples.

Some S. sequence are related to number-theoretic topics. Others, at first glance (second, third, …) seem downright silly.

An example is the progressive concatenation of the digits of the Fibonacci numbers:

1, 11, 112, 1123, 11235, …

Sequences like this one, based on digit manipulation, do not have any natural important mathematical properties, since they depend on base-10 representation of numbers and not on the properties of the numbers themselves.

On the other hand, weave design depends on patterns and not on the actual values of numbers. The numbering scheme used for shafts and treadles does not reply on any mathematical properties of the numbers.

With this said, it is worth exploring S. sequences to see what patterns emerge, starting with concatenation sequences and going on to other kinds of S. sequences in subsequent articles.

Florentin Smarandache was born in Romania in 1954. He describes himself as poet, playwright, novelist, writer of prose, tales for children, translator from many languages, experimental painter, philosopher, physicist, and mathematician.

He presently is associate professor of computer science at the University of New Mexico.

In 1980 he set up the "paradoxism" movement, which has many advocates in the world. It is based on an excessive use of antitheses, antinomies, contradictions, paradoxes in creation paradoxes — both at the small level and the entire level of the work — mathematics, philosophy, and literature.

Concatenation Sequences

Concatenation sequences have the property that terms are derived from other sequences by concatenation

Examples of concatenation sequences found in the literature [1] include the natural number repetition sequence

1, 22, 333, 4444, 55555, 666666, 7777777,
88888888, 999999999,
10101010101010101010,
1111111111111111111111, …

the prime concatenation sequence

2, 23, 235, 2357, 235711, 23571113, …

and the cube concatenation sequence

1, 18, 1827, 182764, 1827784125, …

The terms in repetition sequences are given by the rule

$$t_i = s_i^{\,i}$$

where s_i is the ith term of the *base sequence* on which the repetition sequences in built, t_i is the ith term in the repetition sequence, and a^i denotes i copies of a.

Another possibility that produces the same results as the one above for the natural numbers is

$$t_i = s_i^{\,s_i}$$

That is, replicate the ith term of the base sequence by its value. For this rule, the repetition of the primes would be

22, 333, 55555, 7777777,…

instead of

2, 33, 555, 7777, …

by the first rule.

For concatenation sequences, the terms are given by

$t_i = t_{i-1}s_i$

There are endless possibilities for other rules of these general types, such as

$t_i = s_i t_{i-1}$

which for the primes produces

2, 32, 532, 7532, 117532, 13117532, …

And, of course, repetition and concatenation sequences can be used as base sequences, and so on, although in most cases the size of terms gets out of hand.

Yes, this is all silly — digit play, not mathematics. But can any interesting weave designs come from it?

From S. Sequences to T-Sequences

For t-sequences (threading and treading sequences) [2], values need to be limited to the number of shafts / treadles available. Direct conversion of an S. concatenated sequences to a t-sequences can be done by modular reduction [3].

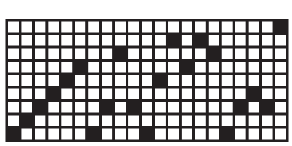
Since the terms in S. concatenated sequences get longer and longer (at least for the rules shown above), another alternative is to interpret a term as a sequence of digits. For example, the ninth term in the concatenated cube sequence is

