A GENERAL RESULT ON THE SMARANDACHE STAR FUNCTION

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ABSTRCT: In this paper ,the result (theorem-2.6) Derived in REF. [2], the paper "Generalization Of Partition Function, Introducing 'Smarandache Factor Partition' which has been observed to follow a beautiful pattern has been generalized.

DEFINITIONS In [2] we define SMARANDACHE FACTOR PARTITION FUNCTION , as follows:

Let $\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r$ be a set of r natural numbers and $p_1 , p_2, p_3 ,\ldots p_r$ be arbitrarily chosen distinct primes then $F(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r)$ called the Smarandache Factor Partition of $(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r)$ is defined as the number of ways in which the number

$$N = p_1^{\alpha 1} \; p_2^{\alpha 2} \; p_3^{\alpha 3} \ldots p_r^{\alpha r}$$

could be expressed as the product of its' divisors. For simplicity , we denote $F(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r) = F'(N)$,where

$$N = p_1^{\alpha_1} \; p_2^{\alpha_2} \; p_3^{\alpha_3} \ldots p_r^{\alpha_r} \ldots p_n^{\alpha_n}$$

and p_r is the r^{th} prime. $p_1 =2$, $p_2 = 3$ etc.

Also for the case

$$\alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_r = \ldots = \alpha_n = 1$$

we denote

$$F(\underbrace{1, 1, 1, 1, 1 \ldots}_{\leftarrow \text{ n - ones } \rightarrow}) = F(1\#n)$$

Smarandache Star Function

$$\textbf{(1)} \quad F^{'*}(N) = \sum_{d/N} F'(d_r) \qquad \text{where } d_r \mid N$$

$$\textbf{(2)} \quad F'^{**}(N) = \sum_{d_r/N} F'^{*}(d_r)$$

d_r ranges over all the divisors of N.

If N is a square free number with n prime factors , let us denote

$$F'^{**}(N) = F^{**}(1\#n)$$

Here we generalise the above idea by the following definition

Smarandache Generalised Star Function

$$\textbf{(3)} \quad F'^{n*}(N) = \sum_{d_r/N} F'^{(n-1)*}(d_r) \qquad n > 1$$

and d_r ranges over all the divisors of N.

For simplicity we denote

$$\mathbf{F'(Np_1p_2 \ldots p_n) = F'(N@1\#n)} \quad \text{, where}$$

$$(N, p_i) = 1 \text{ for } i = 1 \text{ to } n \text{ and each } p_i \text{ is a prime.}$$

F'(N@1#n) is nothing but the Smarandache factor partition of (a number N multiplied by n primes which are coprime to N).

In [3] a proof of the following result is given:

$$F'(Np_1p_2p_3) = F'^*(N) + 3F'^{2*}(N) + F'^{3*}(N)$$

The present paper aims at generalising the abve result.

DISCUSSION:

THEOREM(3.1)

$$F'(N@1\#n) = F'(Np_1p_2...p_n) = \sum_{m=0}^{n} [\ a_{(n,m)}\ F'^{m*}(N)]$$

where

$$a_{(n,m)} = (1/m!) \sum_{k=1}^{m} (-1)^{m-k}\ .{}^{m}C_k\ .k^n$$

PROOF:

Let the divisors of N be

$d_1, \quad d_2, \quad ..., \quad d_k$

Consider the divisors of $(Np_1p_2...p_n)$ arranged as follows

$d_1, \quad d_2, \quad ..., \quad d_k$ -------say type (0)

$d_1p_i, \quad d_2p_i, \quad ..., \quad d_kp_i$ -------say type (1)

$d_1p_ip_j, \quad d_2p_ip_j, \quad ..., \quad d_kp_ip_j$ -------say type (2)

$d_1p_ip_j..., d_2p_ip_j..., ..., d_kp_ip_j...$ -------say type (t)

(there are t primes in the term $d_1p_ip_j...$ apart from d_1)

$d_1p_1p_2...p_n, \quad d_2p_1p_2...p_n, \quad d_np_1p_2...p_n,$ -------say type (n)

There are ${}^{n}C_0$ divisors sets of the type (0)

There are ${}^{n}C_1$ divisors sets of the type (1)

There are ${}^{n}C_2$ divisors sets of the type (2) and so on

There are ${}^{n}C_t$ divisors sets of the type (t)

There are ${}^{n}C_{n}$ divisors sets of the type (n)

Let $Np_1p_2...p_n = M$ .Then

$F^*(M) = {}^{n}C_0$[sum of the factor partitions of all the divisors of row (0)]

+ ${}^{n}C_1$[sum of the factor partitions of all the divisors of row (1)]

+ ${}^{n}C_2$[sum of the factor partitions of all the divisors of row (2)]

+ . . .

