Some new problems about the Smarandache function and related problems[1]

Yulin Lu[†] [‡]

† Department of Mathematics, Northwest University
Xi'an, Shaanxi, P.R.China

‡ Department of Mathematics, Weinan Teacher's University
Weinan, Shaanxi, P.R.China

Abstract For any positive integer n, the famous F.Smarandache function $S(n)$ is defined as the smallest positive integer m such that $n|m!$. That is, $S(n) = \min\{m : m \in N, n|m!\}$. The main purpose of this paper is to introduce some new unsolved problems involving the Smarandache function and the related functions.

Keywords Smarandache function, F.Smarandache LCM function, F.Smarandache dual function, Pseudo-F.Smarandache function.

§1. Introduction and Results

For any positive integer n, the famous F.Smarandache function $S(n)$ is defined as the smallest positive integer m such that $n|m!$. That is, $S(n) = \min\{m : m \in N, n|m!\}$. About the properties of $S(n)$, there are many people had studied it, and obtained a series conclusions, see references [1], [2], [3] and [4]. Here we introduce two unsolved problems about the Smarandache function, they are:

Problem 1. If $n > 1$ and $n \neq 8$, then $\displaystyle\sum_{d|n} \frac{1}{S(d)}$ is not a positive integer, where $\displaystyle\sum_{d|n}$ denotes the summation over all positive divisors of n.

Problem 2. Find all positive integer solutions of the equation $\displaystyle\sum_{d|n} S(d) = \phi(n)$, where $\phi(n)$ is the Euler function.

For any positive integer n, the F.Smarandache LCM function $SL(n)$ is defined as the smallest positive integer k such that $n|[1, 2, \cdots, k]$, where $[1, 2, \cdots, k]$ is the smallest common multiple of $1, 2, \cdots, k$. About this function, there are three unsolved problems as follows：

Problem 3. If $n > 1$ and $n \neq 36$, then $\displaystyle\sum_{d|n} \frac{1}{SL(d)}$ is not a positive integer.

Problem 4. Find all positive integer solutions of the equation $\displaystyle\sum_{d|n} SL(d) = \phi(n)$.

Problem 5. Study the mean value properties of $\ln SL(n)$, and give an asymptotic formula for $\sum_{n \leq x} \ln (SL(n))$.

For any positive integer $n > 1$, let $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ denotes the factorization of n into prime power, we define function $\bar{S}(n) = \max\{\alpha_1 p_1, \ \alpha_2 p_2, \cdots, \alpha_k p_k\}$, and $\bar{S}(1) = 1$. There are two unsolved problems about this function as follows:

Problem 6. If $n > 1$ and $n \neq 24$, then $\sum_{d|n} \dfrac{1}{\bar{S}(d)}$ is not a positive integer.

Problem 7. Find all positive integer solutions of the equation $\sum_{d|n} \bar{S}(d) = \phi(n)$.

For any positive integer n, the dual function $S^*(n)$ of the Smarandache function is defined as the largest positive integer m such that $m!|n$. That is, $S^*(n) = \max\{m : \ m \in N, \ m!|n\}$. About this function, there are two unsolved problems as follows:

Problem 8. Find all positive integer solutions of the equation $\sum_{d|n} S^*(d) = \phi(n)$.

Problem 9. Study the calculating problem of the product $\prod_{d|n} S^*(d)$, and give an exact calculating formula for it.

For any positive integer n, the Pseudo-F.Smarandache function $Z(n)$ is defined as the largest positive integer m such that $(1 + 2 + 3 + \cdots + m)|n$. That is, $Z(n) = \max\{m : \ m \in N, \ \frac{m(m+1)}{2}|n\}$. For this function, there is an unsolved problem as follows:

Problem 10. Study the mean value properties of $Z(n)$, and give an asymptotic formula for $\sum_{n \leq x} Z(n)$.

References

[1] F.Smarandache, Only Problems, Not Solutions, Chicago, Xiquan Publishing House, 1993.

[2] Liu Yaming, On the solutions of an equation involving the Smarandache function, Scientia Magna, **2**(2006), No. 1, 76-79.

[3] Fu Jing, An equation involving the Smarandache function, Scientia Magna, **2**(2006), No. 4, 83-86.

[4] Jozsef Sandor, On additive analogues of certain arithmetical function, Smarandache Notions Journal, **14** (2004), 128-132.

[5] Le Maohua, An equation concerning the Smarandache LCM function, Smarandache Notions Journal, **14** (2004), 186-188.

[6] Jozsef Sandor, On certain inequalities involving the Smarandache function, Scientia Magna, **2**(2006), No. 3, 78-80.

[7] Xu Zhefeng, The mean value of the Smarandache function, Mathematic Sinica，**49**(2006), No. 5, 1009-1012.

[8] Tom M.Apostol, Introduction to analytical number theory, Spring-Verlag, New York, 1976.