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Surfaces Family With Common Smarandache Asymptotic Curve

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ABSTRACT: In this paper, we analyzed the problem of consructing a family of surfaces from a given some special Smarandache curves in Euclidean 3-space. Using the Frenet frame of the curve in Euclidean 3-space, we express the family of surfaces as a linear combination of the components of this frame, and derive the necessary and sufficient conditions for coefficents to satisfy both the asymptotic and isoparametric requirements. Finally, examples are given to show the family of surfaces with common Smarandache curve.

Contents

1. Introduction

In differential geometry, there are many important consequences and properties of curves [1], [2], [3]. Researches follow labours about the curves. In the light of the existing studies, authors always introduce new curves. Special Smarandache curves are one of them. Special Smarandache curves have been investigated by some differential geometers [4,5,6,7,8,9,10]. This curve is defined as, a regular curve in Minkowski space-time, whose position vector is composed by Frenet frame vectors on another regular curve, is Smarandache curve [4]. A.T. Ali has introduced some special Smarandache curves in the Euclidean space [5]. Special Smarandache curves according to Bishop Frame in Euclidean 3-space have been investigated by Çetin et al [6]. In addition, Special Smarandache curves according to Darboux Frame in Euclidean 3-space has introduced in [7]. They found some properties of these special curves and calculated normal curvature, geodesic curvature and geodesic torsion of these curves. Also, they investigate special Smarandache curves in Minkowski 3-space, [8]. Furthermore, they find some properties of these special curves and they calculate curvature and torsion of these curves. Special Smarandache curves such as -Smarandache curves according to Sabban frame in Euclidean unit sphere has introduced in [9]. Also, they give some characterization of Smarandache curves and illustrate examples of their results.On the Quaternionic Smarandache Curves in Euclidean 3-Space have been investigated in [10].

One of the most significant curve on a surface is asymptotic curve. Asymptotic curve on a surface has been a long-term research topic in Differential Geometry, [3,11,12]. A curve on a surface is called an asymptotic curve provided its velocity always points in an asymptotic direction, that is the direction in which the normal curvature is zero. Another criterion for a curve in a surface M to be asymptotic is that its acceleration always be tangent to M, [2]. Asymptotic curves are also encountered in astronomy, astrophysics and CAD in architecture. The concept of family of surfaces having a given characteristic curve was first introduced by Wang et.al. [13] in Euclidean 3-space. Kasap et.al. [14] generalized the work of Wang by introducing new types of marching-scale functions, coefficients of the Frenet frame appearing in the parametric representation of surfaces. With the inspiration of work of Wang, Li et.al. [15] changed the characteristic curve from geodesic to line of curvature and defined the surface pencil with a common line of curvature.Recently, in [16] Bayram et.al. defined the surface pencil with a common asymptotic curve. They introduced three types of marching-scale functions and derived the necessary and sufficient conditions on them to satisfy both parametric and asymptotic requirements.

In this paper, we study the problem: given a curve (with Frenet frame), how to characterize those surfaces that posess this curve as a common isoasymptotic and Smarandache curve in Euclidean 3-space. In section 2, we give some preliminary information about Smarandache curves in Euclidean 3-space and define isoasymptotic curve. We express surfaces as a linear combination of the Frenet frame of the given curve and derive necessary and sufficient conditions on marching-scale functions to satisfy both isoasymptoticand Smarandache requirements in Section 3. We illustrate the method by giving some examples.

2. Preliminaries

Let be a 3-dimensional Euclidean space provided with the metric given by $\langle , \rangle = dx_1^2 + dx_2^2 + dx_3^2$ where (x_1, x_2, x_3) is a rectangular coordinate system of E^3. Recall that, the norm of a arbitrary vector $X \epsilon E^3$ is given by $\parallel X \parallel = \sqrt{\langle X, X \rangle}$. Let $\alpha = \alpha(s) : I \subset IR \longrightarrow E^3$ is an arbitrary curve of arc-length parameter s. The curve α is called a unit speed curve if velocity α' vector of a satisfies $\parallel \alpha' \parallel = 1$. Let $\{T(s), N(s), B(s)\}$be the moving Frenet frame along α, Frenet formulas is given by

$$\frac{d}{ds}\begin{pmatrix} T(s) \\ N(s) \\ B(s) \end{pmatrix} = \begin{pmatrix} 0 & \kappa(s) & 0 \\ -\kappa(s) & 0 & \tau(s) \\ 0 & -\tau(s) & 0 \end{pmatrix} \begin{pmatrix} T(s) \\ N(s) \\ B(s) \end{pmatrix}$$

where the function $\kappa(s)$ and $\tau(s)$ are called the curvature and torsion of the curve $\alpha(s)$, respectively.