+ ${}^{n}C_t$[sum of the factor partitions of all the divisors of row (t)]

+ . . .

+ ${}^{n}C_n$[sum of the factor partitions of all the divisors of row (n)]

Let us consider the contributions of divisor sets one by one.

Row (0) or type (0) contributes

$$F'(d_1) + F'(d_2) + F'(d_3) + ... + F'(d_n) = F'^*(N)$$

Row (1) or type (1) contributes

$$[F'(d_1p_1) + F'(d_2p_1) + ... F'(d_kp_1)]$$

$$= [F'^*(d_1) + F'^*(d_2) + ... + F'^*(d_k)]$$

$$= F'^{2*}(N)$$

Row (2) or type (2) contributes

$$[F'(d_1p_1p_2) + F'(d_2p_1p_2) + ... + F'(d_kp_1p_2)$$

Applying theorem (5) on each of the terms

$$F'(d_1p1p2) = F'^*(d_1) + F'^{**}(d_1) \qquad \text{----(1)}$$

$$F'(d_2p_1p_2) = F'^*(d_2) + F'^{**}(d_2) \qquad \text{----(2)}$$

.

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$$F'(d_kp1p2) = F'^*(d_k) + F'^{**}(d_k) \qquad \text{----(k)}$$

on summing up (1) , (2) . . . upto (n) we get

$$F'^{2*}(N) + F'^{3*}(N)$$

At this stage let us denote the coefficients as $a_{(n,r)}$ etc. say

$F'(N@1\#r) = a_{(r,1)}F'^{*}(N) + a_{(r,2)}F'^{2*}(N) +. . .+ a_{(r,t)}F'^{t*}(N) + \ldots +$

$a_{(r,r)}F'^{r*}(N)$

Consider row (t) , one divisor set is

$d_1p_1p_2\ldots p_t$, $d_2p_1p_2\ldots p_t$,. . . $d_kp_1p_2\ldots p_t$,

and we have

$F'(d_1@1\#t) = a_{(t,1)}F'^{*}(d_1) + a_{(t,2)}F'^{2*}(d_1) +. . . + a_{(t,t)}F'^{t*}(d_1)$

$F'(d_2@1\#t) = a_{(t,1)}F'^{*}(d_2) + a_{(t,2)}F'^{2*}(d_2) +. . . + a_{(t,t)}F'^{t*}(d_2)$

.

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$F'(d_k@1\#t) = a_{(t,1)}F'^{*}(d_k) + a_{(t,2)}F'^{2*}(d_k) +. . . + a_{(t,t)}F'^{t*}(d_k)$

summing up both the sides columnwise we get for row (t) or divisors of type (t) one of the ${}^nC_t$ divisor sets contributes

$a_{(t,1)}F'^{2*}(N) + a_{(t,2)}F'^{3*}(N) +. . . + a_{(t,t)}F'^{(t+1)*}(N)$

similarly for row (n) we get

$a_{(n,1)}F'^{2*}(N) + a_{(n,2)}F'^{3*}(N) +. . . + a_{(n,n)}F'^{(n+1)*}(N)$

All the divisor sets of type (0) contribute

${}^nC_0\ a_{(0,0)}F'^{*}(N)$ factor partitions.

All the divisor sets of type (1) contribute

${}^nC_1\ a_{(1,1)}F'^{2*}(N)$ factor partitions.

All the divisor sets of type (2) contribute

${}^nC_2\ \{a_{(2,1)}F'^{2*}(N) + a_{(2,2)}F'^{3*}(N)\}$ factor partitions.

