

**MBJW - FILTERS OF LATTICE WAJSBERG ALGEBRAS**T. ANITHA<sup>1</sup>, V. AMARENDRABABU, AND G. BHANU VINOLIA

ABSTRACT. In this paper we define the  $\mathcal{MBJW}$  – filters of Lattice wajsberg algebras and proved the properties of  $\mathcal{MBJW}$  – filters. We derive some relation between fuzzy ideals, interval valued fuzzy ideals to neutrosophic ideals. Further we prove that cut sets of  $\mathcal{MBJ}$ – sets formed  $\mathcal{MBJW}$  – filter. Finally define the  $\mathcal{MBJW}$ – lattice filters and proved every  $\mathcal{MBJW}$  – filter is a  $\mathcal{MBJW}$  – lattice filter and converse is not true.

## 1. INTRODUCTION

In 1935, [20] Wajsberg introduced the concept of wajsberg algebra. In 1984, [5] Front, Antonio and Torrens led the lattice wajsberg algebra and define filters, properties of filters. Ibrahim and Saravan [1] introduced the strong implicative filters of lattice wajsberg algebras and derived some properties. B.Ahamed introduced [2] the concept of fuzzy implicative filter and obtained some properties of lattice wajsberg algebra. At first L. A. Zadeh introduced the Fuzzy sets to handle the real life problems with uncertainty. After that several researchers [2, 7, 8, 14, 15, 19] applied the fuzzy theory to different algebras, differential equations and derived some results. Later Gaw derived the vague set as a generalization of fuzzy set. Vague theory applied to several streams by researchers [3, 4, 12, 13]. After that Smarandache [6, 18] introduced the concept of neutrosophic sets. Later Monoranjan and Madhumangal [9] recall some

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definitions and introduced the truth value based neutrosophic sets and neutrosophic sets and define new operations with examples. S.T. Rao, S.B. Kumar, H.S. Rao [16, 17] studied the gamma neutrosophic soft sets. Y.B. Jun, R.A. Borzooei and M. Mohseni [11] introduced the MBJ-neutrosophic sets and BMBJ-neutrosophic sets and applied to BCK algebra.

In this paper we consider MBJ-neutrosophic sets  $(M_B^J)$  defined by Y.B. Jun and introduce the concept  $(M_B^J)W$ -filter of lattice wajsberg algebra and obtain some results on them. For further information of lattice wajsberg algebra refer the wajsberg algebra [5] by Front, Antonio and Torrens and for MBJ-neutrosophic sets refer the [10] MBJ-neutrosophic structures.

## 2. PRELIMINARIES

**Definition 2.1.** [5] Let  $(w, \rightarrow, ', 1_m)$  be a wajsberg algebra if it satisfies the following axioms for all  $x_m, y_m, z_m \in w$

- (i)  $1_m \rightarrow x_m = x_m$
- (ii)  $(x_m \rightarrow y_m) \rightarrow ((y_m \rightarrow z_m) \rightarrow (x_m \rightarrow z_m)) = 1_m$
- (iii)  $(x_m \rightarrow y_m) \rightarrow y_m = (y_m \rightarrow x_m) \rightarrow x_m$
- (iv)  $(x'_m \rightarrow y'_m) \rightarrow (y_m \rightarrow x_m) = 1_m$

**Definition 2.2.** [5] The wajsberg algebra  $W$  is called a lattice wajsberg algebra with the bounds  $0_m, 1_m$  if it satisfies the following axioms for all  $x_m, y_m \in W$ : A partial ordering  $\leq$  on  $W$ , such that  $x_m \leq y_m$  if and only if  $x_m \rightarrow y_m = 1_m$ ,  $(x_m \vee y_m) = (x_m \rightarrow y_m) \rightarrow y_m$  and  $(x_m \wedge y_m) = ((x'_m \rightarrow y'_m) \rightarrow y'_m)$ .

Let  $I$  denote the family of all intervals numbers of  $[0, 1]$ . If  $I_1 = [a_1, b_1]$ ,  $I_2 = [a_2, b_2]$  are two elements of  $I[0, 1]$ , we call  $I_1 \geq^* I_2$  if  $a_1 \geq a_2$  and  $b_1 \geq b_2$ . we define the term rmax to mean the maximum of two interval as  $\text{rmax}[I_1, I_2] = [\max(a_1, a_2), \max(b_1, b_2)]$ . Similarly, we can define the term rmin of any two intervals.

**Definition 2.3.** [10] A neutrosophic set  $(N^s)$ , if the structure  $A_m = \langle y_m, w_T^A(y_m), w_I^A(y_m), w_F^A(y_m) \rangle$ ,  $y_m \in X$  where  $(w_T^A)$  is truth membership function,  $(w_I^A)$  is an indeterminate membership function and  $(w_F^A)$  is false membership function, on a nonempty set  $X$ .

**Definition 2.4.** [10] A *MBJ neutrosophic set* ( $M_B^J$  - set) is of the structure  $A_m = \langle y_m, M_T^A(y_m), B_I^A(y_m), J_F^A(y_m) \rangle, y_m \in x$  where  $M_T^A$  is truth membership function,  $B_I^A$  is an indeterminate interval -valued membership function and  $J_F^A$  is false membership function, on a nonempty set  $X$ . The  $M_B^J$  set is simply denoted by  $A_m = (M_T^A, B_I^A, J_F^A)$ . Throughout this paper  $W$  denotes the lattice wajsberg algebra and  $M_B^J$ - set denotes the *MBJ-neutrosophic set*.

