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MATHEMATICAL ELEMENTS ON NATURAL REALITY

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Abstract : Actually, one establishes mathematical model for understanding a natural thing or matter T by its mathematical property $\widehat{T}$ characterized by model, called mathematical reality. *Could we always conclude the equality* $\widehat{T}=T$ *in nature*? The answer is disappointing by Godel's incomplete theorem which claims that any formal mathematical axiom system is incomplete because it always has one proposition that can neither be proved, nor disproved in this system. Thus, we can not determine $\widehat{T}=T$ or $\neq T$ sometimes by the boundary of mathematics. Generally, a natural thing or matter is complex, even hybrid with other things. Unlike purely thinking, physics and life science determine natural things by subdividing them into irreducible but detectable units such as those of quarks, gluons or cells, i.e., the composition theory of T in the microcosmic level, which concludes the reality of T is the whole behavior of a complex network induced by local units. However, all mathematical elements can only determines the character of T locally and usually brings about a contradictory system in mathematics. *Could we establish a mathematics on complex networks avoiding Godel's incomplete theorem for science, i.e., mathematical combinatorics*? The answer is positive motivated by the traditional Chinese medicine, in which a living person is completely reflected by 12 meridians with balance of Yin (Y^-) and Yang (Y^+) on his body, which alludes that there is a new kind of mathematical elements, called *harmonic flows* $\overrightarrow{G}^{L^2}$ with edge labeled by $L^2:(v,u)\in E\left(\overrightarrow{G}\right)\to L(v,u)-iL(v,u))$, where $i^2=-1$, $L(v,u)\in\mathscr{B}$ and 2 end-operators A^+_{vu},A^-_{vu} on Banach space $\mathscr{B}$ holding with the continuity equation on vertices $v\in V\left(\overrightarrow{G}\right)$. The dynamic behavior of $\overrightarrow{G}^{L^2}$ can be characterized by Euler-Lagrange equations

$$\frac{\partial\overrightarrow{G}^{\mathscr{L}}}{\partial x_i}-\frac{d}{dt}\frac{\partial\overrightarrow{G}^{\mathscr{L}}}{\partial\dot{x}_i}=\boldsymbol{O},\qquad 1\le i\le n$$

in the microscopic level, where $\mathbf{O}$ is the zero flow with $\{\mathbf{0},\mathbf{0}\}$ on (v,u), $\mathscr{L}\big[L^2(t,\boldsymbol{x}(t),\dot{\mathbf{x}}(t))\big]$ is a differential functional and $L^2(t,\boldsymbol{x}(t),\dot{\boldsymbol{x}}(t))(v,u)$ is a Lagrangian for $\forall(v,u)\in E\left(\overrightarrow{G}\right)$. We establish mathematics on $\overrightarrow{G}^{L^2}$ with dynamics and generalized famous theorems in functional analysis, also discuss the composing matter and antimatter by this kind of mathematical elements.

§**1. Introduction.** All matters are in colorful, mystery and also with a complex mechanism to humans even if vegetables or animals, we are embarrassed hardly know their true face unless images. Usually, we understand things by the reality for promoting the survival and development of humans ourselves and then, construct a harmonious system of humans with the nature. Then, *what is the reality of a matter*? Certainly, the word *reality* of a matter T is its state as it actually exist, including everything that is and has been, no matter it is observable or comprehensible by humans. *Could we really hold* *on the reality of matters*? Usually, a matter T is multilateral or complex one and so, hold on its reality is difficult for humans in logic. For the reality of matters, we have learned a few claims in classical scriptures of China or India. For example, the well-known sentences *Tao that be told is not the eternal Tao, Name that be called is not the eternal Name; Unnamed is the beginning of things in the universe, but naming is the origin of all things* in the first chapter of *Tao Te Ching*, a well-known Chinese ancient book, and also the sentences *Form is emptiness, Emptiness is form; emptiness does not differ from form and form does not differ from emptiness. Whatever is form, it is emptiness, whatever is emptiness, it is form* in Indian ancient book *Heart Sutra* (Prajnaparamita), also implied in the *Diamond Sutra* (Arya Vajracidaka). All of these words claimed that the reality of matters are our own understanding. For example, Newton's spacetime is the union of a Euclidean 3-dimensional space $\mathbb{R}^3$ and a 1-dimensional Euclidean space $\mathbb{R}$, which is not dependent on the human's will. Certainly, it can not be verified by humans but maybe the real nature of spacetime even though the space and time are also the forms of humans. However, the Einstein's spacetime is a 4-dimensional Euclidean space $\mathbb{R}^{\times}\mathbb{R}$, which is dependent on the human's will, and also be verified by humans but it maybe not the nature of spacetime, only our own understanding on spacetime, i.e., the relativity theory is a right theory on the universe only in the eyes of humans.

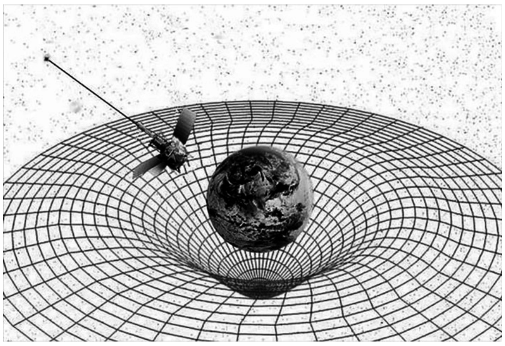
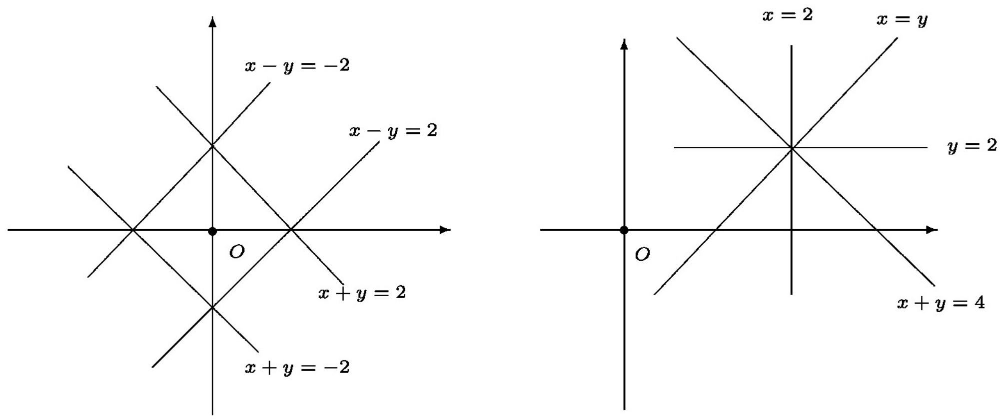


Fig. 1

Thus, the reality of a matter is its nature known by a special living, and we can not separate the word *reality* from that living. Then, *how do we hold on the reality of matters in the universe*? We all known that a matter T is consisting of elements, understand T by its elements, particularly, mathematical elements and then, get the mathematical reality. Certainly, there are many mathematical elements for characterizing local or partial behaviors of matters such as those of numbers, maps, functions, vectors, matrices, points, lines, opened or closed sets, $\cdots$, also with mathematical relations. But, *are these mathematical elements enough for us understanding reality of matters in the universe or is the mathematical reality nothing else but the universe*? For example, let x, y be the populations in a self-system of cats and rats, such as the Tom and Jerry in the cartoon TV series, then they were continuously characterized by Lotka-Volterra model (Brauer and Castillo-Chaver, 2012)) with differential equations

$$\begin{cases} \dot{x} = x(\lambda - by) \\ \dot{y} = y(-\mu + cx) \end{cases}$$

where, $\lambda - by$ is the growth rate of rats and $-\mu + cx$ is the death rate of cats with constants b and c. Even in this mathematical model, *are these* b, c, λ, μ *really constants in the universe*? The answer is certainly not because they are only an assumption of humans ourselves, which results also the next question into beings, i.e., *is the mathematical reality equal to the reality of things* T? Why we ask this question is because of the mathematical universe hypothesis of Max Tegmark (Tegmark, 2003)), which claims that our external physical reality is a mathematical structure, a duplication of *Theory of Everything*. Even for the mathematical reality, it has also questions on the completeness of itself, i.e., *is a mathematical system complete or not in logic*? The answer is discouraged by the Godel's incomplete theorem, i.e.,



