On a problem concerning the Smarandache friendly prime pairs

Felice Russo
Via A. Infante 7
67051 Avezzano (Aq) Italy
felice.russo@katamail.com

Abstract

In this paper a question posed in [1] and concerning the Smarandache friendly prime pairs is analysed.

Introduction

In [1] the Smarandache friendly prime pairs are defined as those prime pairs (p,q) such that:

$$\sum_{x=p}^{q} x = p \cdot q \qquad (1)$$

where x denote the primes between p and q. In other words the Smarandache friendly prime pairs are the pairs (p,q) such that the sum of the primes between p and q is equal to the product of p and q.
As example let's consider the pair (2,5). In this case $2+3+5=2\cdot 5$ and then 2 and 5 are friendly primes. The other three pairs given in the mentioned paper are: (3,13), (5,31) and (7,53). Then the following open questions have been posed:

Are there infinitely many friendly prime pairs?

Is there for every prime p a prime q such that (p,q) is a Smarandache friendly prime pair?

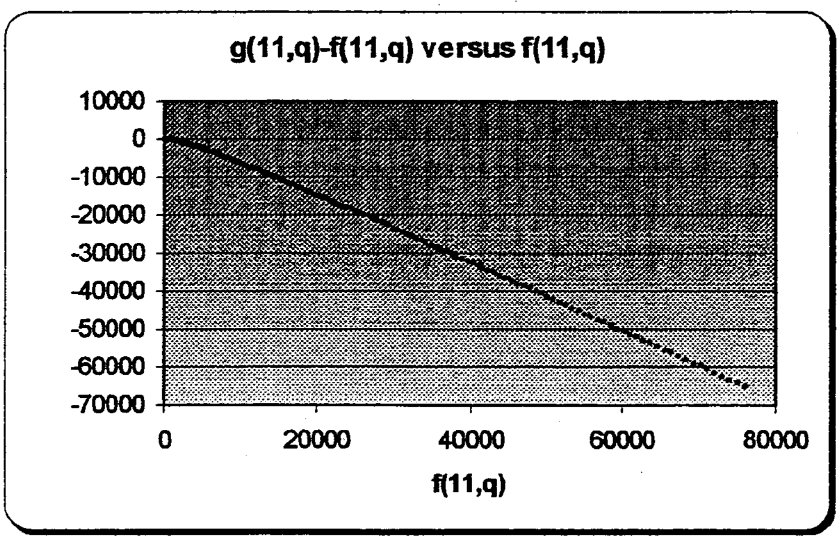
In this paper we analyse the last question and a shortcut to explore the first conjecture is reported.

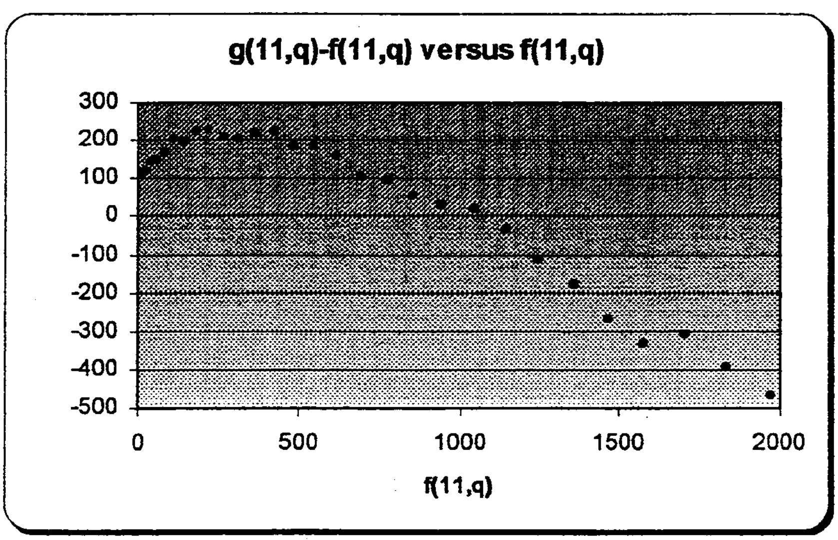
Results

First of all let's analyse the case p=11. Let's indicate:

$$f(11,q)=\sum_{x=11}^{q} x \quad and \qquad g(11,q)=11\cdot q$$

where x denotes always the primes between 11 and q.
A computer program with Ubasic software package has been written to calculate the difference between g(11,q) and f(11,q) for the 164 primes q subsequent to 11. Here below the trend of that difference.





As we can see the difference starts to increase, arrives to a maximum and then starts to decrease and once pass the x axis decrease in average linearly. The same thing is true for all the other primes p.
So for every prime p the search of its friend q can be performed up to:

$$g(p,q) - f(p,q) \le -M$$

where M is a positive constant.
For the first 1000 primes M has been choosen equal to 10^5.
No further friendly prime pair besides those reported in [1] has been found. According to those experimental results we are enough confident to pose the following conjecture:

Not all the primes have a friend, that is there are prime p such that there isn't a prime q such that the (1) is true.

Moreover a furter check of friendly prime pairs for all primes larger than 1000 and smaller than 10000 has been performed choosing M=1000000.
No further friendly prime pair has been found. Those results seem to point out that the number of friendly prime pairs is finite.

Question:

Are (2,5), (3,13), (5,31) and (7,53) the only Smarandache friendly prime pairs?

References.

[1] A. Murthy, *Smarandache friendly numbers and a few more sequences*, Smarandache Notions Journal, Vol. 12 N. 1-2-3 Spring 2001