### 3. $M_B^J$ -FILTERS

**Definition 3.1.** A  $M_B^J$ - set  $A_m = (M_T^A, B_I^A, J_F^A)$  on  $W$  is called a  $M_B^J w$  - filter if it satisfies for all  $x_m, y_m \in W$ ,

(3.1)  $M_T^A(1_m) \geq M_T^A(x_m), B_I^A(1_m) \geq^* B_I^A(x_m)$  and  $J_F^A(1_m) \leq J_F^A(x_m)$ .

(3.2)  $M_T^A(y_m) \geq \min \{M_T^A(x_m \rightarrow y_m), M_T^A(x_m)\}$ ,  
 $B_I^A(y_m) \geq^* \text{rmin} \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\}$   
 and  $J_F^A(y_m) \leq \max \{J_F^A(x_m \rightarrow y_m), J_F^A(x_m)\}$ .

**Example 1.** Let  $W = \{0_m, x_m, y_m, 1_m\}$  with the binary operation  $\rightarrow$  as follows: The  $M_B^J$ - set  $A_m = (M_T^A, B_I^A, J_F^A)$  defined on  $W$  as follows is  $M_B^J$ -filter of  $W$ .

TABLE 1. W-Algebra

Col1	Col2	Col3	Col4	col5
$\rightarrow$	$0_m$	$x_m$	$y_m$	$1_m$
$0_m$	$1_m$	$1_m$	$1_m$	$1_m$
$x_m$	$y_m$	$1_m$	$y_m$	$1_m$
$y_m$	$x_m$	$x_m$	$1_m$	$1_m$
$1_m$	$0_m$	$x_m$	$y_m$	$1_m$

Col1	Col2	Col3	Col4
	$M_T^A$	$B_I^A$	$J_F^A$
$0_m$	.551	[.557, .7]	.451
$x_m$	.551	[.557, .7]	.41
$y_m$	.71	[.61, .72]	.231
$1_m$	.71	[.61, .72]	.231

**Example 2.** Let  $W = \{0_m, x_m, y_m, z_m, v_m, 1_m\}$  with the binary operation  $\rightarrow$  as follows:

TABLE 2. W-Algebra

Col1	Col2	Col3	Col4	col5	col6	col7
$\rightarrow$	$0_m$	$x_m$	$y_m$	$z_m$	$v_m$	$1_m$
$0_m$	$1_m$	$1_m$	$1_m$	$1_m$	$1_m$	$1_m$
$x_m$	$z_m$	$1_m$	$y_m$	$z_m$	$y_m$	$1_m$
$y_m$	$v_m$	$x_m$	$1_m$	$y_m$	$x_m$	$1_m$
$z_m$	$x_m$	$x_m$	$1_m$	$1_m$	$x_m$	$1_m$
$v_m$	$y_m$	$1_m$	$1_m$	$y_m$	$1_m$	$1_m$
$1_m$	$0_m$	$x_m$	$y_m$	$x_m$	$y_m$	$1_m$

The  $M_B^J$ -set  $A_m = (M_T^A, B_I^A, J_F^A)$  defined on  $W$  as follows is  $M_B^J$ -filter of  $W$ .

TABLE 3. MBJW-filter

Col1	Col2	Col3	Col4
	$M_T^A$	$B_I^A$	$J_F^A$
$0_m$	.451	[.5, .557]	.51
$x_m$	.671	[.6, .641]	.445
$y_m$	.451	[.5, .557]	.51
$z_m$	.451	[.5, .557]	.51
$v_m$	.451	[.5, .557]	.51
$1_m$	.671	[.6, .641]	.445

**Theorem 3.1.** Let  $A_m = (M_T^A, B_I^A, J_F^A)$  is  $M_B^J$ -set of  $W$ . If  $(M_T^A, J_F^A)$  is an intuitionistic fuzzy filter of  $W$  and  $B_I^{A+}$  and  $B_I^{A-}$  are fuzzy filters of  $W$  then  $A_m = (M_T^A, B_I^A, J_F^A)$  is a  $M_B^J$ w-filter of  $W$ .

*Proof.* For any  $x_m, y_m \in W$ , we have

$$\begin{aligned}
 B_I^A(1_m) &= [B_I^{A-}(1_m), B_I^{A+}(1_m)] \geq^* [B_I^{A-}(x_m), B_I^{A+}(x_m)] = B_I^A(x_m) \text{ and} \\
 B_I^A(y_m) &= [B_I^{A-}(y_m), B_I^{A+}(y_m)] \\
 &\geq^* [\min \{B_I^{A-}(x_m \rightarrow y_m), B_I^{A-}(x_m)\}, \min \{B_I^{A+}(x_m \rightarrow y_m), B_I^{A+}(x_m)\}] \\
 &= \text{rmin} \{[B_I^{A-}(x_m \rightarrow y_m), B_I^{A+}(x_m \rightarrow y_m)], [B_I^{A-}(x_m), B_I^{A+}(x_m)]\} \\
 &= \text{rmin} \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\}.
 \end{aligned}$$