(LES_4^N) (LES_4^S)

Fig. 2

Any consistent formal system F within which a certain amount of arithmetic can be carried out, there are statements of the language of F which can neither be proved nor disproved in F. Then, *what is the reality about of matters that can be understanding by humans*? Einstein once complained on mathematics with words that *as far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.* Until today, we can not conclude that the mathematical reality is equal to the reality of matters even in our daily life. For example, let

$$A = \{H_1, H_2, H_3, H_4\} \text{ and } B = \{H_1', H_2', H_3', H_4'\}$$

be 2 groups of horses constraint with running on respectively 4 straight lines in Euclidean space $\mathbb{R}^2$ (Mao, 2019), such as those shown in Fig. 2.

Then, *how do we characterize the running orbits of these horses*? We apply the equation systems

$$\left(LES_4^N\right) \begin{cases} x+y = 2 \\ x+y = -2 \\ x-y = -2 \\ x-y = 2 \end{cases} \qquad \left(LES_4^S\right) \begin{cases} x = y \\ x+y = 4 \\ x = 2 \\ y = 2 \end{cases}$$

Clearly, the first one is non-solvable but the second has solution $(2,2)$. *Could we conclude the running behavior of group A is nothing but B is* $(2,2)$? Certainly not because the fact is all of the horses are running on straight lines of Euclidean plane $\mathbb{R}^2$. Certainly, the orbits of the horses can be characterized by the equations but never be the solution of the system, i.e., the orbits $Orb(A)$, $Orb(B)$ of group A, B should be respectively the union of point sets

$$\begin{aligned} Orb(A) &= \{(x,y):\ x+y=2\} \bigcup \{(x,y):\ x+y=-2\} \\ &\quad \bigcup \{(x,y):\ x-y=2\} \bigcup \{(x,y):\ x-y=-2\}, \\ Orb(B) &= \{(x,y):\ x=2\} \bigcup \{(x,y):\ x+y=4\} \\ &\quad \bigcup \{(x,y):\ x=y\} \bigcup \{(x,y):\ x=2\}, \end{aligned}$$

which are nothing else but the *Smarandache multispaces.* This example also implies that the reality of a matter T maybe characterized by a contradictory system of equations, i.e., non-mathematics (Mao, 2014) in classical mathematics. We therefore conclude the inequality *mathematical reality* $\neq$ *reality* on matters in general, i.e., the mathematical reality can be only the local or partial true on matters in the universe. Then, *how to hold on the reality of matters*? We have only one way, i.e., summarizing practice to theory with continuous improving and then guiding the practice of humans. For this objective, a general thinking patter in science is

$$\text{Matter} \xrightarrow{\text{Decompose}} \text{Microcosmic Particles} \xrightarrow{\text{Abstract}} \text{Complex Network}$$

For example, physics determines the nature of matters by irreducibly smallest detectable particles called *elementary particles* (Ho-Kim and Xuam Yem, 1998), such as those of *fundamental fermions* including quarks, antiquarks, leptons, antileptons and *fundamental bosons* including gauge bosons, Higgs boson and the *fundamental interactions* such as the meson, baryon shown in Fig. 3,

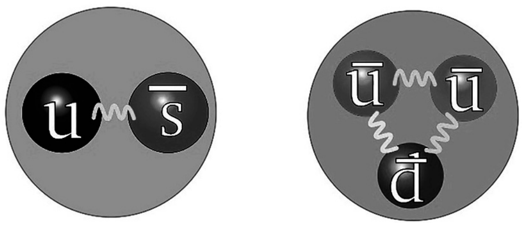


Fig. 3

and biology holds on the life and heredity by cells and genes.

The essence of subdividing on a matter is to determine the nature of irreducibly smallest detectable units and then, holds on reality of the matter. However, a matter can be always divided into submatters, then sub-submatters and so on. A natural question on this notion is *whether it has a terminal point or not*? Certainly, it can be

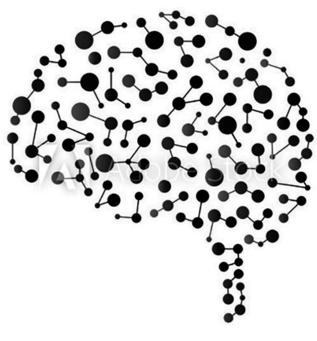


Fig. 4

done infinite times depends on the technology of humans but concludes that *a matter is equal to a complex network*, a very large network in general such as the molecular structure network of brain shown in Fig. 4.

Notice that the cell network of a human consists of $5 \times 10^{14} - 6 \times 10^{14}$ cells, a very large complex network. *Are we always need such a large and complex network for the reality of humans*? The answer is certainly Not by the traditional Chinese medicine. There are 12 meridians which completely reflects the physical condition of human body in traditional Chinese medicine, i.e., LU, LI, ST, SP, HT, SI, BL, KI, PC, SJ, GB, LR meridians. For example, the LI and GB meridians are shown in Fig. 5.

Fig. 5

By the traditional Chinese medicine (Zhang, 2007), if there exists an imbalanced acupoint on one of the 12 meridians, this person must be illness and in turn, there must be imbalance acupoints on the 12 meridians for a patient. Thus, finding out which acupoint on which meridian is in imbalance with Yin (Y^-) more than Yang (Y^+) or Yang (Y^+) more than Yin $()Y^-$ is the main duty of a Chinese doctor. Then, *how to heal the patient*? the doctor regulates the meridian by acupuncture or drugs so that the balance on the imbalance acupoints recovers again, and then the patient recovers. Thus,

$$\begin{aligned} \text{A body} &= \text{A union of } LU, LI, SP, HT, SI, KI, PC, LR, GB, ST, SJ, BL \text{ meridians} \\ &= \text{A non-connected graph of order 362,} \end{aligned}$$

a small one comparing with the complex network of cells such as those shown in Fig. 6 where, all edges are labeled by (Y^+, Y^-).

Then, *what is the significance of this treatment theory in traditional Chinese medicine to modern science*? It implies that we can introduce mathematical elements on complex networks globally, i.e., continuity or harmonic flows over topological graphs because the Yin and Yang in traditional Chinese medicine are flows on the 12 meridians. For holding on the true colors of matters, *whether or not classically mathematical elements enough for understanding complex networks, i.e., matters in the universe*? Certainly not because all of them can be only characterizing matters locally or partically. And then, *could we establish a mathematics over elements underlying combinatorial structures*? The answer is definite, i.e., *mathematical combinatorics* on global elements of complex network following.

