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Possibility neutrosophic soft sets and PNS-decision making method

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Abstract In this paper, we define concept of possibility neutrosophic soft set and investigate their related properties. We then construct a decision making method called possibility neutrosophic soft decision making method (PNS-decision making method) which can be applied to the decision making problems involving uncertainty. We finally give a numerical example to show the method can be successfully applied to the problems.

Keywords Soft set · neutrosophic set · neutrosophic soft set · possibility neutrosophic soft set · decision making.

1 Introduction

Many problems in engineering, medical sciences, economics and social sciences involve uncertainty. To cope with these problems, researchers proposed some theories such as the theory of fuzzy set [27], the theory of intuitionistic fuzzy set [3], the theory of rough set [19], the theory of vague set [13]. However, all of these theories have their own difficulties which are pointed out in by Molodtsov [15]. Molodtsov proposed a completely new approach for modeling uncertainty, free from these difficulties. Also in [15], Molodtsov showed that a wide range of applications of soft sets have been developed in many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. After Molodtsov [15], the operations of soft sets and their properties were given by Maji et al. [17]. To make some modifications to the operations

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of soft sets some researchers such as Ali *et al.* [1], Çağman and Enginoğlu [7], Sezgin and Atagün [23], Zhu and Wen [29], Çağman [8] give their contributions.

Application of soft set theory in decision making problems is first studied by Maji et al. [16] subsequently works on soft set theory and its applications have been progressing rapidly. For examples; Çağman and Enginoğlu [7] defined the uni-int decision making to reduce the alternatives. Feng *et al.* [11] generalized the uni-int decision making based on choice value soft sets, Qin *et al.* [20] improved some algorithms which require relatively fewer calculations compared with the existing decision making algorithms, Zhi *et al.* [28] presented an efficient decision making approach in incomplete soft set.

Neutrosophic logic and neutrosophic set were proposed by Smarandache [24,25], as a new mathematical tool for dealing with problems involving incomplete, indeterminacy, inconsistent knowledge. Neutrosophy is a new branch of philosophy and generalization of fuzzy logic, intuitionistic fuzzy logic, paraconsistent logic. Fuzzy sets and intuitionistic fuzzy sets are characterized by membership functions, membership and non-membership functions, respectively. In some real life problems for proper description of an object in uncertain and ambiguous environment, we need to handle the indeterminate and incomplete information. But fuzzy sets and intuitionistic fuzzy sets don't handle the indeterminant and inconsistent information.

Maji [18] introduced concept of neutrosophic soft set and some operations of neutrosophic soft sets. Karaaslan [14] redefined concept and operations on neutrosophic soft sets. He also gave a decision making method and group decision making method. Recently, on properties of neutrosophic soft sets and applications of this theory in decision making problems have been studied increasingly. For examples; Broumi [5] defined concept of generalized neutrosophic soft set, Broumi *et al.* [6] gave a decision making method on the neutrosophic parameterized soft sets, Şahin and Küçük [22] introduced a new kind of decision making method based on the generalized neutrosophic soft sets and its integration, Deli [9] defined concept of interval-valued neutrosophic soft set and its operations, Deli and Broumi [10] introduced neutrosophic soft matrices and gave a decision making method based on neutrosophic soft matrices.

Alkhazaleh et al [2] were firstly introduced concept of possibility fuzzy soft sets and their operations and they gave applications of this theory in solving a decision making problem. They also introduced a similarity measure of two possibility fuzzy soft sets and presented its application in a medical diagnosis problem. In 2012, Bashir et al. [4] introduced concept of possibility intuitionistic fuzzy soft set and its operations and discussed similarity measure of two possibility intuitionistic fuzzy sets. They also gave an application of this similarity measure.

In this paper, after given the concept of neutrosophic soft set and some operations required throughout study, we define concept of possibility neutrosophic soft set based on idea that each of elements of initial universal set has got a possibility degree related to each element of parameter set. To explain the idea of possibility neutrosophic soft set, let us give an example; let us consider in the last ten days of April and parameter "rainy". First day of last ten days, the possibility of action of rainfall can be 0.8. But amount of water can be $(0.5, 0.3, 0.6)$. As similarly, according to the parameter "rainy" other nine days can be expressed possibility neutrosophic values. For last ten days and for more parameters than one, to show all of possibility neutrosophic values, we need possibility neutrosophic soft sets.

Possibility neutrosophic soft sets are generalization of possibility fuzzy soft sets and possibility intutionistic fuzzy soft sets. Possibility neutrosophic soft set provides to modeling some problems that can not to be expressed possibility fuzzy soft sets and possibility intutionistic fuzzy soft sets. Furthermore, in this study, we define some operations on possibility neutrosophic soft sets and investigate properties related to these operations. We then construct a decision making method enable to make more effective and realistic to possibility neutrosophic soft set called possibility neutrosophic soft neutrosophic soft decision making method. We finally give a numerical example to show the method can be successfully applied to the problems.

2 Preliminary

In this section, we present basic definitions, operations and properties related to the neutrosophic soft set [14] required in next sections.

Throughout paper U is an initial universe, E is a set of parameters and Λ is an index set.

Definition 1 [14] A neutrosophic soft set (or namely ns-set) f over U is a neutrosophic set valued function from E to $\mathcal{N}(U)$. It can be written as

$$f = \Big\{\big(e, \{\langle u, t_{f(e)}(u), i_{f(e)}(u), f_{f(e)}(u)\rangle : u \in U\}\big) : e \in E\Big\}$$

where, $\mathcal{N}(U)$ denotes set of all neutrosophic sets over U. Note that if $f(e) = \big\{\langle u, 0, 1, 1\rangle : u \in U\big\}$, the element $(e, f(e))$ is not appeared in the neutrosophic soft set f. Set of all ns-sets over U is denoted by $\mathcal{NS}$.

Definition 2 [14] Let $f, g \in \mathcal{NS}$. f is said to be neutrosophic soft subset of g, if $t_{f(e)}(u) \leq t_{g(e)}(u)$, $i_{f(e)}(u) \geq i_{g(e)}(u)$ $f_{f(e)}(u) \geq f_{g(e)}(u)$, $\forall e \in E$, $\forall u \in U$. We denote it by $f \sqsubseteq g$. f is said to be neutrosophic soft super set of g if g is a neutrosophic soft subset of f. We denote it by $f \sqsupseteq g$.

If f is neutrosophic soft subset of g and g is neutrosophic soft subset of f. We denote it $f = g$

Definition 3 [14] Let $f \in \mathcal{NS}$. If $t_{f(e)}(u) = 0$ and $i_{f(e)}(u) = f_{f(e)}(u) = 1$ for all $e \in E$ and for all $u \in U$, then f is called null ns-set and denoted by $\tilde{\Phi}$.

Definition 4 [14] Let $f \in \mathcal{NS}$. If $t_{f(e)}(u) = 1$ and $i_{f(e)}(u) = f_{f(e)}(u) = 0$ for all $e \in E$ and for all $u \in U$, then f is called universal ns-set and denoted by $\tilde{U}$.