Therefore  $A_m = (M_T^A, B_I^A, J_F^A)$  is a  $M_B^J$ w - filter of  $W$ . If  $A_m = (M_T^A, B_I^A, J_F^A)$  is a  $M_B^J$ w - filter of  $W$ , then for all  $x_m, y_m \in W$ ,

$$\begin{aligned} [B_I^{A-}(y_m), B_I^{A+}(y_m)] &= B_I^A(y_m) \geq^* \text{rmin} \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\} \\ &= \text{rmin} \{[B_I^{A-}(x_m \rightarrow y_m), B_I^{A+}(x_m \rightarrow y_m)], [B_I^{A-}(x_m), B_I^{A+}(x_m)]\} \\ &= \min \{B_I^{A-}(x_m \rightarrow y_m), B_I^{A-}(x_m)\}, \min \{B_I^{A+}(x_m \rightarrow y_m), B_I^{A+}(x_m)\} \end{aligned}$$

It follows that

$$\begin{aligned} B_I^{A-}(y_m) &\geq \min \{B_I^{A-}(x_m \rightarrow y_m), B_I^{A-}(x_m)\} \text{ and} \\ B_I^{A+}(y_m) &\geq \min \{B_I^{A+}(x_m \rightarrow y_m), B_I^{A+}(x_m)\}. \end{aligned}$$

Thus  $B_I^{A-}$  and  $B_I^{A+}$  are fuzzy filters of  $W$ . But  $(M_T^A, J_F^A)$  is need not to be an intuitionistic fuzzy filter of  $W$ .

For example the  $M_B^J$  - sets  $A_m = (M_T^A, B_I^A, J_F^A)$  and  $B_m = (M_T^B, B_I^B, J_F^B)$  in the example 3.3 are  $M_B^J$ w - filters of  $W$  but  $(M_T^A, J_F^A)$  is an intuitionistic fuzzy filter of  $W$  and  $(M_T^B, J_F^B)$  is not an intuitionistic fuzzy filter of  $W$ . □

**Theorem 3.2.** *If  $A_m = (M_T^A, B_I^A, J_F^A)$  is a  $M_B^J$ w - filter of  $W$  then the sets*

$$(M_T^A, B_I^{A-}, J_F^A)(M_T^A, B_I^{A+}, J_F^A)$$

*are  $Nw$ - filters of  $W$ .*

*Proof.* Let  $A_m = (M_T^A, B_I^A, J_F^A)$  is a  $M_B^J$ w - filter of  $W$ . Then  $B_I^A(1_m) \geq^* B_I^A(x_m)$  then clearly  $B_I^{A-}(1_m) \geq B_I^{A-}(x_m)$  and  $B_I^{A+}(1_m) \geq B_I^{A+}(x_m)$  for all  $x_m \in W$ . And

$$B_I^A(y_m) \geq^* \text{rmin} \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\}$$

that is

$$\begin{aligned} B_I^{A-}(y_m) &\geq \min \{B_I^{A-}(x_m \rightarrow y_m), B_I^{A-}(x_m)\}, \\ B_I^{A+}(y_m) &\geq \min \{B_I^{A+}(x_m \rightarrow y_m), B_I^{A+}(x_m)\}. \end{aligned}$$

$B_I^{A-}$  and  $B_I^{A+}$  satisfies the necessary conditions. So the sets  $(M_T^A, B_I^{A-}, J_F^A)$  and  $(M_T^A, B_I^{A+}, J_F^A)$  are  $Nw$ - filters of  $W$ . □

**Theorem 3.3.** *Let  $A_m = (M_T^A, B_I^A, J_F^A)$  is  $M_B^J$ w - filter of  $W$ . If  $x_m \leq y_m$  then  $\{M_T^A(x_m) \leq M_T^A(y_m), B_I^A(x_m) \leq^* B_I^A(y_m)$  and  $J_F^A(x_m) \geq J_F^A(y_m)\}$  for all  $x_m, y_m \in W$ .*

*Proof.* Since  $x_m \leq y_m$ , then  $x_m \rightarrow y_m = 1$ . By  $A_m$  is  $M_B^J$ w -filter of  $W$ , We have

$$\begin{aligned} M_T^A(y_m) &\geq \min \{M_T^A(x_m \rightarrow y_m), M_T^A(x_m)\} \\ &= \min \{M_T^A(1_m), M_T^A(x_m)\} = M_T^A(x_m), \\ B_I^A(y_m) &\geq^* \text{rmin} \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\} \\ &= \min \{B_I^A(1_m), B_I^A(x_m)\} = B_I^A(x_m) \end{aligned}$$

and

$$\begin{aligned} J_F^A(y_m) &\leq \max \{ J_F^A(x_m \rightarrow y_m), J_F^A(x_m) \} \\ &= \max \{ J_F^A(1_m), J_F^A(x_m) \} = J_F^A(x_m). \end{aligned} \quad \square$$

