A NOTE ON THE SMARANDACHE DIVISOR SEQUENCES

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ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION (**SFP**) , as follows:

Let $\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r$ be a set of r natural numbers and $p_1 , p_2, p_3 ,\ldots p_r$ be arbitrarily chosen distinct primes then $F(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r)$ called the Smarandache Factor Partition of $(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r)$ is defined as the number of ways in which the number

$$N = p_1^{\alpha 1} \; p_2^{\alpha 2} \; p_3^{\alpha 3} \ldots p_r^{\alpha r}$$ could be expressed as the

product of its' divisors. For simplicity , we denote $F(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r) = F'(N)$,where

$$N = p_1^{\alpha_1} \; p_2^{\alpha_2} \; p_3^{\alpha_3} \ldots p_r^{\alpha_r} \ldots p_n^{\alpha_n}$$

and p_r is the r^{th} prime. $p_1 =2$, $p_2 = 3$ etc.

In [2] we have defined SMARANDACHE DIVISOR SEQUENCES as follows

$\mathbf{P_n = \{ x \mid d(x) = n \}}$, d(x) = number of divisors of n.

$P_1 = \{1\}$

$P_2 = \{ x \mid x \text{ is a prime} \}$

$P_3 = \{ x \mid x = p^2 , \; p \text{ is a prime} \}$

$P_4 = \{ x \mid x = p^3 \text{ or } x = p_1p_2 , \; p ,p_1 , p_2 \text{ are primes} \}$

Let F_1 be a SFP of N . Let $\Psi_{F1} = \{ y | d(y) = N \}$, generated by the SFP F_1 of N. It has been shown in Ref. [3] that each SFP generates some elements of Ψ or P_n. Here each SFP generates infinitely many elements of P_n. Similarly Ψ_{F1}, Ψ_{F2}, Ψ_{F3}, . . . $\Psi_{F'(N)}$, are defined. It is evident that all these F_k's are disjoint and also

$P_N = \cup \Psi_{Fk} \quad 1 \le k \le F'(N)$.

THEOREM(7.1) There are F'(N) disjoint and exhaustive subsets in which P_N can be decomposed.

PROOF: Let $\theta \in P_N$, and let it be expressed in canonical form as follows

$$\theta = p_1^{\alpha_1} \; p_2^{\alpha_2} \; p_3^{\alpha_3} \; . . . \; p_r^{\alpha_r}$$

Then $d(\theta) = (\alpha_1+1)(\alpha_2+1)(\alpha_3+1) . . .(\alpha_r+1)$

Hence $\theta \in \Psi_{Fk}$ for some k where F_k is given by

$$N = (\alpha_1+1)(\alpha_2+1)(\alpha_3+1) . . .(\alpha_r+1)$$

Again if $\theta \in \Psi_{Fs}$, and $\theta \in \Psi_{Ft}$ then from unique factorisation theorem F_s and F_t are identical SFPs of N.

REFERENCES:

[1] "Amarnath Murthy" , 'Generalization Of Partition Function, Introducing 'Smarandache Factor Partition', SNJ, Vol. 11, No. 1-2-3, 2000.

[2] "Amarnath Murthy" , 'Some New Smarandache Sequences, Functions And Partitions ', SNJ, Vol. 11, No. 1-2-3, 2000.

[3] "Amarnath Murthy" , ' Some more Ideas on SFPS. SNJ, Vol. 11, No. 1-2-3, 2000.

[4] " The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texas at Austin, USA.