International J.Math. Combin. Vol.2 (2010), 65-67

A Note On Jump Symmetric n-Sigraph

H. A.Malathi and H. C.Savithri

Department of Computer Science & Engineering of Rajeev Institute of Technology,

Industrial Area, B-M Bypass Road, Hassan 573 201, India

Email: malathisharmas@gmail.com, savithriathreyas@gmail.com

Abstract: A *Smarandachely k-signed graph* (*Smarandachely k-marked graph*) is an ordered pair $S = (G, \sigma)$ $(S = (G, \mu))$ where $G = (V, E)$ is a graph called *underlying graph of* S and $\sigma : E \rightarrow (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k)$ $(\mu : V \rightarrow (\overline{e}_1, \overline{e}_2, ..., \overline{e}_k))$ is a function, where each $\overline{e}_i \in \{+, -\}$. Particularly, a Smarandachely 2-signed graph or Smarandachely 2-marked graph is called abbreviated a *signed graph* or a *marked graph*. In this note, we obtain a structural characterization of jump symmetric n-sigraphs. The notion of jump symmetric n-sigraphs was introduced by E. Sampathkumar, P. Siva Kota Reddy and M. S. Subramanya [Proceedings of the Jangjeon Math. Soc., 11(1) (2008), 89-95].

Key Words: Smarandachely symmetric n-sigraph, Smarandachely symmetric n-marked graph, Balance, Jump symmetric n-sigraph.

AMS(2000): 05C22

§1. Introduction

For standard terminology and notion in graph theory we refer the reader to West [6]; the non-standard will be given in this paper as and when required. We treat only finite simple graphs without self loops and isolates.

Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *Smarandachely symmetric n-sigraph (Smarandachely symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \rightarrow H_n$ $(\mu : V \rightarrow H_n)$ is a function.

A *sigraph (marked graph)* is an ordered pair $S = (G, \sigma)$ $(S = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S and $\sigma : E \rightarrow \{+, -\}$ $(\mu : V \rightarrow \{+, -\})$ is a function. Thus a Smarandachely symmetric 1-sigraph (Smarandachely symmetric 1-marked graph) is a sigraph (marked graph).

The *line graph* $L(G)$ of graph G has the edges of G as the vertices and two vertices of $L(G)$

[1]Received April 21, 2010. Accepted June 12, 2010.

are adjacent if the corresponding edges of G are adjacent.

The *jump graph* $J(G)$ of a graph $G = (V, E)$ is $\overline{L(G)}$, the complement of the line graph $L(G)$ of G (See [1] and [2]).

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/Smarandachely symmetric n-sigraph/Smarandachely symmetric n-marked graph.

An n-tuple $(a_1, a_2, ..., a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [4], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P. Siva Kota Reddy [3]):

Definition 1.1 *Let $S_n = (G, \sigma)$ be an n-sigraph. Then,*

(i) S_n is identity balanced *(or* i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and*

(ii) S_n is balanced*, if every cycle in S_n contains an even number of non-identity edges.*

Note An i-balanced n-sigraph need not be balanced and conversely.

The following characterization of i-balanced n-sigraphs is obtained in [4].

Proposition 1.1(E. Sampathkumar et al. [4]) *An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.*

The *line n-sigraph* $L(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is defined as follows (See [5]): $L(S_n) = (L(G), \sigma')$, where for any edge ee'in $L(G)$, $\sigma'(ee') = \sigma(e)\sigma(e')$.

The *jump n-sigraph* of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph $J(S_n) = (J(G), \sigma')$, where for any edge ee' in $J(S_n)$, $\sigma'(ee') = \sigma(e)\sigma(e')$. This concept was introduced by E. Sampathkumar et al. [4]. This notion is analogous to the line n-sigraph defined above. Further, an n-sigraph $S_n = (G, \sigma)$ is called *jump n-sigraph*, if $S_n \cong J(S'_n)$ for some signed graph S'. In the following section, we shall present a characterization of jump n-sigraphs. The following result indicates the limitations of the notion of jump n-sigraphs defined above, since the entire class of i-unbalanced n-sigraphs is forbidden to be jump n-sigraphs.

Proposition 1.2(E. Sampathkumar et al. [4]) *For any n-sigraph $S_n = (G, \sigma)$, its jump n-sigraph $J(S_n)$ is i-balanced.*

§2. Characterization of Jump n-Sigraphs

The following result characterize n-sigraphs which are jump n-sigraphs.

Proposition 2.1 *An n-sigraph $S_n = (G, \sigma)$ is a jump n-sigraph if, and only if, S_n is i-balanced*

n-sigraph and its underlying graph G is a jump graph.

Proof Suppose that S_n is i-balanced and G is a jump graph. Then there exists a graph H such that $J(H) \cong G$. Since S_n is i-balanced, by Proposition 1.1, there exists a marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n-sigraph $S'_n = (H, \sigma')$, where for any edge e in H, $\sigma'(e)$ is the marking of the corresponding vertex in G. Then clearly, $J(S'_n) \cong S_n$. Hence S_n is a jump n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a jump n-sigraph. Then there exists a n-sigraph $S'_n = (H, \sigma')$ such that $J(S'_n) \cong S_n$. Hence G is the jump graph of H and by Proposition 1.2, S_n is i-balanced. □

References

[1] G. T. Chartrand, H. Hevia, E. B. Jaretre, and M. Schutz, Subgraph distance in graphs defined by edge transfers, *Discrete mathematics,* 170 (1997), 63-79.

[2] F. Harary, *Graph Theory*, Addison-Wesley Publishing Co., 1969.

[3] R. Rangarajan and P. Siva Kota Reddy, Notions of balance in symmetric n-sigraphs, *Proceedings of the Jangjeon Math. Soc.*, 11(2) (2008), 145-151.

[4] E. Sampathkumar, P. Siva Kota Reddy, and M. S. Subramanya, Jump symmetric n-sigraph, *Proceedings of the Jangjeon Math. Soc.* 11(1) (2008), 89-95.

[5] E. Sampathkumar, P. Siva Kota Reddy, and M. S. Subramanya, The Line n-sigraph of a symmetric n-sigraph, *Southeast Asian Bull. Math.*(Springer-Verlag), to appear.

[6] D. B. West, *Introduction to Graph Theory*, Prentice-Hall of India Pvt. Ltd., 1996.