**Theorem 3.4.** A  $M_B^J$  set  $A_m = (M_T^A, B_I^A, J_F^A)$  is  $M_B^J$ w- filter of  $W$  if and only if it holds (3.1) and for all  $x_m, y_m, z_m \in W$ ,

$$(3.3) \quad \begin{aligned} M_T^A(x_m \rightarrow y_m) &\geq \min \{ M_T^A(y_m \rightarrow (x_m \rightarrow z_m)), M_T^A(y_m) \}, \\ B_I^A(x_m \rightarrow z_m) &\geq^* \text{rmin} \{ B_I^A(y_m \rightarrow (x_m \rightarrow z_m)), B_I^A(y_m) \} \end{aligned}$$

and

$$J_F^A(x_m \rightarrow z_m) \leq \max \{ J_F^A(y_m \rightarrow (x_m \rightarrow z_m)), J_F^A(y_m) \}.$$

*Proof.* Let  $A_m$  is a  $M_B^J$ w –filter of  $W$ , perceptibly it hold (3.1) and (3.3).

Conversely suppose that  $A_m$  is a  $M_B^J$ - set with (3.1) and (3.3). Taking  $x_m = 1_m$  in (3.3), we get

$$\begin{aligned} M_T^A(1_m \rightarrow z_m) &\geq \min \{ M_T^A(y_m \rightarrow (1_m \rightarrow z_m)), M_T^A(y_m) \} \\ M_T^A(z_m) &\geq \min \{ M_T^A(y_m \rightarrow z_m), M_T^A(y_m) \}, \\ B_I^A(1_m \rightarrow z_m) &\geq^* \text{rmin} \{ B_I^A(y_m \rightarrow (1_m \rightarrow z_m)), B_I^A(y_m) \} \\ B_I^A(z_m) &\geq^* \text{rmin} \{ B_I^A(y_m \rightarrow z_m), B_I^A(y_m) \} \\ J_F^A(1_m \rightarrow z_m) &\leq \max \{ J_F^A(y_m \rightarrow (1_m \rightarrow z_m)), J_F^A(y_m) \} \\ J_F^A(z_m) &\leq \max \{ J_F^A(y_m \rightarrow z_m), J_F^A(y_m) \}. \end{aligned}$$

Hence  $A_m$  is a  $M_B^J$ w-filter of  $W$ . □

**Theorem 3.5.** A  $M_B^J$  set  $A_m = (M_T^A, B_I^A, J_F^A)$  is  $M_B^J$ w- filter of  $W$  if and only if it hold (3.1) and

$$(3.4) \quad \begin{aligned} M_T^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) &\geq \min \{ M_T^A(x_m), M_T^A(y_m) \}, \\ B_I^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) &\geq^* \text{rmin} \{ B_I^A(x_m), B_I^A(y_m) \} \end{aligned}$$

and

$$J_F^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) \leq \max \{ J_F^A(x_m), J_F^A(y_m) \},$$

for all  $x_m, y_m, z_m \in W$ .

*Proof.* Suppose that  $A_m$  is a  $M_B^J$ w- filter of  $W$  and  $x_m, y_m, z_m \in W$ . Clearly

$$\begin{aligned} &M_T^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) \\ &\geq \min \{ M_T^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow (y_m \rightarrow z_m)), M_T^A(y_m) \} \end{aligned}$$

and

$$((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow (y_m \rightarrow z_m)) = (x_m(y_m \rightarrow z_m) \geq x_m.$$

So,  $M_T^A(((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow (y_m \rightarrow z_m))) \geq M_T^A(x_m).$

From above we get,

$$M_T^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) \geq \min \{M_T^A(x_m), M_T^A(y_m)\}.$$

Clearly,

$$B_I^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) \geq \min \{B_I^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow (y_m \rightarrow z_m)), B_I^A(y_m)\}$$

and

$$B_I^A(((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow (y_m \rightarrow z_m))) \geq B_I^A(x_m).$$

From above we get

$$B_I^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) \geq^* \text{rmin} \{B_I^A(x_m), B_I^A(y_m)\}.$$

Clearly

$$J_F^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) \leq \min \{J_F^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow (y_m \rightarrow z_m)), J_F^A(y_m)\}$$

and

$$J_F^A(((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow (y_m \rightarrow z_m))) \leq J_F^A(x_m).$$

From above we get,  $J_F^A((x_m \rightarrow (y_m \rightarrow z_m)) \rightarrow z_m) \leq \max \{J_F^A(x_m), J_F^A(y_m)\}.$

Conversely suppose that  $A_m$  is a  $M_B^J$ -set with (3.1) and (3.4).

$$M_T^A(y_m) = M_T^A(1_m \rightarrow y_m) = M_T^A(((x_m \rightarrow y_m) \rightarrow (x_m \rightarrow y_m)) \rightarrow y_m) \geq \min \{M_T^A(x_m \rightarrow y_m), M_T^A(x_m)\}.$$

$$B_I^A(y_m) = B_I^A(1_m \rightarrow y_m) = B_I^A(((x_m \rightarrow y_m) \rightarrow (x_m \rightarrow y_m)) \rightarrow y_m) \geq^* \min \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\}.$$

$$J_F^A(y_m) = J_F^A(1_m \rightarrow y_m) = J_F^A(((x_m \rightarrow y_m) \rightarrow (x_m \rightarrow y_m)) \rightarrow y_m) \leq \max \{J_F^A(x_m \rightarrow y_m), J_F^A(x_m)\}.$$