Definition 5 [14] Let $f, g \in \mathcal{NS}$. Then union and intersection of ns-sets f and g denoted by $f \sqcup g$ and $f \sqcap g$ respectively, are defined by as follow

$$f \sqcup g = \Big\{\big(e, \{\langle u, t_{f(e)}(u) \vee t_{g(e)}(u), i_{f(e)}(u) \wedge i_{g(e)}(u), f_{f(e)}(u) \wedge f_{g(e)}(u)\rangle : u \in U\}\big) : e \in E\Big\}.$$

and ns-intersection of f and g is defined as

$$f \sqcap g = \Big\{\big(e, \{\langle u, t_{f(e)}(u) \wedge t_{g(e)}(u), i_{f(e)}(u) \vee i_{g(e)}(u), f_{f(e)}(u) \vee f_{g(e)}(u)\rangle : u \in U\}\big) : e \in E\Big\}.$$

Definition 6 [14] Let $f, g \in \mathcal{NS}$. Then complement of ns-set f, denoted by $f^{\tilde{c}}$, is defined as follow

$$f^{\tilde{c}} = \Big\{\big(e, \{\langle u, f_{f(e)}(u), 1 - i_{f(e)}(u), t_{f(e)}(u)\rangle : u \in U\}\big) : e \in E\Big\}.$$

Proposition 1 *[14] Let $f, g, h \in \mathcal{NS}$. Then,*

(1) $\tilde{\Phi} \sqsubseteq f$
(2) $f \sqsubseteq \tilde{U}$
(3) $f \sqsubseteq f$
(4) $f \sqsubseteq g$ *and* $g \sqsubseteq h \Rightarrow f \sqsubseteq h$

Proposition 2 *[14] Let $f \in \mathcal{NS}$. Then,*

(1) $\tilde{\Phi}^{\tilde{c}} = \tilde{U}$
(2) $\tilde{U}^{\tilde{c}} = \tilde{\Phi}$
(3) $(f^{\tilde{c}})^{\tilde{c}} = f$.

Proposition 3 *[14] Let $f, g, h \in \mathcal{NS}$. Then,*

(1) $f \sqcap f = f$ *and* $f \sqcup f = f$
(2) $f \sqcap g = g \sqcap f$ *and* $f \sqcup g = g \sqcup f$
(3) $f \sqcap \tilde{\Phi} = \tilde{\Phi}$ *and* $f \sqcap \tilde{U} = f$
(4) $f \sqcup \tilde{\Phi} = f$ *and* $f \sqcup \tilde{U} = \tilde{U}$
(5) $f \sqcap (g \sqcap h) = (f \sqcap g) \sqcap h$ *and* $f \sqcup (g \sqcup h) = (f \sqcup g) \sqcup h$
(6) $f \sqcap (g \sqcup h) = (f \sqcap g) \sqcup (f \sqcap h)$ *and* $f \sqcup (g \sqcap h) = (f \sqcup g) \sqcap (f \sqcup h)$.

Theorem 1 *[14] Let $f, g \in \mathcal{NS}$. Then, De Morgan's law is valid.*

(1) $(f \sqcup g)^{\tilde{c}} = f^{\tilde{c}} \sqcap g^{\tilde{c}}$
(2) $(f \sqcup g)^{\tilde{c}} = f^{\tilde{c}} \sqcap g^{\tilde{c}}$

Definition 7 [14] Let $f, g \in \mathcal{NS}$. Then 'OR' product of ns-sets f and g denoted by $f \wedge g$, is defined as follow

$$f \vee g = \Big\{\big((e, e'), \{\langle u, t_{f(e)}(u) \vee t_{g(e)}(u), i_{f(e)}(u) \wedge i_{g(e)}(u), f_{f(e)}(u) \wedge f_{g(e)}(u)\rangle : u \in U\}\big) : (e, e') \in E \times E\Big\}.$$

Definition 8 [14] Let $f, g \in \mathcal{NS}$. Then 'AND' product of ns-sets f and g denoted by $f \vee g$, is defined as follow

$$f \wedge g = \Big\{ \big((e, e'), \{ \langle u, t_{f(e)}(x) \wedge t_{g(e)}(u), i_{f(e)}(u) \vee i_{g(e)}(u), \\ f_{f(e)}(u) \vee f_{g(e)}(u) \rangle : u \in U \} \big) : (e, e') \in E \times E \Big\}.$$

Proposition 4 *[14] Let $f, g \in \mathcal{NS}$. Then,*

(1) $(f \vee g)^{\tilde{c}} = f^{\tilde{c}} \wedge g^{\tilde{c}}$
(2) $(f \wedge g)^{\tilde{c}} = f^{\tilde{c}} \vee g^{\tilde{c}}$

Definition 9 [2] Let $U = \{u_1, u_2, ..., u_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, that is $\mu : E \to I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to I^U \times I^U$ be a function defined as follows:

$$F_\mu(e) = (F(e)(u), \mu(e)(u)), \forall u \in U.$$

Then F_μ is called a possibility fuzzy soft set (PFSS in short) over the soft universe (U, E). For each parameter e_i, $F_\mu(e_i) = (F(e_i)(u), \mu(e_i)(u))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $\mu(e_i)$.

Definition 10 [4] Let $U = \{u_1, u_2, ..., u_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to (I \times I)^U \times I^U$ where $(I \times I)^U$ is the collection of all intuitionistic fuzzy subsets of U and I^U is the collection of all fuzzy subsets of U. Let p be a fuzzy subset of E, that is, $p : E \to I^U$ and let $F_p : E \to (I \times I)^U \times I^U$ be a function defined as follows:

$$F_p(e) = (F(e)(u), p(e)(u)), F(e)(u) = (\mu(u), \nu(u)), \forall u \in U.$$

Then F_p is called a possibility intuitionistic fuzzy soft set (PIFSS in short) over the soft universe (U, E). For each parameter e_i, $F_p(e_i) = (F(e_i)(u), p(e_i)(u))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $p(e_i)$.

3 Possibility neutrosophic soft sets

In this section, we introduced the concepts of possibility neutrosophic soft set, possibility neutrosophic soft subset, possibility neutrosophic soft null set, possibility neutrosophic soft universal set and possibility neutrosophic soft set operations.

Definition 11 Let U be an initial universe, E be a parameter set, $\mathcal{N}(\mathcal{U})$ be the collection of all neutrosophic sets of U and I^U is collection of all fuzzy subset of U. A possibility neutrosophic soft set (PNS-set) f_μ over U is a set of ordered pairs defined by

$$f_\mu = \Big\{\big(e_i, \{(\frac{u_j}{f(e_i)(u_j)}, \mu(e_i)(u_j)) : u_j \in U\}\big) : e_i \in E\Big\}$$

or a mapping defined by

$$f_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$$

where, $i, j \in \Lambda$, f is a mapping given by $f : E \to \mathcal{N}(\mathcal{U})$ and $\mu(e_i)$ is a fuzzy set such that $\mu : E \to I^U$.

For each parameter $e_i \in E$, $f(e_i) = \big\{\langle u_j, t_{f(e_i)}(u_j), i_{f(e_i)}(u_j), f_{f(e_i)}(u_j)\rangle : u_j \in U\big\}$ indicates neutrosophic value set of parameter e_i and where $t, i, f : U \to [0, 1]$ are the membership functions of truth, indeterminacy and falsity respectively of the element $u_j \in U$. For each $u_j \in U$ and $e_i \in E$, $0 \leq t_{f(e_i)}(u_j) + i_{f(e_i)}(u_j) + f_{f(e_i)}(u_j) \leq 3$. Also $\mu(e_i)$, degrees of possibility of belongingness of elements of U in $f(e_i)$. So we can write

$$f_\mu(e_i) = \Big\{\Big(\frac{u_1}{f(e_i)(u_1)}, \mu(e_i)(u_1)\Big), \Big(\frac{u_2}{f(e_i)(u_2)}, \mu(e_i)(u_2)\Big), ..., \Big(\frac{u_n}{f(e_i)(u_n)}, \mu(e_i)(u_n)\Big)\Big\}$$

From now on, we will show set of all possibility neutrosophic soft sets over U with $\mathcal{PN}(U, E)$ such that E is parameter set.

