Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multicriteria Decision Making

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Abstract

Neutrosophic set is a powerful general formal framework, which generalizes the concept of the classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, and interval-valued intuitionistic fuzzy set from philosophical point of view. However, it will be difficult to apply in real science and engineering areas, while a simplified neutrosophic set (SNS) is a subclass of a neutrosophic set and includes the concepts of a single valued neutrosophic set (SVNS) and an interval neutrosophic set (INS), which can be used in real science and engineering applications with the incomplete, indeterminate and inconsistent information which exists commonly in real situations. Therefore, the main purposes of the paper are to present three vector similarity measures between SNSs as a generalization of the Jaccard, Dice, and cosine similarity measures in vector space and to apply them to the multicriteria decision-making problem with simplified neutrosophic information. Through the similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected as well. Finally, an illustrative example demonstrates the application of the proposed measure methods in the simplified neutrosophic multicriteria decision making.

Keywords: Decision making, neutrosophic set, simplified neutrosophic set, vector similarity measure.

1. Introduction

Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra [1]. Then, neutrosophic set is a powerful general formal framework, which generalizes the concepts of the classic set, fuzzy set [2], interval valued fuzzy set [3], intuitionistic fuzzy set [4], interval-valued intuitionistic fuzzy set [5], paraconsistent set, dialetheist set, paradoxical set, and tautological set [1]. In a neutrosophic set, its indeterminacy is quantified explicitly and its truth-membership, indeterminacy- membership, and falsity-membership are represented independently. This assumption is extremely important in many applications such as information fusion in which the data are combined from different sensors. Primarily, neutrosophic sets had mainly been applied to image processing [6, 7].

Intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets can only handle incomplete information but not the indeterminate and inconsistent information which exists commonly in real situations. For example, when we ask the opinion of an expert about certain statement, he or she may say that the possibility in which the statement is true is between 0.5 and 0.7, the statement is false is between 0.2 and 0.4, and the statement is unsure is between 0.1 and 0.3. For neutrosophic notation, it can be expressed as $x([0.5, 0.7], [0.1, 0.3], [0.2, 0.4])$. That is beyond the scope of the interval-valued intuitionistic fuzzy set. As another example, suppose there are 10 voters to vote for an alternative. During a voting process, four vote "pro", three vote "con", and three are undecided. For neutrosophic notation, it can be expressed as $x(0.4, 0.3, 0.3)$. That is beyond the scope of the intuitionistic fuzzy set. Therefore, the notion of a neutrosophic set is more general, and then it can overcome the aforementioned issues on the intuitionistic fuzzy set and the interval-valued intuitionistic fuzzy set.

The neutrosophic set generalizes the aforementioned sets from philosophical point of view. Thus, it will be difficult to apply in real science and engineering fields. Therefore, Wang et al. [8, 9] proposed interval neutrosophic sets (INSs) and single valued neutrosophic sets (SVNSs) and provided the set-theoretic operators and various properties of INSs and SVNSs. Then, SVNS and INS are the subclasses of a neutrosophic set and can represent the uncertain, imprecise, incomplete and inconsistent information which exists in real world. Recently, Ye [10] presented the information energy of SVNS, correlation of SVNSs, correlation coefficient of SVNSs and proved that the cosine similarity measure of SVNSs is a special case of the correlation coefficient of SVNSs, and then applied the correlation coefficient to

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single valued neutrosophic decision-making problems. Ye [11] further proposed another form of correlation coefficient between SVNSs for single valued neutrosophic multiple attribute decision-making problems. Moreover, Ye [12] developed the cross- entropy measure between SVNSs and applied it to single valued neutrosophic multiple attribute decision-making problems. In interval neutrosophic setting, Ye [13] defined the Hamming and Euclidean distances between INSs and developed the similarity measures based on the distances of INSs, and then applied the similarity measures to interval neutrosophic multicriteria decision-making problems. Furthermore, Ye [14] introduced the concept of a simplified neutrosophic set (SNS), which is a subclass of a neutrosophic set and includes the concepts of INS and SVNS, and defined some operational laws of SNSs, and then he proposed simplified neutrosophic weighted averaging (SNWA) operator and simplified neutrosophic weighted geometric (SNWG) operator and applied them to multicriteria decision-making problems under the simplified neutrosophic environment. On the other hand, Majumdar and Samanta [15] introduced several similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Broumi and Smarandache [16] defined the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets. As we know, the vector similarity measure is one of important tools for decision making, pattern recognition, and medical diagnosis [17-19]. Therefore, the main purposes of this paper are to present three vector similarity measures for SNSs based on the extension of the Jaccard, Dice, and cosine similarity measures between vectors [17-19] and to apply them to multicriteria decision-making problems with simplified neutrosophic information. Through the similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected as well. Then, an illustrative example demonstrates the application of the proposed similarity measure methods in the decision-making problem with the simplified neutrosophic information.