Let r be a regular curve in a surface P passing through $s\ \epsilon P$, κ the curvature of r at s and $\cos\theta = n.N$,where N is the normal vector to r and is n normal vector to P at s and " ." denotes the standard inner product. The number $k_n = \kappa \cos\theta$ is then called the normal curvature of at s [2].

Let s be a point in a surface P. An asymptotic direction of P at s is a direction of the tangent plane of P for which the normal curvature is zero. An asymptotic curve of P is a regular curve $r \subset P$ such that for each $s\epsilon r$ the tangent line of r at s is an asymptotic direction [2].

An isoparametric curve $\alpha(s)$ is a curve on a surface $\Psi = \Psi(s,t)$is that has a constant s or t-parameter value. In other words, there exist a parameter or such that $\alpha(s) = \Psi(s,t_0)$ or $\alpha(t) = \Psi(s_0,t)$.

Given a parametric curve $\alpha(s)$, we call $\alpha(s)$ an isoasymptotic of a surface Ψif it is both a asymptotic and an isoparametric curve on Ψ.

Let $\alpha = \alpha(s)$ be a unit speed regular curve in E^3 and $\{T(s), N(s), B(s)\}$ be its moving Frenet-Serret frame . Smarandache TN curves are defined by

$\beta = \beta(s^*) = \frac{1}{\sqrt{2}}\left(T(s) + N(s)\right)$

Smarandache NB curves are defined by

$\beta = \beta(s^*) = \frac{1}{\sqrt{2}}\left(N(s) + B(s)\right)$

Smarandache TNB curves are defined by

$\beta = \beta(s^*) = \frac{1}{\sqrt{3}}\left(T(s) + N(s) + B(s)\right)$, [5].

3. Surfaces with common Smarandache asymptotic curve

Let $\varphi = \varphi(s,v)$ be a parametric surface. The surface is defined by a given curve $\alpha = \alpha(s)$ as follows:

$$\varphi(s,v) = \alpha(s) + [x(s,v)T(s) + y(s,v)N(s) + z(s,v)B(s)],\ L_1 \leq s \leq L_2\ ,\ T_1 \leq v \leq T_2\ , \tag{3.1}$$

where $x(s,v)$, $y(s,v)$ and $z(s,v)$ are C^1 functions. The values of the marching-scale functions $x(s,v)$, $y(s,v)$ and $z(s,v)$ indicate, respectively; the extension-like, flexion-like and retortion-like effects, by the point unit through the time v, starting from $\alpha(s)$ and $\{T(s), N(s), B(s)\}$ is the Frenet frame associated with the curve $\alpha(s)$.

Our goal is to find the necessary and sufficient conditions for which the some special Smarandache curves of the unit space curve $\alpha(s)$ is an parametric curve and an asymptotic curve on the surface $\varphi(s,v)$.

Firstly, since Smarandache curve of $\alpha(s)$ is an parametric curve on the surface $\varphi(s,v)$, there exists a parameter $v_0\epsilon[T_1,T_2]$ such that

$$x(s,v_0) = y(s,v_0) = z(s,v_0) = 0,\ L_1 \leq s \leq L_2\ ,\ T_1 \leq v \leq T_2\ . \tag{3.2}$$

Secondly, according to the above definitions, the curve $\alpha(s)$ is an asymptotic curve on the surface $\varphi(s,v)$ if and only if the normal curvature $k_n = \kappa\cos\theta\ = 0$, where θ is the angle between the surface normal $n(s,v_0)$ and the principal normal $N(s)$ of the curve $\alpha(s)$. Since $n(s,v_0).T(s) = 0,\ L_1 \leq s \leq L_2$, by derivating this equation with respect to the arc length parameter s, we have the equivalent constraint

$$\frac{dn}{ds}(s,v_0).T(s) = 0 \tag{3.3}$$

for the curve $\alpha(s)$ to be an asymptotic curve on the surface $\varphi(s,v)$, where " ." denotes the standard inner product.