Example 1 Let $U = \{u_1, u_2, u_3\}$ be a set of three cars. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where e_1 =cheap, e_2 =equipment, e_3 =fuel consumption and let $\mu : E \to I^U$. We can define a function $f_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ as follows:

$$f_\mu = \left\{\begin{array}{l} f_\mu(e_1) = \left\{\left(\frac{u_1}{(0.5,0.2,0.6)}, 0.8\right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.4\right), \left(\frac{u_3}{(0.4,0.5,0.8)}, 0.7\right)\right\} \\ f_\mu(e_2) = \left\{\left(\frac{u_1}{(0.8,0.4,0.5)}, 0.6\right), \left(\frac{u_2}{(0.5,0.7,0.2)}, 0.8\right), \left(\frac{u_3}{(0.7,0.3,0.9)}, 0.4\right)\right\} \\ f_\mu(e_3) = \left\{\left(\frac{u_1}{(0.6,0.7,0.5)}, 0.2\right), \left(\frac{u_2}{(0.5,0.3,0.7)}, 0.6\right), \left(\frac{u_3}{(0.6,0.5,0.4)}, 0.5\right)\right\} \end{array}\right\}$$

also we can define a function $g_\nu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ as follows:

$$g_\nu = \left\{\begin{array}{l} g_\nu(e_1) = \left\{\left(\frac{u_1}{(0.6,0.3,0.8)}, 0.4\right), \left(\frac{u_2}{(0.6,0.5,0.5)}, 0.7), \left(\frac{u_3}{(0.2,0.6,0.4)}, 0.8\right)\right\} \\ g_\nu(e_2) = \left\{\left(\frac{u_1}{(0.5,0.4,0.3)}, 0.3\right), \left(\frac{u_2}{(0.4,0.6,0.5)}, 0.6\right), \left(\frac{u_3}{(0.7,0.2,0.5)}, 0.8\right)\right\} \\ g_\nu(e_3) = \left\{\left(\frac{u_1}{(0.7,0.5,0.3)}, 0.8\right), \left(\frac{u_2}{(0.4,0.4,0.6)}, 0.5\right), \left(\frac{u_3}{(0.8,0.5,0.3)}, 0.6\right)\right\} \end{array}\right\}$$

For the purpose of storing a possibility neutrosophic soft set in a computer, we can use matrix notation of possibility neutrosophic soft set f_μ. For example, matrix notation of possibility neutrosophic soft set f_μ can be written as follows: for $m, n \in \Lambda$,

$$f_\mu = \begin{pmatrix} (\langle 0.5, 0.2, 0.6\rangle, 0.8) & (\langle 0.7, 0.3, 0.5\rangle, 0.4) & (\langle 0.4, 0.5, 0.8\rangle, 0.7) \\ (\langle 0.8, 0.4, 0.5\rangle, 0.6) & (\langle 0.5, 0.7, 0.2\rangle, 0.8) & (\langle 0.7, 0.3, 0.9\rangle, 0.4) \\ (\langle 0.6, 0.7, 0.5\rangle, 0.2) & (\langle 0.5, 0.3, 0.7\rangle, 0.6) & (\langle 0.6, 0.5, 0.4\rangle, 0.5) \end{pmatrix}$$

where the $m-$th row vector shows $f(e_m)$ and $n-$th column vector shows u_n.

Definition 12 Let f_μ, $g_\nu \in \mathcal{PN}(U,E)$. Then, f_μ is said to be a possibility neutrosophic soft subset (PNS-subset) of g_ν, and denoted by $f_\mu \subseteq g_\nu$, if

(1) $\mu(e)$ is a fuzzy subset of $\nu(e)$, for all $e \in E$
(2) f is a neutrosophic subset of g,

Example 2 Let $U = \{u_1, u_2, u_3\}$ be a set of tree houses, and let $E = \{e_1, e_2, e_3\}$ be a set of parameters where e_1 =modern, e_2 =big and e_3 =cheap. Let f_μ be a PNS-set defined as follows:

$$f_\mu = \left\{ \begin{array}{l} f_\mu(e_1) = \left\{ \left(\frac{u_1}{(0.5,0.2,0.6)}, 0.8\right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.4\right), \left(\frac{u_3}{(0.4,0.5,0.9)}, 0.7\right) \right\} \\ f_\mu(e_2) = \left\{ \left(\frac{u_1}{(0.8,0.4,0.5)}, 0.6\right), \left(\frac{u_2}{(0.5,0.7,0.2)}, 0.8\right), \left(\frac{u_3}{(0.7,0.3,0.9)}, 0.4\right) \right\} \\ f_\mu(e_3) = \left\{ \left(\frac{u_1}{(0.6,0.7,0.5)}, 0.2\right), \left(\frac{u_2}{(0.5,0.3,0.8)}, 0.6\right), \left(\frac{u_3}{(0.6,0.5,0.4)}, 0.5\right) \right\} \end{array} \right\}$$

$g_\nu : E \to \mathcal{N}(U) \times I^U$ be another PNS-set defined as follows:

$$g_\nu = \left\{ \begin{array}{l} g_\nu(e_1) = \left\{ \left(\frac{u_1}{(0.6,0.1,0.5)}, 0.9\right), \left(\frac{u_2}{(0.8,0.2,0.3)}, 0.6\right), \left(\frac{u_3}{(0.7,0.5,0.8)}, 0.8\right) \right\} \\ g_\nu(e_2) = \left\{ \left(\frac{u_1}{(0.9,0.2,0.4)}, 0.7\right), \left(\frac{u_2}{(0.9,0.5,0.1)}, 0.9\right), \left(\frac{u_3}{(0.8,0.1,0.9)}, 0.5\right) \right\} \\ g_\nu(e_3) = \left\{ \left(\frac{u_1}{(0.6,0.5,0.4)}, 0.4\right), \left(\frac{u_2}{(0.7,0.1,0.7)}, 0.9\right), \left(\frac{u_3}{(0.8,0.2,0.4)}, 0.7\right) \right\} \end{array} \right\}$$

it is clear that f_μ is $PNS-subset$ of g_ν.

Definition 13 Let $f_\mu, g_\nu \in \mathcal{PN}(U,E)$. Then, f_μ and g_ν are called possibility neutrosophic soft equal set and denoted by $f_\mu = g_\nu$, if $f_\mu \subseteq g_\nu$ and $f_\mu \supseteq g_\nu$.

Definition 14 Let $f_\mu \in \mathcal{PN}(U,E)$. Then, f_μ is said to be possibility neutrosophic soft null set denoted by ϕ_μ, if $\forall e \in E$, $\phi_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ such that $\phi_\mu(e) = \{(\frac{u}{\phi(e)(u)}, \mu(e)(u)) : u \in U\}$, where $\phi(e) = \{\langle u, 0, 1, 1\rangle : u \in U\}$ and $\mu(e) = \{(u, 0) : u \in U\})$.