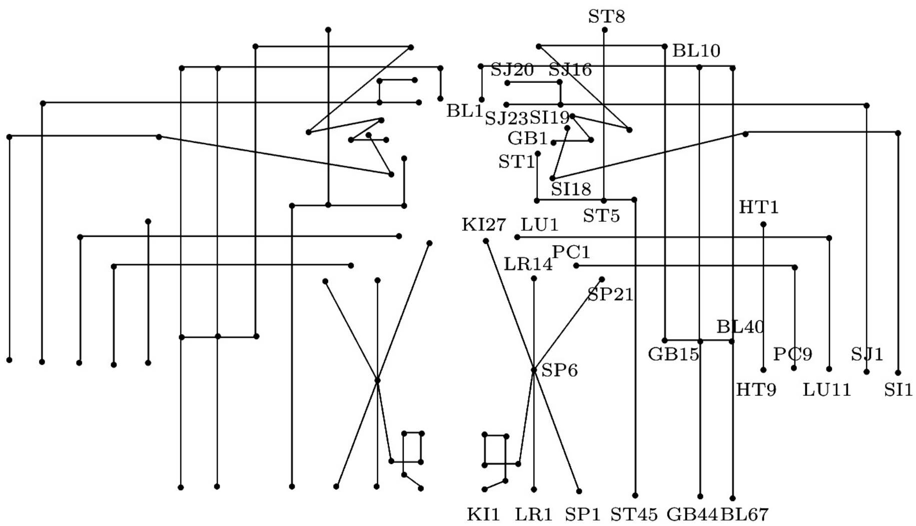


Fig. 6. 12 Meridian graph on a human body

DEFINITION 1. (Element 1) *A continuity flow* $\overrightarrow{G}^L$ *is an oriented embedded graph* $\overrightarrow{G}$ *in a topological space* $\mathscr{S}$ *associated with a mapping* $L : v \to L(v),\ (v,u) \to L(v,u)$, *2 end-operators* $A^+_{vu} : L(v,u) \to L^{A^+_{vu}}(v,u)$ *and* $A^+_{uv} : L(u,v) \to L^{A^+_{uv}}(u,v)$ *on a Banach space* $\mathscr{B}$ *over a field* $\mathscr{F}$ *such as those shown in Fig.* 7,

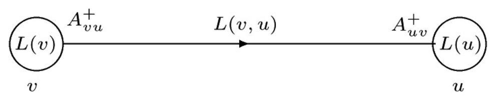


Fig. 7

with $L(v,u)=-L(u,v)$, $A^+_{vu}(-L(v,u))=-L^{A^+_{vu}}(v,u)$ *for* $\forall(v,u)\in E\left(\overrightarrow{G}\right)$ *and holding with continuity equation*

$$\sum_{u\in N_G(v)} L^{A^+_{vu}}(v,u)=L(v)\ \ \text{for}\forall v\in V\left(\overrightarrow{G}\right).$$

DEFINITION 1.2 (Element 2) *A harmonic flow* $\overrightarrow{G}^L$ *is an oriented embedded graph* $\overrightarrow{G}$ *in a topological space* aS *associated with a mapping* $L: v\to L(v)-iL(v)$ *for* $v\in E\left(\overrightarrow{G}\right)$ *and* $L:(v,u)\to L(v,u)-iL(v,u)$, *2 end-operators* $A^+_{vu}: L(v,u)-iL(v,u)\to L^{A^+_{vu}}(v,u)-iL^{A^+_{vu}}(v,u)$ *and* $A^+_{uv}: L(v,u)-iL(v,u)\to L^{A^+_{uv}}(v,u)-iL^{A^+_{uv}}(v,u)$ *on a Banach space* $\mathscr{B}$ *over a field* $\mathscr{F}$ *such as those shown in Fig.* 8,

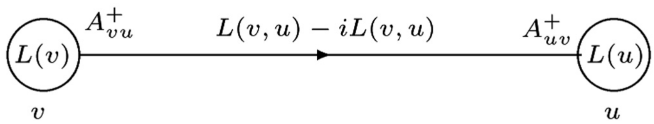


Fig. 8

where $i^2=-1$, $L(v,u)=-L(u,v)$ *for* $\forall(v,u)\in E\left(\overrightarrow{G}\right)$ *and holding with continuity equation*

$$\sum_{u\in N_G(v)}\left(L^{A^+_{vu}}(v,u)-iL^{A^+_{vu}}(v,u)\right)=L(v)-iL(v)\ \ for\ \ \forall v\in V\left(\overrightarrow{G}\right).$$

An element 1 is usually denoted by $\overrightarrow{G}^L$ and an element 2 is denoted by $\overrightarrow{G}^{L^2}$ for emphasizing L^2 mapping edges to $\mathscr{B}\times\mathscr{B}$, where $L_1(v,u),L_2(v,u)\in\mathscr{B}$. Let $\mathscr{G}$ be a closed family of graphs $\overrightarrow{G}$ under the union operation and let $\mathscr{B}$ be a linear space $(\mathscr{B};+,\cdot)$, or furthermore, a commutative ring, a Banach or Hilbert space $(\mathscr{B};+,\cdot)$ over a field aF. Denoted by $\left(\mathscr{G}_{\mathscr{B}};+,\cdot\right)$ and $\left(\mathscr{G}^{\pm}_{\mathscr{B}};+,\cdot\right)$ the respectively elements 1 and 2 form over graphs $\overrightarrow{G}\in\mathscr{G}$.

Could we establish mathematics on elements 1 *and* 2 *for a given graph family and a Banach space, i.e., view elements 1 and 2 as mathematical elements*? The answer is positive. The main purpose of this paper is to report such a mathematical theory on elements 2, i.e., harmonic flows because elements 1 have been extensively discussed in references (Mao, 2015, 2016, 2017, 2018). For such an objective, a dynamic theory, including Banach harmonic flow space closed under action of differential, integral operators is introduced. A few well-known results such as those of Banach theorem, closed graph theorem and Hahn-Banach theorem are generalized with extended Euler-Lagrange equation, i.e., establish mathematical theory on elements 1 and 2 and show such elements can be viewed as vectors underlying topological structures, which can be also applied to analyse the structure of matters and antimatter (Mao, 2019), and also the n-body problem including both particles and antiparticles.

For terminologies and notations not mentioned here, we follow references (Abraham and Marsden, 1978) for mechanics, (Conway, 1990) for functional analysis, (Brauer and Castillo-Chaver, 2012) for biological systems, (Chen, Wang and Li, 2015) for complex network, (Mao, 2011) for combinatorial geometry, and (Mao, 2011 and Smarandache, 1997) for Smarandache systems and multispaces.

§2. Harmonic Flow Space with Operators

2.1 Commutative Rings over Graphs. Let $n \geq 1$ be an integer. Then, *whether or not a vector* $\mathbf{v} = (x_1, x_2, \cdots, x_n)$ *in Euclidean space* $\mathbb{R}^n$ *has a topological structure*? If it has, *what is its underlying topological structure*? Usually, one views a vector $\mathbf{v} \in \mathbb{R}^n$ underlies a linear structure, i.e., a topological line such as those shown in Fig. 9.

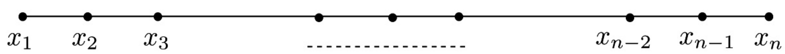


Fig. 9

i.e., $\mathbf{v}$ underlies a path P_n.

Could we view these new elements 1 *and* 2 *as vectors of linear space underlying topological graph* G? The answer is affirmative, which establishes mathematical foundation of complex networks.