All the divisor sets of type (3) contribute

${}^nC_3\{a_{(3,1)}F'^{2*}(N) + a_{(3,2)}F'^{3*}(N) + a_{(3,3)}F'^{4*}(N\ \}$ factor partitions.

All the divisor sets of row (t) or type (t) contribute

$^{n}C_{t}\{ a_{(t,1)}F'^{2*}(N) + a_{(t,2)}F'^{3*}(N) + \ldots + a_{(t,t)}F'^{(t+1)*}(N)\}$

.

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All the divisor sets of row (n) or type (n) contribute

$^{n}C_{n}\{a_{(n,1)}F'^{2*}(N) + a_{(n,2)}F'^{3*}(N) + \ldots + a_{(n,n)}F'^{(n+1)*}(N)\}$

Summing up the contributions from the divisor sets of all the types and considering the coefficient of $F'^{m*}(N)$ for $m = 1$ to $(n+1)$ we get, coefficient of $F'^{*}(N) = a_{(0,0)} = 1 = \mathbf{a_{(n+1,1)}}$

coefficient of $F'^{2*}(N)$

$= {}^{n}C_{1}\, a_{(1,1)} + {}^{n}C_{2}\, a_{(2,1)} + {}^{n}C_{3}\, a_{(3,1)} + \ldots\ {}^{n}C_{t}\, a_{(t,1)} + \ldots + {}^{n}C_{n}\, a_{(n,1)}$

$= \mathbf{a_{(n+1,2)}}$

coefficient of $F'^{3*}(N)$

$= {}^{n}C_{2}\, a_{(2,2)} + {}^{n}C_{3}\, a_{(3,2)} + {}^{n}C_{4}\, a_{(4,2)} + \ldots\ {}^{n}C_{t}\, a_{(t,2)} + \ldots + {}^{n}C_{n}\, a_{(n,2)}$

$= \mathbf{a_{(n+1,3)}}$

coefficient of $F'^{m*}(N) =$

$a_{(n+1,m)} = {}^{n}C_{m-1}.a_{(m-1,m-1)} + {}^{n}C_{m}.a_{(m,m-1)} + \ldots + {}^{n}C_{n}.a_{(n,m-1)}$

coefficient of $F'^{(n+1)*}(N) =$

$a_{(n+1,n+1)} = {}^{n}C_{n}.a_{(n,n)} = {}^{n}C_{n}.{}^{n-1}C_{n-1}\,.a_{(n-1,n-1)} = {}^{n}C_{n}.{}^{n-1}C_{n-1}.\ldots$

${}^{2}C_{2}.a_{(1,1)}$

$= 1$

Consider $a_{(n+1,2)}$

$= {}^{n}C_{1}\, a_{(1,1)} + {}^{n}C_{2}\, a_{(2,1)} + \ldots\ {}^{n}C_{t}\, a_{(t,1)} + \ldots + {}^{n}C_{n}\, a_{(n,1)}$

$= {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n$

$= 2^n - 1$

$= (2^{n+1} - 2)/2$.

Consider $a_{(n+1,3)}$

$= {}^nC_2\, a_{(2,2)} + {}^nC_3\, a_{(3,2)} + {}^nC_4\, a_{(4,2)} + \ldots\ {}^nC_t\, a_{(t,2)} + \ldots + {}^nC_n\, a_{(n,2)}$

$= {}^nC_2(2^1-1) + {}^nC_3(2^2-1) + {}^nC_4(2^3-1) + \ldots + {}^nC_n(\, 2^{n-1} - 1)$

$= {}^nC_2 2^1 + {}^nC_3 2^2 + \ldots + {}^nC_n\, 2^{n-1} - \{\, {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n \,\}$

$=(1/2)\, \{\, {}^nC_2 2^2 + {}^nC_3 2^3 + \ldots + {}^nC_n\, 2^n\} - \{\, \sum_{r=0}^{n} {}^nC_r - {}^nC_1 - {}^nC_0 \,\}$

$= (1/2)\, \{\, \sum_{r=0}^{n} {}^nC_r\, 2^r - {}^nC_1.2^1 - {}^nC_0.2^0 \,\} - \{\, 2^n - n - 1 \,\}$

