International J.Math. Combin. Vol.3 (2011), 107-113

New Mean Graphs

S.K.Vaidya

(Saurashtra University, Rajkot-360005, Gujarat, India)

Lekha Bijukumar

(Shanker Sinh Vaghela Bapu Institute of Technology, Gandhinagar, Gujarat, India)

E-mail: samirkvaidya@yahoo.co.in, dbijuin@yahoo.co.in

Abstract: A vertex labeling of G is an assignment $f : V(G) \to \{1, 2, 3, \ldots, p + q\}$ be an injection. For a vertex labeling f, the induced Smarandachely edge m-labeling f_S^* for an edge $e = uv$, an integer $m \geq 2$ is defined by $f_S^*(e) = \left\lceil \frac{f(u) + f(v)}{m} \right\rceil$. Then f is called a Smarandachely super m-mean labeling if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p+q\}$. Particularly, in the case of $m = 2$, we know that

$$f^*(e) = \begin{cases} \frac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even;} \\ \frac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a mean labeling. A graph that admits a Smarandachely super mean m-labeling is called a Smarandachely super m-mean graph, particularly, a mean graph if $m = 2$. In this paper, some new families of mean graphs are investigated. We prove that the graph obtained by two new operations called mutual duplication of a pair of vertices each from each copy of cycle C_n as well as mutual duplication of a pair of edges each from each copy of cycle C_n admits mean labeling. More over that mean labeling for shadow graphs of star $K_{1,n}$ and bistar $B_{n,n}$ are derived.

Key Words: Smarandachely super m-mean labeling, mean labeling, Smarandachely super m-mean graph, mean graphs; mutual duplication.

AMS(2010): 05C78

§1. Introduction

We begin with simple,finite,connected and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. For all other standard terminology and notations we follow Harary [3]. We will provide brief summary of definitions and other information which serve as prerequisites for the present investigations.

[1]Received February 21, 2011. Accepted September 8, 2011.

Definition 1.1 *Consider two copies of cycle C_n. Then the mutual duplication of a pair of vertices v_k and v'_k respectively from each copy of cycle C_n produces a new graph G such that $N(v_k) = N(v'_k)$.*

Definition 1.2 *Consider two copies of cycle C_n and let $e_k = v_k v_{k+1}$ be an edge in the first copy of C_n with $e_{k-1} = v_{k-1}v_k$ and $e_{k+1} = v_{k+1}v_{k+2}$ be its incident edges. Similarly let $e'_m = u_m u_{m+1}$ be an edge in the second copy of C_n with $e'_{m-1} = u_{m-1}u_m$ and $e'_{m+1} = u_{m+1}u_{m+2}$ be its incident edges. The mutual duplication of a pair of edges e_k, e'_m respectively from two copies of cycle C_n produces a new graph G in such a way that $N(v_k) - v_{k+1} = N(u_m) - u_{m+1}$ $=\{v_{k-1}, u_{m-1}\}$ and $N(v_{k+1}) - v_k = N(u_{m+1}) - u_m$ $=\{v_{k+2}, u_{m+2}\}$.*

Definition 1.3 *The shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''.*

Definition 1.4 *Bistar is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$ by an edge.*

Definition 1.5 *If the vertices are assigned values subject to certain conditions then it is known as graph labeling.*

Graph labeling is one of the fascinating areas of research with wide ranging applications. Enough literature is available in printed and electronic form on different types of graph labeling and more than 1200 research papers have been published so far in past four decades. Labeled graph plays vital role to determine optimal circuit layouts for computers and for the representation of compressed data structure. For detailed survey on graph labeling we refer to *A Dynamic Survey of Graph Labeling* by Gallian [2]. A systematic study on various applications of graph labeling is carried out in Bloom and Golomb [1].

Definition 1.6 *A vertex labeling of G is an assignment $f : V(G) \to \{1, 2, 3, \dots, p+q\}$ be an injection. For a vertex labeling f, the induced Smarandachely edge m-labeling f^*_S for an edge $e = uv$, an integer $m \geq 2$ is defined by $f^*_S(e) = \left\lceil \frac{f(u)+f(v)}{m} \right\rceil$. Then f is called a Smarandachely super m-mean labeling if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p+q\}$. Particularly, in the case of $m = 2$, we know that*

$$f^*(e) = \begin{cases} \dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even;} \\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

Such a labeling is usually called a mean labeling. A graph that admits a Smarandachely super mean m-labeling is called a Smarandachely super m-mean graph, particularly, a mean graph if $m = 2$.

The mean labeling was introduced by Somasundaram and Ponraj [4] and they proved the graphs P_n, C_n, $P_n \times P_m$, $P_m \times C_n$ etc. admit mean labeling. The same authors in [5] have discussed the mean labeling of subdivision of $K_{1,n}$ for $n < 4$ while in [6] they proved that the wheel W_n does not admit mean labeling for $n > 3$. Mean labeling in the context of some graph operations is discussed by Vaidya and Lekha[7] while in [8] the same authors have investigated some new families of mean graphs. In the present work four new results corresponding to mean labeling are investigated.

