

The Smarandache-Coman congruence on primes and four conjectures on Poulet numbers based on this new notion

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Abstract. In two previous articles I defined the Smarandache-Coman divisors of order k of a composite integer n with m prime factors and I made few conjectures about few possible infinite sequences of Poulet numbers, characterized by a certain set of Smarandache-Coman divisors. In this paper I define a very related notion, the Smarandache-Coman congruence on primes, and I also make five conjectures regarding Poulet numbers based on this new notion.

Definition 1:

We define in the following way the *Smarandache-Coman congruence on primes*: we say that two primes p and q are congruent sco n and we note $p \equiv q(\text{sco } n)$ if $S(p - n) = S(q - n) = k$, where n is a positive non-null integer and S is the Smarandache function (obviously k is also a non-null integer). We also may say that k is equal to $p \text{ sco } n$ respectively k is also equal to $q \text{ sco } n$ and note $k = p \text{ sco } n = q \text{ sco } n$.

Note:

The notion of *Smarandache-Coman congruence* is very related with the notion of *Smarandache-Coman divisors*, which we defined in previous papers in the following way (Definitions 2-4):

Definition 2:

We call the set of *Smarandache-Coman divisors of order 1* of a composite positive integer n with m prime factors, $n = d_1 * d_2 * \dots * d_m$, where the least prime factor of n , d_1 , is greater than or equal to 2, the set of numbers defined in the following way: $SCD_1(n) = \{S(d_1 - 1), S(d_2 - 1), \dots, S(d_m - 1)\}$, where S is the Smarandache function.

Definition 3:

We call the set of *Smarandache-Coman divisors of order 2* of a composite positive integer n with m prime factors, $n = d_1 * d_2 * \dots * d_m$, where the least prime factor of n , d_1 , is greater than or equal to 3, the set of numbers defined in the following way: $SCD_2(n) = \{S(d_1 - 2), S(d_2 - 2), \dots, S(d_m - 2)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is $SCD_1(6) = \{S(2 - 1), S(3 - 1)\} = \{S(1), S(2)\} = \{1, 2\}$;
2. The set of SC divisors of order 2 of the number 21 is $SCD_2(21) = \{S(3 - 2), S(7 - 2)\} = \{S(1), S(5)\} = \{1, 5\}$.

Definition 4:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors, $n = d_1 * d_2 * \dots * d_m$, where the least prime factor of n, d_1 , is greater than or equal to $k + 1$, the set of numbers defined in the following way: $SCD_k(n) = \{S(d_1 - k), S(d_2 - k), \dots, S(d_m - k)\}$, where S is the Smarandache function.*

Note:

As I said above, in two previous articles I applied the notion of *Smarandache-Coman divisors* in the study of Fermat pseudoprimes; now I will apply the notion of *Smarandache-Coman congruence* in the study of the same class of numbers.

Conjecture 1:

There is at least one non-null positive integer n such that the prime factors of a Poulet number P, where P is not divisible by 3 or 5 and also P is not a Carmichael number, are, all of them, congruent sco n.

Verifying the conjecture:

(for the first five Poulet numbers not divisible by 3 or 5; see the sequence A001567 in OEIS for a list of these numbers; see also the sequence A002034 for the values of Smarandache function)

: For $P = 341 = 11 * 31$, we have $S(11 - 1) = S(31 - 1) = 5$, so the prime factors 11 and 31 are congruent sco 1, which is written $11 \equiv 31(\text{sco } 1)$, or, in other words, $11 \text{ sco } 1 = 31 \text{ sco } 1 = 5$; we also have $S(11 - 7) = S(31 - 7) = 4$, so $11 \equiv 31(\text{sco } 7)$;

: For $P = 1387 = 19 * 73$, we have $S(19 - 1) = S(73 - 1) = 6$, so the prime factors 19 and 73 are congruent sco 1, or, in other words, 6 is equal to 19 sco 1 and also with 73 sco 1;

: For $P = 2047 = 23 * 89$, we have $S(23 - 1) = S(89 - 1) = 11$, so the prime factors 19 and 73 are congruent sco 1;

: For $P = 2701 = 37 * 73$, we have $S(37 - 1) = S(73 - 1) = 6$, so the prime factors 19 and 73 are congruent sco 1;

: For $P = 3277 = 29 * 113$, we have $S(29 - 1) = S(113 - 1) = 7$, so the prime factors 29 and 113 are congruent sco 1.