$= (1/2)\, \{\, 3^n - 2n - 1\} - 2^n + n + 1$

$= (1/2)\, \{\, 3^n - 2^{n+1} + 1\}$ ----------(3.1)

$= \{1/3!\}\, \{\, (1).3^{n+1} - (3).\ 2^{n+1} + (3).\ (1)^{n+1} - (1)\ (0)^{n+1} \,\}$

Evaluating $a_{(n+1,4)}$

$a_{(n+1,4)} = {}^nC_3\, a_{(3,3)} + {}^nC_4\, a_{(4,3)} + \ldots + {}^nC_n\, a_{(n,3)}$

$= {}^nC_3\{3^2 + 1 - 2^3\}/2 + {}^nC_4\{\, 3^3 + 1 - 2^4 \,\}/2 + \ldots + {}^nC_n\{\, 3^{n-1} + 1 - 2^n \,\}/2$

$= (1/2)[\{\, 3^2.{}^nC_3 + 3^3\, {}^nC_4 + \ldots + 3^{n-1}.{}^nC_n \,\} + \{\, {}^nC_3 + {}^nC_4 + \ldots + {}^nC_n \,\}$

$- \{\, {}^nC_3 2^3 + {}^nC_4 2^4 + \ldots + {}^nC_n\, 2^n\}]$

$= (1/2)[\ (1/3)\ \{\, \sum_{r=0}^{n} {}^nC_r\, 3^r - 3^2\, {}^nC_2 - 3\, {}^nC_1 - {}^nC_0\} + \{\, \sum_{r=0}^{n} {}^nC_r - {}^nC_2 - {}^nC_1$

$- {}^nC_0 \,\} - \{\, \sum_{r=0}^{n} {}^nC_r.2^r - 2^2\, {}^nC_2 - 2\, {}^nC_1 - {}^nC_0\}]$

$= (1/2)\, [(1/3)\{\, 4^n - 9n(n-1)/2 - 3n - 1 \,\} + \{\, 2^n - n(n-1)/2 - n - 1 \,\}$

$- \{ 3^n - 4n(n-1)/2 - 2n - 1\}$

$$a_{(n+1,4)} = (1/4!)\,[\,(1)\,4^{n+1} - (4)\,3^{n+1} + (6)\,2^{n+1} - (4)\,1^{n+1} + 1(0)^{n+1}\,]$$

Observing the pattern we can explore the possibility of

$$\mathbf{a_{(n,r)} = (1/r!) \sum_{k=0}^{r} (-1)^{r-k} \,.{}^{r}C_k \,.k^n \quad \text{-------}(3.2)}$$

which is yet to be established. Now we shall apply induction.

Let the following proposition (3.3) be true for r and all n > r.

$$a_{(n+1,r)} = (1/r!) \sum_{k=1}^{r} (-1)^{r-k} \,.{}^{r}C_k \,.k^{n+1} \quad \text{------}(3.3)$$

Given (3.3) our aim is to prove that

$$a_{(n+1,r+1)} = (1/(r+1)!) \sum_{k=1}^{r+1} [\,(-1)^{(r+1)-k} \,.{}^{r+1}C_k\,(k)^{n+1}\,]$$

we have

$$a_{(n+1,r+1)} = {}^{n}C_r\, a_{(r,r)} + {}^{n}C_{r+1}\, a_{(r+1,r)} + {}^{n}C_{r+2}\, a_{(r+2,r)} + \ldots + {}^{n}C_n\, a_{(n,r)}$$

$$a_{(n+1,r+1)} = {}^{n}C_r \{(1/r!) \sum_{k=0}^{r} (-1)^{r-k} \,.{}^{r}C_k \,.k^r \} + {}^{n}C_{r+1} \{(1/r!) \sum_{k=0}^{r} (-1)^{r-k} \,.{}^{r}C_k\, k^{r+1}$$

$$+ \ldots + {}^{n}C_n \{(1/r!) \sum_{k=0}^{r} (-1)^{r-k} \,.{}^{r}C_k \,.k^n \}$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} \,.{}^{r}C_k \{ {}^{n}C_r\, k^r + {}^{n}C_{r+1}\, k^{r+1} + \ldots + {}^{n}C_n\, k^n \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} \,.{}^{r}C_k \{ \sum_{q=0}^{n} {}^{n}C_q\, k^q - \sum_{q=0}^{r-1} {}^{n}C_q\, k^q \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} \,.{}^{r}C_k\, (1+k)^n\,] - (1/r!) \sum_{q=0}^{r-1} [(-1)^{r-k} \,.{}^{r}C_k \{\sum {}^{n}C_q\, k^q \}]$$