§2. Main Results

Theorem 2.1 *The graph obtained by the mutual duplication of a pair of vertices in cycle C_n admits mean labeling.*

Proof Let v_1,v_2,…, v_n be the vertices of the first copy of cycle C_n and let u_1,u_2,…, u_n be the vertices of the second copy of cycle C_n. Let G be the graph obtained by the mutual duplication of a pair of vertices each respectively from each copy of cycle C_n. To define $f : V(G) \rightarrow \{0, 1, 2, \ldots, q\}$ two cases are to be considered.

Case 1. n is odd.

Without loss of generality assume that the vertex $v_{\frac{n+3}{2}}$ from the first copy of cycle C_n and the vertex u_1 from the second copy of cycle C_n are mutually duplicated.

$$f(v_i) = 2i - 2 \text{ for } 1 \leq i \leq \frac{n+1}{2},;$$
$$f(v_i) = 2(n - i) + 3 \text{ for } \frac{n+3}{2} \leq i \leq n;$$
$$f(u_1) = n + 4;$$
$$f(u_i) = n + 2i + 3 \text{ for } 2 \leq i \leq \frac{n+1}{2};$$
$$f(u_i) = 3n - 2i + 6 \text{ for } \frac{n+3}{2} \leq i \leq n.$$

Case 2: n is even.

Without loss of generality assume that the vertex $v_{\frac{n+2}{2}}$ from the first copy of cycle C_n and the vertex u_1 from the second copy of cycle C_n are mutually duplicated.

$$f(v_i) = 2i - 2 \text{ for } 1 \leq i \leq \frac{n+2}{2};$$
$$f(v_i) = 2(n - i) + 3 \text{ for } \frac{n+4}{2} \leq i \leq n;$$
$$f(u_1) = n + 4;$$
$$f(u_i) = n + 2i + 3 \text{ for } 2 \leq i \leq \frac{n}{2};$$
$$f(u_i) = 3n - 2i + 6 \text{ for } \frac{n+2}{2} \leq i \leq n.$$

In view of the above defined labeling pattern f is a mean labeling for the graph obtained by the mutual duplication of a pair of vertices in cycle C_n. □

Illustration 2.2 The following Fig.1 shows the pattern of mean labeling of the graph obtained by the mutual duplication of a pair of vertices of cycle C_{10}.

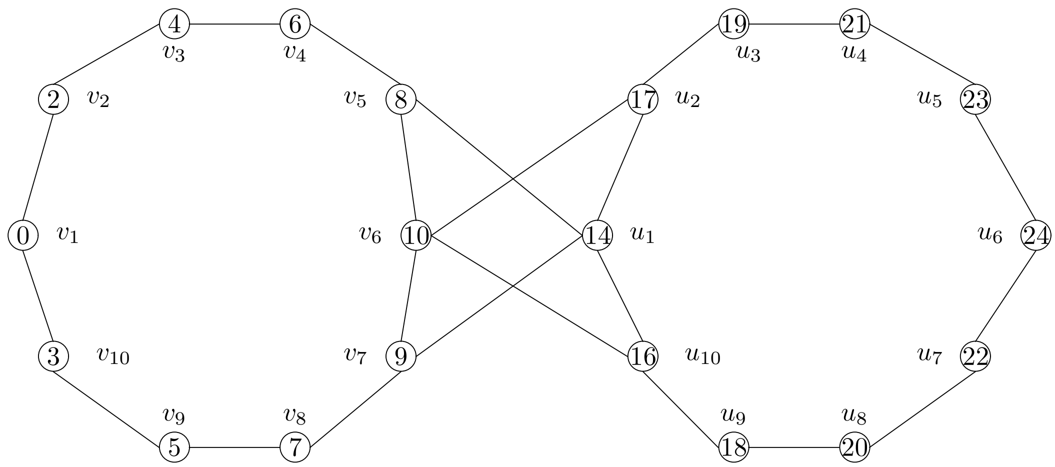

Fig.1

Theorem 2.3 *The graph obtained by the mutual duplication of a pair of edges in cycle C_n admits mean labeling.*

Proof Let v_1,v_2,..., v_n be the vertices of the first copy of cycle C_n and let u_1,u_2,..., u_n be the vertices of the second copy of cycle C_n. Let G be the graph obtained by the mutual duplication of a pair of edges each respectively from each copy of cycle C_n. To define $f\colon V(G) \to \{0, 1, 2, \ldots, q\}$ two cases are to be considered.

Case 1. n is odd.