So,  $A_m$  is a  $M_B^J$ w-filter of  $W$ . □

**Theorem 3.6.** Every  $M_B^J$ w-filter  $A_m = (M_T^A, B_I^A, J_F^A)$  fulfills the following result:

If  $x_m \rightarrow (y_m \rightarrow z_m) = 1_m$  then for all  $x_m, y_m, z_m \in W$ ,

$$M_T^A(z_m) \geq \min \{M_T^A(x_m), M_T^A(y_m)\}, B_I^A(z_m) \geq^* \text{rmin} \{B_I^A(x_m), B_I^A(y_m)\}$$

and  $J_F^A(z_m) \leq \max \{J_F^A(x_m), J_F^A(y_m)\}$

*Proof.* Suppose  $A_m$  is  $M_B^J$ w-filter of  $W$  and  $x_m \rightarrow (y_m \rightarrow z_m) = 1_m$  and  $x_m, y_m, z_m \in W$ .

We get

$$M_T^A(z_m) \geq \min \{M_T^A(y_m \rightarrow z_m), M_T^A(y_m)\}$$

$$\begin{aligned} &\geq \min \{ \min \{ M_T^A(x_m), M_T^A(x_m \rightarrow (y_m \rightarrow z_m)) \}, M_T^A(y_m) \} \\ &\geq \min \{ \min \{ M_T^A(x_m), M_T^A(1_m) \}, M_T^A(y_m) \} \\ &\geq \min \{ M_T^A(x_m), M_T^A(y_m) \} \end{aligned}$$

$$\begin{aligned} B_I^A(z_m) &\geq^* \text{rmin} \{ B_I^A(y_m \rightarrow z_m), B_I^A(y_m) \} \\ &\geq^* \text{rmin} \{ \min \{ B_I^A(x_m), B_I^A(x_m \rightarrow (y_m \rightarrow z_m)) \}, B_I^A(y_m) \} \\ &\geq^* \text{rmin} \{ \min \{ B_I^A(x_m), B_I^A(1_m) \}, B_I^A(y_m) \} \\ &\geq^* \text{rmin} \{ B_I^A(x_m), B_I^A(y_m) \} \end{aligned}$$

and

$$\begin{aligned} J_F^A(z_m) &\leq \max \{ J_F^A(y_m \rightarrow z_m), J_F^A(y_m) \} \\ &\leq \max \{ \max \{ J_F^A(x_m), J_F^A(x_m \rightarrow (y_m \rightarrow z_m)) \}, J_F^A(y_m) \} \\ &\leq \max \{ \max \{ J_F^A(x_m), J_F^A(1_m) \}, J_F^A(y_m) \} \\ &\leq \max \{ J_F^A(x_m), J_F^A(y_m) \}. \end{aligned} \quad \square$$

**Lemma 3.1.** Every  $M_B^J$  set  $A_m = (M_T^A, B_I^A, J_F^A)$  of  $W$  fulfills the following result for all  $x((n_w), \dots, x(1_w), y_m \in W$ :

If  $x(n_w) \rightarrow (x(n-1)_w) \rightarrow \dots \rightarrow (x(1_w) \rightarrow y_m) = 1_m$  then

$$M_T^A(y_m) \geq \min \{ M_T^A(x(n_w)), \dots, M_T^A(x(1_w)) \},$$

$$B_I^A(y_m) \geq^* \text{rmin} \{ B_I^A(x(n_w)), \dots, B_I^A(x(1_w)) \}.$$

And  $J_F^A(y_m) \leq \max \{ J_F^A(x(n_w)), \dots, J_F^A(x(1_w)) \}.$

**Theorem 3.7.** Let  $A_m$  and  $B_m$  are two  $M_B^J w$ -filters of  $W$ , then  $A_m \cap B_m$  is also a  $M_B^J w$ -filter of  $W$ .

*Proof.* Let  $x_m, y_m, z_m \in W$  such that  $x_m \leq (y_m \rightarrow z_m)$ , then  $x_m \rightarrow (y_m \rightarrow z_m) = 1_m$ . Since  $A_m$  and  $B_m$  are two  $M_B^J w$ -filters of  $W$ , we have

$$M_T^A(z_m) \geq \min \{ M_T^A(x_m), M_T^A(y_m) \}, B_I^A(z_m) \geq^* \text{rmin} \{ B_I^A(x_m), B_I^A(y_m) \}$$

and

$$J_F^A(z_m) \leq \max \{ J_F^A(x_m), J_F^A(y_m) \}.$$

$$M_T^B(z_m) \geq \min \{ M_T^B(x_m), M_T^B(y_m) \},$$

$$B_I^B(z_m) \geq^* \text{rmin} \{ B_I^B(x_m), B_I^B(y_m) \}$$

and

$$J_F^B(z_m) \leq \max \{ J_F^B(x_m), J_F^B(y_m) \}.$$

$$\begin{aligned} M_T^A \cap B(z_m) &= \min \{ M_T^A(z_m), M_T^B(z_m) \} \\ &= \min \{ \min \{ M_T^A(x_m), M_T^A(y_m) \}, \min \{ M_T^B(x_m), M_T^B(y_m) \} \} \\ &= \min \{ \min \{ M_T^A(x_m), M_T^B(x_m) \}, \min \{ M_T^A(y_m), M_T^B(y_m) \} \} \\ &= \min \{ M_T^A \cap B(x_m), M_T^A \cap B(y_m) \} \end{aligned}$$



