NEW SMARANDACHE ALGEBRAIC STRUCTURES

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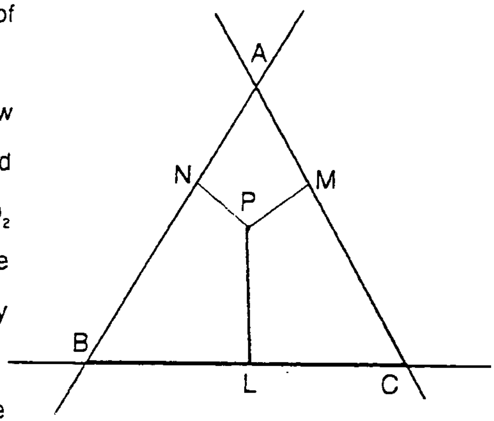
ABSTRACT

Generally in R^3 any plane with equation $x + y + z = a$, where a is nonzero number, is not a linear space under the usual vector addition and scalar multiplication. If we define new algebraic operations on the plane $x + y + z = a$ it will become a linear space in R^3. The additive identity of this linear space has nonzero components.

1. The plane $x + y + z = a$ touches the x-axis at point A (a ,0 ,0), y- axis at point B (0,a,0) and z-axis at point C (0,0,a). Take triangle ABC as a fixed equilateral triangle known as " triangle of reference."



From any point P in its plane draw perpendiculars PM, PN and PL to AC, AB and BC respectively. Let $\ell(PM) = p_1$, $\ell(PN) = p_2$ and $\ell(PL) = p_3$. These p_1, p_2 and p_3 are called the trilinear coordinater of point P [Loney 1, Smith 2, Sen 3].

The coordinate p_1 is positive if P and the vertex B of the triangle are on the same side of AC and p_1 is negative if P and B are on the opposite sides of AC. So for the other coordinates p_2 and p_3.

2. Length of each side of the triangle is $\sqrt{2}\,|a| = b$ (say).

$1/2.\, b .p_1 + 1/2\ b .p_2 + 1/2 .\, b\ p_3 = 1/2\, b .\sqrt{3} / 2.\, b$

$$p_1 + p_2 + p_3 = \sqrt{3} / 2.\, b = k \quad (\text{say}).$$

The trilinear coordinates p_1, p_2, p_3 of any point P in the plane whether it is within the triangle or outside the triangle ABC satisfy the relation

$$p_1 + p_2 + p_3 = k \qquad (2.1)$$

Thus trilinear coordinates of points A, B and C are (0,0,k), (k,0,0) and (0,k,0) respectively. Trilinear coordinetes of the centroid of triangle are (k/3, k/3, k/3).

3. Now the plane $x+y+z=a$ is a set T of all points p whose trilinear coordinates p_1, p_2, p_3 satisfy the relation $p_1+p_2+p_3=k$.

Let $p=(p_1, p_2, p_3)$ and $q=(q_1, q_2, q_3)$ be in T.

By usuel addition $p+q=(p_1+q_1, p_2+q_2, p_3+q_3) \notin T$, (3.1)

since $(p_1+q_1)+(p_2+q_2)+(p_3+q_3)=(p_1+p_2+p_3)+(q_1+q_2+q_3)=2k$ (3.2)

By usual scalar multiplication by α, $\alpha p=(\alpha p_1, \alpha p_2, \alpha p_3) \notin T$, (3.3)

since $\alpha p_1+\alpha p_2+\alpha p_3=\alpha(p_1+p_2+p_3)=\alpha k$. (3.4)

In view of (3.1), (3.2), (3.3) and (3.4) the set T is not closed with respect to the usual vector addition and scalar multiplication. Hence it can not become a linear space.

4. Now we shall prove, by defining following new algebraic operations, T is a linear space in which components of additive identity are nonzero.

4.1 **Definition** Let $p=(p_1, p_2, p_3)$ & $q=(q_1, q_2, q_3)$ be in T.

We define :

a. **Equality :**

$$p=q \text{ if and only if } p_1=q_1,\ p_2=q_2,\ p_3=q_3.$$

b. **Sum :**

$$p+q=(-k/3+p_1+q_1,\ -k/3+p_2+q_2,\ -k/3+p_3+q_3)$$

c. **Multiplication by real numbers :**

$$\alpha p=((1-\alpha)k/3+\alpha p_1,\ (1-\alpha)k/3+\alpha p_2,\ (1-\alpha)k/3+\alpha p_3)$$

(α real)

d. **Difference :**

$$p-q=p+(-1)q.$$

e. **Zero vector (centroid of the triangle):**

$$0=(k/3,\ k/3,\ k/3).$$

5.1 To every pair of elements p and q in T there corresponds an element p+q, in such a way that

$$p+q=q+p \quad \text{and} \quad p+(q+r)=(p+q)+r.$$

$$p+\mathbf{0}=p \quad \text{for every } p \in T.$$

To each $p \in T$ there exists a unique element $-p$ such that $p+(-p)=\mathbf{0}$

T is an abelien group with respect to vector addition.

5.2 For every $\alpha, \beta \in R$ and $p, q \in T$ we have

i) $\alpha(\beta p)=(\alpha\beta)p$

ii) $\alpha(p+q)=\alpha p+\alpha q$,

iii) $(\alpha+\beta)q=\alpha q+\beta q$

iv) $1p=p$,

Therefore T is a real linear space.

Remark .1. The real number k is related with the position of the plane x+y+z= a in R^3

2. There are infinite number of linear spaces of above kind in R^3

References.

1. Loney S.L. (1952): The Elements of coordinate Geometry Part II, Macmillan and Co. Ltd.
2. Smith Charles (1948): An Elementary Treatise on conic sections" Macmillan and Co. Ltd.
3. Sen B. (1968): Trilinear Coordinates and Boundry Value Problems, Bull. Cal. Math. Soc., 60(1-2), pp. 25-30.

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