Without loss of generality assume that the edge $e= v_{\frac{n+1}{2}}v_{\frac{n+3}{2}}$ from the first copy of cycle C_n and the edge $e'= u_1u_2$ from the second copy of cycle C_n are mutually duplicated.

$f(v_1) = 0;$

$f(v_i) = 2i - 1$ for $2 \le i \le \dfrac{n+1}{2};$

$f(v_i) = 2(n - i) + 2$ for $\dfrac{n+3}{2} \le i \le n;$

$f(u_i) = n + 2i + 2$ for $1 \le i \le \dfrac{n+1}{2};$

$f(u_i) = 3n - 2i + 7$ for $\dfrac{n+3}{2} \le i \le n.$

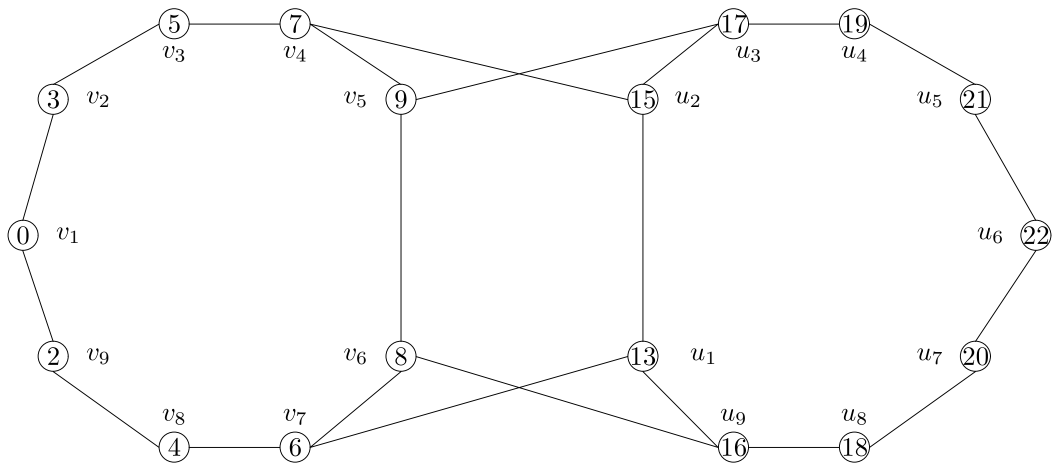

Fig.2

Case 2. n is even, $n \neq 4$.

Without loss of generality assume that the edge $e= v_{\frac{n}{2}+1}v_{\frac{n}{2}+2}$ from the first copy of cycle C_n and the edge $e'= u_1u_2$ from the second copy of cycle C_n are mutually duplicated.

$f(v_i) = 2i - 2$ for $1 \leq i \leq \dfrac{n}{2} + 1$;
$f(v_i) = 2(n - i) + 3$ for $\dfrac{n}{2} + 2 \leq i \leq n$;
$f(u_i) = n + 2i + 2$ for $1 \leq i \leq \dfrac{n}{2} + 1$;
$f(u_i) = 3n - 2i + 7$ for $\dfrac{n}{2} + 2 \leq i \leq n$.

Then above defined function f provides mean labeling for the graph obtained by the mutual duplication of a pair of edges in C_n. □

Illustration 2.4 The following Fig.2 shows mean labeling for the graph obtained by the mutual duplication of a pair of edges in cycle C_9.

Theorem 2.5 *$D_2(K_{1,n})$ is a mean graph.*

Proof Consider two copies of $K_{1,n}$. Let v,v_1,v_2,..., v_n be the vertices of the first copy of $K_{1,n}$ and v',v'_1,v'_2,..., v'_n be the vertices of the second copy of $K_{1,n}$ where v and v' are the respective apex vertices. Let G be $D_2(K_{1,n})$. Define $f\colon V(G) \to \{0, 1, 2, \ldots, q\}$ as follows.

$f(v) = 0$;
$f(v_i) = 2i$ for $1 \leq i \leq n$;
$f(v') = 4n$;
$f(v'_1) = 4n - 1$;
$f(v'_i) = 4n - 2i + 2$ for $2 \leq i \leq n$.

The above defined function provides the mean labeling of the graph $D_2(K_{1,n})$. □

Illustration 2.6 The labeling pattern for $D_2(K_{1,4})$ is given in Fig.3.