Theorem 3.1. *: Smarandache TN curve of the curve $\alpha(s)$ is isoasymptotic on a surface $\varphi(s,v)$ if and only if the following conditions are satisfied:*

$$\begin{cases} x(s,v_0) = y(s,v_0) = z(s,v_0) = 0, \\ \frac{\partial z}{\partial v}(s,v_0) = -\frac{\tau(s)}{\kappa(s)}\frac{\partial x}{\partial v}(s,v_0). \end{cases}$$

Proof: From (3.1) , $\varphi(s,v)$, parametric surface is defined by a given Smarandache TN curve of curve $\alpha(s)$ as follows:

$$\varphi(s,v) = \frac{1}{\sqrt{2}}(T(s)+N(s)) + [x(s,v)T(s) + y(s,v)N(s) + z(s,v)B(s)]$$

Let $\alpha(s)$ be a Smarandache TN curve on surface $\varphi(s,v)$, . If Smarandache TN curve is an parametric curve on this surface , then there exist a parameter $v = v_0$ such that , $\frac{1}{\sqrt{2}}(T(s)+N(s)) = \varphi(s,v_0)$, that is,

$$x(s,v_0) = y(s,v_0) = z(s,v_0) = 0 \tag{3.4}$$

The normal vector of $\varphi(s,v)$ can be written as

$$n(s,v) = \frac{\partial\varphi(s,v)}{ds} \times \frac{\partial\varphi(s,v)}{\partial v}$$

Since

$$\begin{aligned}
\frac{\partial\varphi(s,v)}{ds} &= (-\frac{\kappa(s)}{\sqrt{2}} + \frac{\partial x(s,v)}{ds} - \kappa(s)y(s,v))T(s) \\
&\quad +(\frac{\kappa(s)}{\sqrt{2}} + \frac{\partial y(s,v)}{ds} + \kappa(s)x(s,v) - \tau(s)z(s,v))N(s) \\
&\quad +(\frac{\tau(s)}{\sqrt{2}} + \frac{\partial z(s,v)}{ds} + \tau(s)y(s,v))B(s) \\
\frac{\partial\varphi(s,v)}{dv} &= \frac{\partial x(s,v)}{dv}T(s) + \frac{\partial y(s,v)}{dv}N(s) + \frac{\partial z(s,v)}{dv}B(s)
\end{aligned}$$

The normal vector can be expressed as

$$\begin{aligned}
n(s,v) &= [\frac{\partial z(s,v)}{dv}(\frac{\kappa(s)}{\sqrt{2}} + \frac{\partial y(s,v)}{ds} + \kappa(s)x(s,v) - \tau(s)z(s,v)) \\
&\quad -\frac{\partial y(s,v)}{dv}(\frac{\tau(s)}{\sqrt{2}} + \frac{\partial z(s,v)}{ds} + \tau(s)y(s,v))]T(s) \\
&\quad +[\frac{\partial x(s,v)}{dv}(\frac{\tau(s)}{\sqrt{2}} + \frac{\partial z(s,v)}{ds} + \tau(s)y(s,v)) \\
&\quad -\frac{\partial z(s,v)}{dv}(-\frac{\kappa(s)}{\sqrt{2}} + \frac{\partial x(s,v)}{ds} - \kappa(s)y(s,v))]N(s) \\
&\quad +[\frac{\partial y(s,v)}{dv}(-\frac{\kappa(s)}{\sqrt{2}} + \frac{\partial x(s,v)}{ds} - \kappa(s)y(s,v)) \\
&\quad -\frac{\partial x(s,v)}{dv}(\frac{\kappa(s)}{\sqrt{2}} + \frac{\partial y(s,v)}{ds} + \kappa(s)x(s,v) \\
&\quad -\tau(s)z(s,v))]B(s)
\end{aligned} \tag{3.5}$$