If we denote the I[st] and the second term as T_1 and T_2 , we have

$$a_{(n+1,r+1)} = T_1 - T_2 \qquad \text{----------(3.4)}$$

consider $T_1 = (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^{r}C_k (1+k)^n]$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} \{ r!/((k!)(r-k)!)\} (1+k)^n]$$

$$= (1/(r+1)!) \sum_{k=0}^{r} [(-1)^{r-k} \{(r+1)!/((k+1)!(r-k)!)\}(1+k)^{n+1}]$$

$$= (1/(r+1)!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^{r+1}C_{k+1} (1+k)^{n+1}]$$

$$= (1/(r+1)!) \sum_{k=0}^{r} [(-1)^{(r+1)-(k+1)} . {}^{r+1}C_{k+1} (1+k)^{n+1}]$$

Let $k + 1 = s$, we get , $s = 1$ at $k = 0$ and $s = r + 1$ at $k = r$

$$= (1/(r+1)!) \sum_{s=1}^{r+1} [(-1)^{(r+1)-s} . {}^{r+1}C_s (s)^{n+1}].$$

replacing s by k we get

$$= (1/(r+1)!) \sum_{k=1}^{r+1} [(-1)^{(r+1) - k} . {}^{r+1}C_k (k)^{n+1}]$$

in this if we include $k = 0$ case we get

$$\mathbf{T_1 = (1/(r+1)!) \sum_{k=0}^{r+1} [(-1)^{(r+1) - k} . {}^{r+1}C_k (k)^{n+1}]} \qquad \textbf{----(3.5)}$$

T_1 is nothing but the right hand side member of (3.3).

To prove (3.3) we have to prove $a_{(n+1,r+1)} = T_1$

In view of (3.4) our next step is to prove that $T_2 = 0$

$$T_2 = (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k \{ \sum_{q=0}^{r-1} {}^nC_q k^q \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k \{ {}^nC_0 k^0 + {}^nC_1 k^1 + {}^nC_2 k^2 + . . . + {}^nC_{r-1} k^{r-1} \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k] + {}^nC_1[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k k \}] +$$

$${}^nC_2[(1/r!) \sum_{k=0}^{r}\{(-1)^{r-k} {}^rC_k k^2\}] + . . . + {}^nC_{r-1}[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k k^{r-1} \}]$$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k] + {}^nC_1[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k k \}] +$$

$$[{}^nC_2 .a_{(2,r)} + {}^nC_3 .a_{(3,r)} .+ . . . + {}^nC_{r-1} .a_{(r-1,r)}]$$

$= \quad X \quad + Y \quad + Z$ say where

$$X = (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k] \quad , \quad Y = {}^nC_1[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k k \}]$$

$$Z = [{}^nC_2 .a_{(2,r)} + {}^nC_3 .a_{(3,r)} + . . . + {}^nC_{r-1} .a_{(r-1,r)}]$$

We shall prove that $X = 0$, $Y = 0$, $Z = 0$ seperately.

(1) $X = (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_k]$

$$= (1/r!) \sum_{k=0}^{r} [(-1)^{r-k} . {}^rC_{r-k}]$$

let $r - k = w$ then we get at $k = 0$ $w = r$ and at $k = r$ $w = 0$.