$$\begin{aligned}
 B_I^{\langle A \cap B \rangle}(z_m) &= \min \{ B_I^A(z_m), B(z_m) \} \\
 &= \min \{ \min \{ B_I^A(x_m), B_I^A(y_m) \}, \min \{ B_I^B(x_m), B_I^B(y_m) \} \} \\
 &= \min \{ \min \{ B_I^A(x_m), B_I^B(x_m) \}, \min \{ B_I^A(y_m), B_I^B(y_m) \} \} \\
 &= \min \{ B_I^{\langle A \cap B \rangle}(x_m), B_I^{\langle A \cap B \rangle}(y_m) \}. \\
 J_F^{\langle A \cap B \rangle}(z_m) &= \max \{ J_F^A(z_m), J_F^B(z_m) \} \\
 &= \max \{ \max \{ J_F^A(x_m), J_F^A(y_m) \}, \max \{ J_F^B(x_m), J_F^B(y_m) \} \} \\
 &= \max \{ \max \{ J_F^A(x_m), J_F^B(x_m) \}, \max \{ J_F^A(y_m), J_F^B(y_m) \} \} \\
 &= \max \{ J_F^{\langle A \cap B \rangle}(x_m), J_F^{\langle A \cap B \rangle}(y_m) \}.
 \end{aligned}$$

So  $A_m \cap B_m$  is a  $M_B^J$ w – filter of  $W$ .

□

**Theorem 3.8.** *The  $M_B^J$  -set  $A_m = (M_T^A, B_I^A, J_F^A)$  is  $M_B^J$ w- filter of  $W$  if and only if its nonempty  $M_B^J$  cut sets  $M_T^{\langle A_\alpha \rangle}$  and  $J_F^{\langle A_\gamma \rangle}$  are implicative filters of  $W$  and  $B_I^{\langle A_\beta \rangle}$  is an intuitionistic fuzzy filter of  $W$  for all  $\alpha, \gamma \in [0, 1]$  and  $[\beta_1, \beta_2] \in I$ .*

*Proof.* Suppose  $A_m$  is  $M_B^J$ w-filter of  $W$  and  $\alpha, \gamma \in [0, 1]$  and  $[\beta_1, \beta_2] \in I$ .

Let  $M_T^{\langle A_\alpha \rangle}, B_I^{\langle A_\beta \rangle}$  and  $J_F^{\langle A_\gamma \rangle}$  are nonempty. Obviously  $1_m \in M_T^{\langle A_\alpha \rangle}, 1_m \in B_I^{\langle A_\beta \rangle}$  and  $1_m \in J_F^{\langle A_\gamma \rangle}$ . Let  $x_1, x_2, y_1, y_2, z_1$  and  $z_2 \in W$  such that  $(x_1 \rightarrow x_2, x_1 \in M_T^{\langle A_\alpha \rangle}), (y_1 \rightarrow y_2, y_1 \in B_I^{\langle A_\beta \rangle})$  and  $(z_1 \rightarrow z_2, z_1 \in J_F^{\langle A_\gamma \rangle})$ . Then:

$$\begin{aligned}
 M_T^{\langle A_\alpha \rangle}(x_2) &\geq \min \{ M_T^{\langle A_\alpha \rangle}(x_1 \rightarrow x_2), M_T^{\langle A_\alpha \rangle}(x_1) \} \geq \alpha \text{ implies } x_2 \in M_T^{\langle A_\alpha \rangle} \\
 B_I^{\langle A_\beta \rangle}(y_2) &\geq^* \text{rmin} \{ B_I^{\langle A_\beta \rangle}(y_1 \rightarrow y_2), B_I^{\langle A_\beta \rangle}(y_1) \} \geq [\beta_1, \beta_2] \text{ implies } y_2 \in B_I^{\langle A_\beta \rangle}. \\
 J_F^{\langle A_\gamma \rangle}(z_2) &\leq \max \{ J_F^{\langle A_\gamma \rangle}(z_1 \rightarrow z_2), J_F^{\langle A_\gamma \rangle}(z_1) \} \leq \gamma \text{ implies } z_2 \in J_F^{\langle A_\gamma \rangle}.
 \end{aligned}$$

So,  $M_T^{\langle A_\alpha \rangle}$  and  $J_F^{\langle A_\gamma \rangle}$  are implicative filters of  $W$  and  $B_I^{\langle A_\beta \rangle}$  is an intuitionistic fuzzy filter of  $W$ .