Thus, if we let

$$\begin{cases} \Phi_1(s,v_0) = \frac{\partial z(s,v_0)}{dv}\frac{\kappa(s)}{\sqrt{2}} - \frac{\partial y(s,v_0)}{dv}\frac{\tau(s)}{\sqrt{2}}, \\ \Phi_2(s,v_0) = \frac{\partial x(s,v_0)}{dv}\frac{\tau(s)}{\sqrt{2}} + \frac{\partial z(s,v_0)}{dv}\frac{\kappa(s)}{\sqrt{2}}, \\ \Phi_3(s,v_0) = -\frac{\partial y(s,v_0)}{dv}\frac{\kappa(s)}{\sqrt{2}} - \frac{\partial x(s,v_0)}{dv}\frac{\kappa(s)}{\sqrt{2}}. \end{cases} \tag{3.6}$$

we obtain

$$n(s,v_0) = \Phi_1(s,v_0)T(s) + \Phi_2(s,v_0)N(s) + \Phi_3(s,v_0)B(s).$$

From the eqn. (3.2), we should have

$$\begin{aligned} \frac{\partial n}{\partial s}(s,v_0).T(s) = 0 \Leftrightarrow & \frac{\partial\left(\Phi_1(s,v_0)T(s) + \Phi_2(s,v_0)N(s) + \Phi_3(s,v_0)B(s)\right)}{\partial s}.T(s) = 0 \\ \Leftrightarrow & \frac{\partial \Phi_1}{\partial s}(s,v_0) - \kappa(s)\Phi_2(s,v_0) = 0. \end{aligned} \tag{3.7}$$

We using eqn. (3.6) in eqn. (3.7), since $\kappa(s) \neq 0$, we get

$$\frac{\partial z(s,v_0)}{dv} = -\frac{\tau(s)}{\kappa(s)}\frac{\partial x(s,v_0)}{dv} \tag{3.8}$$

which completes the proof. □

Theorem 3.2. *: Smarandache NB curve of the curve $\alpha(s)$ is isoasymptotic on a surface $\varphi(s,v)$ if and only if the following conditions are satisfied:*

$$\begin{cases} x(s,v_0) = y(s,v_0) = z(s,v_0) = 0, \\ \frac{\partial z}{\partial v}(s,v_0) = -\frac{\tau(s)}{\kappa(s)}\frac{\partial x}{\partial v}(s,v_0). \end{cases}$$

Proof: From (3.1) , $\varphi(s,v)$ parametric surface is defined by a given Smarandache NB curve of curve $\alpha = \alpha(s)$ as follows:

$$\varphi(s,v) = \frac{1}{\sqrt{2}}\left(N(s) + B(s)\right) + [x(s,v)T(s) + y(s,v)N(s) + z(s,v)B(s)].$$

Let $\alpha(s)$ be a Smarandache NB curve on surface $\varphi(s,v)$. If Smarandache NB curve is an parametric curve on this surface , then there exist a parameter $v = v_0$ such that ,$\frac{1}{\sqrt{2}}\left(N(s) + B(s)\right) = \varphi(s,v_0)$, that is,

$$x(s,v_0) = y(s,v_0) = z(s,v_0) = 0 \tag{3.9}$$

The normal vector can be expressed as

$$\begin{aligned} n(s,v) &= [\frac{\partial z(s,v)}{dv}(\kappa(s)x(s,v)+\frac{\partial y(s,v)}{ds}-\tau(s)z(s,v)) \\ &\quad -\frac{\partial y(s,v)}{dv}(\frac{\tau(s)}{\sqrt{2}}+\frac{\partial z(s,v)}{ds}+\tau(s)y(s,v))]T(s) \\ &\quad +[\frac{\partial x(s,v)}{dv}(\frac{\tau(s)}{\sqrt{2}}+\frac{\partial z(s,v)}{ds}+\tau(s)y(s,v)) \\ &\quad -\frac{\partial z(s,v)}{dv}(-\frac{\kappa(s)}{\sqrt{2}}+\frac{\partial x(s,v)}{ds}-\kappa(s)y(s,v))]N(s) \\ &\quad +[\frac{\partial y(s,v)}{dv}(-\frac{\kappa(s)}{\sqrt{2}}+\frac{\partial x(s,v)}{ds}-\kappa(s)y(s,v))-\frac{\partial x(s,v)}{dv}(\kappa(s)x(s,v) \\ &\quad +\frac{\partial y(s,v)}{ds}+\kappa(s)x(s,v)-\tau(s)z(s,v))]B(s) \end{aligned} \tag{3.10}$$