Let $\mathscr{G}$ be a closed family of graphs $\overrightarrow{G}$ under the union operation and let $\mathscr{B}$ be a linear space $(\mathscr{B}; +, \cdot)$, or furthermore, a commutative ring $(\mathscr{B}; +, \cdot)$ over a field $\mathscr{F}$. Define

$$\overrightarrow{G}^{L^2} + \overrightarrow{G'}^{L'^2} = \left(\overrightarrow{G} \setminus \overrightarrow{G'}\right)^{L^2} \bigcup \left(\overrightarrow{G} \bigcap \overrightarrow{G'}\right)^{L^2+L'^2} \bigcup \left(\overrightarrow{G'} \setminus \overrightarrow{G}\right)^{L'^2}, \tag{2.1}$$

$$\overrightarrow{G}^{L^2} \cdot \overrightarrow{G'}^{L'^2} = \left(\overrightarrow{G} \setminus \overrightarrow{G'}\right)^{L^2} \bigcup \left(\overrightarrow{G} \bigcap \overrightarrow{G'}\right)^{L^2 \cdot L'^2} \bigcup \left(\overrightarrow{G'} \setminus \overrightarrow{G}\right)^{L'^2}, \tag{2.2}$$

$$\lambda \cdot \overrightarrow{G}^{L^2} = \overrightarrow{G}^{\lambda \cdot L^2}, \tag{2.3}$$

where $\lambda \in \mathscr{F}$ and

$$L^2 : (v,u) \to (L_1(v,u), L_2(v,u)), \quad {L'}^2 : (v,u) \to ({L'}_1(v,u), {L'}_2(v,u)),$$
$$L^2 + {L'}^2 : (v,u) \to \left(L_1(v,u) + L'_1(v,u),\ L_2(v,u) + L'_2(v,u)\right),$$
$$L^2 \cdot {L'}^2 : (v,u) \to \left(L_1(v,u) \cdot L'_1(v,u), L_2(v,u) \cdot L'_2(v,u)\right),$$
$$\lambda \cdot L^2(v,u) = (\lambda \cdot L_1(v,u), \lambda \cdot L_2(v,u))$$

with substituting end-operator $A : (v,u) \to A^+_{vu}(v,u) + (A')^+_{vu}(v,u)$ or $A : (v,u) \to A^+_{vu}(v,u) \cdot (A')^+_{vu}(v,u)$ for $(v,u) \in E\left(\overrightarrow{G} \bigcap \overrightarrow{G'}\right)$ in $\overrightarrow{G}^{L^2} + \overrightarrow{G'}^{{L'}^2}$ or $\overrightarrow{G}^{L^2} \cdot \overrightarrow{G'}^{{L'}^2}$ and $L_1(v,u)$, $L_2(v,u)$, $L'_1(v,u)$, $L'_2(v,u) \in \mathscr{B}$ for $\forall (v,u) \in E\left(\overrightarrow{G}\right)$ or $E\left(\overrightarrow{G'}\right)$. For example, let all end-operators $A = \mathbf{1}_{aB}$. Then, the operation $+$ and $\cdot$ are shown in Fig. 10.

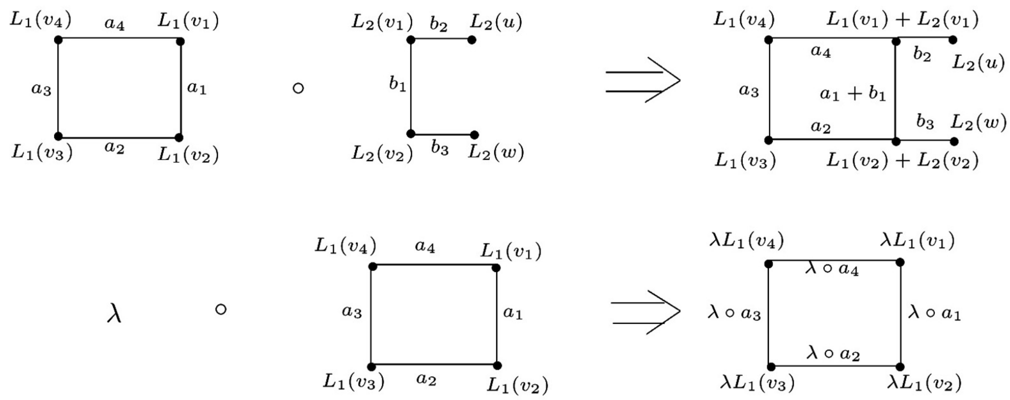


Fig. 10

Let

$$L^{\circ}_{kl}(e) = \begin{cases} L^2_k(e), & \text{if } e \in E\left(\overrightarrow{G}_k \setminus \overrightarrow{G}_l\right) \\ L^2_l(e), & \text{if } e \in E\left(\overrightarrow{G}_l \setminus \overrightarrow{G}_k\right) \\ L^2_k(e) \circ L^2_l(e) & \text{if } e \in E\left(\overrightarrow{G}_k \bigcap \overrightarrow{G}_l\right) \end{cases}, \tag{2.4}$$

and

$$L^{\circ}_{kls}(e)=\begin{cases} L_k^2(e), & \text{if } e\in E\left(\overrightarrow{G}_k\setminus\left(\overrightarrow{G}_l\bigcup\overrightarrow{G}_s\right)\right)\\ L_l^2(e), & \text{if } e\in E\left(\overrightarrow{G}_l\setminus\left(\overrightarrow{G}_k\bigcup\overrightarrow{G}_s\right)\right)\\ L_s^2(e), & \text{if } e\in E\left(\overrightarrow{G}_s\setminus\left(\overrightarrow{G}_k\bigcup\overrightarrow{G}_l\right)\right)\\ L^{\circ}_{kl}(e), & \text{if } e\in E\left(\left(\overrightarrow{G}_k\bigcap\overrightarrow{G}_l\right)\setminus\overrightarrow{G}_s\right)\\ L^{\circ}_{ks}(e), & \text{if } e\in E\left(\left(\overrightarrow{G}_k\bigcap\overrightarrow{G}_s\right)\setminus\overrightarrow{G}_l\right)\\ L^{\circ}_{ls}(e), & \text{if } e\in E\left(\left(\overrightarrow{G}_l\bigcap\overrightarrow{G}_s\right)\setminus\overrightarrow{G}_k\right)\\ L_k^2(e)^{\circ}L_l^2(e)^{\circ}L_s^2(e) & \text{if } e\in E\left(\overrightarrow{G}_k\bigcap\overrightarrow{G}_l\bigcap\overrightarrow{G}_s\right)\end{cases}, \tag{2.5}$$

where $\circ$ is the operation $+$, $-$ or $\cdot$ and $\overrightarrow{G}_k, \overrightarrow{G}_l, \overrightarrow{G}_s\in\mathscr{G}$. Then, we have

THEOREM 2.1 (Mao, 2015 2019) *If $\mathscr{G}$ is a closed family of graphs under the union operation and $\mathscr{B}$ a linear space $(\mathscr{B};+,\cdot)$, then, all pair flows $\left(\mathscr{G}_{\mathscr{B}^2};+,\cdot\right)$ form a linear space, and furthermore, a commutative ring if $\mathscr{B}$ is a commutative ring $(\mathscr{B};+,\cdot)$ over a field $\mathscr{F}$ with a zero flow $\mathbf{O}$, i.e., $\{\mathbf{0},\mathbf{0}\}$ on edges (v,u) in $\left(\mathscr{G}_{\mathscr{B}^2};+\right)$ and a unit $\mathbf{1}$, i.e., $\{\mathbf{1},\mathbf{1}\}$ on edges (v,u) in $\left(\mathscr{G}_{\mathscr{B}^2};\cdot\right)$ for $\forall(v,u)\in E\left(\overrightarrow{G}\right)$, $\overrightarrow{G}\in\mathscr{G}$.*

2.2 Banach Harmonic Flow Space For $\forall\overrightarrow{G}^{L^2}\in\mathscr{G}_{\mathscr{B}^2}$ with $L^2(e)=(L_1(e),L_2(e))$, $e\in E\left(\overrightarrow{G}\right)$ define

$$\left\|\overrightarrow{G}^{L^2}\right\|=\sum_{e\in E\left(\overrightarrow{G}\right)}\left(\|L_1(e)\|+\|L_2(e)\|\right), \tag{2.6}$$

where $\mathscr{B}$ is a Banach space $(\mathscr{B};+,\cdot)$ over a field $\mathscr{F}$ with a norm $\|\cdot\|$. Then we know

THEOREM 2.2 (Mao, 2019) *If $\mathscr{G}$ is a closed family of graphs under the union operation and $\mathscr{B}$ a Banach space $(\mathscr{B};+,\cdot)$, then, $\mathscr{G}_{\mathscr{B}^2}$ with linear operators A^+_{vu}, A^+_{uv} for $\forall(v,u)\in E\left(\bigcup\limits_{G\in\mathscr{G}}\overrightarrow{G}\right)$ is a Banach space, and furthermore, if $\mathscr{B}$ is a Hilbert space, $\mathscr{G}_{\mathscr{B}^2}$ is a Hilbert space too.*

Thus, all elements in $\mathscr{G}_{\mathscr{B}^2}$ can be viewed as vectors underlying a graph $\overrightarrow{G}\in\mathscr{G}$.