$$= (1/r!) \sum_{w=r}^{0} [(-1)^{w} . {}^rC_w]$$

$$= (1/r!) \sum_{w=0}^{r} [(-1)^w . {}^rC_w]$$

$$=(1 - 1)^r /r!$$

$$= 0$$

We have proved that $X = 0$

(2)

$$Y = {}^nC_1[(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} . {}^rC_k\ k \}]$$

$$= {}^nC_1[(1/(r-1)!) \sum_{k=1}^{r} \{(-1)^{r-1-(k-1)} . {}^{r-1}C_{k-1} \}]$$

$$= {}^nC_1[(1/(r-1)!) \sum_{k-1=0}^{r-1} \{(-1)^{r-1-(k-1)} . {}^{r-1}C_{k-1} \}]$$

$$= {}^nC_1[(1/(r-1)!)(1 - 1)^{r-1}$$

$$= 0$$

We have proved that $Y = 0$

(3) To prove

$$Z = [{}^nC_2 .a_{(2,r)} + {}^nC_3 .a_{(3,r)} + . . . + {}^nC_{r-1} .a_{(r-1,r)}] = 0 \quad \text{----(3.6)}$$

Proof:

Refer the matrix

$$\begin{array}{llllll}
\underline{a_{(1,1)}} & a_{(1,2)} & a_{(1,3)} & a_{(1,4)} & \cdots & a_{(1,r)} \\
a_{(2,1)} & \underline{a_{(2,2)}} & a_{(2,3)} & a_{(2,4)} & \cdots & a_{(2,r)} \\
a_{(3,1)} & a_{(3,2)} & \underline{a_{(3,3)}} & a_{(3,4)} & \cdots & a_{(3,r)} \\
a_{(4,1)} & a_{(4,2)} & a_{(4,3)} & \underline{a_{(4,4)}} & a_{(4,5)} \cdots & a_{(4,r)} \\
 & & & \cdots\cdots & &
\end{array}$$

. . . . $\underline{a_{(r-1,r-1)}}$ $\mathbf{a_{(r-1,r)}}$ . . .

$a_{(r,1)}$ $a_{(r,2)}$ $a_{(r,3)}$ . . . $a_{(r,r-1)}$ $\underline{a_{(r,r)}}$

The Diagonal elements are underlined . And the the elements

above the leading diagonal are shown with bold face.

We have

$$a_{(1,r)} = [\,(1/r!) \sum_{k=0}^{r} \{(-1)^{r-k} \,.\, {}^{r}C_k \, k\}] = Y/{}^{n}C_1 = 0 \text{ for } r > 1$$

All the elements of the first row except $a_{(1,1)}$ (the one on the leading diagonal) are zero.
Also

$$a_{(n+1,r)} = a_{(n,r-1)} + r \,.\, a_{(n,r)} \qquad \text{--------(3.7)}$$

(This can be easily established by simplifying the right hand side.)

(7) gives us

$$a_{(2,r)} = a_{(1,r-1)} + r \,.\, a_{(1,r)} = 0 \text{ for } r > 2$$

i.e. $a_{(2,r)}$ can be expresssed as a linear combination of two

elements of the first row (except the one on the leading diagonal)

$\Rightarrow a_{(2,r)} = 0 \quad r > 2$

Similarly $a_{(3,r)}$ can be expresssed as a linear combination of two

elements of the second row of the type $a_{(2,r)}$ with $r > 3$

$\Rightarrow a_{(2,r)} = 0 \quad r > 3$

and so on $a_{(r-1,r)} = 0$

substituting

$a_{(2,r)} =.a_{(3,r)} = . . . = a_{(r-1,r)} = 0$ in (6)

we get $Z = 0$

With $X = Y = Z = 0$ we get $T_2 = 0$
or $a_{(n+1,r+1)} = T_1 - T_2 = T_1$

from (5) we have

$$T_1 = (1/(r+1)!) \sum_{k=0}^{r+1} [(-1)^{(r+1)-k} \; {}^{r+1}C_k \, (k)^{n+1}]$$

which gives

$$a_{(n+1,r+1)} = (1/(r+1)!) \sum_{k=0}^{r+1} [(-1)^{(r+1)-k} \; {}^{r+1}C_k \, (k)^{n+1}]$$

We have proved ,if the propposition (3.3) is true for r it is true for (r+1) as well .We have already verified it for 1, 2, 3 etc. Hence by induction (3.3) is true for all n.

This completes the proof of theorem (3.1) .

Remarks: This proof is quite lengthy , clumsy and heavy in algebra. The readers can try some analytic , combinatorial approach.

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[4] 'Smarandache Notion Journal' Vol. 10 ,No. 1-2-3, Spring 1999. Number Theory Association of the UNIVERSITY OF CRAIOVA.