Thus, if we let

$$\begin{cases} \Phi_1(s,v_0) = -\frac{\partial y(s,v_0)}{dv}\frac{\tau(s)}{\sqrt{2}}, \\ \Phi_2(s,v_0) = \frac{\partial x(s,v_0)}{dv}\frac{\tau(s)}{\sqrt{2}}+\frac{\partial z(s,v_0)}{dv}\frac{\kappa(s)}{\sqrt{2}}, \\ \Phi_3(s,v_0) = -\frac{\partial y(s,v_0)}{dv}\frac{\kappa(s)}{\sqrt{2}}. \end{cases} \tag{3.11}$$

From the eqn. (3.2), we should have

$$\begin{aligned} \frac{\partial n}{\partial s}(s,v_0).T(s)=0 &\Leftrightarrow \frac{\partial\left(\Phi_1(s,v_0)T(s)+\Phi_2(s,v_0)N(s)+\Phi_3(s,v_0)B(s)\right)}{\partial s}.T(s)=0 \\ &\Leftrightarrow \frac{\partial \Phi_1}{\partial s}(s,v_0)-\kappa(s)\Phi_2(s,v_0)=0. \end{aligned} \tag{3.12}$$

We using eqn. (3.11) in eqn. (3.12),since $\kappa(s)\neq 0$, we get

$$\frac{\partial z(s,v_0)}{dv} = -\frac{\tau(s)}{\kappa(s)}\frac{\partial x(s,v_0)}{dv} \tag{3.13}$$

which completes the proof. □

Theorem 3.3. *: Smarandache TNB curve of the curve $\alpha(s)$ is isoasymptotic on a surface $\varphi(s,v)$ if and only if the following conditions are satisfied:*

$$\begin{cases} x(s,v_0)=y(s,v_0)=z(s,v_0)=0, \\ \frac{\partial z}{\partial v}(s,v_0)=-\frac{\tau(s)}{\kappa(s)}\frac{\partial x}{\partial v}(s,v_0). \end{cases}$$

Proof: From (3.1) , $\varphi(s,v)$ parametric surface is defined by a given Smarandache TNB curve of curve $\alpha=\alpha(s)$ as follows:

$$\varphi(s,v)=\frac{1}{\sqrt{3}}\left(T(s)+N(s)+B(s)\right)+[x(s,v)T(s)+y(s,v)N(s)+z(s,v)B(s)].$$

Let $\alpha(s)$ be a Smarandache TNB curve on surface $\varphi(s,v)$. If Smarandache TNB curve is an parametric curve on this surface , then there exist a parameter $v = v_0$ such that ,$\frac{1}{\sqrt{3}}(T(s)+N(s)+B(s)) = \varphi(s,v_0)$, that is,

$$x(s,v_0) = y(s,v_0) = z(s,v_0) = 0.$$

The normal vector can be expressed as

$$\begin{aligned} n(s,v) \quad = \quad & [\frac{\partial z(s,v)}{dv}(\frac{\kappa(s)-\tau(s)}{\sqrt{3}} + \frac{\partial y(s,v)}{ds} + \kappa(s)x(s,v) - \tau(s)z(s,v)) \\ & -\frac{\partial y(s,v)}{dv}(\frac{\tau(s)}{\sqrt{3}} + \frac{\partial z(s,v)}{ds} + \tau(s)y(s,v))]T(s) \\ & +[\frac{\partial x(s,v)}{dv}(\frac{\tau(s)}{\sqrt{3}} + \frac{\partial z(s,v)}{ds} + \tau(s)y(s,v)) \\ & -\frac{\partial z(s,v)}{dv}(-\frac{\kappa(s)}{\sqrt{3}} + \frac{\partial x(s,v)}{ds} - \kappa(s)y(s,v))]N(s) \\ & +[\frac{\partial y(s,v)}{dv}(-\frac{\kappa(s)}{\sqrt{3}} + \frac{\partial x(s,v)}{ds} - \kappa(s)y(s,v)) \\ & -\frac{\partial x(s,v)}{dv}(\frac{\kappa(s)-\tau(s)}{\sqrt{3}} + \frac{\partial y(s,v)}{ds} + \kappa(s)x(s,v) - \tau(s)z(s,v))]B(s) \end{aligned}$$