2.3 Operators on Banach Harmonic Flow Space.

DEFINITION 2.3 *Let* $\boldsymbol{T} : \mathscr{G}^{\pm}_{\mathscr{B}} \to \mathscr{B}^{\pm}$ *be an operator on Banach harmonic flow space* $\mathscr{G}^{\pm}_{\mathscr{B}}$ *over a field* $\mathscr{F}$. *Then,* $\boldsymbol{T}$ *is linear if*

$$\boldsymbol{T}\left(\lambda\overrightarrow{G}_k^{L_k^2} + \mu\overrightarrow{G}_l^{L_l^2}\right) = \lambda\boldsymbol{T}\left(\overrightarrow{G}_k^{L_k^2}\right) + \mu\boldsymbol{T}\left(\overrightarrow{G}_l^{L_l^2}\right)$$

for $\forall\overrightarrow{G}_k^{L_k^2}, \overrightarrow{G}_l^{L_l^2} \in \mathscr{G}^{\pm}_{\mathscr{B}}$ *and* $\lambda, \mu \in \mathscr{F}$, *is continuous at* $\overrightarrow{G}_0^{L_0^2}$ *if there always exist a number* $\delta(\varepsilon)$ *for* $\forall\epsilon > 0$ *such that*

$$\left\|\boldsymbol{T}\left(\overrightarrow{G}^{L^2}\right) - \boldsymbol{T}\left(\overrightarrow{G}_0^{L_0^2}\right)\right\| < \varepsilon$$

if $\left\|\overrightarrow{G}^{L^2} - \overrightarrow{G}_0^{L_0^2}\right\| < \delta(\varepsilon)$, *bounded if* $\left\|\boldsymbol{T}\left(\overrightarrow{G}^{L^2}\right)\right\| \leq \xi\left\|\overrightarrow{G}^{L^2}\right\|$ *for* $\forall\overrightarrow{G}^{L^2} \in \mathscr{G}^{\pm}_{\mathscr{B}}$ *with a constant* $\xi \in [0, \infty)$ *and furthermore, a contractor if*

$$\left\|\boldsymbol{T}\left(\overrightarrow{G}_k^{L_k^2}\right) - \boldsymbol{T}\left(\overrightarrow{G}_l^{L_l^2}\right)\right\| \leq \xi\left\|\overrightarrow{G}_k^{L_k^2} - \overrightarrow{G}_l^{L_l^2}\right\|$$

for $\forall\overrightarrow{G}_k^{L_k^2}, \overrightarrow{G}_l^{L_l^2} \in \mathscr{G}^{\pm}_{\mathscr{B}}$ *with* $\xi \in [0, 1)$.

Then, the fixed point theorem and Banach theorem in functionals are generalized to harmonic flows following.

THEOREM 2.4 (Fixed Harmonic Flow Theorem, (Mao, 2019) *If* $\boldsymbol{T} : \mathscr{G}^{\pm}_{\mathscr{B}} \to \mathscr{G}^{\pm}_{\mathscr{B}}$ *is a linear continuous contractor, then there is a uniquely harmonic flow* $\overrightarrow{G}^{L^2} \in \mathscr{G}^{\pm}_{\mathscr{B}}$ *such that*

$$\boldsymbol{T}\left(\overrightarrow{G}^{L^2}\right) = \overrightarrow{G}^{L^2}.$$

THEOREM 2.5 (Mao, 2019) *A linear operator* $\boldsymbol{T} : \overrightarrow{\mathscr{G}}^{\pm} \to \mathscr{G}^{\pm}_{\mathscr{B}}$ *is continuous if and only if it is bounded.*

THEOREM 2.6 (Banach, (Mao, 2019)) *Let* $\boldsymbol{T} : \mathscr{G}^{\pm}_{\mathscr{B}_1} \to \mathscr{G}^{\pm}_{\mathscr{B}_2}$ *be a linear continuous operator with Banach spaces* $\mathscr{B}_1$ *and* $\mathscr{B}_2$. *If* $\boldsymbol{T}$ *is bijective then its inverse operator* $\boldsymbol{T}^{-1}$ *is continuous.*

Similarly, we define graph of an operator $\boldsymbol{T}$ in $\mathscr{G}^{\pm}_{\mathscr{B}_2}$ following.

DEFINITION 2.7 *Let* $\boldsymbol{T}: \mathscr{G}^{\pm}_{\mathscr{B}_1} \to \mathscr{G}^{\pm}_{\mathscr{B}_2}$ *be a linear continuous operator with Banach spaces* $\mathscr{B}_1$, $\mathscr{B}_2$. *The graph of* $\boldsymbol{T}$ *in* $\mathscr{G}^{\pm}_{\mathscr{B}_2}$ *is defined by*

$$\mathrm{Grap}\boldsymbol{T} = \left\{ \left(\overrightarrow{G}^{L^2}, \boldsymbol{T}\left(\overrightarrow{G}^{L^2}\right)\right) \middle| \overrightarrow{G}^{L^2} \in \mathscr{G}^{\pm}_{\mathscr{B}_1} \right\}$$

and $\boldsymbol{T}$ *is closed if* $Cl\left(\mathrm{Grap}\boldsymbol{T}\right) = \mathrm{Grap}\boldsymbol{T}$, *i.e., a closed subspace.*

Then, we generalize the closed graph theorem in functionals as follows.

THEOREM 2.8 (Mao, 2019) *Let* $\boldsymbol{T}: \mathscr{G}^{\pm}_{\mathscr{B}_1} \to \mathscr{G}^{\pm}_{\mathscr{B}_2}$ *be a linear operator with Banach spaces* $\mathscr{B}_1$, $\mathscr{B}_2$. *Then* $\boldsymbol{T}$ *is closed if and only if for any harmonic flow sequence* $\left\{\overrightarrow{G}^{L^2_n}_n\right\} \in \mathscr{G}^{\pm}_{\mathscr{B}_1}$ *with* $\lim\limits_{n\to\infty} \overrightarrow{G}^{L^2_n}_n = \overrightarrow{G}^{L^2_0}_0 \in \mathscr{G}^{\pm}_{\mathscr{B}_1}$, $\lim\limits_{n\to\infty} \boldsymbol{T}\left(\overrightarrow{G}^{L^2_n}_n\right) = \overrightarrow{G}^{L^2} \in \mathscr{G}^{\pm}_{\mathscr{B}_2}$ *and* $\boldsymbol{T}\left(\overrightarrow{G}^{L^2_0}_0\right) = \overrightarrow{G}^{L^2}$.

THEOREM 2.9 (Closed Graph Theorem, (Mao, 2019)) *If* $\boldsymbol{T}: \mathscr{G}^{\pm}_{\mathscr{B}_1} \to \mathscr{G}^{\pm}_{\mathscr{B}_2}$ *is a closed linear operator with Banach spaces* $\mathscr{B}_1$, $\mathscr{B}_2$, *then* $\boldsymbol{T}$ *is continuous.*

Particularly, we generalize the Hahn-Banach theorem following.