Conversely, suppose that  $M_T^{\langle A_\alpha \rangle}$  and  $J_F^{\langle A_\gamma \rangle}$  are implicative filters of  $W$  and  $B_I^{\langle A_\beta \rangle}$  is an intuitionistic fuzzy filter of  $W$  for all  $\alpha, \gamma \in [0, 1]$  and  $[\beta_1, \beta_2] \in I$ . For any  $x_m, y_m, z_m \in W$  such that  $M_T^{\langle A_\alpha \rangle}(x_m) = \alpha, B_I^{\langle A_\beta \rangle}(y_m) = [\beta_1, \beta_2]$  and  $J_F^{\langle A_\gamma \rangle}(z_m) = \gamma$ . Then  $x_m \in M_T^{\langle A_\alpha \rangle}, y_m \in B_I^{\langle A_\beta \rangle}$  and  $z_m \in J_F^{\langle A_\gamma \rangle}$ , so  $M_T^{\langle A_\alpha \rangle}, B_I^{\langle A_\beta \rangle}$  and  $J_F^{\langle A_\gamma \rangle}$  are nonempty.

For any  $x_1, x_2 \in W$ , let  $\alpha = \min \{ M_T^{\langle A_\alpha \rangle}(x_1 \rightarrow x_2), M_T^{\langle A_\alpha \rangle}(x_1) \}, [\beta_1, \beta_2] = \min \{ B_I^{\langle A_\beta \rangle}(x_1 \rightarrow x_2), B_I^{\langle A_\beta \rangle}(x_1) \}$  and  $\gamma = \{ J_F^{\langle A_\gamma \rangle}(x_1 \rightarrow x_2), J_F^{\langle A_\gamma \rangle}(x_1) \}$ .

Then clearly:

$$\begin{aligned}
 M_T^{\langle A_\alpha \rangle}(x_2) &\geq \alpha = \min \{ M_T^{\langle A_\alpha \rangle}(x_1 \rightarrow x_2), M_T^{\langle A_\alpha \rangle}(x_1) \} \\
 B_I^{\langle A_\beta \rangle}(y_2) &\geq^* [\beta_1, \beta_2] = \min \{ B_I^{\langle A_\beta \rangle}(x_1 \rightarrow x_2), B_I^{\langle A_\beta \rangle}(x_1) \}
 \end{aligned}$$

and

$$J_F^A(z_2) \leq \gamma = \max \{ J_F^A(x_1 \text{ Re } x_2), J_F^A(x_1) \}.$$

So,  $A_m = (M_T^A, B_I^A, J_F^A)$  is a  $M_B^J w$  – filter of  $W$ . □

**Lemma 3.2.** *If  $A_m$  is a  $M_B^J w$ -filter of  $W$  then  $M_T^A A_\alpha \cap B_I^A A_\beta \cap J_F^A A_\gamma$  are implicative filters of  $W$ .*

**Theorem 3.9.** *Any implicative filter  $A$  of  $w$  is a  $(\alpha, [\alpha, \alpha], \alpha)$  cut-  $M_B^J$  of  $W$ .*

*Proof.* Let  $A$  is implicative filter of  $W$  and  $\alpha \in [0, 1]$ . Consider a  $M_B^J$ - set:

$$A_m = (M_T^A(y_m), [B_I^{A-}(y_m) B_I^{A+}(y_m)]),$$

$$J_F^A(y_m) = (\alpha, [\alpha, \alpha], \alpha) \text{ if } y_m \in A_m \text{ and}$$

$$A_m = (0_m, [0_m, 0_m], 0_m) \text{ if } y_m \text{ not in } A_m. \text{ Let } x_m, y_m \in W. \text{ If } y_m \in A \text{ then}$$

$$M_T^A(y_m) = \alpha \geq \min \{ M_T^A(x_m \rightarrow y_m), M_T^A(x_m) \},$$

$$B_I^A(y_m) = [\alpha, \alpha] \geq^* \min \{ B_I^A(x_m \rightarrow y_m), B_I^A(x_m) \}$$

and

$$J_F^A(y_m) = \alpha \leq \max \{ J_F^A(x_m \rightarrow y_m), J_F^A(x_m) \}.$$

Suppose  $y_m$  not in  $A$  then  $x$  not in  $A$  or  $x_m \rightarrow y_m$  not in  $A$ . So

$$M_T^A(y_m) = 0_m = \min \{ M_T^A(x_m \rightarrow y_m), M_T^A(x_m) \}$$

$$B_I^A(y_m) = [0_m, 0_m] = \min \{ B_I^A(x_m \rightarrow y_m), B_I^A(x_m) \}$$

and

$$J_F^A(y_m) = 0_m = \max \{ J_F^A(x_m \rightarrow y_m), J_F^A(x_m) \}. \text{ So, } A_m \text{ is } M_B^J w \text{ – filter of}$$

$W$ . □

**Theorem 3.10.** *If  $A_m$  is  $M_B^J w$  - filter of  $W$  then the set*

$$A = \{ x_m \in W / (M_T^A(y_m), B_I^A(y_m, y_m), J_F^A(y_m)) = (M_T^A(1_m), B_I^A[1_m, 1_m], J_F^A(1_m)) \}$$