The rest of paper is structured as follows. Section 2 introduces some concepts of neutrosophic sets and SNSs. Section 3 proposes three vector similarity measures based on the extension of the Jaccard, Dice, and cosine similarity measures in vector space and indicates their properties. In Section 4, we apply the three vector similarity measures to multicriteria decision-making problems under simplified neutrosophic environment. In Section 5, an illustrative example demonstrates the application and effectiveness of the proposed similarity measure methods. Finally, some final remarks and future research are provided in Section 6.

2. Some Concepts of Neutrosophic Sets and SNSs

A. *Some concepts of neutrosophic sets*

To deal with indeterminate and inconsistent information, Smarandache [1] originally proposed a neutrosophic set from philosophical point of view and gave some definitions of neutrosophic sets.

Definition 1 [1]: Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$. That is $T_A(x)$: $X \to]0^-, 1^+[$, $I_A(x)$: $X \to]0^-, 1^+[$, and $F_A(x)$: $X \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Definition 2 [1]: The complement of a neutrosophic set A is denoted by A^c and is defined as $T_A^c(x) = \{1^+\} \ominus T_A(x)$, $I_A^c(x) = \{1^+\} \ominus I_A(x)$, and $F_A^c(x) = \{1^+\} \ominus F_A(x)$ for every x in X.

Definition 3 [1]: A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$, if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$, and $\sup F_A(x) \ge \sup F_B(x)$ for every x in X.

B. *Some Concepts of SNSs*

To apply a neutrosophic set to science and engineering areas, Ye [14] introduced a SNS, which is a subclass of the neutrosophic set, and gave the following definition of SNS.

Definition 4 [14]: Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. If the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x)$: $X \to [0, 1]$, $I_A(x)$: $X \to [0, 1]$, and $F_A(x)$: $X \to [0, 1]$. Then, a simplification of the neutrosophic set A is denoted by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X\}$$

which is called a SNS. It is a subclass of the neutrosophic set and includes the concepts of INS and SVNS.

On the one hand, if we only use the SNS A whose $T_A(x)$, $I_A(x)$ and $F_A(x)$ values are single points in the real standard [0, 1] instead of subintervals/subsets in the real standard [0, 1], the SNS A can be described by three real numbers in the real unit interval [0, 1]. Therefore, the sum of $T_A(x) \in [0, 1]$, $I_A(x) \in [0, 1]$ and $F_A(x) \in [0, 1]$

satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. In this case, the SNS A is reduced to the SVNS A. From this concept, we can give the following definitions [14].

Definition 5: A SNS A is contained in the other SNS B, written as $A \subseteq B$, if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, and $F_A(x) \ge F_B(x)$ for every x in X.

Definition 6: The complement of a SNS A is denoted by A^c and is defined as $T_A{}^c(x) = F_A(x)$, $I_A{}^c(x) = 1 - I_A(x)$, $F_A{}^c(x) = T_A(x)$ for any x in X.

Definition 7: Two SNSs A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

On the other hand, if we only consider three membership degrees in a SNS A as the subunit interval of the real unit interval [0, 1], the SNS can be described by three interval numbers in the real unit interval [0, 1]. For each point x in X, we have that $T_A(x) = [\inf T_A(x), \sup T_A(x)]$, $I_A(x) = [\inf I_A(x), \sup I_A(x)]$, $F_A(x) = [\inf F_A(x), \sup F_A(x)] \subseteq [0, 1]$, and $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$ for any $x \in X$. In this case, the SNS A is reduced to the INS A. From this concept, we can give the following definitions [14].

Definition 8: The complement of an SNS A is denoted by A^c and is defined as $T_A{}^c(x) = F_A(x) = [\inf F_A(x), \sup F_A(x)]$, $I_A{}^c(x) = [1 - \sup I_A(x), 1 - \inf I_A(x)]$, $F_A{}^c(x) = T_A(x) = [\inf T_A(x), \sup T_A(x)]$ for any x in X.

Definition 9: A SNS A is contained in the other SNS B, written as $A \subseteq B$, if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$, and $\sup F_A(x) \ge \sup F_B(x)$ for any x in X.

Definition 10: Two SNSs A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

3. Vectors Similarity Measures between SNSs

The vector similarity measure is one of important tools for the degree of similarity between objects. However, the Jaccard, Dice, and cosine similarity measures are often used for this purpose. In the following, the Jaccard, Dice, and cosine similarity measures between two vectors are introduced from [17-19].