Definition 15 Let $f_\mu \in \mathcal{PN}(U,E)$. Then, f_μ is said to be possibility neutrosophic soft universal set denoted by U_μ, if $\forall e \in E$, $U_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ such that $U_\mu(e) = \{(\frac{u}{U(e)(u)}, \mu(e)(u)) : u \in U\}$, where $U(e) = \{\langle u, 1, 0, 0\rangle : u \in U\}$ and $\mu(e) = \{(u, 1) : u \in U\})$.

Proposition 5 *Let f_μ, g_ν and $h_\delta \in \mathcal{PN}(U,E)$. Then,*

(1) $\phi_\mu \subseteq f_\mu$
(2) $f_\mu \subseteq U_\mu$
(3) $f_\mu \subseteq f_\mu$
(4) $f_\mu \subseteq g_\nu$ *and* $g_\nu \subseteq h_\delta \Rightarrow f_\mu \subseteq h_\delta$

Proof It is clear from Definition 13, 14 and 15.

Definition 16 Let $f_\mu \in \mathcal{PN}(U,E)$, where $f_\mu(e_i) = \{(f(e_i)(u_j), \mu(e_i)(u_j)) : e_i \in E, u_j \in U\}$ and $f(e_i) = \left\{\langle u, t_{f(e_i)}(u_j), i_{f(e_i)}(u_j), f_{f(e_i)}(u_j)\rangle\right\}$ for all $e_i \in E$, $u \in U$. Then for $e_i \in E$ and $u_j \in U$,

(1) f_μ^t is said to be truth-membership part of f_μ,

$$f_\mu^t = \{(f_{ij}^t(e_i), \mu_{ij}(e_i))\}$$

and

$$f_{ij}^t(e_i) = \{(u_j, t_{f(e_i)}(u_j))\}, \mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}$$

(2) f_μ^i is said to be indeterminacy-membership part of f_μ,

$$f_\mu^i = \{(f_{ij}^t(e_i), \mu_{ij}(e_i))\}$$

and

$$f_{ij}^i(e_i) = \{(u_j, i_{f(e_i)}(u_j))\}, \mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}$$

(3) f_μ^f is said to be falsity-membership part of f_μ,

$$f_\mu^i = \{(f_{ij}^f(e_i), \mu_{ij}(e_i))\}$$

and

$$f_{ij}^f(e_i) = \{(u_j, f_{f(e_i)}(u_j))\}, \ \mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}$$

We can write a possibility neutrosophic soft set in form $f_\mu = (f_\mu^t, f_\mu^i, f_\mu^f)$.

If considered the possibility neutrosophic soft set f_μ in Example 1, f_μ can be expressed in matrix form as follow:

$$f_\mu^t = \begin{pmatrix} (0.5, 0.8) & (0.7, 0.4) & (0.4, 0.7) \\ (0.8, 0.6) & (0.5, 0.8) & (0.7, 0.4) \\ (0.6, 0.2) & (0.5, 0.6) & (0.6, 0.5) \end{pmatrix}$$

$$f_\mu^i = \begin{pmatrix} (0.2, 0.8) & (0.3, 0.4) & (0.5, 0.7) \\ (0.4, 0.6) & (0.7, 0.8) & (0.3, 0.4) \\ (0.7, 0.2) & (0.3, 0.6) & (0.5, 0.5) \end{pmatrix}$$

$$f_\mu^f = \begin{pmatrix} (0.6, 0.8) & (0.5, 0.4) & (0.8, 0.7) \\ (0.5, 0.6) & (0.2, 0.8) & (0.9, 0.4) \\ (0.5, 0.2) & (0.7, 0.6) & (0.4, 0.5) \end{pmatrix}$$

Definition 17 [21] A binary operation $\otimes : [0,1] \times [0,1] \to [0,1]$ is continuous $t-$norm if $\otimes$ satisfies the following conditions

(1) $\otimes$ is commutative and associative,
(2) $\otimes$ is continuous,
(3) $a \otimes 1 = a$, $\forall a \in [0,1]$,
(4) $a \otimes b \leq c \otimes d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 18 [21] A binary operation $\oplus : [0,1] \times [0,1] \to [0,1]$ is continuous $t-$conorm (s-norm) if $\oplus$ satisfies the following conditions

(1) $\oplus$ is commutative and associative,
(2) $\oplus$ is continuous,
(3) $a \oplus 0 = a$, $\forall a \in [0,1]$,
(4) $a \oplus b \leq c \oplus d$ whenever $a \leq c, b \leq d$ and $a, b, c, d \in [0,1]$.

Definition 19 Let $I^3 = [0,1] \times [0,1] \times [0,1]$ and $N(I^3) = \{(a,b,c) : a,b,c \in [0,1]\}$. Then $(N(I^3), \oplus, \otimes)$ be a lattices together with partial ordered relation $\preceq$, where order relation $\preceq$ on $N(I^3)$ can be defined by for $(a,b,c),(d,e,f) \in N(I^3)$

$$(a,b,c) \preceq (e,f,g) \Leftrightarrow a \leq e, b \geq f, c \geq g$$

Definition 20 A binary operation

$$\tilde{\otimes} : \Big([0,1] \times [0,1] \times [0,1]\Big)^2 \to [0,1] \times [0,1] \times [0,1]$$

is continuous $n-$norm if $\tilde{\otimes}$ satisfies the following conditions

(1) $\tilde{\otimes}$ is commutative and associative,
(2) $\tilde{\otimes}$ is continuous,
(3) $a\tilde{\otimes}\hat{0} = \hat{0}$, $a\tilde{\otimes}\hat{1} = a$, $\forall a \in [0,1] \times [0,1] \times [0,1]$, $(\hat{1} = (1,0,0))$ and $(\hat{0} = (0,1,1))$
(4) $a\tilde{\otimes}b \leq c\tilde{\otimes}d$ whenever $a \preceq c, b \preceq d$ and $a,b,c,d \in [0,1] \times [0,1] \times [0,1]$.

Here,

$$a\tilde{\otimes}b = \tilde{\otimes}(\langle t(a), i(a), f(a)\rangle, \langle t(b), i(b), f(b)\rangle) = \langle t(a)\otimes t(b), i(a)\oplus i(b), f(a)\oplus f(b)$$

Definition 21 A binary operation

$$\tilde{\oplus} : \Big([0,1] \times [0,1] \times [0,1]\Big)^2 \to [0,1] \times [0,1] \times [0,1]$$

is continuous $n-$conorm if $\tilde{\oplus}$ satisfies the following conditions

(1) $\tilde{\oplus}$ is commutative and associative,
(2) $\tilde{\oplus}$ is continuous,
(3) $a\tilde{\oplus}\hat{0} = a$, $a\tilde{\oplus}\hat{1} = \hat{1}$, $\forall a \in [0,1] \times [0,1] \times [0,1]$, $(\hat{1} = (1,0,0))\, and\, (\hat{0} = (0,1,1))$
(4) $a\tilde{\oplus}b \leq c\tilde{\oplus}d$ whenever $a \leq c, b \leq d$ and $a,b,c,d \in [0,1] \times [0,1] \times [0,1]$.