Thus, if we let

$$\begin{cases} \Phi_1(s,v_0) = \frac{\partial z(s,v_0)}{dv}\left(\frac{\kappa(s)-\tau(s)}{\sqrt{3}}\right) - \frac{\partial y(s,v_0)}{dv}\frac{\tau(s)}{\sqrt{3}}, \\ \Phi_2(s,v_0) = \frac{\partial x(s,v_0)}{dv}\frac{\tau(s)}{\sqrt{3}} + \frac{\partial z(s,v_0)}{dv}\frac{\kappa(s)}{\sqrt{3}}, \\ \Phi_3(s,v_0) = -\frac{\partial y(s,v_0)}{dv}\frac{\kappa(s)}{\sqrt{3}} - \frac{\partial x(s,v_0)}{dv}\left(\frac{\kappa(s)-\tau(s)}{\sqrt{3}}\right). \end{cases} \tag{3.14}$$

We obtain

$$n(s,v_0) = \Phi_1(s,v_0)T(s) + \Phi_2(s,v_0)N(s) + \Phi_3(s,v_0)B(s).$$

From the Eqn. (3.2), we should have

$$\begin{aligned} \frac{\partial n}{\partial s}(s,v_0).T(s) = 0 \Leftrightarrow & \frac{\partial\left(\Phi_1(s,v_0)T(s) + \Phi_2(s,v_0)N(s) + \Phi_3(s,v_0)B(s)\right)}{\partial s}.T(s) = 0 \\ \Leftrightarrow & \frac{\partial \Phi_1}{\partial s}(s,v_0) - \kappa(s)\Phi_2(s,v_0) = 0. \end{aligned} \tag{3.15}$$

We using eqn. (3.14) in eqn. (3.15),since $\kappa(s) \neq 0$, we get

$$\frac{\partial z(s,v_0)}{dv} = -\frac{\tau(s)}{\kappa(s)}\frac{\partial x(s,v_0)}{dv} \tag{3.16}$$

which completes the proof. □

Now let us consider other types of the marching-scale functions. In the Eqn. (3.1) marching-scale functions $x(s,v)$, $y(s,v)$ and $z(s,v)$ can be choosen in two different forms:

1) If we choose

$$\begin{cases} x(s,v) = \sum\limits_{k=1}^{p} a_{1k} l(s)^k x(v)^k, \\ y(s,v) = \sum\limits_{k=1}^{p} a_{2k} m(s)^k y(v)^k, \\ z(s,v) = \sum\limits_{k=1}^{p} a_{3k} n(s)^k z(v)^k. \end{cases}$$

then we can simply express the sufficient condition for which the curve $\alpha(s)$ is an Smarandache asymptotic curve on the surface $\varphi(s,v)$ as

$$\begin{cases} x(v_0) = y(v_0) = z(v_0) = 0, \\ a_{21} = 0 \text{ or } m(s) = 0 \text{ or } \frac{dy(v_0)}{dv} = 0, \\ a_{31} n(s) \frac{dz(v_0)}{dv} = -\frac{\tau(s)}{\kappa(s)} a_{11} l(s) \frac{dx(v_0)}{dv}. \end{cases} \tag{3.17}$$

where $l(s)$, $m(s)$, $n(s)$, $x(v)$, $y(v)$ and $z(v)$ are C^1 functions, $a_{ij} \epsilon\ IR$, $i = 1,2,3$, $j = 1,2,...,p$.