Let $\boldsymbol{X} = (x_1, x_2,..., x_n)$ and $\boldsymbol{Y} = (y_1, y_2,..., y_n)$ be the two vectors of length n where all the coordinates are positive. The Jaccard index of these two vectors (measuring the "similarity" of these vectors) [19] is defined as

$$J(\boldsymbol{X},\boldsymbol{Y}) = \frac{\boldsymbol{X}\cdot\boldsymbol{Y}}{\|\boldsymbol{X}\|_2^2 + \|\boldsymbol{Y}\|_2^2 - \boldsymbol{X}\cdot\boldsymbol{Y}} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2 - \sum_{i=1}^{n} x_i y_i} \quad (1)$$

where $\boldsymbol{X}\cdot\boldsymbol{Y} = \sum_{i=1}^{n} x_i y_i$ is the inner product of the vectors $\boldsymbol{X}$ and $\boldsymbol{Y}$, $\|\boldsymbol{X}\|_2 = \sqrt{\sum_{i=1}^{n} x^2}$ and $\|\boldsymbol{Y}\|_2 = \sqrt{\sum_{i=1}^{n} y^2}$ are the Euclidean norms of $\boldsymbol{X}$ and $\boldsymbol{Y}$ (also called the L_2 norms).

Then the Dice similarity measure [18] is defined as follows:

$$D(\boldsymbol{X},\boldsymbol{Y}) = \frac{2\boldsymbol{X}\cdot\boldsymbol{Y}}{\|\boldsymbol{X}\|_2^2 + \|\boldsymbol{Y}\|_2^2} = \frac{2\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2} \quad (2)$$

Cosine formula is then defined as the inner product of these two vectors divided by the product of their lengths. This is nothing but the cosine of the angle between the vectors. The cosine similarity measure [17] is defined as

$$C(\boldsymbol{X},\boldsymbol{Y}) = \frac{\boldsymbol{X}\cdot\boldsymbol{Y}}{\|\boldsymbol{X}\|_2 \|\boldsymbol{Y}\|_2} = \frac{\sum_{i=1}^{n} x_i y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}} \quad (3)$$

These three formulae are similar in the sense. Then, the cosine measure is undefined if x_i and/or y_i ($i = 1, 2, \ldots, n$) are equal to zero, while the Jaccard and Dice measures are undefined if x_i and y_i ($i = 1, 2, \ldots, n$) are all equal to zero. For these cases, only let the similarity measures be equal to zero if undefined.

It is obvious that the Jaccard, Dice, and cosine similarity measures satisfy the following properties [17-19]:

(P1) $0 \le J(\boldsymbol{X}, \boldsymbol{Y}), D(\boldsymbol{X}, \boldsymbol{Y}), C(\boldsymbol{X}, \boldsymbol{Y}) \le 1$;

(P2) $J(\boldsymbol{X}, \boldsymbol{Y}) = J(\boldsymbol{Y}, \boldsymbol{X})$, $D(\boldsymbol{X}, \boldsymbol{Y}) = D(\boldsymbol{Y}, \boldsymbol{X})$, and $C(\boldsymbol{X}, \boldsymbol{Y}) = C(\boldsymbol{Y}, \boldsymbol{X})$;

(P3) $J(\boldsymbol{X}, \boldsymbol{Y}) = 1$, $D(\boldsymbol{X}, \boldsymbol{Y}) = 1$, and $C(\boldsymbol{X}, \boldsymbol{Y}) = 1$ if $\boldsymbol{X} = \boldsymbol{Y}$, i.e., $x_i = y_i$ ($i = 1, 2,\ldots, n$) for every $x_i \in \boldsymbol{X}$ and $y_i \in \boldsymbol{Y}$.

Assume that there are two SNSs $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle | x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle | x_i \in X\}$. If we only use INSs in the SNSs A and B, then the six elements in three functions $T_A(x_i) = [\inf T_A(x_i), \sup T_A(x_i)]$, $I_A(x_i) = [\inf I_A(x_i), \sup I_A(x_i)]$, $F_A(x_i) = [\inf F_A(x_i), \sup F_A(x_i)] \subseteq [0, 1]$ for any $x_i \in X$ in A or $T_B(x_i) = [\inf T_B(x_i), \sup T_B(x_i)]$, $I_A(x_i) = [\inf I_B(x_i), \sup I_B(x_i)]$, $F_B(x_i) = [\inf F_B(x_i), \sup F_B(x_i)] \subseteq [0, 1]$ for any $x_i \in X$ in B can be considered as a vector representation (i.e. 6-D vector space). Based on the extension of the above three vector similarity measures, the three similarity measures between SNSs A and B are proposed in the 6-D vector space as follows:

$$S_J(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\left(\begin{array}{l} \inf T_A(x_i)\inf T_B(x_i) + \sup T_A(x_i)\sup T_B(x_i) \\ + \inf I_A(x_i)\inf I_B(x_i) + \sup I_A(x_i)\sup I_B(x_i) \\ + \inf F_A(x_i)\inf F_B(x_i) + \sup F_A(x_i)\sup F_B(x_i) \end{array}\right)}{\left(\begin{array}{l} \left[\inf T_A(x_i)\right]^2 + (\left[\inf I_A(x_i)\right]^2 + \left[\inf F_A(x_i)\right]^2 \\ + \left[\sup T_A(x_i)\right]^2 + \left[\sup I_A(x_i)\right]^2 + \left[\sup F_A(x_i)\right]^2 \\ + \left[\inf T_B(x_i)\right]^2 + (\left[\inf I_B(x_i)\right]^2 + \left[\inf F_B(x_i)\right]^2 \\ + \left[\sup T_B(x_i)\right]^2 + \left[\sup I_B(x_i)\right]^2 + \left[\sup F_B(x_i)\right]^2 \\ - \left[\inf T_A(x_i)\inf T_B(x_i) + \inf I_A(x_i)\inf I_B(x_i) \right. \\ \left. + \inf F_A(x_i)\inf F_B(x_i)\right] - \left[\sup T_A(x_i)\sup T_B(x_i) \right. \\ \left. + \sup I_A(x_i)\sup I_B(x_i) + \sup F_A(x_i)\sup F_B(x_i)\right] \end{array}\right)} \quad (4)$$

$$S_D(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{2\left(\begin{array}{l}\inf T_A(x_i)\inf T_B(x_i)+\inf I_A(x_i)\inf I_B(x_i)\\+\inf F_A(x_i)\inf F_B(x_i)+\sup T_A(x_i)\sup T_B(x_i)\\+\sup I_A(x_i)\sup I_B(x_i)+\sup F_A(x_i)\sup F_B(x_i)\end{array}\right)}{\left(\begin{array}{l}[\inf T_A(x_i)]^2+[\inf I_A(x_i)]^2+[\inf F_A(x_i)]^2\\+[\sup T_A(x_i)]^2+[\sup I_A(x_i)]^2+[\sup F_A(x_i)]^2\\+[\inf T_B(x_i)]^2+[\inf I_B(x_i)]^2+[\inf F_B(x_i)]^2\\+[\sup T_B(x_i)]^2+[\sup I_B(x_i)]^2+[\sup F_B(x_i)]^2\end{array}\right)} \quad (5)$$

$$S_C(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{\left(\begin{array}{l}\inf T_A(x_i)\inf T_B(x_i)+\inf I_A(x_i)\inf I_B(x_i)\\+\inf F_A(x_i)\inf F_B(x_i)+\sup T_A(x_i)\sup T_B(x_i)\\+\sup I_A(x_i)\sup I_B(x_i)+\sup F_A(x_i)\sup F_B(x_i)\end{array}\right)}{\left(\begin{array}{l}\sqrt{\begin{array}{l}[\inf T_A(x_i)]^2+[\inf I_A(x_i)]^2+[\inf F_A(x_i)]^2\\+[\sup T_A(x_i)]^2+[\sup I_A(x_i)]^2+[\sup F_A(x_i)]^2\end{array}}\\ \sqrt{\begin{array}{l}[\inf T_B(x_i)]^2+[\inf I_B(x_i)]^2+[\inf F_B(x_i)]^2\\+[\sup T_B(x_i)]^2+[\sup I_B(x_i)]^2+[\sup F_B(x_i)]^2\end{array}}\end{array}\right)} \quad (6)$$

According to the properties of the Jaccard, Dice, and cosine similarity measures [17-19], each similarity measure $S_k(A, B)$ (k = J, D, C) also satisfies the following properties:

(P1) $0 \le S_k(A, B) \le 1$;

(P2) $S_k(A, B) = S_k(B, A)$;

(P3) $S_k(A, B) = 1$ if $A = B$, i.e., $T_A(x_i), = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$ for every $x_i \in X$.

*Proof***:**

(P1) It is obvious that the property is true according to the inequality $a^2+b^2 \ge 2ab$ for Eqs. (4) and (5), and the cosine value for Eq. (6).

(P2) It is obvious that the property is true.

(P3) When $A = B$, there are $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$ and $F_A(x_i) = F_B(x_i)$, i.e. $\inf T_A(x_i) = \inf T_B(x_i)$, $\sup T_A(x_i) = \sup T_B(x_i)$, $\inf I_A(x_i) = \inf I_B(x_i)$, $\sup I_A(x_i) = \sup I_B(x_i)$, $\inf F_A(x_i) = \inf F_B(x_i)$, and $\sup F_A(x_i) = \sup F_B(x_i)$ for $i = 1, 2, \ldots, n$. So there are $S_J(A, B) = 1$, $S_D(A, B) = 1$, and $S_C(A, B) = 1$. □

However, $S_k(A, B)$ is undefined if $T_A(x_i), = I_A(x_i) = F_A(x_i) = 0$ and/or $T_B(x_i), = I_B(x_i) = F_B(x_i) = 0$ for every $x_i \in X$. In this case, let the measure value $S_k(A, B) = 0$ for k = J, D, C.