THEOREM 2.10 (Hahn-Banach, (Mao, 2019)) *Let* $\mathscr{H}^{\pm}_{\mathscr{B}}$ *be a harmonic flow subspace of* $\mathscr{G}^{\pm}_{\mathscr{B}}$ *and let* $F: \mathscr{H}^{\pm}_{\mathscr{B}} \to \boldsymbol{C}$ *be a linear continuous functional on* $\mathscr{H}^{\pm}_{\mathscr{B}}$. *Then, there is a linear continuous functional* $\widetilde{F}: \mathscr{G}^{\pm}_{\mathscr{B}} \to \boldsymbol{C}$ *hold with*

(1) $\widetilde{F}\left(\overrightarrow{G}^{L^2}\right) = F\left(\overrightarrow{G}^{L^2}\right)$ *if* $\overrightarrow{G}^{L^2} \in \mathscr{H}^{\pm}_{\mathscr{B}}$;

(2) $\left\|\widetilde{F}\right\| = \|F\|$.

COROLLARY 2.11 (Mao, 2019) *For* $\overrightarrow{G}^{L^2} \in \mathscr{G}^{\pm}_{\mathscr{B}}$, *if* $F\left(\overrightarrow{G}^{L^2}\right) = 0$ *hold with all linear functionals* F *on* $\mathscr{G}^{\pm}_{\mathscr{B}}$ *then* $\overrightarrow{G}^{L^2} = \boldsymbol{O}$

Clearly, $\mathscr{G}^{\pm}_{\mathscr{B}_1}$ and $\mathscr{G}'^{\pm}_{\mathscr{B}_2}$ are both labeled graph families by definition. Consequently, we define a harmonic flow space $\mathscr{G}^{\pm}_{\mathscr{B}_1}$ *isomorphic* to $\mathscr{G}'^{\pm}_{\mathscr{B}_2}$ if there is a linear continuous operator $\boldsymbol{T}: \mathscr{G}^{\pm}_{\mathscr{B}_1} \to \mathscr{G}'^{\pm}_{\mathscr{B}_2}$ of bijection with $\boldsymbol{T}: \overrightarrow{G}^{L^2} \in \mathscr{G}^{\pm}_{\mathscr{B}_1} \to \overrightarrow{G'}^{L'^2} \in \mathscr{G}^{\pm}_{\mathscr{B}_2}$ such that

$$\boldsymbol{T}\left(A^{+}_{vu}, (L(v,u), -L(v,u)), A^{+}_{uv}\right) = \left(A'^{+}_{vu}, (L'(v,u), -L'(v,u)), A'^{+}_{uv}\right)$$

for $\forall (v,u) \in E\left(\overrightarrow{G}\right)$. We can therefore classify harmonic flow spaces following.

THEOREM 2.12 (Mao, 2019) *A harmonic flow spaces* $\mathscr{G}^{\pm}_{\mathscr{B}_1}$ *is isomorphic to* $\mathscr{G}'^{\pm}_{\mathscr{B}_2}$ *with* $\boldsymbol{T}: \overrightarrow{G}^{L^2} \to \overrightarrow{G'}^{L'^2}$ *if and only if* $\mathscr{G} = \mathscr{G}'$ *and* $\mathscr{B}_1$ *is isomorphic to* $\mathscr{B}_2$.

§3. Harmonic Flow Dynamics

3.1 Harmonic Flow Calculus

DEFINITION 3.1 *Let* D *be a boundary subset of* $\boldsymbol{C}^n = \{(x_1, x_2, \cdots, x_n) | x_i \in \boldsymbol{C}, 1 \leq i \leq n\}$, $\mathscr{B} = \boldsymbol{C}(D)$ *of differentiable functions on* D *and all end-operators in* $\mathscr{A}$ *satisfying* $[A, \frac{\partial}{\partial x_i}] = \mathbf{0}$ *for* $\forall A \in \mathscr{A}$. *Define* n *differential operators* $\partial_i : \mathscr{G}^2_{\mathscr{B}} \to \mathscr{G}^2_{\mathscr{B}}$, $1 \leq i \leq n$ *by*

$$\partial_i \overrightarrow{G}^{L^2} = \overrightarrow{G}^{\frac{\partial L^2}{\partial x_i}},$$

and denoted by $\frac{d\overrightarrow{G}^{L^2}}{dz}$ *if* $D \subset \boldsymbol{C}$ *for* $\overrightarrow{G}^{L^2} \in \mathscr{G}^2_{\mathscr{B}}$ *in which the integral flow of* $\overrightarrow{G}^{L^2} \in \mathscr{G}^2_{\mathscr{B}}$ *along a curve* $C = \{z(t)\ |\alpha \leq t \leq \beta\}$ *of length* $< +\infty$ *is defined by*

$$\int_C \overrightarrow{G}^{L^2} dz = \overrightarrow{G}^{\int_C L^2 dz}.$$

A calculation immediately shows the following result.

THEOREM 3.2 (Mao, 2019) *All partial differential operators* ∂_i *and the integral operator* $\int_C$ *are linear continuous on* $\mathscr{G}^2_{\mathscr{B}}$, *and furthermore, on* $\mathscr{G}^{\pm}_{\mathscr{B}}$ *for integers* $1 \leq i \leq n$.

Similar to the calculus, if $\frac{d\overrightarrow{G}^{L^2}}{dz} = \overrightarrow{G'}^{L'^2}$, i.e., $d\overrightarrow{G}^{L^2} = \overrightarrow{G'}^{L'^2} dz$ then $\overrightarrow{G}^{L^2}$ is called the *primitive flow* of $\overrightarrow{G'}^{L'^2}$ and denoted by $\int \overrightarrow{G}^{L^2} dz$. We know easily that

$$\int_C \overrightarrow{G}^{L^2} dz = \overrightarrow{G}^{\int_C L^2 dz} = \overrightarrow{G}^{\int L^2 dz|_\beta - \int L^2 dz|_\alpha} = \int \overrightarrow{G}^{L^2}\bigg|_\beta - \int \overrightarrow{G}^{L^2}\bigg|_\alpha$$

and if C is the boundary curve of a simply connected domain on $\mathbb{R}^2$,

$$\int_C \overrightarrow{G}^{L^2} dz = \boldsymbol{O}.$$

Furthermore,

$$\overrightarrow{G}^{L^2}(z) = \frac{1}{2\pi i}\int_C \frac{\overrightarrow{G}^{L^2}(\zeta)}{\zeta - z}d\zeta$$

with $z \in D$ if $\overrightarrow{G}^{L^2}$ is differentiable on D and continuous on $Cl(D) = D+C$ by definition. We generalize a few well-known results of complex analysis to $\mathscr{G}^2_{\boldsymbol{C}}$, for instance the following result.

THEOREM 3.3 (Cauchy Integral Formula, (Mao, 2019)) *Let $D \subset \boldsymbol{C}$ be a domain with boundary curve C and $\mathscr{B} = \boldsymbol{C}(D)$. If $\overrightarrow{G}^{L^2} \in \mathscr{G}^2_{\mathscr{B}}$ or $\mathscr{G}^{\pm}_{\mathscr{B}}$ is differentiable on D and continuous on $Cl(D) = D + C$, then*

$$\overrightarrow{G}^{L^2}(z) = \frac{1}{2\pi i}\int_C \frac{\overrightarrow{G}^{L^2}(\zeta)}{\zeta - z}d\zeta, \tag{3.1}$$

where, $z \in D$.

3.2 Harmonic Flow Dynamics. Let $\mathscr{L}[L^2(t, \boldsymbol{x}(t), \dot{\boldsymbol{x}}(t))] : (v, u) \in E\left(\overrightarrow{G}\right) \to \mathscr{L}[L^2(t, \boldsymbol{x}(t), \dot{\boldsymbol{x}}(t))(v, u)]$ be a differentiable functional with $[\mathscr{L}, A] = \mathbf{0}$ for $A \in \mathscr{A}$. Then, there must be $\overrightarrow{G}^{\mathscr{L}[L^2(t,\boldsymbol{x}(t),\dot{\boldsymbol{x}}(t))]} \in \mathscr{G}^{\pm}_{\mathscr{B}}$.