Here,

$$a\tilde{\oplus}b = \tilde{\oplus}(\langle t(a), i(a), f(a)\rangle, \langle t(b), i(b), f(b)\rangle) = \langle t(a)\oplus t(b), i(a)\otimes i(b), f(a)\otimes f(b)$$

Definition 22 Let $f_\mu, g_\nu \in \mathcal{PN}(U,E)$. The union of two possibility neutrosophic soft sets f_μ and g_ν over U, denoted by $f_\mu \cup g_\nu$, is defined by

$$f_\mu \cup g_\nu = \Big\{\Big(e_i, \Big\{\big(\alpha, \mu_{ij}(e_i) \oplus \nu_{ij}(e_i)\big) : u_j \in U\Big\}\Big) : e_i \in E\Big\}$$

where

$$\alpha = \frac{u_j}{(f^t_{ij}(e_i) \oplus g^t_{ij}(e_i), f^i_{ij}(e_i) \otimes g^i_{ij}(e_i), f^f_{ij}(e_i) \otimes g^f_{ij}(e_i))}$$

Definition 23 Let $f_\mu, g_\nu \in \mathcal{PN}(U,E)$. The intersection of two possibility neutrosophic soft sets f_μ and g_ν over U, denoted by $f_\mu \cap g_\nu$ is defined by

$$f_\mu \cap g_\nu = \Big\{\Big(e_i, \Big\{\big(\theta, \mu_{ij}(e_i) \otimes \nu_{ij}(e_i)\big) : u_j \in U\Big\}\Big) : e_i \in E\Big\}$$

where

$$\theta = \frac{u_j}{(f^t_{ij}(e_i) \otimes g^t_{ij}(e_i), f^i_{ij}(e_i) \oplus g^i_{ij}(e_i), f^f_{ij}(e_i) \oplus g^f_{ij}(e_i))}$$

Example 3 Let us consider the possibility neutrosophic soft sets f_μ and g_ν defined as in Example 1. Let us suppose that $t-$norm is defined by $a \otimes b = min\{a,b\}$ and the $t-$conorm is defined by $a \oplus b = max\{a,b\}$ for $a, b \in [0,1]$. Then,

$$f_\mu \cup g_\nu = \left\{ \begin{array}{l} (f_\mu \cup g_\nu)(e_1) = \left\{\left(\frac{u_1}{(0.6,0.2,0.6)}, 0.8\right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.7\right), \left(\frac{u_3}{(0.4,0.5,0.4)}, 0.8\right)\right\} \\ (f_\mu \cup g_\nu)(e_2) = \left\{\left(\frac{u_1}{(0.8,0.4,0.3)}, 0.6\right), \left(\frac{u_2}{(0.5,0.6,0.2)}, 0.8\right), \left(\frac{u_3}{(0.7,0.2,0.5)}, 0.8\right)\right\} \\ (f_\mu \cup g_\nu)(e_3) = \left\{\left(\frac{u_1}{(0.7,0.3,0.3)}, 0.8\right), \left(\frac{u_2}{(0.5,0.3,0.6)}, 0.6\right), \left(\frac{u_3}{(0.8,0.5,0.3)}, 0.6\right)\right\} \end{array} \right\}$$

and

$$f_\mu \cap g_\nu = \left\{ \begin{array}{l} (f_\mu \cap g_\nu)(e_1) = \left\{\left(\frac{u_1}{(0.5,0.3,0.8)}, 0.4\right), \left(\frac{u_2}{(0.6,0.5,0.5)}, 0.4\right), \left(\frac{u_3}{(0.2,0.6,0.8)}, 0.7\right)\right\} \\ (f_\mu \cap g_\nu)(e_2) = \left\{\left(\frac{u_1}{(0.5,0.4,0.5)}, 0.3\right), \left(\frac{u_2}{(0.4,0.7,0.5)}, 0.6\right), \left(\frac{u_3}{(0.7,0.3,0.9)}, 0.4\right)\right\} \\ (f_\mu \cap g_\nu)(e_3) = \left\{\left(\frac{u_1}{(0.6,0.7,0.5)}, 0.2\right), \left(\frac{u_2}{(0.4,0.5,0.7)}, 0.5\right), \left(\frac{u_3}{(0.6,0.5,0.4)}, 0.5\right)\right\} \end{array} \right\}$$

Proposition 6 *Let $f_\mu, g_\nu, h_\delta \in \mathcal{PN}(U,E)$. Then,*

(1) $f_\mu \cap f_\mu = f_\mu$ *and* $f_\mu \cup f_\mu = f_\mu$
(2) $f_\mu \cap g_\nu = g_\nu \cap f_\mu$ *and* $f_\mu \cup g_\nu = g_\nu \cup f_\mu$
(3) $f_\mu \cap \phi_\mu = \phi_\mu$ *and* $f_\mu \cap U_\mu = f_\mu$
(4) $f_\mu \cup \phi = f_\mu$ *and* $f_\mu \cup U_\mu = U_\mu$
(5) $f_\mu \cap (g_\nu \cap h_\delta) = (f_\mu \cap g_\nu) \cap h_\delta$ *and* $f_\mu \cup (g_\nu \cup h_\delta) = (f_\mu \cup g_\nu) \cup h_\delta$
(6) $f_\mu \cap (g_\nu \cup h_\delta) = (f_\mu \cap g_\nu) \cup (f_\mu \cap h_\delta)$ *and* $f_\mu \cup (g_\nu \cap h_\delta) = (f_\mu \cup g_\nu) \cap (f_\mu \cup h_\delta)$.

Proof The proof can be obtained from Definitions 22. and 23.

Definition 24 [12,26] A function $N : [0,1] \to [0,1]$ is called a negation if $N(0) = 1$, $N(1) = 0$ and N is non-increasing ($x \leq y \Rightarrow N(x) \geq N(y)$). A negation is called a strict negation if it is strictly decreasing ($x < y \Rightarrow N(x) > N(y)$) and continuous. A strict negation is said to be a strong negation if it is also involutive, i.e. $N(N(x)) = x$

Definition 25 [24] A function $n_N : [0,1] \times [0,1] \times [0,1] \to [0,1] \times [0,1] \times [0,1]$ is called a negation if $n_N(\hat{0}) = \hat{1}$, $n_N(\hat{1}) = \hat{0}$ and n_N is non-increasing ($x \preceq y \Rightarrow n_N(x) \succeq n_N(y)$). A negation is called a strict negation if it is strictly decreasing ($x \prec y \Rightarrow N(x) \succ N(y)$) and continuous.

Definition 26 Let $f_\mu \in \mathcal{PN}(U,E)$. Complement of possibility neutrosophic soft set f_μ, denoted by f^c_μ, is defined by

$$f_\mu^c = \Big\{\Big(e, \big\{\big(\frac{u_j}{n_N(f(e_i))}, N(\mu_{ij}(e_i)(u_j))\Big) : u_j \in U\big\}\Big) : e \in E\Big\}$$

where

$$(n_N(f_{ij}(e_i)) = (N(f_{ij}^t(e_i)), N(f_{ij}^i(e_i)), N(f_{ij}^f(e_i)) \text{ for all } i, j \in \Lambda.$$