Furthermore, the differences of importance are considered in the elements in the universe. Thus, we need to take the weight of each element x_i ($i = 1, 2,\ldots, n$) into account. In the following, we develop weighted similarity measures between SNSs.

Let w_i be the weight for each element x_i ($i = 1, 2,\ldots, n$), $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$, then we have the following three weighted similarity measures:

$$WS_J(A,B)=\sum_{i=1}^{n} w_i\frac{\left(\begin{array}{l}\inf T_A(x_i)\inf T_B(x_i)+\sup T_A(x_i)\sup T_B(x_i)\\+\inf I_A(x_i)\inf I_B(x_i)+\sup I_A(x_i)\sup I_B(x_i)\\+\inf F_A(x_i)\inf F_B(x_i)+\sup F_A(x_i)\sup F_B(x_i)\end{array}\right)}{\left(\begin{array}{l}[\inf T_A(x_i)]^2+([\inf I_A(x_i)]^2+[\inf F_A(x_i)]^2\\+[\sup T_A(x_i)]^2+[\sup I_A(x_i)]^2+[\sup F_A(x_i)]^2\\+[\inf T_B(x_i)]^2+([\inf I_B(x_i)]^2+[\inf F_B(x_i)]^2\\+[\sup T_B(x_i)]^2+[\sup I_B(x_i)]^2+[\sup F_B(x_i)]^2\\-[\inf T_A(x_i)\inf T_B(x_i)+\inf I_A(x_i)\inf I_B(x_i)\\+\inf F_A(x_i)\inf F_B(x_i)]-[\sup T_A(x_i)\sup T_B(x_i)\\+\sup I_A(x_i)\sup I_B(x_i)+\sup F_A(x_i)\sup F_B(x_i)]\end{array}\right)}, \quad (7)$$

$$WS_D(A,B)=\sum_{i=1}^{n} w_i\frac{2\left(\begin{array}{l}\inf T_A(x_i)\inf T_B(x_i)+\inf I_A(x_i)\inf I_B(x_i)\\+\inf F_A(x_i)\inf F_B(x_i)+\sup T_A(x_i)\sup T_B(x_i)\\+\sup I_A(x_i)\sup I_B(x_i)+\sup F_A(x_i)\sup F_B(x_i)\end{array}\right)}{\left(\begin{array}{l}[\inf T_A(x_i)]^2+[\inf I_A(x_i)]^2+[\inf F_A(x_i)]^2\\+[\sup T_A(x_i)]^2+[\sup I_A(x_i)]^2+[\sup F_A(x_i)]^2\\+[\inf T_B(x_i)]^2+[\inf I_B(x_i)]^2+[\inf F_B(x_i)]^2\\+[\sup T_B(x_i)]^2+[\sup I_B(x_i)]^2+[\sup F_B(x_i)]^2\end{array}\right)}, \quad (8)$$

$$WS_C(A,B)=\sum_{i=1}^{n} w_i\frac{\left(\begin{array}{l}\inf T_A(x_i)\inf T_B(x_i)+\inf I_A(x_i)\inf I_B(x_i)\\+\inf F_A(x_i)\inf F_B(x_i)+\sup T_A(x_i)\sup T_B(x_i)\\+\sup I_A(x_i)\sup I_B(x_i)+\sup F_A(x_i)\sup F_B(x_i)\end{array}\right)}{\left(\begin{array}{l}\sqrt{\begin{array}{l}[\inf T_A(x_i)]^2+[\inf I_A(x_i)]^2+[\inf F_A(x_i)]^2\\+[\sup T_A(x_i)]^2+[\sup I_A(x_i)]^2+[\sup F_A(x_i)]^2\end{array}}\\ \sqrt{\begin{array}{l}[\inf T_B(x_i)]^2+[\inf I_B(x_i)]^2+[\inf F_B(x_i)]^2\\+[\sup T_B(x_i)]^2+[\sup I_B(x_i)]^2+[\sup F_B(x_i)]^2\end{array}}\end{array}\right)}. \quad (9)$$

If $\mathbf{w} = (1/n, 1/n,\ldots, 1/n)^T$, then Eqs. (7)-(9) are reduced to Eqs. (4)-(6).