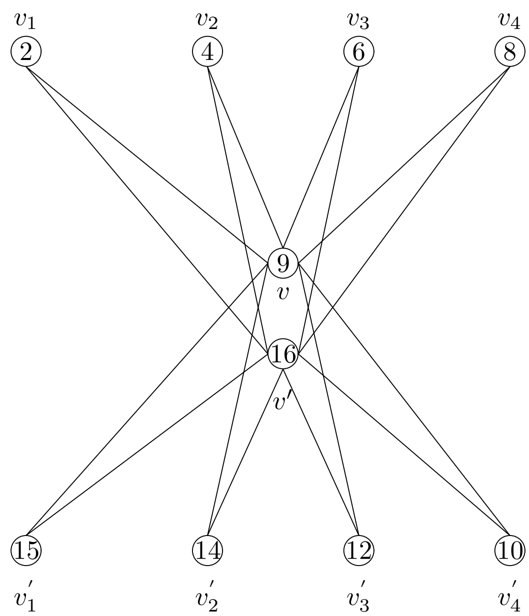

Fig.3

Theorem 2.7 *$D_2(B_{n,n})$ is a mean graph.*

Proof Consider two copies of $B_{n,n}$. Let $\{u, v, u_i, v_i, 1 \leq i \leq n\}$ and $\{u', v', u_i', v_i', 1 \leq i \leq n\}$ be the corresponding vertex sets of each copy of $B_{n,n}$. Let G be $D_2(B_{n,n})$. Define $f : V(G) \to \{0, 1, 2, \ldots, q\}$ as follows.

$f(u) = 0$;

$f(u_i) = 2i$ for $1 \leq i \leq n$;

$f(v) = 8n + 1$;

$f(v_i) = 4i + 1$ for $1 \leq i \leq n - 1$;

$f(v_n) = 4n + 5$;

$f(u') = 4n$;

$f(u_i') = 2(n + i)$ for $1 \leq i \leq n - 1$;

$f(u_n') = 4n - 1$;

$f(v') = 8n + 3$;

$f(v_i') = 8(n + 1) - 4i$ for $1 \leq i \leq n$.

In view of the above defined labeling pattern G admits mean labeling. □

Illustration 2.8 The labeling pattern for $D_2(B_{3,3})$ is given in Fig.4.

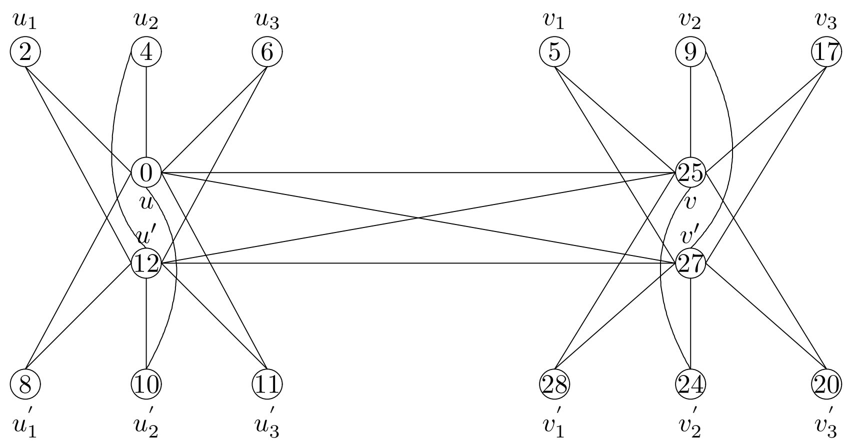

Fig.4

§3. Concluding Remarks

As all the graphs are not mean graphs it is very interesting to investigate graphs or graph families which admit mean labeling. Here we contribute two new graph operations and four new families of mean graphs. Somasundaram and Ponraj have proved that star $K_{1,n}$ is mean graph for $n \leq 2$ and bistar $B_{m,n}$ $(m > n)$ is mean graph if and only if $m < n + 2$ while in this paper we have investigated that the shadow graphs of star $K_{1,n}$ and bistar $B_{n,n}$ also admit mean labeling.

To investigate similar results for other graph families and in the context of different labeling techniques is an open area of research.

References

[1] G.S.Bloom and S.W.Golomb, Applications of numbered undirected graphs, *Proceedings of IEEE*, 65(4) (1977), 562-570.

[2] J. A. Gallian, A dynamic survey of graph labeling, *The Electronics Journal of Combinatorics*, 17 (2010) $\sharp DS6$.

[3] F. Harary, *Graph Theory*, Addison Wesley, Reading, Massachusetts, 1972.

[4] S. Somasundaram and R. Ponraj, Some results on mean graphs, *Pure and Applied Mathematical Sciences*, 58 (2003),29-35.

[5] S. Somasundaram and R. Ponraj, On mean graphs of order < 5, J.Decision and Mathematical Sciences, 9 (2004),47-58.

[6] S. Somasundaram and R. Ponraj, Non-existance of mean labeling for a wheel, *Bulletin of Pure and Applied Sciences*, 22E (2003),103-111.

[7] S. K.Vaidya and Lekha Bijukumar, Mean labeling in the context of some graph operations, *Int. J of Algorithms, Comp. and Math.*,3(1) (2010), 1-8.

[8] S. K.Vaidya and Lekha Bijukumar, Some new families of mean graphs, *J. Math. Res.*, 2(1) (2010), 169-176.