Example 4 Let us consider the possibility neutrosophic soft set f_μ define in Example 1. Suppose that the negation is defined by $N(f_{ij}^t(e_i)) = f_{ij}^f(e_i)$, $N(f_{ij}^f(e_i)) = f_{ij}^t(e_i)$, $N(f_{ij}^i(e_i)) = 1 - f_{ij}^i(e_i)$ and $N(\mu_{ij}(e_i)) = 1 - \mu_{ij}(e_i)$, respectively. Then, f_μ^c is defined as follow:

$$f_\mu^c = \left\{\begin{array}{l} f_\mu^c(e_1) = \Big\{\Big(\frac{u_1}{(0.6,0.8,0.5)}, 0.2\Big), \Big(\frac{u_2}{(0.5,0.7,0.7)}, 0.6\Big), \Big(\frac{u_3}{(0.8,0.5,0.4)}, 0.3\Big)\Big\} \\ f_\mu^c(e_2) = \Big\{\Big(\frac{u_1}{(0.5,0.6,0.8)}, 0.4\Big), \Big(\frac{u_2}{(0.2,0.3,0.5)}, 0.2\Big), \Big(\frac{u_3}{(0.9,0.7,0.7)}, 0.6\Big)\Big\} \\ f_\mu^c(e_3) = \Big\{\Big(\frac{u_1}{(0.5,0.3,0.6)}, 0.8\Big), \Big(\frac{u_2}{(0.7,0.7,0.5)}, 0.4\Big), \Big(\frac{u_3}{(0.4,0.5,0.6)}, 0.5\Big)\Big\} \end{array}\right\}$$

Proposition 7 *Let $f_\mu \in \mathcal{PN}(U, E)$. Then,*

(1) $\phi_\mu^c = U_\mu$
(2) $U_\mu^c = \phi_\mu$
(3) $(f_\mu^c)^c = f_\mu$.

Proof It is clear from Definition 26.

Proposition 8 *Let $f_\mu, g_\nu \in \mathcal{PN}(U, E)$. Then, De Morgan's law is valid.*

(1) $(f_\mu \cup g_\nu)^c = f_\mu^c \cap g_\nu^c$
(2) $(f_\mu \cap g_\nu)^c = f_\mu^c \cup g_\nu^c$

Proof (1) Let $i, j \in \Lambda$

$$\begin{aligned} &(f_\mu \cup g_\nu)^c \\ &= \Big\{\Big(e_i, \big\{\big(\frac{u_j}{(f_{ij}^t(e_i) \oplus g_{ij}^t(e_i), f_{ij}^i(e_i) \otimes g_{ij}^i(e_i), f_{ij}^f(e_i) \otimes g_{ij}^f(e_i))}, \\ &\quad \mu_{ij}(e_i) \oplus \nu_{ij}(e_i)\big) : u_j \in U\big\}\Big) : e_i \in E\Big\}^c \\ &= \Big\{\Big(e_i, \big\{\big(\frac{u_j}{(f_{ij}^f(e_i) \otimes g_{ij}^f(e_i), N(f_{ij}^i(e_i) \otimes g_{ij}^i(e_i)), f_{ij}^t(e_i) \oplus g_{ij}^t(e_i))}, \\ &\quad N(\mu_{ij}(e_i) \oplus \nu_{ij}(e_i))\big) : u_j \in U\big\}\Big) : e_i \in E\Big\} \\ &= \Big\{\Big(e_i, \big\{\big(\frac{u_j}{(f_{ij}^f(e_i) \otimes g_{ij}^f(e_i), N(f_{ij}^i(e_i)) \oplus N(g_{ij}^i(e_i))), f_{ij}^t(e_i) \oplus g_{ij}^t(e_i))}, \\ &\quad N(\mu_{ij}(e_i)) \otimes N(\nu_{ij}(e_i))\big) : u_j \in U\big\}\Big) : e_i \in E\Big\} \\ &= \Big\{\Big(e_i, \big\{\big(\frac{u_j}{(f_{ij}^f(e_i), N(f_{ij}^i(e_i)), f_{ij}^t(e_i)}, N(\mu_{ij}(e_i))\big) : u_j \in U\big\}\Big) : e_i \in E\Big\} \end{aligned}$$

$$\cap \Big\{\Big(e_i, \{(\frac{u_j}{g^f_{ij}(e_i), N(g^i_{ij}(e_i)), g^t_{ij}(e_i))}, N(\nu_{ij}(e_i))) : u_j \in U\Big\}\Big) : e_i \in E\Big\}$$

$$= \Big\{\Big(e_i, \{(\frac{u_j}{(f^t_{ij}(e_i), N(f^i_{ij}(e_i)), f^f_{ij}(e_i)}, \mu_{ij}(e_i)) : u_j \in U\}\Big) : e_i \in E\Big\}^c$$

$$\cap \Big\{\Big(e_i, \{(\frac{u_j}{g^t_{ij}(e_i), g^i_{ij}(e_i), g^f_{ij}(e_i))}, \nu_{ij}(e_i)) : u_j \in U\}\Big) : e_i \in E\Big\}^c$$

$$= f^c_\mu \cap g^c_\nu.$$

(2) By using similar techniques used to prove (i), (ii) can be shown, too, therefore we skip the proof.

Definition 27 Let f_μ and $g_\nu \in \mathcal{PN}(U, E)$. Then 'AND' product of PNS-set f_μ and g_ν denoted by $f_\mu \wedge g_\nu$, is defined as follow:

$$f_\mu \wedge g_\nu = \Big\{\Big((e_k, e_l), (f^t_{kj}(e_k) \wedge g^t_{lj}(e_l), f^i_{kj}(e_k) \vee g^i_{lj}(e_l), f^f_{kj}(e_k) \vee g^f_{lj}(e_l)), \mu_{kj}(e_k) \wedge \nu_{lj}(e_l)\Big) : (e_k, e_l) \in E \times E, j, k, l \in \Lambda\Big\}$$

Definition 28 Let f_μ and $g_\nu \in \mathcal{PN}(U, E)$. Then 'OR' product of PNS-set f_μ and g_ν denoted by $f_\mu \vee g_\nu$, is defined as follow:

$$f_\mu \vee g_\nu = \Big\{\Big((e_k, e_l), (f^t_{kj}(e_k) \vee g^t_{lj}(e_l), f^i_{kj}(e_k) \wedge g^i_{lj}(e_l), f^f_{kj}(e_k) \wedge g^f_{lj}(e_l)), \mu_{kj}(e_k) \vee \nu_{lj}(e_l)\Big) : (e_k, e_l) \in E \times E, j, k, l \in \Lambda\Big\}$$

4 PNS-decision making method

In this section, we construct a decision making method over the possibility neutrosophic soft set that is called possibility neutrosophic soft decision making method (PNS-decision making method).

Definition 29 Let $f_\mu \in \mathcal{PN}(U, E)$, f^t_μ, f^i_μ and f^f_μ be the truth, indeterminacy and falsity matrices of $\wedge$-product matrix, respectively. Then, weighted matrices of f^t_μ, f^i_μ and f^f_μ, denoted by $\wedge^t, \wedge^i$ and $\wedge^f$, are defined by, respectively

$\wedge^t(e_{ij}, u_k) = t_{(f_\mu \wedge g_\nu)(e_{ij})}(u_k) + (\mu_{ik}(e_i) \wedge \nu_{jk}(e_j)) - t_{(f_\mu \wedge g_\nu)(e_{ij})}(u_k) \times (\mu_{ik}(e_i) \wedge \nu_{jk}(e_j))$

$\wedge^i(e_{ij}, u_k) = i_{(f_\mu \wedge g_\nu)(e_{ij})}(u_k) \times (\mu_{ik}(e_i) \wedge \nu_{jk}(e_j))$

$\wedge^f(e_{ij}, u_k) = f_{(f_\mu \wedge g_\nu)(e_{ij})}(u_k) \times (\mu_{ik}(e_i) \wedge \nu_{jk}(e_j))$

for $i, j, k \in \Lambda$.