It is obvious that each weighted similarity measure $WS_k(A, B)$ for k = J, D, C also satisfies the following properties:

(P1) $0 \le WS_k(A, B) \le 1$;

(P2) $WS_k(A, B) = WS_k(B, A)$;

(P3) $WS_k(A, B) = 1$ if $A = B$, i.e., $T_A(x_i), = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$ for every $x_i \in X$.

Similar to the previous proof method, we can prove that the properties (P1)-(P3).

Assume that there are two SNSs $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle| x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle| x_i \in X\}$. If we only use SVNSs in the SNSs A and B, then three functions $T_A(x_i), I_A(x_i), F_A(x_i) \in [0, 1]$ for any $x_i \in X$ in A or $T_B(x_i), I_B(x_i), F_B(x_i) \in [0, 1]$ for any $x_i \in X$ in B can be considered as a vector representation with three elements (i.e. 3-D vector space). Then, the three vector similarity measures Eqs. (4)-(6) are reduced, respectively, to the following three similarity measures of SVNSs:

$$S_J(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\left[\begin{array}{l}\left(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)\right)+\left(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)\right)\\-\left(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)\right)\end{array}\right]} \quad (10)$$

$$S_D(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{2\left(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)\right)}{\left(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)\right)+\left(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)\right)} \quad (11)$$

$$S_C(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\sqrt{T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)}\sqrt{T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)}} \quad (12)$$

Therefore, we can see that Eqs. (10)-(12) are special cases of Eqs. (4)-(6) when the upper and lower ends of the interval numbers of $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $T_B(x_i)$, $I_A(x_i)$, and $F_B(x_i)$ are equal.

Similarly, the three weighted similarity measures Eqs. (7)-(9) are also reduced, respectively, to the following three weighted similarity measures of SVNSs:

$$WS_J(A,B)=\sum_{i=1}^{n}w_i\frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\left[\begin{array}{l}\left(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)\right)+\left(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)\right)\\-\left(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)\right)\end{array}\right]} \quad (13)$$

$$WS_D(A,B)=\sum_{i=1}^{n}w_i\frac{2\left(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)\right)}{\left(T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)\right)+\left(T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)\right)} \quad (14)$$

$$WS_C(A,B)=\sum_{i=1}^{n}w_i\frac{T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i)}{\sqrt{T_A^2(x_i)+I_A^2(x_i)+F_A^2(x_i)}\sqrt{T_B^2(x_i)+I_B^2(x_i)+F_B^2(x_i)}} \quad (15)$$

Meanwhile, we can also see that Eqs. (13)-(15) are special cases of Eqs. (7)-(9).

4. Application of the Vector Similarity Measures in Decision-Making

In this section, we apply the three similarity measures between SNSs to the multicriteria decision-making problem with simplified neutrosophic information.

Let $A = \{A_1, A_2,\ldots, A_m\}$ be a set of alternatives and $C = \{C_1, C_2,\ldots, C_n\}$ be a set of criteria. Assume that the weight of the criterion C_j ($j = 1, 2,\ldots, n$), entered by the decision-maker, is w_j, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. In this case, the characteristic of the alternative A_i ($i = 1, 2,\ldots, m$) on criterion C_j ($j = 1, 2,\ldots, n$) is represented by the following form of a SNS:

$$A_i = \{\langle C_j, T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j)\rangle \mid C_j \in C\}$$

Here, we only consider that the three interval pairs $T_{A_i}(C_j) = [\inf T_{A_i}(C_j),\ \sup T_{A_i}(C_j)]$, $I_{A_i}(C_j) = [\inf I_{A_i}(C_j),\ \sup I_{A_i}(C_j)]$, $F_{A_i}(C_j) = [\inf F_{A_i}(C_j),\ \sup F_{A_i}(C_j)] \subseteq [0, 1]$ are given in an INS A_i, where $0 \le \sup T_{A_i}(C_j) + \sup I_{A_i}(C_j) + \sup F_{A_i}(C_j) \le 3$ for $C_j \in C$, $j = 1, 2, \ldots, n$, and $i = 1, 2, \ldots, m$, because an INS A_i is reduced to a SVNS A_i when $T_{A_i}(C_j) = \inf T_{A_i}(C_j) = \sup T_{A_i}(C_j) \in [0, 1]$, $I_{A_i}(C_j) = \inf I_{A_i}(C_j) = \sup I_{A_i}(C_j) \in [0, 1]$, $F_{A_i}(C_j) = \inf F_{A_i}(C_j) = \sup F_{A_i}(C_j) \in [0, 1]$ are held as three real numbers in the real unit interval [0, 1].