182764125216343512729

Converting the digits to terms gives

1, 8, 2, 7, 6, 4, 1, 2, 5, 2, 1, 6, 3, 4, 3, 5, 1, 2, 7, 2, 9

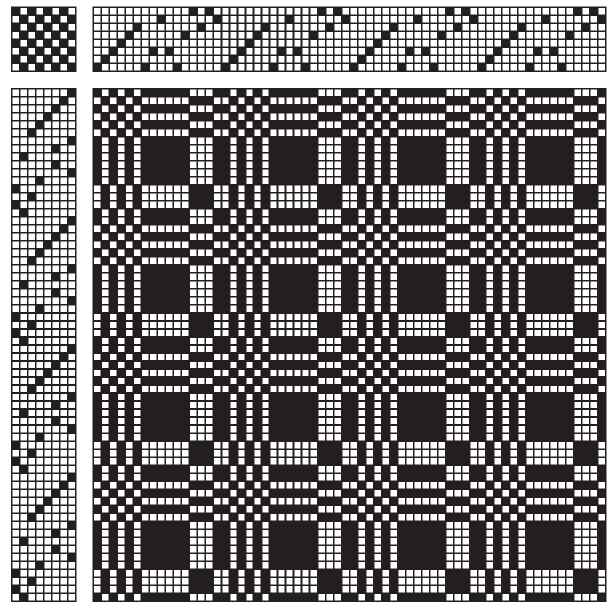
Normalizing this [4] and representing the result graphically produces:



With a tabby tie-up and treadled as drawn in

2, 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 9, 9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 2, 2, 2, 2, 2, 2,

the draft is



This interpretation of terms in an S. sequence can produce t-sequences with at most 10 values (0 can be converted to 10 arbitrarily, or one can be added to all values).

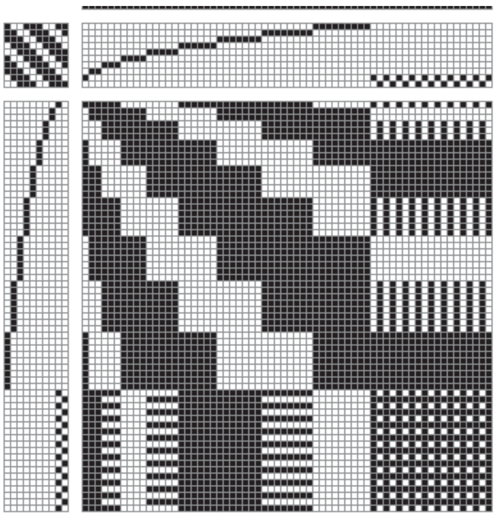
To get good results with this method, S. sequences and their terms need to selected with care. Even then, some will need to be modified.

What about the more conventional approach of using modular reduction to bring large values into the domain of t-sequences?

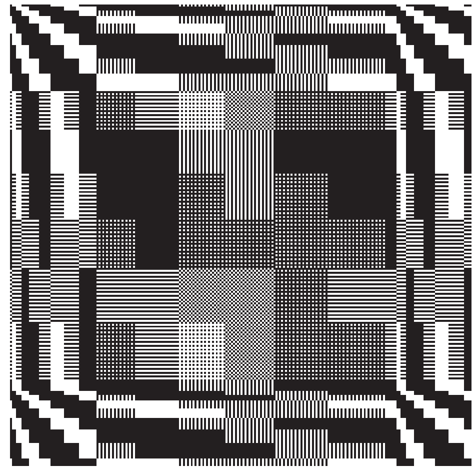
The natural number repetition sequence is about as unpromising a candidate for design as one might find among concatenation sequences. But modular reduction for 10 shafts produces this sequence:

2, 3, 3, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 9, 9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 2, 2, 2, 2, 2, 2, …

Here is one possible draft:



and the weave pattern is



Conclusion

S. concatenation sequences are clearly artificial from a mathematical point of view, but they do produce patterns and among them there are possibilities for novel weave designs.

The next article will look at a more promising kind of S. sequence: S. palindromes.

References

1. *Some Notions and Questions in Number Theory, Volume 2*, Mihaly Bencze and Lucian Tutescu. Self published.

2. Ralph E. Griswold, "T-Sequences, Part 1: Introduction", 2004: http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_ts01.pdf

3. *Drafting with Sequences*, Ralph E. Griswold, 1999: http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_seqd.pdf

4. Ralph E. Griswold, "Sequence Normalization", 2004: http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_norm.pdf

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