Definition 30 Let $f_\mu \in \mathcal{PN}(U,E)$, $\wedge^t, \wedge^i$ and $\wedge^f$ be the weighed matrices of f_μ^t, f_μ^i and f_μ^f, respectively. Then, for all $u_t \in U$ such that $t \in \Lambda$, scores of u_t is in the weighted matrices $\wedge^t, \wedge^i$ and $\wedge^f$, denoted by $s^t(u_k)$, $s^i(u_k)$ and $s^f(u_k)$, defined by, respectively,

$$s^t(u_t) = \sum_{i,j\in\Lambda} \delta_{ij}^t(u_t)$$

$$s^i(u_t) = \sum_{i,j\in\Lambda} \delta_{ij}^i(u_t)$$

$$s^f(u_t) = \sum_{i,j\in\Lambda} \delta_{ij}^f(u_t)$$

where

$$\delta_{ij}^t(u_t) = \begin{cases} \wedge^t(e_{ij}, u_t), & \wedge^t(e_{ij}, u_t) = max\{\wedge^t(e_{ij}, u_k) : u_k \in U\} \\ 0, & otherwise \end{cases}$$

$$\delta_{ij}^i(u_t) = \begin{cases} \wedge^i(e_{ij}, u_t), & \wedge^i(e_{ij}, u_t) = max\{\wedge^i(e_{ij}, u_k) : u_k \in U\} \\ 0, & otherwise \end{cases}$$

$$\delta_{ij}^f(u_t) = \begin{cases} \wedge^f(e_{ij}, u_t), & \wedge^f(e_{ij}, u_t) = max\{\wedge^f(e_{ij}, u_k) : u_k \in U\} \\ 0, & otherwise \end{cases}$$

Definition 31 Let $s^t(u_t), s^i(u_t)$ and $s^f(u_t)$ be scores of $u_t \in U$ in the weighted matrices $\wedge^t, \wedge^i$ and $\wedge^f$. Then, decision score of $u_t \in U$, denoted by $ds(u_t)$, is defined by

$$ds(u_t) = s^t(u_t) - s^i(u_t) - s^f(u_t)$$

Now, we construct a PNS-decision making method by the following algorithm;

Algorithm:

Step 1: Input the possibility neutrosophic soft set f_μ,

Step 2: Construct the matrix $\wedge$-product

Step 3: Construct the matrices f_μ^t, f_μ^i and f_μ^f

Step 4: Construct the weighted matrices $\wedge^t, \wedge^i$ and $\wedge^f$,

Step 5: Compute score of $u_t \in U$, for each of the weighted matrices,

Step 6: Compute decision score, for all $u_t \in U$,

Step 7: The optimal decision is to select $u_t = maxds(u_i)$.

Example 5 Assume that $U = \{u_1, u_2, u_3\}$ is a set of houses and $E = \{e_1, e_2, e_3\} = \{cheap, large, moderate\}$ is a set of parameters which is attractiveness of houses. Suppose that Mr.X want to buy a most suitable house according to to himself depending on three of the parameters only.

Step 1:Based on the choice parameters of $Mr.X$, let there be two observations f_μ and g_ν by two experts defined as follows:

$$f_\mu = \left\{ \begin{array}{l} f_\mu(e_1) = \left\{ \left(\frac{u_1}{(0.5,0.3,0.7)}, 0.6 \right), \left(\frac{u_2}{(0.6,0.2,0.5)}, 0.2 \right), \left(\frac{u_3}{(0.7,0.6,0.5)}, 0.4 \right) \right\} \\ f_\mu(e_2) = \left\{ \left(\frac{u_1}{(0.35,0.2,0.6)}, 0.4 \right), \left(\frac{u_2}{(0.7,0.8,0.3)}, 0.5 \right), \left(\frac{u_3}{(0.2,0.4,0.4)}, 0.6 \right) \right\} \\ f_\mu(e_3) = \left\{ \left(\frac{u_1}{(0.7,0.2,0.5)}, 0.5 \right), \left(\frac{u_2}{(0.4,0.5,0.2)}, 0.3 \right), \left(\frac{u_3}{(0.5,0.3,0.6)}, 0.2 \right) \right\} \end{array} \right\}$$

$$g_\nu = \left\{ \begin{array}{l} g_\nu(e_1) = \left\{ \left(\frac{u_1}{(0.3,0.4,0.5)}, 0.2 \right), \left(\frac{u_2}{(0.7,0.3,0.4)}, 0.5 \right), \left(\frac{u_3}{(0.4,0.5,0.2)}, 0.3 \right) \right\} \\ g_\nu(e_2) = \left\{ \left(\frac{u_1}{(0.4,0.6,0.2)}, 0.3 \right), \left(\frac{u_2}{(0.2,0.5,0.3)}, 0.7 \right), \left(\frac{u_3}{(0.4,0.6,0.2)}, 0.8 \right) \right\} \\ g_\nu(e_3) = \left\{ \left(\frac{u_1}{(0.2,0.1,0.6)}, 0.7 \right), \left(\frac{u_2}{(0.8,0.4,0.5)}, 0.4 \right), \left(\frac{u_3}{(0.6,0.4,0.3)}, 0.4 \right) \right\} \end{array} \right\}$$

Step 2: Let us consider possibility neutrosophic soft set $\wedge$-product which is the mapping $\wedge : E \times E \to \mathcal{N}(\mathcal{U}) \times I^U$ given as follows:

$$\left(\begin{array}{c|ccc} \wedge & u_1, \mu & u_2, \mu & u_3, \mu \\ \hline e_{11} & (\langle 0.3, 0.4, 0.7 \rangle, 0.2) & (\langle 0.6, 0.3, 0.5 \rangle, 0.2) & (\langle 0.4, 0.6, 0.5 \rangle, 0.3) \\ e_{12} & (\langle 0.4, 0.6, 0.7 \rangle, 0.3) & (\langle 0.2, 0.5, 0.5 \rangle, 0.2) & (\langle 0.4, 0.6, 0.5 \rangle, 0.4) \\ e_{13} & (\langle 0.2, 0.3, 0.7 \rangle, 0.6) & (\langle 0.6, 0.4, 0.5 \rangle, 0.2) & (\langle 0.6, 0.6, 0.5 \rangle, 0.4) \\ e_{21} & (\langle 0.3, 0.4, 0.6 \rangle, 0.2) & (\langle 0.7, 0.8, 0.4 \rangle, 0.5) & (\langle 0.2, 0.5, 0.5 \rangle, 0.3) \\ e_{22} & (\langle 0.35, 0.6, 0.6 \rangle, 0.3) & (\langle 0.2, 0.8, 0.3 \rangle, 0.5) & (\langle 0.2, 0.6, 0.5 \rangle, 0.6) \\ e_{23} & (\langle 0.2, 0.2, 0.6 \rangle, 0.4) & (\langle 0.7, 0.8, 0.5 \rangle, 0.4) & (\langle 0.2, 0.4, 0.5 \rangle, 0.4) \\ e_{31} & (\langle 0.3, 0.4, 0.5 \rangle, 0.2) & (\langle 0.4, 0.5, 0.4 \rangle, 0.3) & (\langle 0.4, 0.5, 0.6 \rangle, 0.2) \\ e_{32} & (\langle 0.4, 0.6, 0.5 \rangle, 0.3) & (\langle 0.2, 0.5, 0.3 \rangle, 0.3) & (\langle 0.4, 0.6, 0.6 \rangle, 0.2) \\ e_{33} & (\langle 0.2, 0.2, 0.6 \rangle, 0.5) & (\langle 0.4, 0.5, 0.5 \rangle, 0.3) & (\langle 0.5, 0.4, 0.6 \rangle, 0.2) \end{array} \right)$$