For convenience, the interval pairs $T_{A_i}(C_j) = [\inf T_{A_i}(C_j),\ \sup T_{A_i}(C_j)]$, $I_{A_i}(C_j) = [\inf I_{A_i}(C_j),\ \sup I_{A_i}(C_j)]$, $F_{A_i}(C_j) = [\inf F_{A_i}(C_j),\ \sup F_{A_i}(C_j)] \subseteq [0, 1]$ are denoted by a simplified neutrosophic value (SNV) $\alpha_{ij} = \langle [a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}]\rangle$ ($i = 1, 2, \ldots, m$; $j = 1, 2,\ldots, n$), which is usually derived from the evaluation of an alternative A_i with respect to a criterion C_j by the expert or decision maker. Hence, we can elicit a simplified neutrosophic decision matrix $D = (\alpha_{ij})_{m\times n}$.

In multicriteria decision-making environment, the concept of ideal point has been used to help identify the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives [13].

Generally, the evaluation criteria can be categorized into two types: benefit criteria and cost criteria. Let H be a collection of benefit criteria and L be a collection of cost criteria. In the presented decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit criteria and a minimum operator for the cost criteria to determine the best value of each criterion among all alternatives. Therefore, we define an ideal SNV for a benefit criterion in the ideal alternative A^* as

$$\alpha_j^* = \left\langle [a_j^*, b_j^*], [c_j^*, d_j^*], [e_j^*, f_j^*]\right\rangle = \left\langle \left[\max_i(a_{ij}), \max_i(b_{ij})\right], \left[\min_i(c_{ij}), \min_i(d_{ij})\right], \left[\min_i(e_{ij}), \min_i(f_{ij})\right]\right\rangle$$

for $j \in H$,

while for a cost criterion, we define an ideal SNV in the ideal alternative A^* as

$$\alpha_j^* = \left\langle [a_j^*, b_j^*], [c_j^*, d_j^*], [e_j^*, f_j^*]\right\rangle = \left\langle \left[\min_i(a_{ij}), \min_i(b_{ij})\right], \left[\max_i(c_{ij}), \max_i(d_{ij})\right], \left[\max_i(e_{ij}), \max_i(f_{ij})\right]\right\rangle$$

for $j \in L$.

Thus, by applying Eqs. (7), (8) and (9) three vector similarity measures between an alternative A_i and the ideal alternative A^* are rewritten respectively as

$$WS_J(A_i, A^*) = \sum_{j=1}^{n} w_j \frac{a_{ij}a_j^* + b_{ij}b_j^* + c_{ij}c_j^* + d_{ij}d_j^* + e_{ij}e_j^* + f_{ij}f_j^*}{\left(\begin{array}{l} a_{ij}^2 + b_{ij}^2 + c_{ij}^2 + d_{ij}^2 + e_{ij}^2 + f_{ij}^2 + \left(a_j^*\right)^2 \\ + \left(b_j^*\right)^2 + \left(c_j^*\right)^2 + \left(d_j^*\right)^2 + \left(e_j^*\right)^2 + \left(f_j^*\right)^2 \\ - \left(a_{ij}a_j^* + b_{ij}b_j^* + c_{ij}c_j^* + d_{ij}d_j^* + e_{ij}e_j^* + f_{ij}f_j^*\right) \end{array}\right)} \quad (16)$$

$$WS_D(A_i, A^*) = \sum_{j=1}^{n} w_j \frac{2\left(a_{ij}a_j^* + b_{ij}b_j^* + c_{ij}c_j^* + d_{ij}d_j^* + e_{ij}e_j^* + f_{ij}f_j^*\right)}{\left(\begin{array}{l} a_{ij}^2 + b_{ij}^2 + c_{ij}^2 + d_{ij}^2 + e_{ij}^2 + f_{ij}^2 + \left(a_j^*\right)^2 \\ + \left(b_j^*\right)^2 + \left(c_j^*\right)^2 + \left(d_j^*\right)^2 + \left(e_j^*\right)^2 + \left(f_j^*\right)^2 \end{array}\right)} \quad (17)$$

$$WS_C(A_i, A^*) = \sum_{j=1}^{n} w_j \frac{a_{ij}a_j^* + b_{ij}b_j^* + c_{ij}c_j^* + d_{ij}d_j^* + e_{ij}e_j^* + f_{ij}f_j^*}{\left(\begin{array}{l} \sqrt{a_{ij}^2 + b_{ij}^2 + c_{ij}^2 + d_{ij}^2 + e_{ij}^2 + f_{ij}^2} \\ \sqrt{\left(a_j^*\right)^2 + \left(b_j^*\right)^2 + \left(c_j^*\right)^2 + \left(d_j^*\right)^2 + \left(e_j^*\right)^2 + \left(f_j^*\right)^2} \end{array}\right)} \quad (18)$$

Through the similarity measure $WS_k(A_i, A^*)$ (k = J, D, C) between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best one can be easily selected as well.

