FROM BOLYAI'S GEOMETRY TO SMARANDACHE ANTI-GEOMETRY

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ABSTRACT

It is considered the notion of absolute Geometry in its evolution, from the first Non-euclidean Geometry of Lobacewski, Bolyai and Gauss till that of Smarandache Anti-Geometry.

Key words: Euclidean Geometry, Non-euclidean Geometry, Hilbert's axioms and incidence structures deduced from them, Smarandache Geometries, Hjelmslev-Barbilian structures

Any theory or deductive system has two distinguishable parts:

1. the specifical part
2. the logical part.

When we formalize one, or both of parts of a theory, we obtain next classification for axiomatical theories:

1. nonformalized,
2. semiformalized, or
3. formalized axiomatical theory.

When J. Bolyai in [3] in 1831, and N. Lobacewski in [12], in 1826 began the studies about non-euclidean Geometries the formalization in mathematics was not yet introduced. Their contributions should be considered as more important as at that moment the formalized system of axioms of Geometry of D. Hilbert was not given.

The way we can establish the metamathematical analyse of a theory are two:

1. sintactical (that is directly)
2. semantical, by the interpretations and models.

J. Bolyai and N. Lobacevski worked only sintactically and not semantically in the study of the metamathematical analyse of their theory.

On the first way, non-contradiction of their Geometry given in [3] and [12] could not be proved, because in a such a way they could not convince that the set of correct affirmations of their theory were exhausted and that has excluded the possibility to meet a proposition p correct constructed such that p and $\neg p$ to be verified in their Geometry. Later were given semantically the proofs that their non-euclidean Geometry is consistent, and so is non-contradictory, by the models of Berltrami in 1868, after that of Cayley-Klein in 1871 and of H. Poincaré in 1882.

In Bolyai-Lobacewski's absolute Geometry from a point A to a line a, A is not incident with a, there exists a parallel. In this absolute Geometry we have only two possibilities:

1. the Euclidean Geometry
2. the Hyperbolical Geometry.

The elliptical Geometry with none parallel through a point to a line, are excluded from this absolute Geometry. Also this absolute Geometry contained only continuous Geometry.

In 1903 in [6], D. Hilbert proved that the hyperbolical plane geometry can be introduced without to use the tridimensional space, and that is possible to renounce to the axioms of continuity. This is an important moment for research in Geometry because from that moment the notion of absolute Geometry changes its meaning and begins to be different considered to different moments and to different authors. The absolute plane of Bolyai becomes a particular case of the absolute plane in recently researches of Geometry.

From 1889, when D. Hilbert in [5] gave a formalized system of axioms for absolute Geometry, appeared more directions of investigation in Geometry. The incidence structures are largely used and so are introduced a great variety of affine and projective planes and affine and projective spaces.

The great importance of geometrical transformations for geometrical problems was put by F. Klein in "The programme from Erlangen" in 1872, when he began to consider the Geometry as the study of invariant properties to a group of transformations. From that moment the system of axioms of many Geometries are based on the notions of theory of groups. This group is given as an abstract group, and geometrical structure is a consequence of structure of group. This fact was possible, after that it was proved that the geometry can be transposed in the group of its automorphisms generated by axial symmetries. A such a system of axioms is more simple than a classical one, it is easily adopted to the special qualities of non-euclidean Geometries. Compared with a such a system of axioms, the system of axioms of D. Hilbert is more complicated.

As it is the calcullus in a field for Analytical Geometry a method of work, as the calcullus in the group generated by the axial symmetries becomes a method for proofs in Geometry, after J. Hjelmslev in [7] introduced it. In [16] Thompsen proved that this can become an efficient method of demonstration for the theorems of Euclidean Geometry. This is an attrative method because the hypothesis and conclusions of a theorem can be written simply as relations of group.

The first system of axioms of absolute plane geometry formulated in theory of groups was given by A. Schmidt in [13], and after that F. Bachman in [1]. From that date this method is largely used in Geometry as in [9], [10], [11], [17] and many others works.

In 1954 after E. Sperner gives a group proof of theorem of Desargues for a large classes of Geometries, in absolute geometry are included new-types of geometry, as geometry with centre, with perpendicular nuclei [10], and many others.

The classical Geometries are extended, because are not made hypothesis of order, of continuity or mobility [1], [9], [11], [17].

In 1967 H. Wolf in [21], includes near Euclidean, elliptical, and hyperbolical Geometry, also Minkowski's Geometry.

The geometries constructed over a field of characteristic 2 are included later, by a more general system of axioms of R. Lingenberg in [11].

Another generalization of incidence structures was that in which it were considered geometrical structures to which the line incident with two different points is not unique. A such a theory is consistent and as a model for it we have the Geometry over a ring. Such structures were introduced by J. Hjelmslev in [8] and D. Barbilian in [2]. A new direction of study in Geometry begins from this moment, in which we have also some results.

The researches of absolute Geometry have a natural continuity, the notion of absolute Geometry is a notion in evolution in the modern literature of speciality. This help us to understand better the life, the transformations in the life, and finally this could bring us more wisdom and increasing degree of understanding of human condition, and such to answer to the deep desire of their creatores: that the mathematics to become also a force of life.

Such Florentin Smarandache even in 1969 said that it is natural to consider a new Geometry denying not only one axiom from the axioms of D.H. Hilbert from [5] but more or even all of them, what he did in 1985 in [14] and in 1997 in [15].

So he introduced so called "Smarandache Anti-Geometry". It seems strange but it is natural. We should remember that when J. Bolyai the genial discoverer of first non-euclidean Geometry was deeply implied in his great work even the great Gauss said that the people are not prepared to receive a new Geometry, a such a new theory. And that was the truth. J. Bolyai suffered very much at that time seing that he can not be understood, but he was convinced that not only in Mathematics, but in the whole history of thinking his conception represents a crucial point. Besides the value of his discoveries in Mathematics, J. Bolyai must be discovered and then, inevitable loved, as a great thinker preoccupied of the problems of harmonious integration in the life. As we showed in [18], [19] J. Bolyai always was thirsty of harmony and with a stoical wisdom he supported his ideas until the end of his life, a life full of misunderstanding. In spite of all what he met as nonunderstanding he continued to believe in what he created and he felt them to be true.

As J. Bolyai, N. Lobacevski was not understood during his life and his work was not recognized at that time. Their contributions today have to be appreciate even more as at their time the formalized theories has not been introduced.

Beyond the mathematical contribution their works represent an opening meditation of human condition which have not been enough exploited. Feeling the potential of this opening in the understanding of the human complexity we suggested it as a direction of research and to try to imply, we all scientists, to get an amelioration of the human condition as in [18], [19], [20] we did. This research can be done by the utilisation of mathematical ideas and theories to the construction of a model of self-knowledge. Have we ever put the question which are the axioms which stay at the base of the existence? As any theory the human existence should have some axioms, propositions, theorems, conjectures, false affirmations etc. The consequences of false affirmations in our behaviour can be clearly observed: the pollution of the mind, of the nature, ecological perturbations etc.

We all can realize that the elimination or diminuation of false affirmations about the existence and man, could bring harmony and peace. Taking in consideration the profoundness and credibility of scientists we can hope more and more from us paing attention to this noble work. The incredible technical progress and discoveries of the science have a correspondent in the science of selfknowledge.

The Anti-Geometry introduced by Florentin Smarandache in [14], [15] would correspond to the understanding of the degradation of human condition. Even this "Anti-Geometry" could be a model for this kind of "inner Geometry", in the sense that the degree of degradation represents the different levels of negation of our inner possibilities, of our natural qualities.

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