Matrix representation of $\wedge$-product

Step 3: We construct matrices f^t_μ, f^i_μ and f^f_μ as follows:

$$\left(\begin{array}{c|ccc} \wedge & u_1, \mu & u_2, \mu & u_3, \mu \\ \hline e_{11} & (0.3, 0.2) & (0.6, 0.2) & (0.4, 0.3) \\ e_{12} & (0.4, 0.3) & (0.2, 0.2) & (0.4, 0.4) \\ e_{13} & (0.2, 0.6) & (0.6, 0.2) & (0.6, 0.4) \\ e_{21} & (0.3, 0.2) & (0.7, 0.5) & (0.2, 0.3) \\ e_{22} & (0.35, 0.3) & (0.2, 0.5) & (0.2, 0.6) \\ e_{23} & (0.2, 0.4) & (0.7, 0.4) & (0.2, 0.4) \\ e_{31} & (0.3, 0.2) & (0.4, 0.3) & (0.4, 0.2) \\ e_{32} & (0.4, 0.3) & (0.2, 0.3) & (0.4, 0.2) \\ e_{33} & (0.2, 0.5) & (0.4, 0.3) & (0.5, 0.2) \end{array} \right)$$

Matrix f^t_μ of $\wedge$-product

$$\left(\begin{array}{c|ccc}
\wedge & u_1,\mu & u_2,\mu & u_3,\mu \\
\hline
e_{11} & (0.4,0.2) & (0.3,0.2) & (0.6,0.3) \\
e_{12} & (0.6,0.3) & (0.5,0.2) & (0.6,0.4) \\
e_{13} & (0.3,0.6) & (0.4,0.2) & (0.6,0.4) \\
e_{21} & (0.4,0.2) & (0.8,0.5) & (0.5,0.3) \\
e_{22} & (0.6,0.3) & (0.8,0.5) & (0.6,0.6) \\
e_{23} & (0.2,0.4) & (0.8,0.4) & (0.4,0.4) \\
e_{31} & (0.4,0.2) & (0.5,0.3) & (0.5,0.2) \\
e_{32} & (0.6,0.3) & (0.5,0.3) & (0.6,0.2) \\
e_{33} & (0.2,0.5) & (0.5,0.3) & (0.4,0.2)
\end{array}\right)$$

Matrix f^i_μ of $\wedge$-product

$$\left(\begin{array}{c|ccc}
\wedge & u_1,\mu & u_2,\mu & u_3,\mu \\
\hline
e_{11} & (0.7,0.2) & (0.5,0.2) & (0.5,0.3) \\
e_{12} & (0.7,0.3) & (0.5,0.2) & (0.5,0.4) \\
e_{13} & (0.7,0.6) & (0.5,0.2) & (0.5,0.4) \\
e_{21} & (0.6,0.2) & (0.4,0.5) & (0.5,0.3) \\
e_{22} & (0.6,0.3) & (0.3,0.5) & (0.5,0.6) \\
e_{23} & (0.6,0.4) & (0.5,0.4) & (0.5,0.4) \\
e_{31} & (0.5,0.2) & (0.4,0.3) & (0.6,0.2) \\
e_{32} & (0.5,0.3) & (0.3,0.3) & (0.6,0.2) \\
e_{33} & (0.6,0.5) & (0.5,0.3) & (0.6,0.2)
\end{array}\right)$$

Matrix f^f_μ of $\wedge$-product

Step 4: We obtain weighted matrices $\wedge^t, \wedge^i$ and $\wedge^f$ using Definition 29 as follows:

$$\left(\begin{array}{c|ccc}
\wedge^t & u_1 & u_2 & u_3 \\
\hline
e_{11} & 0.44 & \underline{0.64} & 0.58 \\
e_{12} & 0.58 & 0.36 & \underline{0.64} \\
e_{13} & 0.68 & 0.68 & \underline{0.76} \\
e_{21} & 0.44 & \underline{0.85} & 0.44 \\
e_{22} & 0.55 & 0.60 & \underline{0.68} \\
e_{23} & 0.52 & \underline{0.82} & 0.48 \\
e_{31} & 0.44 & \underline{0.58} & 0.52 \\
e_{32} & \underline{0.58} & 0.44 & 0.52 \\
e_{33} & \underline{0.60} & 0.58 & \underline{0.60}
\end{array}\right),
\left(\begin{array}{c|ccc}
\wedge^i & u_1 & u_2 & u_3 \\
\hline
e_{11} & 0.08 & \underline{0.16} & 0.18 \\
e_{12} & 0.18 & 0.10 & \underline{0.24} \\
e_{13} & 0.18 & 0.08 & \underline{0.24} \\
e_{21} & 0.08 & \underline{0.40} & 0.15 \\
e_{22} & 0.18 & \underline{0.40} & 0.36 \\
e_{23} & 0.08 & \underline{0.32} & 0.16 \\
e_{31} & 0.08 & \underline{0.15} & 0.10 \\
e_{32} & \underline{0.18} & 0.15 & 0.12 \\
e_{33} & 0.10 & \underline{0.15} & 0.08
\end{array}\right),
\left(\begin{array}{c|ccc}
\wedge^f & u_1 & u_2 & u_3 \\
\hline
e_{11} & 0.14 & 0.10 & \underline{0.15} \\
e_{12} & \underline{0.21} & 0.10 & 0.20 \\
e_{13} & \underline{0.42} & 0.10 & 0.20 \\
e_{21} & 0.12 & \underline{0.20} & 0.15 \\
e_{22} & 0.18 & 0.15 & \underline{0.30} \\
e_{23} & \underline{0.24} & 0.20 & \underline{0.20} \\
e_{31} & 0.10 & \underline{0.12} & \underline{0.12} \\
e_{32} & \underline{0.15} & 0.09 & 0.12 \\
e_{33} & \underline{0.30} & 0.15 & 0.12
\end{array}\right)$$

Weighed matrices of f^t_μ, f^i_μ and f^f_μ from left to right, respectively.

Step 5: For all $u \in U$, we find scores using Definition 30 as follow:

$$s^t(u_1) = 1,18, \quad s^t(u_2) = 2,89, \quad s^t(u_3) = 2,68$$

$$s^i(u_1) = 0,18 \quad s^i(u_2) = 1,42 \quad s^i(u_3) = 0,66$$

$$s^f(u_1) = 1,32 \quad s^f(u_2) = 0,32 \quad s^f(u_3) = 0,57$$

Step 5: For all $u \in U$, we find scores using Definition 30 as follows:

$$ds(u_1) = 1,18 - 0,18 - 1,32 = -0,32$$
$$ds(u_2) = 2,89 - 1,42 - 0,32 = 0,90$$
$$ds(u_3) = 2,68 - 0,66 - 0,57 = 1,45$$

Step 5: Then the optimal selection for $Mr.X$ is u_3.

5 Conclusion

In this paper, we introduced the concept of possibility neutrosophic soft set and studied some of the related properties. Also we presented a decision making method based on possibility neutrosophic soft set and gave an application of this method to solve a decision making problem. The method should be more applicable in the future to solve the related problems.

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