Consider the variational action

$$J\left[\overrightarrow{G}^{L^2}[t]\right] = \int_{t_1}^{t_2} \overrightarrow{G}^{\mathscr{L}[L^2(t,\boldsymbol{x}(t),\dot{\boldsymbol{x}}(t))]}dt. \tag{3.2}$$

on a harmonic flow $\overrightarrow{G}^{L^2}[t] \in \mathscr{G}^{\pm}_{\mathscr{B}}$ where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$. According to the Hamiltonian principle there must be $\delta J\left[\overrightarrow{G}^{L^2}[t]\right] = \boldsymbol{O}$, i.e.,

$$\int_{t_1}^{t_2} \sum_{i=1}^{n} \left(\left(\frac{\partial \mathscr{L}}{\partial x_i} - \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{x}_i} \right)\Bigg|_{(v,u)} \right) \delta x_i dt = 0 \tag{3.3}$$

for $(v, u) \in E\left(\overrightarrow{G}\right)$. We therefore get the Euler-Lagrange equations on $\overrightarrow{G}^{L^2}[t]$ following.

THEOREM 3.4 (Mao, 2019) *If $L^2(t, \boldsymbol{x}(t), \dot{\boldsymbol{x}}(t))(v, u)$ is a Lagrangian on edge (v, u), $\mathscr{L}[L^2(t, \boldsymbol{x}(t), \dot{\boldsymbol{x}}(t))] : (v, u) \to \mathscr{L}[L^2(t, \boldsymbol{x}(t), \dot{\boldsymbol{x}}(t))(v, u)]$ is a differentiable functional on* *a harmonic flow* $\overrightarrow{G}^{L^2}[t]$ *for* $(v,u)\in E\left(\overrightarrow{G}\right)$ *with* $[\mathscr{L},A]=\mathbf{0}$ *for* $A\in\mathscr{A}$, *then*

$$\frac{\partial\overrightarrow{G}^{\mathscr{L}}}{\partial x_i}-\frac{d}{dt}\frac{\partial\overrightarrow{G}^{\mathscr{L}}}{\partial\dot{x}_i}=\boldsymbol{O},\qquad 1\le i\le n. \tag{3.4}$$

Particularly, if $\mathscr{L}$ is linear dependent on L^2, we get a conclusion following.

COROLLARY 3.5 (Mao, 2019) *If* $\mathscr{L}$ *is linear dependent on* L^2, *then*

$$\frac{\partial\overrightarrow{G}^{L^2}}{\partial x_i}-\frac{d}{dt}\frac{\partial\overrightarrow{G}^{L^2}}{\partial\dot{x}_i}=\boldsymbol{O},\quad 1\le i\le n.$$

Furthermore, if all parts on edges are moving in coherence or synchronization, we get the Euler-Lagrange equations.

COROLLARY 3.6 (Euler-Lagrange, (Mao, 2019)) *If the Lagrangian* $\mathscr{L}\left[\overrightarrow{G}^{L^2}[t]\right]$ *of a harmonic flow* $\overrightarrow{G}^{L^2}[t]$ *is independent on* (v,u), *i.e., all Lagrangian* $L^2(t,\boldsymbol{x}(t),\dot{\boldsymbol{x}}(t))(v,u)$, $(v,u)\in E\left(\overrightarrow{G}\right)$ *are synchronized, then the dynamic behavior of* $\overrightarrow{G}^{L^2}[t]$ *can be characterized by* n *equations*

$$\frac{\partial L}{\partial x_i}-\frac{d}{dt}\frac{\partial L}{\partial\dot{x}_i}=0,\qquad 1\le i\le n, \tag{3.5}$$

which are essentially equivalent to the Euler-Lagrange equations of bouquet $\overrightarrow{B}_1^{L^2}\in\overrightarrow{B}_{1\,\mathscr{B}}^{\pm}$, *i.e, dynamic equations on a particle* P.

Corollary 3.6 explains an additional assumption in classical mechanics also, i.e., we can view an object as a geometrical point only in the case that all of its parts are synchronized. However, the non-coherence appears from time to time in eyes of humans, which implies that the non-coherence is general but coherence is special. Whence, for understanding the reality of matters we have to turn our attention on coherence to non-coherence by Smarandache multispaces (Mao, 2011, 2014)).

§4. Applications

4.1 Matter-Antimatter Asymmetry Problem. The universe is made up both by matter and antimatter. Although it was first theoretically considered by Paul A.M.Dirac in 1928, antimatter for instance, positron, antiproton, antineutron, antideuteron, antihydrogen,$\cdots$ were discovered one after another in laboratory since 1932. In fact, the most interesting character of antimatter is it will be completely annihilated into energy if it collides with its normal matter. For example, an electron collides with a positron will completely transforms to 2 photons, an energy form, i.e.,

$$e^{-1} + e^{+} \to \gamma + \gamma.$$

Theoretically, the Big Bang should have created equal amounts of matter and antimatter in the early universe. However, all things we see from the smallest life forms on Earth to the largest stellar objects is made entirely of matter, without antimatter. *Why there is an asymmetry picture for matter and antimatter in today's universe*?

In the Standard Model of Particle (SM), baryons such as those of the proton and neutron are bound of 3 quarks and antiquarks, including gluon such as those shown in Fig. 11. Certainly, such a composition theory on matter by quarks and gluons in SM is essentially the elements 1 on complex networks, which enables one to speculating its behavior by combinatorial speculation. Particularly, we present a question on the behavior of antigluon, i.e., *is it attractive, likewise the behavior of gluon or its reverse, repulsive*? This will enables one explaining why the matter-antimatter picture is asymmetry and understanding well the material constitution of universe.

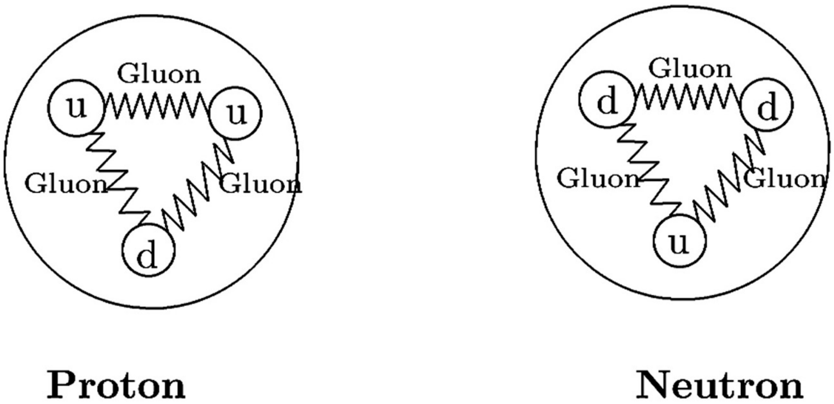


Fig. 11

We usually understand the universe by matter, not including antimatter. However, the universe consists of matter and antimatter, and the matter contributes only 4.6% to the whole matter/energy distribution of the universe as physicist verified. Thus, we always understand the universe by the known matter, i.e., 4.6% not the whole 100% consisting of the universe. However, we can not conclude the universe is dominated by the matter, and can not claim that we have hold on the truth face of the universe because all known of humans are a partial or local true on matters. Now, if we include antimatter distribution in the universe, we would extend our understanding on the universe. For this objective, a central topic is the assumption on the behavior of antigluon.