Note:

If the conjecture doesn't hold in this form might be considered only the 2-Poulet numbers not divisible by 3 or 5.

Conjecture 2:

There is at least one non-null positive integer n such that, for all the prime factors $(d_1, d_2, \dots, d_{k-1})$ beside 3 of a k -Poulet number P divisible by 3 and not divisible by 5 is true that there exist the primes q_1, q_2, \dots, q_n (not necessarily distinct) such that $q_1 = d_1 \text{ sco } n$, $q_2 = d_2 \text{ sco } n$, \dots , $q_{k-1} = d_{k-1} \text{ sco } n$.

Verifying the conjecture:

(for the first four Poulet numbers divisible by 3 and not divisible by 5)

: For $P = 561 = 3 \cdot 11 \cdot 17$, we have $7 = 11 \text{ sco } 4$ and $13 = 17 \text{ sco } 4$;

: For $P = 4371 = 3 \cdot 31 \cdot 47$, we have $31 = 7 \text{ sco } 3$ and $47 = 11 \text{ sco } 3$;

: For $P = 8481 = 3 \cdot 11 \cdot 257$, we have $11 = 7 \text{ sco } 4$ and $257 = 23 \text{ sco } 4$;

: For $P = 12801 = 3 \cdot 17 \cdot 251$, we have $17 = 5 \text{ sco } 2$ and $251 = 83 \text{ sco } 2$.

Conjecture 3:

There is at least one non-null positive integer n such that, for all the prime factors $(d_1, d_2, \dots, d_{k-1})$ beside 5 of a k -Poulet number P divisible by 5 and not divisible by 3 is true that there exist the primes q_1, q_2, \dots, q_n (not necessarily distinct) such that $q_1 = d_1 \text{ sco } n$, $q_2 = d_2 \text{ sco } n$, \dots , $q_{k-1} = d_{k-1} \text{ sco } n$.

Verifying the conjecture:

(for the first four Poulet numbers divisible by 5 and not divisible by 3)

: For $P = 1105 = 5 \cdot 13 \cdot 17$, we have $13 = 11 \text{ sco } 2$ and $17 = 5 \text{ sco } 2$;

: For $P = 10585 = 5 \cdot 29 \cdot 73$, we have $29 = 13 \text{ sco } 3$ and $73 = 7 \text{ sco } 3$;

: For $P = 11305 = 5 \cdot 7 \cdot 17 \cdot 19$, we have $7 = 5 \text{ sco } 2$, $17 = 5 \text{ sco } 2$ and $19 = 17 \text{ sco } 2$;

: For $P = 41665 = 5 \cdot 13 \cdot 641$, we have $13 = 11 \text{ sco } 2$ and $641 = 71 \text{ sco } 2$.

Conjecture 4:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, \dots, d_k) of a k -Poulet number P not divisible by 3 or 5 is true that there exist the primes q_1, q_2, \dots, q_n (not necessarily distinct) such that $q_1 = d_1 \text{ sco } n$, $q_2 = d_2 \text{ sco } n$, \dots , $q_k = d_k \text{ sco } n$.

Note:

In other words, because we defined the Smarandache-Coman congruence only on primes, we can say that for any set of divisors d_1, d_2, \dots, d_k of a k -Poulet number P not divisible by 3 or 5 there exist a non-null positive integer n such that for any d_i (where i from 1 to k) can be defined a Smarandache-Coman congruence $d_i \equiv q_i \text{ (sco } n)$.

References:

1. Coman, Marius, *The math encyclopedia of Smarandache type notions*, Educational publishing, 2013;
2. Coman, Marius, *Two hundred conjectures and one hundred and fifty open problems about Fermat pseudoprimes*, Educational publishing, 2013.