*is a implicative filter of  $W$ .*

*Proof.* Clearly

$$A = \{ x_m \in W / (M_T^A(y_m), B_I^A(y_m, y_m), J_F^A(y_m)) = (M_T^A(1_m), B_I^A[1_m, 1_m], J_F^A(1_m)) \},$$

and  $1_m \in A$ . Let  $x_m, y_m \in w$  such that  $x_m, x_m \rightarrow y_m \in A$ . Then

$$M_T^A(x_m \rightarrow y_m) = M_T^A(x_m) = M_T^A(1_m),$$

$$B_I^A(x_m \rightarrow y_m) = B_I^A(x_m) = B_I^A[1_m, 1_m]$$

and

$$J_F^A(x_m \rightarrow y_m) = J_F^A(x_m) = J_F^A(1_m).$$

So,

$$M_T^A(y_m) \geq \min \{ M_T^A(x_m \rightarrow y - m), M_T^A(x_m) \} = M_T^A(1_m),$$

$$B_I^A(y_m) \geq^* \text{rmin} \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\} = B_I^A(1_m)$$

and

$$J_F^A(y_m) \leq \max J_F^A(x_m \rightarrow y_m), J_F^A(x_m) = J_F^A(1_m).$$

That is  $y_m \in A$ . So  $A$  a implicative filter of  $W$ . □

**Definition 3.2.** A  $M_B^J$  set  $A_m = (M_T^A, B_I^A, J_F^A)$  is on  $W$  is called a  $M_B^J$ -lattice filter if it satisfies for all  $x_m, y_m \in W$ ,

$$(3.5) \quad \begin{aligned} M_T^A(x_m \wedge y_m) &\geq \min \{M_T^A(x_m), M_T^A(y_m)\}, \\ B_I^A(x_m \wedge y_m) &\geq^* \text{rmin} \{B_I^A(x_m), B_I^A(y_m)\} \\ \text{and } J_F^A(x_m \wedge y_m) &\leq \max \{J_F^A(x_m), J_F^A(y_m)\} \end{aligned}$$

**Example 3.** The  $M_B^J$  set  $A_m = (M_T^A, B_I^A, J_F^A)$  defined on  $W$  as follows is  $M_B^J$ -lattice filter of  $W$ .

TABLE 4. MBJW-Lattice filter

Col1	Col2	Col3	Col4
	$M_T^A$	$B_I^A$	$J_F^A$
$0_m$	.547	[.557, .6]	.451
$x_m$	.547	[.557, .6]	.451
$y_m$	.721	[.561, .64]	.331
$z_m$	.721	[.561, .64]	.331
$v_m$	.547	[.557, .6]	.451
$1_m$	.721	[.561, .64]	.331

**Theorem 3.11.** Every  $M_B^J$ -filter  $A_m$  of  $W$  is  $M_B^J$ -lattice filter of  $W$ .

*Proof.* Let  $A_m$  is a  $M_B^J$ -filter of  $W$ .

$$\begin{aligned} M_T^A(x_m \wedge y_m) &\geq \min \{M_T^A(x_m \rightarrow (x_m \wedge y_m)), M_T^A(x_m)\} \\ &= \min \{M_T^A(x_m \rightarrow y_m), M_T^A(x_m)\} \\ &\geq \min \{ \min \{M_T^A(y_m \rightarrow (x_m \wedge y_m)), M_T^A(y_m)\}, M_T^A(x_m) \} \\ &\geq \min \{ \min \{M_T^A(1_m), M_T^A(y_m)\}, M_T^A(x_m) \} \\ &= \min \{M_T^A(y_m), M_T^A(x_m)\} \\ B_I^A(x_m \wedge y_m) &\geq^* \min \{B_I^A(x_m \rightarrow (x_m \wedge y_m)), B_I^A(x_m)\} \\ &= \min \{B_I^A(x_m \rightarrow y_m), B_I^A(x_m)\} \\ &\geq^* \min \{ \min \{B_I^A(y_m \rightarrow (x_m \wedge y_m)), B_I^A(y_m)\}, B_I^A(x_m) \} \\ &\geq^* \min \{ \min \{B_I^A(1_m), B_I^A(y_m)\}, B_I^A(x_m) \} \end{aligned}$$

$$\begin{aligned}
&= \min \{B_I^A(y_m), B_I^A(x_m)\} \\
J_F^A(x_m \wedge y_m) &\leq \min \{J_F^A(x_m \rightarrow (x_m \wedge y_m)), J_F^A(x_m)\} \\
&= \min \{J_F^A(x_m \rightarrow y_m), J_F^A(x_m)\} \\
&\leq \min \{\min \{J_F^A(y_m \rightarrow (x_m \wedge y_m)), J_F^A(y_m)\}, J_F^A(x_m)\} \\
&\leq \min \{\min \{J_F^A(1_m), J_F^A(y_m)\}, J_F^A(x_m)\} \\
&= \min \{J_F^A(y_m), J_F^A(x_m)\}.
\end{aligned}$$

So  $A_m$  of  $W$  is  $M_B^J$ -lattice filter of  $W$ .  $\square$

**Remark 3.1.** The  $M_B^J$ -lattice filter of  $W$  is need not to be a  $M_B^J$ -filter of  $W$ . For example the  $M_B^J$ -lattice filter of  $A_m$  of  $W$  in example 3 is not a  $M_B^J$ -filter of  $W$  because  $M_T^A(z_m) \leq \min \{M_T^A(y_m \rightarrow z_m), M_T^A(y_m)\}$ .

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