5. Illustrative Example

In this section, an example for the multicriteria decision-making problem of alternatives is used as the demonstration of the application of the proposed similarity measure methods.

Let us consider the decision-making problem adapted from [13]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investment company must take a decision according to the three criteria: (1) C_1 is the risk; (2) C_2 is the growth; (3) C_3 is the environmental impact, where C_1 and C_2 are benefit criteria, and C_3 is a cost criterion. The weight vector of the criteria is given by $\boldsymbol{w} = (0.35, 0.25, 0.40)^T$. The four possible alternatives are to be evaluated under the above three criteria by the form of SNVs, as shown in the following simplified neutrosophic decision matrix D:

$$D = \left(\alpha_{ij}\right)_{4\times 3} = \begin{bmatrix} \langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle & \langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle \\ \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle & \langle[0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle \\ \langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle & \langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle \\ \langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle & \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle & \langle[0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle \end{bmatrix}$$

Then, we utilize the developed approaches to obtain the most desirable alternative(s).

From the simplified neutrosophic decision matrix D we can obtain the following ideal alternative:

$$A^* = \{\langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle, \langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle, \langle[0.3,0.5],[0.3,0.5],[0.8,0.9]\rangle\}$$

Then by using Eq. (16) or (17) or (18), we can obtain the various similarity measure values of $WS_k(A_i, A^*)$ (i =1, 2, 3, 4; k = J, D, C), as shown in Table 1. For the comparison of different measure methods, the results in [13] are also shown in Table 1.

Table 1. Results of different measure methods.

Measure method	Measure value	Ranking order
$WS_J(A_i, A^*)$	$WS_J(A_1, A^*) = 0.7579$ $WS_J(A_2, A^*) = 0.9773$ $WS_J(A_3, A^*) = 0.8646$ $WS_J(A_4, A^*) = 0.9768$	$A_2 \succ A_4 \succ A_3 \succ A_1$
$WS_D(A_i, A^*)$	$WS_D(A_1, A^*) = 0.8594$ $WS_D(A_2, A^*) = 0.9884$ $WS_D(A_3, A^*) = 0.9224$ $WS_D(A_4, A^*) = 0.9880$	$A_2 \succ A_4 \succ A_3 \succ A_1$
$WS_C(A_i, A^*)$	$WS_C(A_1, A^*) = 0.8676$ $WS_C(A_2, A^*) = 0.9894$ $WS_C(A_3, A^*) = 0.9276$ $WS_C(A_4, A^*) = 0.9896$	$A_4 \succ A_2 \succ A_3 \succ A_1$
$S_1(A_i, A^*)$ [13]	$S_1(A_1, A^*) = 0.7667$ $S_1(A_2, A^*) = 0.9542$ $S_1(A_3, A^*) = 0.8625$ $S_1(A_4, A^*) = 0.9600$	$A_4 \succ A_2 \succ A_3 \succ A_1$
$S_2(A_i, A^*)$ [13]	$S_2(A_1, A^*) = 0.7370$ $S_2(A_2, A^*) = 0.9323$ $S_2(A_3, A^*) = 0.8344$ $S_2(A_4, A^*) = 0.9034$	$A_2 \succ A_4 \succ A_3 \succ A_1$

From the results of Table 1, the ranking order of the four alternatives is $A_4 \succ A_2 \succ A_3 \succ A_1$ or $A_2 \succ A_4 \succ A_3 \succ A_1$. Obviously, amongst them A_4 or A_2 is the best alternative. Then, the two ranking orders of the four alternatives are in agreement with the results of Ye's methods [13]. As we can see from Table 1, for different measure methods, the ranking orders may be different. Therefore, the three measure methods proposed in this paper can be assigned one of them to satisfy the decision maker's preference.

The example clearly indicates that the proposed measure methods are simple and effective in simplified neutrosophic decision-making. Then, we truly need new types of the vector similarity measures of SNSs for dealing with various intelligent decision making problems.

6. Conclusions

This paper has developed three vector similarity measures between SNSs as a generalization of the Jaccard, Dice, and cosine similarity measures between two vectors. Then the three similarity measures have been applied to a multicriteria decision-making problem in simplified neutrosophic setting. Through the similarity degrees between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. Finally, an illustrative example demonstrated the application and effectiveness of the developed similarity measure methods. Therefore, the proposed similarity measures between SNSs are more suitable for decision-making problems with the incomplete, indeterminate, and inconsistent information which exists commonly in real situations. Furthermore, the techniques proposed in this paper extend existing decision-making methods and can provide a convenient decision-making method for decision-makers. In the future, we shall continue to work in the application of the similarity measures between SNSs to other domains, such as pattern recognition and medical diagnosis.

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