2) If we choose

$$\begin{cases} x(s,v) = f\left(\sum\limits_{k=1}^{p} a_{1k} l(s)^k x(v)^k\right), \\ y(s,v) = g\left(\sum\limits_{k=1}^{p} a_{2k} m(s)^k y(v)^k\right), \\ z(s,v) = h\left(\sum\limits_{k=1}^{p} a_{3k} n(s)^k z(v)^k\right). \end{cases}$$

then we can write the sufficient condition for which the curve $\alpha(s)$ is an Smarandache asymptotic curve on the surface $\varphi(s,v)$ as

$$\begin{cases} x(v_0) = y(v_0) = z(v_0) = f(0) = g(0) = h(0) = 0, \\ a_{21} = 0 \text{ or } m(s) = 0 \text{ or } \frac{dy(v_0)}{dv} = 0 \text{ or } g'(0) = 0, \\ a_{31} n(s) \frac{dz(v_0)}{dv} h'(0) = -\frac{\tau(s)}{\kappa(s)} a_{11} l(s) \frac{dx(v_0)}{dv} f'(0). \end{cases} \tag{3.18}$$

where $l(s)$, $m(s)$, $n(s)$, $x(v)$, $y(v)$, $z(v)$, f, g and h are C^1 functions.

Also conditions for different types of marching-scale functions can be obtained by using the Eqn. (3.12) and (3.16).

Conclusion 3.4. Smarandache curves of the curve $\alpha(s)$ necessary and sufficient conditions to be isoasymptotic on the surface $\varphi(s,v)$ are the same.

4. Examples of generating simple surfaces with common Smarandache asymptotic curve

Example 4.1. *Let* $\alpha(s) = \left(\frac{3}{5}\cos(s), \frac{3}{5}\sin(s), \frac{4}{5}s\right)$ *be a unit speed curve. Then it is easy to show that*

$$\begin{cases} T(s) = \left(-\frac{3}{5}\sin(s), \frac{3}{5}\cos(s), \frac{4}{5}s\right), \\ N(s) = (-\cos(s), -\sin(s), 0), \\ B(s) = \left(\frac{4}{5}\sin(s), -\frac{4}{5}\cos(s), \frac{3}{5}\right). \end{cases}, \ \kappa = \| T' \| = \frac{3}{5}, \ \tau = \langle N', B \rangle = \frac{4}{5}.$$

If we take $x(s,v) = \sin(sv)$, $y(s,v) = 0$, $z(s,v) = -\frac{4}{3}\sin(sv)$ *we obtain a member of the surface with curve* $\alpha(s)$ *as*

$$\varphi(s,v) = \left(\frac{3}{5}\cos(s) - \frac{5}{3}\sin(s)\sin(sv), \frac{3}{5}\sin(s) + \frac{5}{3}\cos(s)\sin(sv), \frac{4}{5}s\right)$$

where $0 \le s \le 2\pi$, $-1 \le v \le 1$ *(Fig. 1).*

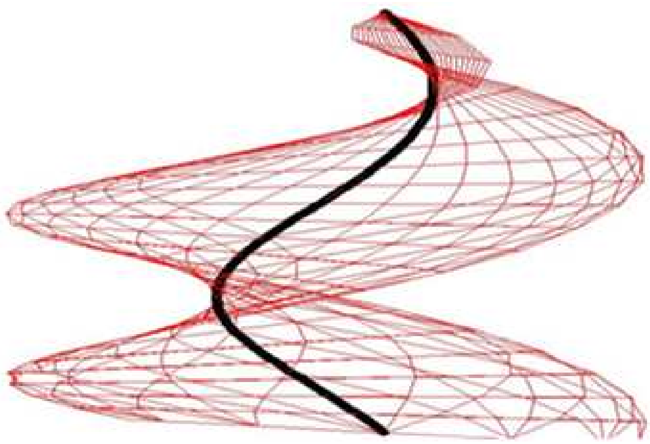

Figure 1: Figure 1: $\varphi(s,v)$ surface with curve $\alpha(s)$.