Attraction Assumption. This is the current notion on antigluon consisting of antiproton, antineutron, antimeson,$\cdots$ within an internal equilibrium between regions of attraction R_2 and repulsion R_1 such as those shown in Fig. 12

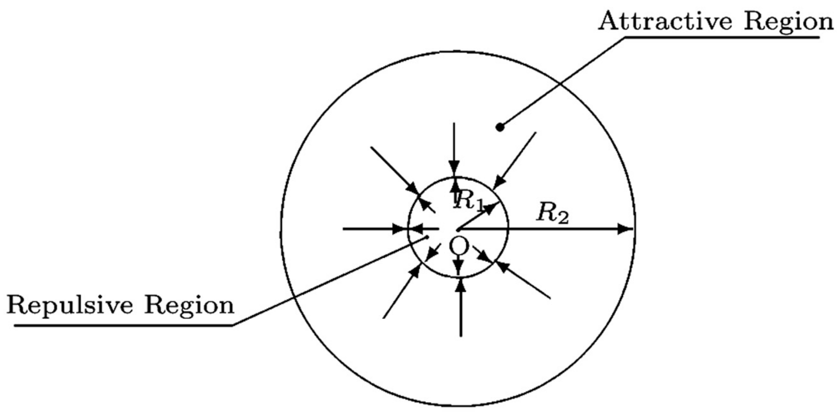


Fig. 12

where r is the distance of 2 quarks, $R_1 = 5 \times 10^{-14}$cm, $R_2 = 4 \times 10^{-12}$cm (Tian, 2014) and results in an attractively residual strong interaction of antiproton or antineutron to form antimatter. However, it is this notion that leads to the asymmetry problem of matter and antimatter, contradicts to all experimental results of humans. If it is true, there must be all antimolecules likewise the molecules, and there are must be all antianimals, particularly, antihumans like animals on the earth. If so, humans ourselves can not being in the universe. Whence, it is only a priori hypothesis on matter and antimatter forming after the Big Bang.

Repulsion Assumption. This is an opposite notion to the current on antigluon within an internal equilibrium between regions of repulsion R_2 and attraction R_1 such as those shown in Fig.13 which finally results in a repulsion of residual strong interaction within antiprotons and antineutrons.

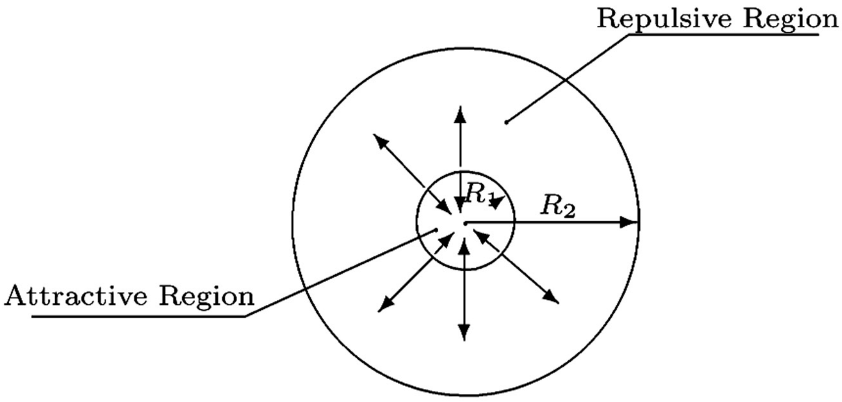


Fig. 13

In this case, the residual strong interaction within an antiproton or antineutron is repulsive, which contributes a different composite theory to the usual theory on antimatters, i.e., an antiproton can not be bound with an antiproton, an antiproton can not be bound with an antineutron, and an antineutron can not also be bound with an antineutron in theory. However, this assumption can explains the asymmetry of matter and antimatter, consistent with known experiments on matter and antimatter, which also implies that the huge investment on searching new elementary particles of matters such as the investment more than 5 billion dollars on the ring electron-positron collider (CEPC) of China is worthless because the matter contributes only 4.6% to the whole matter and we can not understand the reality of the universe by this way. For details, the reader is referred to the reference (Mao, 2019).

4.2 N-Body Problem. Formally, the n-body problem (Abraham and Marsden, 1978) is to predict the individual motions of a group of celestial objects interacting with each other gravitationally, i.e., find solution of differential equations

$$m_i \frac{d^2 \boldsymbol{r}_i}{dt^2} = \sum_{j \neq i} \frac{m_i m_j \left(\boldsymbol{r}_j - \boldsymbol{r}_i\right)}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^3}$$

in classical mechanics, where $\boldsymbol{r}_i = (x_i, y_i, z_i)$ for $1 \leq i \leq n$. This problem is partially solved by Weierstrass in 1880's, H.Poincaré in 1890, K.Sundman in 1912 and then, completely solved in power series by Q.D.Wang (Wang, 1991) in 1991.

We have known that there always exists universal gravitation in 2 normal particles by Newton. But for antimatter, *how about the gravitation between* 2 *antiparticles, is* *it also an attraction*? If so, *why do we not find large antimatter in large mass unless a few elementary antiparticles today*? Certainly, all experimental results point to that the gravitation in 2 antiparticles is not attractive but maybe a repulsive one.

If the gravitation in antimatters is repulsion, the behaviors of gravitation in particles and antiparticles are classified into 3 cases following:

(1) Attractive in 2 normal particles;

(2) Repulsive in 2 antiparticles;

(3) No gravitation in normal particles and antiparticles. Certainly, an electron attracts a positron if they are very close to and then, transforms to 2 photons but this is not a result of gravitation. It is only because of the electromagnetic force between them.

Clearly, this classification on gravitation is an extension of Newton's, i.e., *gravity is everywhere in the universe*. Furthermore, if the n-body problem includes both particles and antiparticles, we can conclude also the existence of power series solution by result of Q.D.Wang in 1991 because in this case, there are 2 independent systems of differential equations respectively on particles and antiparticles, i.e.,

$$m_i \frac{d^2 \boldsymbol{r}_{Mi}}{dt^2} = \sum_{j \neq i} \frac{m_i m_j \left(\boldsymbol{r}_{Mj} - \boldsymbol{r}_{Mi}\right)}{\left|\boldsymbol{r}_{Mj} - \boldsymbol{r}_{Mi}\right|^3}$$

for integers $1 \le i \le n_1$ and

$$\overline{m}_i \frac{d^2 \boldsymbol{r}_{\overline{M}i}}{dt^2} = -\sum_{j \neq i} \frac{\overline{m}_i \overline{m}_j \left(\boldsymbol{r}_{\overline{M}j} - \boldsymbol{r}_{\overline{M}i}\right)}{\left|\boldsymbol{r}_{\overline{M}j} - \boldsymbol{r}_{\overline{M}i}\right|^3}$$

for integers $1 \le i \le n_2$, where, $n_1 + n_2 = n$, m_i, $\overline{m}_i$ denote the masses and $\boldsymbol{r}_{Mi}$, $\boldsymbol{r}_{\overline{M}i}$ are the position vector (x_i, y_i, z_i) of particle or antiparticle in $\mathbb{R}^3$, respectively. Therefore, we get global elements of particles and antiparticles to be a disjoint union of 2 elements 1, i.e., $\overrightarrow{G}^L_M \bigcup \overrightarrow{G}^L_{\overline{M}}$, where $\overrightarrow{G}^L_M$ and $\overrightarrow{G}^L_{\overline{M}}$ are respectively elements 1 of particles and antiparticles in the universe.

§**5. Conclusion.** Certainly, we have classically mathematical elements on reality of matters in the universe. But these elements are not enough for understanding the reality of matters because all these system must be a compatible one in eyes of humans, i.e., classical mathematics. However, contradictions exist everywhere. We need new elements form mathematical systems for hold on the reality of matters in the universe. For such an objective, we introduce new mathematical elements, i.e., elements 1 and 2 which globally characterize complex networks on matters and can be really viewed as mathematical elements, likewise classical mathematical elements, i.e., vectors underlying a combinatorial structure $\overrightarrow{G}$

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