If we take
$x(s,v) = \sin(sv)$, $y(s,v) = 0$, $z(s,v) = -\frac{4}{3}\sin(sv)$ *and* $v_0 = 0$ *then the Eqns.(3.3) and (3.6) are satisfied. Thus, we obtain a member of the surface with common Smarandache TN asymptotic curve as*

$$\begin{aligned} \varphi(s,v) = & \left(-\frac{3}{5\sqrt{2}}\sin(s) - \frac{\cos(s)}{\sqrt{2}} - \frac{5}{3}\sin(s)\sin(sv), \frac{3}{5\sqrt{2}}\cos(s) - \frac{\sin(s)}{\sqrt{2}}\right. \\ & \left. +\frac{5}{3}\cos(s)\sin(sv), \frac{4}{5\sqrt{2}}\right) \end{aligned}$$

where $0 \le s \le 2\pi$, $-1 \le v \le 1$ *(Fig. 2).*

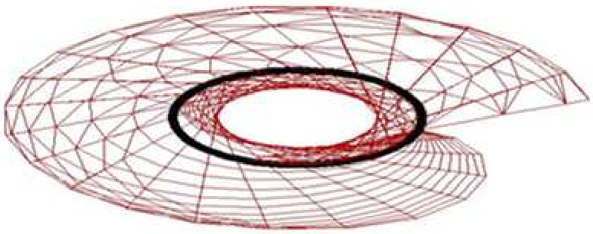

Figure 2: Figure 2: $\varphi(s,v)$ as a member of the surface and its Smarandache NB asymptotic curve of $\alpha(s)$.

Also, we obtain a member of the surface with common Smarandache NB asymptotic curve as

$$\begin{aligned}\varphi(s,v) &= \left(\frac{4}{5\sqrt{2}}\sin(s)-\frac{\cos(s)}{\sqrt{2}}-\frac{5}{3}\sin(s)\sin(sv),-\frac{4}{5\sqrt{2}}\cos(s)-\frac{\sin(s)}{\sqrt{2}}\right.\\ &\quad\left.+\frac{5}{3}\cos(s)\sin(sv),\frac{3}{5\sqrt{2}}\right)\end{aligned}$$

where $-\frac{\pi}{4}\leq s\leq\frac{\pi}{4}$, $-1\leq v\leq 1$ *(Fig. 3).*

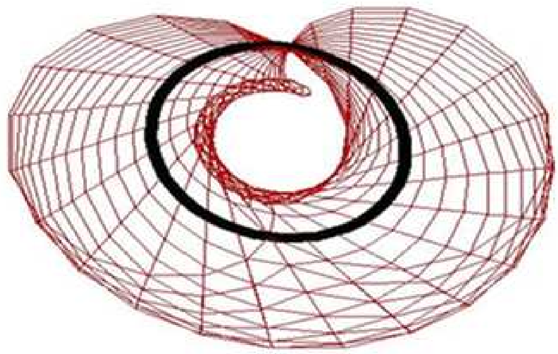

Figure 3: Figure 3: $\varphi(s,v)$ as a member of the surface and its Smarandache NB asymptotic curve of $\alpha(s)$.

A member of the surface and its Smarandache TNB asymptotic curve
$\varphi(s,v)=\left(\frac{\sin(s)}{5\sqrt{3}}-\frac{\cos(s)}{\sqrt{3}}-\frac{5}{3}\sin(s)\sin(sv),-\frac{\cos(s)}{5\sqrt{3}}-\frac{\sin(s)}{\sqrt{3}}+\frac{5}{3}\cos(s)\sin(sv),\frac{7}{5\sqrt{3}}\right)$
where $-\pi\leq s\leq\pi$, $-1\leq v\leq 1$ *(Fig. 4).*

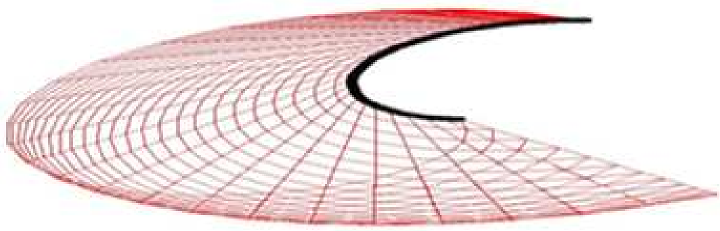

Figure 4: Figure 4: $\varphi(s,v)$ as a member of the surface and its Smarandache TNB asymptotic curve of $\alpha(s)$.

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